Real and virtual $N\bar{N}$ pair production near the threshold

V. F. Dmitriev,* A. I. Milstein,† and S. G. Salnikov‡

Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia and Novosibirsk State University, 630090 Novosibirsk, Russia

(Received 4 December 2015; published 23 February 2016)

Nucleon-antinucleon optical potential, which explains the experimental data for the processes $e^+e^- \to p\bar{p}$ and $e^+e^- \to$ pions near the threshold of $p\bar{p}$ pair production, is suggested. To obtain this potential, we have used the available experimental data for $p\bar{p}$ scattering, $p\bar{p}$ pair production in e^+e^- annihilation, and the ratio of electromagnetic form factors of a proton in the timelike region. It turns out that final-state interaction via the optical potential allows one to reproduce the available experimental data with good accuracy. Our results for the cross sections of the $e^+e^- \to 6\pi$ process near the threshold of $p\bar{p}$ pair production are in agreement with the recent experiments.

DOI: 10.1103/PhysRevD.93.034033

I. INTRODUCTION

At present, the study of nucleon-antinucleon interaction in the low-energy region is an actual topic. Several optical nucleon-antinucleon potentials [1–3] are usually used to describe the interaction in this region. All these nucleon-antinucleon potentials have been proposed to fit the nucleon-antinucleon scattering data. These data include elastic, charge-exchange, and annihilation cross sections of $p\bar{p}$ scattering, as well as some single-spin observables. These observables can be described very well by any of the models [1–3]. To discriminate between different models, one can use other observables. For example, calculations were made for double-spin observables in $p\bar{p}$ scattering [4–7], and the predictions of different optical potentials were indeed different. Unfortunately, the experimental data for these observables are still absent.

There is another set of data that one can hope to describe with the help of potential models—namely, the cross sections of nucleon-antinucleon production in e^+e^- annihilation. It was shown in our previous papers [8,9] that the cross sections of these processes in the energy region close to the threshold can be written in terms of the radial wave functions of a nucleon-antinucleon pair at the origin. These cross sections were measured at *BABAR* [10], CMD-3 [11], and SND [12]. The ratio of electromagnetic form factors of a proton in the timelike region, which was also measured [10,11], can also be expressed via the wave functions. This ratio has quite strong energy dependence near the threshold, and one needs a nontrivial model to describe this. It was shown that some of these observables can be described by slightly modified Paris optical potential [8] or by the Jülich model [13].

In this paper we go further and try to describe, with the help of an optical theorem, the contribution of virtual nucleon-antinucleon pairs to the cross sections of meson production in the energy region close to the $p\bar{p}$ threshold. We refer to the cross section of this process as the inelastic cross section of nucleon-antinucleon pair production, while the cross section of real $N\bar{N}$ pair production is called the elastic cross section. The inelastic cross section can be expressed in terms of the Green's function of the Schrödinger equation in the presence of an optical potential. The e^+e^- annihilation to mesons close to the threshold of $N\bar{N}$ pair production is very interesting, because the cross sections of e^+e^- annihilation to $3(\pi^+\pi^-)$ and $2(\pi^+\pi^-\pi^0)$ have a sharp dip in that energy region [14–16], and this phenomenon is not well understood yet. In the recent paper [17], it was suggested that this feature is a consequence of an annihilation channel $e^+e^- \to N\bar{N}$ opened above threshold. The strong energy dependence of the cross sections of 6π production is expected to be a consequence of the interaction of virtual nucleons, because other contributions should be smooth functions in the energy region under consideration. There is an approach [18], based on the Jülich model [3], which allows one to predict the meson-production cross sections near the threshold of the $e^+e^- \to N\bar{N}$ process. In that approach, calculations are performed in momentum representation, and some qualitative agreement of the theoretical predictions and experimental data has been achieved. It is interesting to compare experimental results with the theoretical predictions, performing calculations in the coordinate space with the use of well-known Paris [1] and Nijmegen [2] potentials. We have found that the Paris optical potential completely fails to describe the process $e^+e^- \to$ mesons via annihilation of the virtual nucleon-antinucleon pair. This problem appears due to the very large imaginary part of the central potential in that model. Such a huge imaginary part results in the significant overestimation of the inelastic cross section

*V.F.Dmitriev@inp.nsk.su
†A.I.Milstein@inp.nsk.su
‡S.G.Salnikov@inp.nsk.su

compared with the elastic one. A strong potential appears as a result of an attempt to describe the data in wide energy and angle regions, i.e., in a large region of momentum transfers. The Nijmegen optical potential has another shortcoming. This model implies that a complicated matching condition should be applied to the wave functions at a radius of about 1 fm, and it is not evident how to apply this condition to calculate the Green's function at the origin.

The interaction of virtual nucleons in the process of e^+e^- annihilation into mesons is very sensitive to the potential at small distances. Unfortunately, the short-range potential cannot be determined very well from $p\bar{p}$ scattering data alone. Therefore, one should also take into account other experimental data. We consider the partial waves with the total angular momentum $J = 1$, contributing to the processes of nucleon-antinucleon production in e^+e^- annihilation, and assume that other partial waves can be described by any of the models mentioned above. For $^3S_1$ and $^3D_1$ partial waves, coupled by the tensor forces, we propose a new potential model based on the fit of experimental data for the cross sections of $p\bar{p}$ scattering, as well as the cross sections of nucleon-antinucleon pair production and the ratio of electromagnetic form factors of the proton. The account for the tensor potential is very important, because it is crucial for the description of the electromagnetic form factor ratio. Our model qualitatively reproduces the features of 6π production in e^+e^- annihilation in the vicinity of the threshold of the $e^+e^- \to p\bar{p}$ process.

II. AMPLITUDE OF THE PROCESS

It is shown in our recent paper [9] that in the nonrelativistic approximation, the amplitude $T^I_{\lambda\mu}$ of $N\bar{N}$ pair production in e^+e^- annihilation near the threshold can be presented for the certain isospin channel $I = 0, 1$ as follows (in units $4\pi\alpha/Q^2$, where α is the fine structure constant, and $\hbar = c = 1$):

$$T^I_{\lambda\mu} = G^I_s\{\sqrt{2}u^I_{1R}(0)(\mathbf{e}_\mu\cdot\boldsymbol{\epsilon}^*_\lambda) + u^I_{2R}(0) \times [(\mathbf{e}_\mu\cdot\boldsymbol{\epsilon}^*_\lambda) - 3(\hat{\boldsymbol{k}}\cdot\mathbf{e}_\mu)(\hat{\boldsymbol{k}}\cdot\boldsymbol{\epsilon}^*_\lambda)]\}, \quad (1)$$

where G^I_s is an energy-independent constant; $\mathbf{e}_\mu$ is a virtual photon polarization vector, corresponding to the projection of spin $J_z = \mu = \pm 1$; $\boldsymbol{\epsilon}_\lambda$ is the spin-1 function of the $N\bar{N}$ pair; $\lambda = \pm 1, 0$ is the projection of spin on the nucleon momentum $\boldsymbol{k}$; and $\hat{\boldsymbol{k}} = \boldsymbol{k}/k$. The radial wave functions $u^I_{nR}(r)$ and $w^I_{nR}(r)$, $n = 1, 2$, are the regular solutions of the equations

$$\frac{p_r^2}{M}\chi_n + \mathcal{V}\chi_n = 2E\chi_n,$$
$$\mathcal{V} = \begin{pmatrix} V^I_S & -2\sqrt{2}V^I_T \\ -2\sqrt{2}V^I_T & V^I_D - 2V^I_T + \frac{6}{Mr^2}\end{pmatrix}, \quad \chi_n = \begin{pmatrix} u^I_n \\ w^I_n\end{pmatrix}. \quad (2)$$

Here M is the proton mass, and $E = k^2/(2M)$, $V^I_S(r)$, $V^I_D(r)$, and $V^I_T(r)$ are the functions in the Hamiltonian H^I of $N\bar{N}$ interaction for the isospin I,

$$H^I = \frac{p_r^2}{M} + V^I_S(r)\delta_{L0} + V^I_D(r)\delta_{L2} + V^I_T(r)S_{12},$$
$$S_{12} = 6(\boldsymbol{S}\cdot\boldsymbol{n})^2 - 4, \quad (3)$$

where $\boldsymbol{S}$ is the spin operator for the spin-1 system of the produced pair, $(-p_r^2)$ is the radial part of the Laplace operator, L denotes the orbital angular momentum, and $\boldsymbol{n} = \boldsymbol{r}/r$. The asymptotic forms of the regular solutions (they have no singularities at $r = 0$) at large distances are [9]

$$u^I_{1R}(r) = \frac{1}{2ikr}[S^I_{11}e^{ikr} - e^{-ikr}],$$
$$w^I_{1R}(r) = -\frac{1}{2ikr}S^I_{12}e^{ikr},$$
$$u^I_{2R}(r) = \frac{1}{2ikr}S^I_{21}e^{ikr},$$
$$w^I_{2R}(r) = \frac{1}{2ikr}[-S^I_{22}e^{ikr} + e^{-ikr}], \quad (4)$$

where S^I_{ij} are some functions of energy, $S^I_{21} = S^I_{12}$, $|S^I_{11}|^2 + |S^I_{12}|^2 \le 1$, and $|S^I_{22}|^2 + |S^I_{21}|^2 \le 1$. For our purpose, we also need to know the nonregular solutions of Eq. (2) which have the asymptotic forms at large distances

$$u^I_{1N}(r) = \frac{1}{kr}e^{ikr}, \qquad \lim_{r\to\infty} rw^I_{1N}(r) = 0,$$
$$\lim_{r\to\infty} ru^I_{2N}(r) = 0, \qquad w^I_{2N}(r) = -\frac{1}{kr}e^{ikr}. \quad (5)$$

III. CROSS SECTION AND THE SACHS FORM FACTORS

Performing summation over the polarization of the nucleon pair and averaging over the polarization of the virtual photon by means of the equations

$$\sum_{\lambda=1,2,3}\epsilon^{i*}_\lambda\epsilon^j_\lambda = \delta^{ij}, \quad \frac{1}{2}\sum_{\mu=1,2}e^{i*}_\mu e^j_\mu = \frac{1}{2}\delta^{ij}_\perp = \frac{1}{2}(\delta^{ij} - P^iP^j/P^2), \quad (6)$$

where $\boldsymbol{P}$ is the electron momentum, we obtain the cross section corresponding to the amplitude (1) in the center-of-mass frame (see, e.g., Ref. [19]):

$$\frac{d\sigma^I}{d\Omega} = \frac{\beta\alpha^2}{4Q^2}\left[|G^I_M(Q^2)|^2(1+\cos^2\theta) + \frac{4M^2}{Q^2}|G^I_E(Q^2)|^2\sin^2\theta\right]. \quad (7)$$

Here $\beta = k/M$, $Q = 2(M + E)$, and θ is the angle between the electron (positron) momentum $\boldsymbol{P}$ and the momentum of the final particle $\boldsymbol{k}$. In terms of the form factor G_s^I, the electromagnetic Sachs form factors have the form

$$G_M^I = G_s^I\left[u_{1R}^I(0) + \frac{1}{\sqrt{2}}u_{2R}^I(0)\right],$$
$$\frac{2M}{Q}G_E^I = G_s^I[u_{1R}^I(0) - \sqrt{2}u_{2R}^I(0)]. \quad (8)$$

Thus, in the nonrelativistic approximation, the ratio G_E^I/G_M^I is independent on the constant G_s^I:

$$\frac{G_E^I}{G_M^I} = \frac{u_{1R}^I(0) - \sqrt{2}u_{2R}^I(0)}{u_{1R}^I(0) + \frac{1}{\sqrt{2}}u_{2R}^I(0)}. \quad (9)$$

Note that the electromagnetic interaction is important only in the narrow region $\beta \sim \pi\alpha$ where the nucleon energy is $E = M\beta^2/2 \sim 0.3$ MeV. In this paper we do not consider this narrow region and neglect the electromagnetic interaction in the potential. The contribution of the isospin I to the total cross section of the nucleon pair production (the elastic cross section) reads

$$\sigma^I = \frac{2\pi\beta\alpha^2}{Q^2}|G_s^I|^2[|u_{1R}^I(0)|^2 + |u_{2R}^I(0)|^2]. \quad (10)$$

Thus, to describe the energy dependence of the ratio G_E^I/G_M^I and the cross section σ^I in the nonrelativistic approximation, it is necessary to know the functions $u_1^I(0)$ and $u_2^I(0)$.

In order to describe the total cross section, a sum of elastic and inelastic cross sections (the production of mesons via annihilation of virtual $N\bar{N}$ pairs), we use the method of the Green's function. Let us introduce the Green's function $\mathcal{D}(r, r'|E)$:

$$\left(\frac{p_r^2}{M} + \mathcal{V} - 2E\right)\mathcal{D}(r, r'|E) = -\frac{1}{rr'}\delta(r - r'). \quad (11)$$

Then the total cross section, σ_{tot}^I, can be written as [20]

$$\sigma_{\text{tot}}^I = -\frac{2\pi\alpha^2}{M^2Q^2}|G_s^I|^2 Sp[\text{Im}\mathcal{D}(0, 0|E)]. \quad (12)$$

The solution of Eq. (11) can be written in the form

$$\mathcal{D}(r, r'|E) = -Mk\sum_{n=1,2}[\vartheta(r' - r)\chi_{nR}(r)\chi_{nN}^T(r') + \vartheta(r - r')\chi_{nN}(r)\chi_{nR}^T(r')], \quad (13)$$

where χ^T denotes transposition of χ, if the following relations hold:

$$\sum_{n=1,2}[\chi_{nR}(r)\chi_{nN}^T(r) - \chi_{nN}(r)\chi_{nR}^T(r)] = \mathbf{0},$$
$$\sum_{n=1,2}[\chi'_{nR}(r)\chi_{nN}^T(r) - \chi'_{nN}(r)\chi_{nR}^T(r)] = \frac{1}{kr^2}\mathbf{1}. \quad (14)$$

Here $\chi'(r) = \partial\chi(r)/\partial r$, and 0 and 1 stand for the zero and unit matrices, respectively. The validity of Eq. (14) is a consequence of the relations

$$\chi_{1R}^T(r)\chi'_{2N}(r) - \chi_{2N}^T(r)\chi'_{2R}(r) = 0,$$
$$\chi_{2R}^T(r)\chi'_{1N}(r) - \chi_{1N}^T(r)\chi'_{2N}(r) = 0,$$
$$\chi_{1N}^T(r)\chi'_{1R}(r) - \chi_{1R}^T(r)\chi'_{1N}(r) = \frac{1}{kr^2},$$
$$\chi_{2N}^T(r)\chi'_{2R}(r) - \chi_{2R}^T(r)\chi'_{2N}(r) = \frac{1}{kr^2}, \quad (15)$$

following from Eq. (2), symmetry of the matrix $\mathcal{V}$ in that equation, and the asymptotic forms (4) and (5).

IV. RESULTS OF THE CALCULATIONS

We propose a simple potential model to describe the nucleon-antinucleon interaction in the state with the total angular momentum $J = 1$. This is the only state that contributes to the processes of nucleon-antinucleon production in e^+e^- annihilation. The interaction in other partial waves can be described very well by the models [1–3]. The optical potential of nucleon-antinucleon interaction in Eq. (16) can be written as

$$V_n(r) = V_{n0}(r) + V_{n1}(r)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2), \quad n = S, D, T, \quad (16)$$

where $\boldsymbol{\tau}_i$ are the Pauli matrices in the isospin space, so that the potentials corresponding to the $I = 0$, 1 channels read

$$V_n^0(r) = V_{n0}(r) - 3V_{n1}(r), \qquad V_n^1(r) = V_{n0}(r) + V_{n1}(r). \quad (17)$$

We use a potential which is the sum of a long-range pion-exchange potential and a short-range potential well:

$$V_{n0}(r) = (U_{n0} - iW_{n0})\theta(a_{n0} - r),$$
$$V_{n1}(r) = (U_{n1} - iW_{n1})\theta(a_{n1} - r) + \tilde{V}_n(r)\theta(r - a_{n1}), \quad (18)$$

where $\theta(x)$ is the Heaviside function; $\tilde{V}_n(r)$ is the pion-exchange potential; and U_{nI}, W_{nI}, and a_{nI} are free parameters fixed by fitting the experimental data. The pion-exchange potential of nucleon-antinucleon interaction for the total spin $S = 1$ is given by the expression (see, e.g., Ref. [21])

$$\tilde{V}_S(r) = \tilde{V}_D(r) = -f_\pi^2\frac{e^{-m_\pi r}}{3r},$$
$$\tilde{V}_T(r) = -f_\pi^2\left(\frac{1}{3} + \frac{1}{m_\pi r} + \frac{1}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r}, \quad (19)$$

where $f_\pi^2 = 0.075$, and m_π is the pion mass. At small r, the tensor potentials V_T^I are regularized by the factor

$$F(r) = \frac{(cr)^2}{1+(cr)^2}$$

with $c = 10\ \mathrm{fm}^{-1}$. Our analysis shows that one can take the radii of real and imaginary parts of the potentials (18) to be the same.

The electromagnetic form factors of the proton and neutron are expressed via the isoscalar and isovector form factors (8) by the relations

$$G_E^p = \frac{G_E^0 + G_E^1}{\sqrt{2}}, \qquad G_E^n = \frac{G_E^0 - G_E^1}{\sqrt{2}},$$
$$G_M^p = \frac{G_M^0 + G_M^1}{\sqrt{2}}, \qquad G_M^n = \frac{G_M^0 - G_M^1}{\sqrt{2}}. \tag{20}$$

Thus, the cross sections of nucleon-antinucleon production read

$$\sigma^{p\bar{p}} = \frac{\pi\beta\alpha^2}{Q^2}[|G_S^0 u_{1R}^0(0) + G_S^1 u_{1R}^1(0)|^2 + |G_S^0 u_{2R}^0(0) + G_S^1 u_{2R}^1(0)|^2],$$
$$\sigma^{n\bar{n}} = \frac{\pi\beta\alpha^2}{Q^2}[|G_S^0 u_{1R}^0(0) - G_S^1 u_{1R}^1(0)|^2 + |G_S^0 u_{2R}^0(0) - G_S^1 u_{2R}^1(0)|^2], \tag{21}$$

and the ratio of electromagnetic form factors of the proton is given by

$$\frac{G_E^p}{G_M^p} = \frac{G_S^0 u_{1R}^0(0) + G_S^1 u_{1R}^1(0) - \sqrt{2}[G_S^0 u_{2R}^0(0) + G_S^1 u_{2R}^1(0)]}{G_S^0 u_{1R}^0(0) + G_S^1 u_{1R}^1(0) + \frac{1}{\sqrt{2}}[G_S^0 u_{2R}^0(0) + G_S^1 u_{2R}^1(0)]}. \tag{22}$$

The data used for fitting the parameters of the potential include the cross sections of $p\bar{p}$ and $n\bar{n}$ production [10–12]; the ratio of electromagnetic form factors of the proton [10]; and the partial contributions of $J = 1$ waves to the elastic, charge-exchange, and total cross sections of $p\bar{p}$ scattering. The partial cross sections of $p\bar{p}$ scattering were calculated from the Nijmegen partial wave S-matrix (Tables VI and VII of Ref. [2]). The results of the fit are shown in Table I. The accuracy of the fit can be seen from Figs. 1–3.

The number of free parameters in our model is $N_{\mathrm{fp}} = 20$. The total number of experimental data points for the cross sections of $p\bar{p}$ and $n\bar{n}$ production and for the ratio $|G_E^p/G_M^p|$ is $N_{\mathrm{dat}} = 35$. Thus, we have $N_{\mathrm{df}} = N_{\mathrm{dat}} - N_{\mathrm{fp}} = 15$ degrees of freedom. The minimum χ^2 per degree of freedom is $\chi^2_{\mathrm{min}}/N_{\mathrm{df}} = 29/15$, and is rather large. However, the large χ^2_{min} value is originated mainly from poor accuracy of some data points for the $n\bar{n}$ production cross section. Excluding two less accurate data points that obviously are not in agreement with other experimental points and probably have some systematic uncertainty gives $\chi^2_{\mathrm{min}}/N_{\mathrm{df}} = 16/13$, which is good enough. The errors in Table I correspond to the values of the parameters that give $\chi^2 = \chi^2_{\mathrm{min}} + 1$.

As soon as the potential is determined, we can calculate the Green's function and the total cross section of pion production through the nucleon-antinucleon intermediate state (12). The elastic cross section of $N\bar{N}$ production, the total cross section, and the cross section of annihilation into mesons for different isospins are shown in Fig. 4. A dip in the total cross section of e^+e^- annihilation into mesons is predicted close to the $N\bar{N}$ threshold. This behavior seems to be the consequence of some quasibound $N\bar{N}$ state near the threshold. To check this hypothesis, we have searched for bound states in the potential considered. Our analysis shows that there are no near-threshold bound states in the $I = 1$ channel. However, we have found a state with energy $E_B = (10 - i32)$ MeV in the $I = 0$ channel. This state is located above the $N\bar{N}$ threshold, but it moves to $E_B = -21$ MeV if the imaginary part of the potential is turned off. This is an unstable bound state in the terminology of Ref. [22]. This result is quite similar to the result obtained

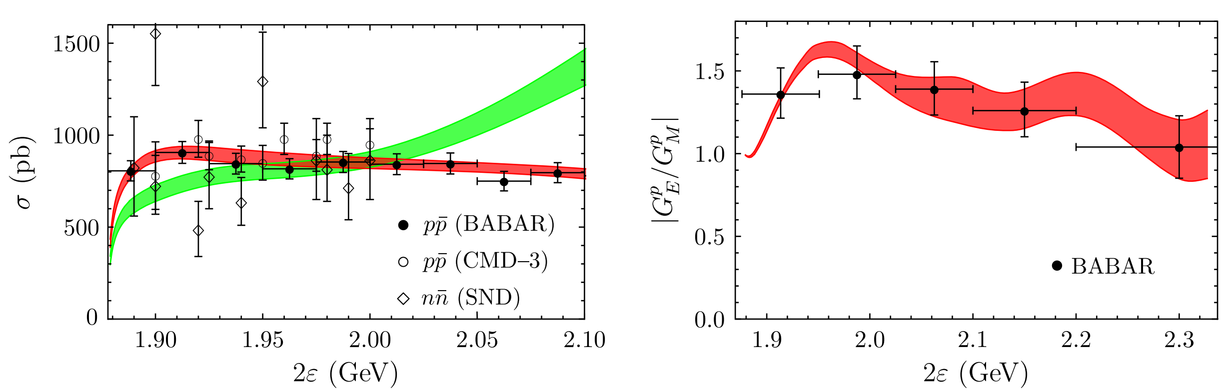

FIG. 1. The cross sections of $p\bar{p}$ (red/dark line) and $n\bar{n}$ (green/light line) production (left) and the ratio of electromagnetic form factors of the proton (right) as a function of total energy $2\varepsilon = 2M + 2E$. The experimental data are from Refs. [10–12].

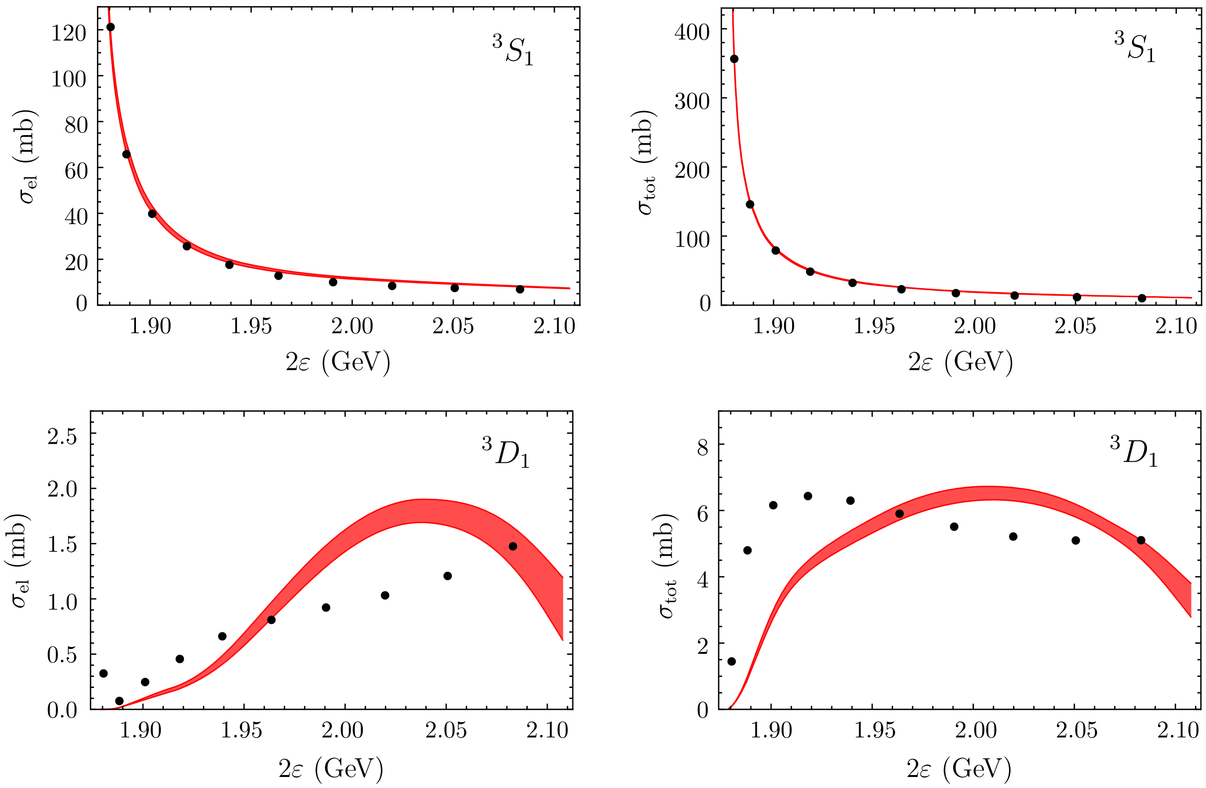

FIG. 2. $^3S_1$ (first row) and $^3D_1$ (second row) contributions to the elastic and total cross sections of $p\bar{p}$ scattering compared with the Nijmegen data [2], $\varepsilon = M + E$.

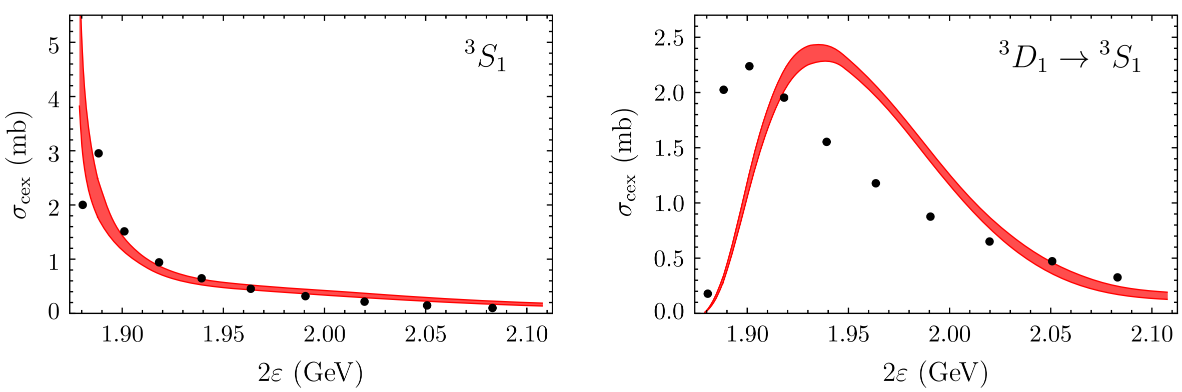

FIG. 3. $^3S_1$ and $^3D_1 \to {}^3S_1$ contributions to the charge-exchange cross section compared with the Nijmegen data [2], $\varepsilon = M + E$.

TABLE I. The results of the fit for the short-range potential (18) and the constants G_S^I.

	V_{S0}	V_{D0}	V_{T0}	V_{S1}	V_{D1}	V_{T1}
U (MeV)	-433 ± 3	-140^{+40}_{-36}	58 ± 4	$2.4^{+0.7}_{-0.6}$	798^{+165}_{-140}	7.1 ± 0.1
W (MeV)	224 ± 10	0^{+28}_{-27}	19 ± 1	$0^{+0.9}_{-1.3}$	456^{+215}_{-107}	-0.3 ± 0.3
a (fm)	0.564 ± 0.002	$1.02^{+0.06}_{-0.09}$	1.03 ± 0.02	$1.86^{+0.08}_{-0.09}$	$0.49^{+0.04}_{-0.02}$	2.4 ± 0.02
G_S	$G_S^0 = 0.179 \pm 0.006$			$G_S^1 = 0.044 + 0.29i \pm 0.014$		

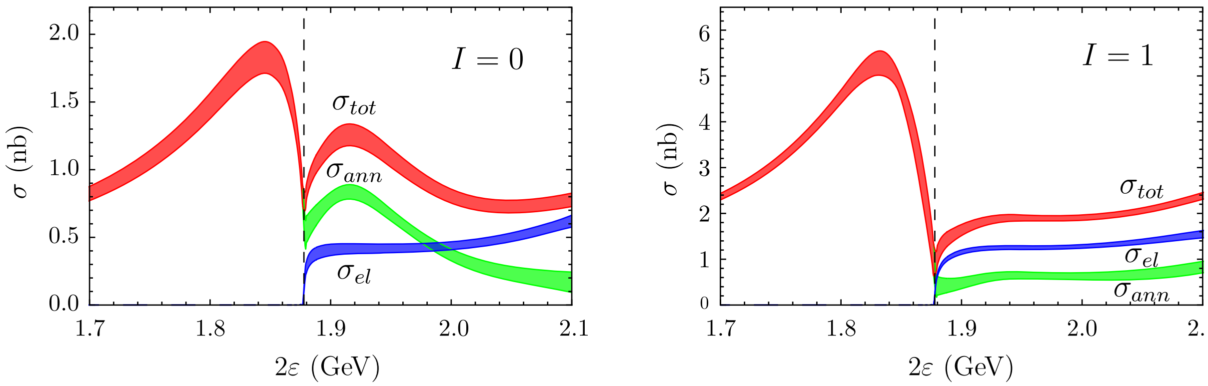

FIG. 4. Elastic (blue/dark line), inelastic (green/light line), and total (red/medium line) cross sections of e^+e^- annihilation through nucleon-antinucleon intermediate states with different isospins, $\varepsilon = M + E$.

in Ref. [3], where 4.8 MeV $< \mathrm{Re}E_B <$ 21.3 MeV and −74.9 MeV $< \mathrm{Im}E_B <$ −60.6 MeV in the $I = 0$ channel.

The total contribution of nucleon-antinucleon intermediate states to the cross section of e^+e^- annihilation into mesons is given by the sum of $I = 0$ and $I = 1$ terms. The states with $I = 0$ contribute to the production of an odd number of pions, while the states with $I = 1$ contribute to the production of an even number of pions. The optical theorem does not allow one to predict the individual contributions to the total cross sections with even or odd numbers of pions. The experimental data on $e^+e^- \to 4\pi$ annihilation do not demonstrate strong energy dependence in the vicinity of the $N\bar{N}$ threshold [23,24], and the data on $e^+e^- \to 5\pi$ are not accurate enough for all channels [25]. The meson multiplicity distribution was analyzed for $p\bar{p}$ annihilation at rest [26,27], where the cross section of six-pion production gives about 55% of the total cross section with $I = 1$. Therefore, in order to perform the qualitative comparison of our predictions with the experimental data available for the cross section of $e^+e^- \to 6\pi$ annihilation [the sum of the cross sections $e^+e^- \to 3(\pi^+\pi^-)$ and $e^+e^- \to 2(\pi^+\pi^-\pi^0)$], we approximate this cross section by the function $A \cdot \sigma^1_{\mathrm{ann}} + B \cdot E + C$, where A, B, and C are some fitting parameters. We fit these parameters in the energy region between 1.7 GeV and 2.1 GeV. We obtain the best coincidence for $A = 0.56$, $B = 0.012$ nb/MeV, $C = 4.96$ nb. The comparison of the experimental data and our fitting formula in Fig. 5 demonstrates good agreement.

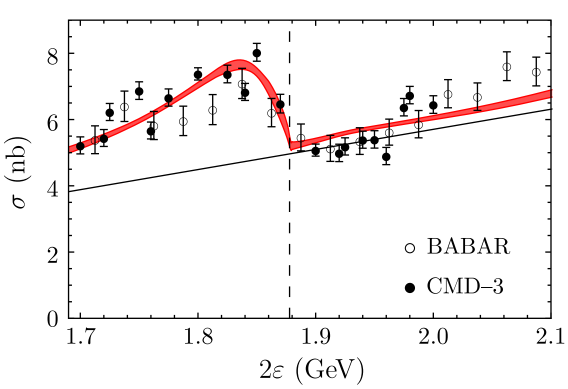

FIG. 5. The prediction for the cross section of 6π production (red thick line). The thin line shows the contribution of non-$N\bar{N}$ channels. The data for total 6π production are calculated from *BABAR* [14] and CMD-3 [15,16] data. The sum of the cross sections $e^+e^- \to 3(\pi^+\pi^-)$ and $e^+e^- \to 2(\pi^+\pi^-\pi^0)$ is shown, $\varepsilon = M + E$.

V. CONCLUSIONS

We have proposed an optical potential describing simultaneously the experimental data for $N\bar{N}$ scattering and e^+e^- annihilation to $N\bar{N}$ and 6π close to the threshold of $N\bar{N}$ production. Our model predicts the dip in the total cross section of e^+e^- annihilation to mesons, which is consistent with the observed behavior of the cross section of 6π production (see Fig. 5). The calculation of the inelastic cross section of the process $e^+e^- \to$ *mesons* is based on the use of the Green's function method. The Green's function of the Schrödinger equation in the optical potential is derived with the tensor forces taken into account.

It is worth noting that we found several sets of potential parameters that fit the experimental data for $N\bar{N}$ scattering and for the cross sections $e^+e^- \to N\bar{N}$ with good χ^2. However, the account for the data for the cross section of $e^+e^- \to 6\pi$ close to the threshold of $N\bar{N}$ production leads to the unique set of parameters presented in Table I. Diminishing the uncertainties of experimental data for the cross sections of other channels of e^+e^- annihilation into mesons would be very important for better determination of the optical potential.

ACKNOWLEDGMENTS

This work was supported in part by RFBR under Grants No. 14-02-00016 and No. 15-02-07893.

[1] B. El-Bennich, M. Lacombe, B. Loiseau, and S. Wycech, Phys. Rev. C **79**, 054001 (2009).
[2] D. Zhou and R. G. E. Timmermans, Phys. Rev. C **86**, 044003 (2012).
[3] X.-W. Kang, J. Haidenbauer, and U.-G. Meißner, J. High Energy Phys. 02 (2014) 113.
[4] V. F. Dmitriev, A. I. Milstein, and V. M. Strakhovenko, Nucl. Instrum. Methods Phys. Res., Sect. B **266**, 1122 (2008).
[5] Y. Uzikov and J. Haidenbauer, Phys. Rev. C **79**, 024617 (2009).
[6] V. F. Dmitriev, A. I. Milstein, and S. G. Salnikov, Phys. Lett. B **690**, 427 (2010).
[7] D. Zhou and R. G. E. Timmermans, Phys. Rev. C **87**, 054005 (2013).
[8] V. Dmitriev and A. Milstein, Phys. Lett. B **658**, 13 (2007).
[9] V. F. Dmitriev, A. I. Milstein, and S. G. Salnikov, Phys. At. Nucl. **77**, 1173 (2014).
[10] J. P. Lees *et al.*, Phys. Rev. D **87**, 092005 (2013).
[11] R. R. Akhmetshin *et al.*, arXiv:1507.08013.
[12] M. N. Achasov *et al.*, Phys. Rev. D **90**, 112007 (2014).
[13] J. Haidenbauer, X.-W. W. Kang, and U.-G. G. Meißner, Nucl. Phys. **A929**, 102 (2014).
[14] B. Aubert *et al.*, Phys. Rev. D **73**, 052003 (2006).
[15] R. Akhmetshin *et al.*, Phys. Lett. B **723**, 82 (2013).
[16] P. A. Lukin *et al.*, Phys. At. Nucl. **78**, 353 (2015).
[17] A. E. Obrazovsky and S. I. Serednyakov, JETP Lett. **99**, 315 (2014).
[18] J. Haidenbauer, C. Hanhart, X.-W. Kang, and U.-G. Meißner, Phys. Rev. D **92**, 054032 (2015).
[19] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics*, 2nd ed. (Pergamon Press, Oxford, 1982).
[20] V. S. Fadin and V. A. Khoze, JETP Lett. **46**, 525 (1987).
[21] T. E. O. Ericson and W. Weise, *Pions and Nuclei* (Clarendon Press, Oxford, 1988).
[22] A. M. Badalyan, L. P. Kok, M. I. Polikarpov, and Y. A. Simonov, Phys. Rep. **82**, 31 (1982).
[23] B. Aubert *et al.*, Phys. Rev. D **71**, 052001 (2005).
[24] M. Achasov *et al.*, EPJ Web Conf. **71**, 00121 (2014).
[25] B. Aubert *et al.*, Phys. Rev. D **76**, 092005 (2007).
[26] C. Amsler *et al.*, Nucl. Phys. **A720**, 357 (2003).
[27] E. Klempt, C. Batty, and J. M. Richard, Phys. Rep. **413**, 197 (2005).