Three-flavor chiral effective model with four baryonic multiplets within the mirror assignment

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In the case of three quark flavors, (pseudo)scalar diquarks transform as antiquarks under chiral transformations. We construct four spin-$1/2$ baryonic multiplets from left- and right-handed quarks as well as left- and right-handed diquarks. The fact that two of these multiplets transform in a "mirror" way allows for chirally invariant mass terms. We then embed these baryonic multiplets into the Lagrangian of the so-called extended linear sigma model, which features (pseudo)scalar and (axial-)vector mesons, as well as glueballs. Reducing the Lagrangian to the two-flavor case, we obtain four doublets of nucleonic states. These mix to produce four experimentally observed states with definite parity: the positive-parity nucleon $N(939)$ and Roper resonance $N(1440)$, as well as the negative-parity resonances $N(1535)$ and $N(1650)$. We determine the parameters of the nucleonic part of the Lagrangian from a fit to masses and decay properties of the aforementioned states. Studying the limit of vanishing quark condensate, we conclude that $N(939)$ and $N(1535)$, as well as $N(1440)$ and $N(1650)$, form pairs of chiral partners.

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I. INTRODUCTION

The strong interaction determines the masses of the baryons and their interactions with mesons. At low energies, chiral effective approaches play an important role to describe these phenomena [1]. Most notably, one can use chiral perturbation theory, which is based on the nonlinear realization of chiral symmetry [2–4], or σ-like models, which are based on the linear realization of chiral symmetry [5–10].

An effective model based on linearly realized chiral symmetry as well as dilatation invariance has been constructed in Refs. [9–14]. This so-called extended linear sigma model (eLSM) also contains anomalous, explicit, and spontaneous symmetry-breaking (SSB) terms in order to reproduce known features of the strong interaction. The mesonic sector of the eLSM, first developed for two flavors ($N_f = 2$) [11] and further extended to $N_f = 3$ [12,13] and $N_f = 4$ [14], includes scalar and pseudoscalar as well as vector and axial-vector degrees of freedom. It is able to describe mesonic masses and decays of quark-antiquark mesons up to 1.7 GeV within reasonable accuracy [for precursory models including (axial-)vector degrees of freedom, see Ref. [15]]. Moreover, in agreement with results from other approaches [16], the model implies that the scalar quark-antiquark states are heavier than 1 GeV and that $f_0(1710)$ is predominantly gluonic [13]. As a consequence, the chiral partner of the pion is the resonance $f_0(1370)$ and not the light scalar state $f_0(500)$ [which, together with the other light scalar mesons, is a state made from (at least) four quarks, either a resonance dynamically generated in the pseudoscalar scattering continuum or a diquark-diquark configuration; see e.g. Refs. [17–21]].

In the standard linear sigma model with nucleons only, chiral symmetry requires that the mass of the nucleon be (apart from explicit symmetry-breaking effects from the current quark masses) solely generated by the chiral condensate, $m_N \propto \langle \bar{q}q \rangle$. However, when one includes the chiral partner of the nucleon, one can either assume that the partner transforms as the nucleon under chiral transformations (the so-called "naive" assignment), or that it transforms in a "mirror" way (the so-called "mirror" assignment) [22–26]. The latter allows for an additional chirally invariant mass term, which physically parametrizes the contribution to the nucleon mass that arises from sources other than the chiral condensate (e.g. a gluon or a four-quark condensate). Nucleons and their chiral partners have been studied within the eLSM in the mirror assignment in Refs. [9,10,27], indicating that the contribution to the nucleon mass from these other sources is sizable.

In this work, we extend the work of Refs. [9,10] to the case of baryons with $N_f = 3$ flavors. This extension will enable us to address in future work important problems in hadron physics, such as scattering processes involving strange hadrons [28–31], and in astrophysics, e.g. the hyperon puzzle for compact stars [32,33].

For baryons, the extension to the $N_f = 3$ case is not as straightforward as for mesons. In the $N_f = 2$ case, the nucleon multiplet is described by a spinor isodoublet, $\psi_N = (p, n)^T$, where p and n are the proton and the neutron, respectively. However, in the $N_f = 3$ case the $J^P = \frac{1}{2}^+$ baryon octet is given by a 3×3 matrix,

$$\begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}. \tag{1}$$

Adding the chiral partner $J^P = \frac{1}{2}^-$ multiplet is also not as straightforward as in the $N_f = 2$ case. Here, we utilize a quark-diquark "quasiparticle" picture for the baryonic substructure. We assume that the diquark is a (pseudo) scalar and lives in the color- and flavor-antitriplet representation (a so-called "good" diquark in the nomenclature of Jaffe [17]), such that it transforms as an antiquark. Then, it is quite natural that $\mathbf{J^P = \frac{1}{2}^\pm}$ baryonic fields, just like quark-antiquark mesonic fields, are parametrized by 3×3 matrices. In a chirally symmetric approach, it is also natural to construct baryons from quarks and diquarks with definite behavior under chiral transformations, i.e., from left- and right-handed quarks and diquarks. If we want to include states that transform in the mirror assignment, such that we can construct chirally invariant mass terms in the Lagrangian, we will show that we are then necessarily led to consider four distinct baryonic multiplets. The possibility to have four multiplets of chiral partners in the mirror assignment was already discussed in the outlook of Ref. [9]. Then, instead of only the ground-state baryon (the nucleon doublet for $N_f = 2$) and its chiral partner, two positive-parity baryons [the nucleon and the Roper $N(1440)$ for $N_f = 2$] and two negative-parity states [$N(1535)$ and $N(1650)$ for $N_f = 2$] occur.

This paper is organized as follows: In Sec. II we present our model and its implications. Namely, in Sec. II A we introduce the baryonic fields for $N_f = 3$, and in Sec. II B the corresponding Lagrangian. A full $N_f = 3$ analysis with $32 = 8 \times 4$ baryonic resonances is very difficult. Therefore, for the present work we decided to study a simplified scenario by considering a reduction of the $N_f = 3$ Lagrangian to the $N_f = 2$ case. This reduction is discussed in Sec. II C. In Sec. II D the mass matrix involving the four nucleonic states $N(939), N(1440)$, $N(1535)$, and $N(1650)$ is determined and diagonalized. In Sec. III we perform a fit of the parameters of our model to experimental data [34] for the masses, decay widths, and axial coupling constants. In Sec. IV we discuss our results and give an outlook to future work. Technical details are relegated to various appendixes.

We use natural units, $\hbar = c = 1$, and the metric tensor is $(g^{\mu\nu}) = \mathrm{diag}(+,-,-,-)$.

II. THE MODEL AND ITS IMPLICATIONS

In this section we first construct the baryonic fields in a chiral quark-diquark picture. We account for the fact that two of the four baryonic fields transform in a "mirror" way as compared to the other two. We then present the complete Lagrangian of the eLSM for $N_f = 3$ flavors. A reduction to $N_f = 2$ flavors is performed, and finally the mass matrix for the four nucleonic states $N(939), N(1440), N(1535)$, and $N(1650)$ is given.

A. Baryonic fields for $N_f = 3$

In the two-flavor case, one works with isospin doublets ψ_i, where the upper field is proton-like, i.e., of the type uud, and the lower field is neutron-like, i.e., of the type udd. The right- and left-handed components ψ_{iR} and ψ_{iL} behave either in a "naive" or in a mirror way under chiral transformations. The naive transformation behavior implies $\psi_{iR} \to U_R \psi_{iR}$ and $\psi_{iL} \to U_L \psi_{iL}$, while the mirror one implies $\psi_{iR} \to U_L \psi_{iR}$ and $\psi_{iL} \to U_R \psi_{iL}$, where the index i labels the nucleonic doublets and the quantities U_R and U_L are 2×2 matrix representations of the chiral group $U(2)_R \times U(2)_L$.

As mentioned in the Introduction, for three flavors $J^P = \frac{1}{2}^+$ baryons are described by 3×3 matrices. In order to construct these fields we use a chiral quark-diquark model (see Ref. [35] and in particular Ref. [36]), i.e., baryons are considered to be made of a quark and a diquark, where a diquark is a (colored) state consisting of two quarks. We are interested in so-called "good" diquarks [17,18,36], which are (pseudo)scalar objects with antisymmetric color- and flavor-wave functions. For $N_f = 3$ there are three scalar, $J^P = 0^+$, and three pseudoscalar diquarks, $J^P = 0^-$. Mathematically, they can be expressed as follows [36]:

$$J^P = 0^+ : \ \mathcal{D}_{ij} = \frac{1}{\sqrt{2}} (q_j^T C\gamma^5 q_i - q_i^T C\gamma^5 q_j) \equiv \sum_{k=1}^{3} D_k \epsilon_{kij}$$

$$\text{with} \quad D_k = \frac{1}{\sqrt{2}} \epsilon_{klm} q_m^T C\gamma^5 q_l,$$

$$J^P = 0^- : \ \tilde{\mathcal{D}}_{ij} = \frac{1}{\sqrt{2}} (q_j^T C q_i - q_i^T C q_j) \equiv \sum_{k=1}^{3} \tilde{D}_k \epsilon_{kij}$$

$$\text{with} \quad \tilde{D}_k = \frac{1}{\sqrt{2}} \epsilon_{klm} q_m^T C q_l, \tag{2}$$

where D_k is the scalar diquark current and $\tilde{D}_k$ is the pseudoscalar diquark current. The indices i, j, k, l, and m are flavor indices. The color structure of these objects is formally identical to the flavor structure and thus suppressed here. From the scalar and pseudoscalar diquarks (2) we can construct left- and right-handed diquarks,

$$\mathcal{D}_R \coloneqq \frac{1}{\sqrt{2}} (\tilde{\mathcal{D}} + \mathcal{D}) = \sum_{i=1}^{3} D_i^R A^i \quad \text{with} \quad D_i^R \equiv \frac{1}{\sqrt{2}} (\tilde{D}_i + D_i),$$

$$\mathcal{D}_L \coloneqq \frac{1}{\sqrt{2}} (\tilde{\mathcal{D}} - \mathcal{D}) = \sum_{i=1}^{3} D_i^L A^i \quad \text{with} \quad D_i^L \equiv \frac{1}{\sqrt{2}} (\tilde{D}_i - D_i),$$

where $(A_i)_{jk} = \epsilon_{ijk}$. Under $U(3)_L \times U(3)_R$ chiral transformations, they behave as

$$D_i^L \to D_i^L U_L^\dagger, \qquad D_i^R \to D_i^R U_R^\dagger, \tag{3}$$

where U_L and U_R are unitary 3×3 matrices. Thus, $D_i^{L(R)}$ transforms as a left-(right-)handed antiquark.

In order to construct baryonic fields as quark-diquark pairs, we have to combine D_j^R or D_j^L with a quark, q_i:

$$N_1 \equiv (N_1)_{ij} \hat{=} D_j^R q_i = \frac{1}{\sqrt{2}}(\tilde{D}_j + D_j)q_i,$$
$$N_2 \equiv (N_2)_{ij} \hat{=} D_j^L q_i = \frac{1}{\sqrt{2}}(\tilde{D}_j - D_j)q_i.$$

These two fields are obviously 3×3 matrices in flavor space.

We now compute the left- and right-handed components of these fields. To this end, one has to take into account that the chiral projection operators act only on the quark fields q_i, because they carry a spinor index, while the diquarks are scalars in Dirac space:

$$N_{1(2)R} = \mathcal{P}_R N_{1(2)} \hat{=} D^{R(L)} q_R,$$
$$N_{1(2)L} = \mathcal{P}_L N_{1(2)} \hat{=} D^{R(L)} q_L.$$

Using the transformation behavior of a quark spinor and Eq. (3), the chiral transformation of the baryonic fields can be computed as

$$N_{1R} \to U_R N_{1R} U_R^\dagger, \qquad N_{1L} \to U_L N_{1L} U_R^\dagger,$$
$$N_{2R} \to U_R N_{2R} U_L^\dagger, \qquad N_{2L} \to U_L N_{2L} U_L^\dagger. \tag{4}$$

One observes that the chiral transformation from the left follows the naive assignment, while the one from the right results from the transformation of the diquark field ($1 \leftrightarrow R$, $2 \leftrightarrow L$). Thus, the presence of two multiplets which transform in a naive way (from the left) is quite natural in the $N_f = 3$ framework.

The behavior under parity and charge-conjugation transformations is given by

	Parity	Charge conjugation
N_{1R}	$-\gamma^0 N_{2L}(t,-\boldsymbol{x})$	$-i\gamma^2(N_{2L})^\star$
N_{1L}	$-\gamma^0 N_{2R}(t,-\boldsymbol{x})$	$-i\gamma^2(N_{2R})^\star$
N_{2R}	$-\gamma^0 N_{1L}(t,-\boldsymbol{x})$	$-i\gamma^2(N_{1L})^\star$
N_{2L}	$-\gamma^0 N_{1R}(t,-\boldsymbol{x})$	$-i\gamma^2(N_{1R})^\star$

, (5)

which shows that the fields N_1 and N_2 are not parity eigenstates and cannot be directly associated with existing resonances (even in the limit of vanishing mixing).

Furthermore, we introduce two baryonic matrices M_1 and M_2 whose chiral transformation from the left is "mirror-like." These fields can be constructed in the same way as N_1 and N_2; however, we need to include an additional Dirac matrix so that a left-(right-)handed projection operator is converted into a right-(left-)handed one (due to the commutation relation $[\gamma^5, \gamma^\mu] = 0$). Only then one can act with a right-(left-)handed chiral transformation $U_{R(L)}$ from the left onto $M_{iR(L)}$. To contract the additional Lorentz index we also include a partial derivative. Consequently, the mathematical structure of the "mirror-like" fields is given by

$$M_1 \equiv (M_1)_{ij} \hat{=} D_j^R \gamma^\mu \partial_\mu q_i = \frac{1}{\sqrt{2}}(\tilde{D}_j + D_j)\gamma^\mu \partial_\mu q_i,$$
$$M_2 \equiv (M_2)_{ij} \hat{=} D_j^L \gamma^\mu \partial_\mu q_i = \frac{1}{\sqrt{2}}(\tilde{D}_j - D_j)\gamma^\mu \partial_\mu q_i.$$

Their chiral transformations are given by

$$M_{1R} \to U_L M_{1R} U_R^\dagger, \qquad M_{1L} \to U_R M_{1L} U_R^\dagger,$$
$$M_{2R} \to U_L M_{2R} U_L^\dagger, \qquad M_{2L} \to U_R M_{2L} U_L^\dagger, \tag{6}$$

where the left transformation is now mirror-like, while the one from the right results from the transformation of the diquark field ($1 \leftrightarrow R$, $2 \leftrightarrow L$). Under parity they transform just as N_1 and N_2, but under charge conjugation they transform with a reversed sign:

	Parity	Charge conjugation
M_{1R}	$-\gamma^0 M_{2L}(t,-\boldsymbol{x})$	$i\gamma^2(M_{2L})^\star$
M_{1L}	$-\gamma^0 M_{2R}(t,-\boldsymbol{x})$	$i\gamma^2(M_{2R})^\star$
M_{2R}	$-\gamma^0 M_{1L}(t,-\boldsymbol{x})$	$i\gamma^2(M_{1L})^\star$
M_{2L}	$-\gamma^0 M_{1R}(t,-\boldsymbol{x})$	$i\gamma^2(M_{1R})^\star$

. (7)

The transformation laws (4)–(7) allow us to write down a baryonic Lagrangian with chirally invariant mass terms; see the next section and Appendix B.

Baryonic fields with definite behavior under parity transformations are introduced as

$$B_N = \frac{N_1 - N_2}{\sqrt{2}}, \qquad B_{N*} = \frac{N_1 + N_2}{\sqrt{2}},$$
$$B_M = \frac{M_1 - M_2}{\sqrt{2}}, \qquad B_{M*} = \frac{M_1 + M_2}{\sqrt{2}}, \tag{8}$$

where now B_N and B_M have positive parity and B_{N*} and B_{M*} have negative parity. In the limit of zero mixing, B_N describes the ground-state baryonic fields of Eq. (1), i.e., $\{N(939), \Lambda(1116), \Sigma(1193), \Xi(1338)\}$; B_M describes the positive-parity fields $\{N(1440), \Lambda(1600), \Sigma(1660), \Xi(1690)\}$; B_{N*} can be assigned to the negative-parity fields $\{N(1535), \Lambda(1670), \Sigma(1620), \Xi(?)\}$; and, finally, B_{M*} to $\{N(1650), \Lambda(1800), \Sigma(1750), \Xi(?)\}$. The detailed study of the mixing will be performed below for the two-flavor case.

B. The eLSM Lagrangian for $N_f = 3$

The mesonic part of the Lagrangian of the eLSM reads [12]

$$\begin{aligned}\mathcal{L}_{\text{meson}} &= \mathrm{Tr}\{(D^\mu\Phi)^\dagger D_\mu\Phi)\} - m_0^2\mathrm{Tr}\{\Phi^\dagger\Phi\} - \lambda_1(\mathrm{Tr}\{\Phi^\dagger\Phi\})^2 - \lambda_2\mathrm{Tr}\{(\Phi^\dagger\Phi)^2\} \\ &\quad -\frac{1}{4}\mathrm{Tr}\{L_{\mu\nu}L^{\mu\nu} + R_{\mu\nu}R^{\mu\nu}\} + \mathrm{Tr}\left\{\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu L^\mu + R_\mu R^\mu)\right\} \\ &\quad + \mathrm{Tr}\{H(\Phi+\Phi^\dagger)\} + c(\det\Phi - \det\Phi^\dagger)^2 \\ &\quad + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ &\quad + \frac{h_1}{2}\mathrm{Tr}\{\Phi^\dagger\Phi\}\mathrm{Tr}\{L_\mu L^\mu + R_\mu R^\mu\} + h_2\mathrm{Tr}\{(L_\mu\Phi)^\dagger(L^\mu\Phi) + (\Phi R_\mu)^\dagger(\Phi R^\mu)\} \\ &\quad + 2h_3\mathrm{Tr}\{\Phi R^\mu\Phi^\dagger L^\mu\} + g_3(\mathrm{Tr}\{L_\mu L_\nu L^\mu L^\nu\} + \mathrm{Tr}\{R_\mu R_\nu R^\mu R^\nu\}) \\ &\quad + g_4(\mathrm{Tr}\{L_\mu L^\mu L_\nu L^\nu\} + \mathrm{Tr}\{R_\mu R^\mu R_\nu R^\nu\}) + g_5\mathrm{Tr}\{L_\mu L^\mu\}\mathrm{Tr}\{R_\nu R^\nu\} \\ &\quad + g_6(\mathrm{Tr}\{L_\mu L^\mu\}\mathrm{Tr}\{L_\nu L^\nu\} + \mathrm{Tr}\{R_\mu R^\mu\}\mathrm{Tr}\{R_\nu R^\nu\}), \end{aligned} \tag{9}$$

with the covariant derivative $D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu)$ and the field-strength tensors $R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$, $L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu$. The matrices Φ, R^μ, and L^μ represent the (pseudo)scalar and (axial-)vector nonets:

$$\Phi = \sum_{i=0}^{8}(S_i + iP_i)T_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N + a_0^0 + i(\eta_N+\pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ \\ a_0^- + i\pi^- & \frac{\sigma_N - a_0^0 + i(\eta_N-\pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 \\ K_0^{*-} + iK^- & \bar{K}_0^{*0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix},$$

$$R^\mu = \sum_{i=0}^{8}(V_i^\mu - A_i^\mu)T_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N^\mu+\rho^{0\mu}}{\sqrt{2}} - \frac{f_{1N}^\mu + a_1^{0\mu}}{\sqrt{2}} & \rho^{+\mu} - a_1^{+\mu} & K^{*+\mu} - K_1^{+\mu} \\ \rho^{-\mu} - a_1^{-\mu} & \frac{\omega_N^\mu-\rho^{0\mu}}{\sqrt{2}} - \frac{f_{1N}^\mu - a_1^{0\mu}}{\sqrt{2}} & K^{0*\mu} - K_1^{0\mu} \\ K^{*-\mu} - K_1^{-\mu} & \bar{K}^{*0\mu} - \bar{K}_1^{0\mu} & \omega_S^\mu - f_{1S}^\mu \end{pmatrix},$$

$$L^\mu = \sum_{i=0}^{8}(V_i^\mu + A_i^\mu)T_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N^\mu+\rho^{0\mu}}{\sqrt{2}} + \frac{f_{1N}^\mu + a_1^{0\mu}}{\sqrt{2}} & \rho^{+\mu} + a_1^{+\mu} & K^{*+\mu} + K_1^{+\mu} \\ \rho^{-\mu} + a_1^{-\mu} & \frac{\omega_N^\mu-\rho^{0\mu}}{\sqrt{2}} + \frac{f_{1N}^\mu - a_1^{0\mu}}{\sqrt{2}} & K^{*0\mu} + K_1^{0\mu} \\ K^{*-\mu} + K_1^{-\mu} & \bar{K}^{*0\mu} + \bar{K}_1^{0\mu} & \omega_S^\mu + f_{1S}^\mu \end{pmatrix}. \tag{10}$$

Here, S_i $(i = 0,\ldots,8)$ represents the scalar, P_i the pseudoscalar, V_i the vector, and A_i the axial-vector mesonic fields. The quantities T_i are the generators of $U(3)$. Under $U(N_f)_R \times U(N_f)_L$, chiral transformations Φ behave as $\Phi \to U_L\Phi U_R^\dagger$, and the left- and right-handed vector fields as $R^\mu \to U_R R^\mu U_R^\dagger$ and $L^\mu \to U_L L^\mu U_L^\dagger$.

For $H = \Delta = c = 0$, the Lagrangian $\mathcal{L}_{\text{meson}}$ is invariant under global chiral $U(3)_R \times U(3)_L (= U(3)_V \times U(3)_A)$ transformations. The $U(1)_A$ anomaly of QCD is parametrized by $c \neq 0$. The explicit breaking of $U(3)_A$ due to the nonzero quark masses in the (pseudo)scalar and (axial-)vector sector is implemented by the terms proportional to H and Δ, respectively. We assume isospin symmetry for the u and d quarks to be exact. As a consequence, only the pure nonstrange scalar-isoscalar field σ_N and the pure strange scalar-isoscalar field σ_S, carrying the same quantum numbers as the vacuum, condense and have nonzero vacuum expectation values (VEVs); for more details and for the values of all relevant parameters, see Ref. [11].

To describe the baryonic degrees of freedom and their interactions with mesons, we use the following Lagrangian, which is invariant under global chiral $U(3)_R \times U(3)_L$ as well as parity and charge-conjugation transformations:

$$\begin{aligned}
\mathcal{L}_{N_f=3} = {} & \mathrm{Tr}\{\bar{N}_{1L} i\gamma_\mu D^\mu_{2L} N_{1L} + \bar{N}_{1R} i\gamma_\mu D^\mu_{1R} N_{1R} + \bar{N}_{2L} i\gamma_\mu D^\mu_{1L} N_{2L} + \bar{N}_{2R} i\gamma_\mu D^\mu_{2R} N_{2R}\} \\
& + \mathrm{Tr}\{\bar{M}_{1L} i\gamma_\mu D^\mu_{4R} M_{1L} + \bar{M}_{1R} i\gamma_\mu D^\mu_{3L} M_{1R} + \bar{M}_{2L} i\gamma_\mu D^\mu_{3R} M_{2L} + \bar{M}_{2R} i\gamma_\mu D^\mu_{4L} M_{2R}\} \\
& - g_N \mathrm{Tr}\{\bar{N}_{1L}\Phi N_{1R} + \bar{N}_{1R}\Phi^\dagger N_{1L} + \bar{N}_{2L}\Phi N_{2R} + \bar{N}_{2R}\Phi^\dagger N_{2L}\} \\
& - g_M \mathrm{Tr}\{\bar{M}_{1L}\Phi^\dagger M_{1R} + \bar{M}_{1R}\Phi M_{1L} + \bar{M}_{2L}\Phi^\dagger M_{2R} + \bar{M}_{2R}\Phi M_{2L}\} \\
& - m_{0,1} \mathrm{Tr}\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\} \\
& - m_{0,2} \mathrm{Tr}\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\} \\
& - \kappa_1 \mathrm{Tr}\{\bar{N}_{1R}\Phi^\dagger N_{2L}\Phi + \bar{N}_{2L}\Phi N_{1R}\Phi^\dagger\} - \kappa_1' \mathrm{Tr}\{\bar{N}_{1L}\Phi N_{2R}\Phi + \bar{N}_{2R}\Phi^\dagger N_{1L}\Phi^\dagger\} \\
& - \kappa_2 \mathrm{Tr}\{\bar{M}_{1R}\Phi M_{2L}\Phi + \bar{M}_{2L}\Phi^\dagger M_{1R}\Phi^\dagger\} - \kappa_2' \mathrm{Tr}\{\bar{M}_{1L}\Phi^\dagger M_{2R}\Phi + \bar{M}_{2R}\Phi M_{1L}\Phi^\dagger\} \\
& - \epsilon_1 (\mathrm{Tr}\{\bar{N}_{1L}\Phi\}\mathrm{Tr}\{N_{2R}\Phi\} + \mathrm{Tr}\{\bar{N}_{2R}\Phi^\dagger\}\mathrm{Tr}\{N_{1L}\Phi^\dagger\}) \\
& - \epsilon_2 (\mathrm{Tr}\{\bar{M}_{1R}\Phi\}\mathrm{Tr}\{M_{2L}\Phi\} + \mathrm{Tr}\{\bar{M}_{2L}\Phi^\dagger\}\mathrm{Tr}\{M_{1R}\Phi^\dagger\}) \\
& - \epsilon_3 \mathrm{Tr}\{\Phi^\dagger\Phi\}\mathrm{Tr}\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\} \\
& - \epsilon_4 \mathrm{Tr}\{\Phi^\dagger\Phi\}\mathrm{Tr}\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\},
\end{aligned} \tag{11}$$

where the covariant derivatives are given by

$$D^\mu_{kR} = \partial^\mu - i c_k R^\mu, \quad D^\mu_{kL} = \partial^\mu - i c_k L^\mu, \quad k = 1, \ldots, 4,$$

with dimensionless coupling constants $c_1, \ldots, c_4$, which determine the strength of baryon-baryon-(axial-)vector interactions. The interactions of the baryonic fields with (pseudo)scalar mesons are parametrized by g_N and g_M, which are also dimensionless. The terms proportional to $\kappa_1, \kappa_2, \kappa_1', \kappa_2'$ (and ϵ_i) are included because otherwise the baryonic fields become pairwise degenerate in mass (see Appendix A). Terms parametrized by ϵ_i are proportional to a product of two traces. Such terms are large-N_c suppressed (OZI rule) and will be neglected in the following discussion. The explicit form of the Lagrangian in terms of the parity eigenstates B_N, B_{N*}, B_M, and B_{M*} is given in Appendix B.

Note that the terms in the first four lines of Eq. (11) have naive scaling dimension 4, and are thus dilatation invariant. The terms in the fifth and sixth lines have naive scaling dimension 3. Thus, they formally break dilatation symmetry, but can be made dilation invariant assuming that $m_{0,1}$ and $m_{0,2}$ are proportional to a gluon and/or a four-quark condensate (with a dimensionless proportionality constant). Such terms arise from the (dilatation-invariant) interaction of a glueball and/or a four-quark state with baryons, assuming that a spontaneous or explicit symmetry-breaking mechanism induces a nonvanishing VEV for the gluon and/or the four-quark field. For a more detailed description of how one can render the mass term dilation invariant by including a tetraquark, see e.g. Ref. [18].

The terms in the seventh to twelfth lines of Eq. (11) have naive scaling dimension 5 and therefore also break dilatation symmetry. However, in this case the coupling constants κ_i, $\kappa_i{}'$, and ϵ_i would need to be proportional to inverse powers of a gluon and/or a four-quark condensate. Such terms can only arise from nonanalytic interaction terms between baryons and glueballs/four-quark states, which should be avoided in a Lagrangian prescription. Nevertheless, these terms may also be considered as effective four-point interactions arising from two (dilatation-invariant) three-point interaction vertices between a meson, a baryon, and a heavier baryonic resonance, where the vertices are connected by a propagator of the latter. If the mass of the baryon resonance is much larger than the typical energy scale where the Lagrangian (11) is applicable, its propagator may be considered to be static and homogeneous, resulting in the four-point interactions proportional to κ_i, $\kappa_i{}'$, and ϵ_i in Eq. (11).

C. The Lagrangian for $N_f = 2$

In this section we reduce the $N_f = 3$ Lagrangian (11) to $N_f = 2$ flavors. In order to achieve this reduction, we set all strange quark fields s to zero. Only the (1 3) and (2 3) elements of the baryonic matrices remain:

$$B_N \overset{s=0}{\longrightarrow} \begin{pmatrix} 0 & 0 & \Psi_N^1 \\ 0 & 0 & \Psi_N^2 \\ 0 & 0 & 0 \end{pmatrix}, \quad B_{N*} \overset{s=0}{\longrightarrow} \begin{pmatrix} 0 & 0 & \Psi_{N*}^1 \\ 0 & 0 & \Psi_{N*}^2 \\ 0 & 0 & 0 \end{pmatrix}, \tag{12}$$

$$B_M \overset{s=0}{\longrightarrow} \begin{pmatrix} 0 & 0 & \Psi_M^1 \\ 0 & 0 & \Psi_M^2 \\ 0 & 0 & 0 \end{pmatrix}, \quad B_{M*} \overset{s=0}{\longrightarrow} \begin{pmatrix} 0 & 0 & \Psi_{M*}^1 \\ 0 & 0 & \Psi_{M*}^2 \\ 0 & 0 & 0 \end{pmatrix}, \tag{13}$$

where $\Psi_i^{1(2)}$ $(i = N, N*, M, M*)$ are fields with quark content $\Psi_i^1 \hat{=} uud$ and $\Psi_i^2 \hat{=} udd$. Applying the same to the meson matrix Φ and to the left- and right-handed (axial-)vector fields, L^μ and R^μ, we obtain

$$\Phi \overset{s=0}{\longrightarrow} \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N+\varphi_N+a_0^0)+i(\eta_N+\pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & 0 \\ a_0^- + i\pi^- & \frac{(\sigma_N+\varphi_N-a_0^0)+i(\eta_N-\pi^0)}{\sqrt{2}} & 0 \\ 0 & 0 & \varphi_S \end{pmatrix} \equiv \begin{pmatrix} (\Phi_{N_f=2}) & & 0 \\ & & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}\varphi_S \end{pmatrix}, \tag{14}$$

$$R^\mu \overset{s=0}{\longrightarrow} \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu+\rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu+a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu+} - a_1^{\mu+} & 0 \\ \rho^{\mu-} - a_1^{\mu-} & \frac{\omega_N^\mu-\rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu-a_1^{\mu 0}}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix} \equiv \begin{pmatrix} (R^\mu_{N_f=2}) & & 0 \\ & & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{15}$$

$$L^\mu \overset{s=0}{\longrightarrow} \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu+\rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu+a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu+} + a_1^{\mu+} & 0 \\ \rho^{\mu-} + a_1^{\mu-} & \frac{\omega_N^\mu-\rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu-a_1^{\mu 0}}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix} \equiv \begin{pmatrix} (L^\mu_{N_f=2}) & & 0 \\ & & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{16}$$

Note that it is crucial to first consider the condensation of both scalar fields σ_N and σ_S and only then set the mesons with s quarks to zero; otherwise one would lose the VEV φ_S of the field σ_S. For $N_f = 2$ it is common to write the 2×2 meson matrices in the basis of the three $SU(2)$ generators $\boldsymbol{T} = \boldsymbol{\tau}/2$, where $\boldsymbol{\tau}$ are the Pauli matrices, and $T^0 = \mathbb{1}_{2x2}/2$:

$$\Phi_{N_f=2} = (\sigma_N + \varphi_N + i\eta_N)T^0 + (\boldsymbol{a}_0 + i\boldsymbol{\pi}) \cdot \boldsymbol{T},$$
$$R^\mu_{N_f=2} = (\omega^\mu - f_1^\mu)T^0 + (\boldsymbol{\rho}^\mu - \boldsymbol{a}_1^\mu) \cdot \boldsymbol{T},$$
$$L^\mu_{N_f=2} = (\omega^\mu + f_1^\mu)T^0 + (\boldsymbol{\rho}^\mu + \boldsymbol{a}_1^\mu) \cdot \boldsymbol{T}.$$

As already indicated in the notation, the fields are identified with the mesons listed in Ref. [34] in the following way. The scalar resonances σ and $\boldsymbol{a}_0$ are assigned to $f_0(1370)$ and $a_0(1450)$. The second possibility $\{\sigma, \boldsymbol{a}_0\} \hat{=} \{f_0(500), a_0(980)\}$ has to be excluded, because then our model cannot describe the scattering lengths and the decay $\sigma \to \pi\pi$ at the same time; for more details, see Ref. [11]. The pseudoscalar $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$ is the $SU(2)$ counterpart of the η meson, and $\boldsymbol{\pi}$ corresponds to the pion triplet. The vectors ω^μ and $\boldsymbol{\rho}^\mu$ represent the resonances $\omega(782)$ and $\rho(770)$, and the axial-vector fields f_1^μ and $\boldsymbol{a}_1^\mu$ are identified with the resonances $f_1(1285)$ and $a_1(1260)$.

The resulting Lagrangian for the case $N_f = 2$ reads (for details, see Appendix B)

$$\begin{aligned}
\mathcal{L}_{N_f=2} = {} & \bar{\Psi}_{NR} i\gamma_\mu D^\mu_{NR} \Psi_{NR} + \bar{\Psi}_{NL} i\gamma_\mu D^\mu_{NL} \Psi_{NL} + \bar{\Psi}_{N*R} i\gamma_\mu D^\mu_{NR} \Psi_{N*R} + \bar{\Psi}_{N*L} i\gamma_\mu D^\mu_{NL} \Psi_{N*L} \\
& + \bar{\Psi}_{MR} i\gamma_\mu D^\mu_{ML} \Psi_{MR} + \bar{\Psi}_{ML} i\gamma_\mu D^\mu_{MR} \Psi_{ML} + \bar{\Psi}_{M*R} i\gamma_\mu D^\mu_{ML} \Psi_{M*R} + \bar{\Psi}_{M*L} i\gamma_\mu D^\mu_{MR} \Psi_{M*L} \\
& + c_{A_N}(\bar{\Psi}_{NR} i\gamma_\mu R^\mu \Psi_{N*R} + \bar{\Psi}_{N*R} i\gamma_\mu R^\mu \Psi_{NR} - \bar{\Psi}_{NL} i\gamma_\mu L^\mu \Psi_{N*L} - \bar{\Psi}_{N*L} i\gamma_\mu L^\mu \Psi_{NL}) \\
& + c_{A_M}(\bar{\Psi}_{MR} i\gamma_\mu L^\mu \Psi_{M*R} + \bar{\Psi}_{M*R} i\gamma_\mu L^\mu \Psi_{MR} - \bar{\Psi}_{ML} i\gamma_\mu R^\mu \Psi_{M*L} - \bar{\Psi}_{M*L} i\gamma_\mu R^\mu \Psi_{ML}) \\
& - g_N(\bar{\Psi}_{NL} \Phi \Psi_{NR} + \bar{\Psi}_{NR} \Phi^\dagger \Psi_{NL} + \bar{\Psi}_{N*L} \Phi \Psi_{N*R} + \bar{\Psi}_{N*L} \Phi^\dagger \Psi_{N*R}) \\
& - g_M(\bar{\Psi}_{ML} \Phi^\dagger \Psi_{MR} + \bar{\Psi}_{MR} \Phi \Psi_{ML} + \bar{\Psi}_{M*L} \Phi^\dagger \Psi_{M*R} + \bar{\Psi}_{M*L} \Phi \Psi_{M*R}) \\
& - \frac{m_{0,1} + m_{0,2}}{2}(\bar{\Psi}_{NL} \Psi_{MR} + \bar{\Psi}_{NR} \Psi_{ML} + \bar{\Psi}_{N*L} \Psi_{M*R} + \bar{\Psi}_{N*R} \Psi_{M*L} \\
& + \bar{\Psi}_{ML} \Psi_{NR} + \bar{\Psi}_{MR} \Psi_{NL} + \bar{\Psi}_{M*L} \Psi_{N*R} + \bar{\Psi}_{M*R} \Psi_{N*L}) \\
& - \frac{m_{0,1} - m_{0,2}}{2}(\bar{\Psi}_{NL} \Psi_{M*R} - \bar{\Psi}_{NR} \Psi_{M*L} - \bar{\Psi}_{ML} \Psi_{N*R} + \bar{\Psi}_{MR} \Psi_{N*L} \\
& - \bar{\Psi}_{N*L} \Psi_{MR} + \bar{\Psi}_{N*R} \Psi_{ML} + \bar{\Psi}_{M*L} \Psi_{NR} - \bar{\Psi}_{M*R} \Psi_{NL})
\end{aligned}$$

$$
\begin{aligned}
&-\frac{\kappa_1'+\kappa_1}{2}\frac{\varphi_S}{\sqrt{2}}(-\bar{\Psi}_{NL}\Phi\Psi_{NR}-\bar{\Psi}_{NR}\Phi^\dagger\Psi_{NL}+\bar{\Psi}_{N*L}\Phi\Psi_{N*R}+\bar{\Psi}_{N*R}\Phi^\dagger\Psi_{N*L})\\
&-\frac{\kappa_1'-\kappa_1}{2}\frac{\varphi_S}{\sqrt{2}}(\bar{\Psi}_{NL}\Phi\Psi_{N*R}-\bar{\Psi}_{NR}\Phi^\dagger\Psi_{N*L}-\bar{\Psi}_{N*L}\Phi\Psi_{NR}+\bar{\Psi}_{N*R}\Phi^\dagger\Psi_{NL})\\
&-\frac{\kappa_2'+\kappa_2}{2}\frac{\varphi_S}{\sqrt{2}}(-\bar{\Psi}_{ML}\Phi^\dagger\Psi_{MR}-\bar{\Psi}_{MR}\Phi\Psi_{ML}+\bar{\Psi}_{M*L}\Phi^\dagger\Psi_{M*R}+\bar{\Psi}_{M*R}\Phi\Psi_{M*L})\\
&-\frac{\kappa_2'-\kappa_2}{2}\frac{\varphi_S}{\sqrt{2}}(\bar{\Psi}_{ML}\Phi^\dagger\Psi_{M*R}-\bar{\Psi}_{MR}\Phi\Psi_{M*L}-\bar{\Psi}_{M*L}\Phi^\dagger\Psi_{MR}+\bar{\Psi}_{M*R}\Phi\Psi_{ML}),
\end{aligned}
\tag{17}
$$

where we suppress the subscript "$N_f = 2$" of the mesonic fields and introduce the isovectors $\Psi_k = (\Psi_k^1, \Psi_k^2)^T$, $k = N, N*, M, M*$. The covariant derivatives are

$$
D^\mu_{NR} = \partial^\mu - ic_N R^\mu, \qquad D^\mu_{NL} = \partial^\mu - ic_N L^\mu,
$$
$$
D^\mu_{MR} = \partial^\mu - ic_M R^\mu, \qquad D^\mu_{ML} = \partial^\mu - ic_M L^\mu,
$$

with

$$
c_N = \frac{c_1+c_2}{2} \quad \text{and} \quad c_M = \frac{c_3+c_4}{2}.
$$

These two constants parametrize the coupling between baryons of equal parity. The constants

$$
c_{A_N} = \frac{c_1-c_2}{2} \quad \text{and} \quad c_{A_M} = \frac{c_3-c_4}{2}
$$

describe the coupling of two baryons with different parity to (axial-)vector mesons.

Interestingly, the number of parameters of this $N_f = 2$ Lagrangian obtained as a reduction of the more general $N_f = 3$ Lagrangian is smaller than what one would obtain by directly writing down the corresponding two-flavor Lagrangian with four multiplets. This is due to the fact that some terms are not allowed because of the more complex parity and charge-conjugation transformations of the baryonic fields in the $N_f = 3$ case. (Some terms which in principle have different coupling constants in the $N_f = 2$ case [9] now have the same constants, as they transform into each other under parity or charge conjugation.)

D. The mass matrix

After SSB in the meson sector (see Appendix C), the following terms contribute to the mass matrix of the four fields Ψ_N, Ψ_{N*}, Ψ_M, and Ψ_{M*}:

$$
\begin{aligned}
\mathcal{L}_{\text{mass}} = &-\left(\frac{g_N\varphi_N}{2}-\frac{\kappa_1'+\kappa_1}{4\sqrt{2}}\varphi_N\varphi_S\right)\bar{\Psi}_N\Psi_N-\left(\frac{g_N\varphi_N}{2}+\frac{\kappa_1'+\kappa_1}{4\sqrt{2}}\varphi_N\varphi_S\right)\bar{\Psi}_{N*}\Psi_{N*}\\
&-\left(\frac{g_M\varphi_N}{2}-\frac{\kappa_2'+\kappa_2}{4\sqrt{2}}\varphi_N\varphi_S\right)\bar{\Psi}_M\Psi_M-\left(\frac{g_M\varphi_N}{2}+\frac{\kappa_2'+\kappa_2}{4\sqrt{2}}\varphi_N\varphi_S\right)\bar{\Psi}_{M*}\Psi_{M*}\\
&-\frac{\kappa_1'-\kappa_1}{4\sqrt{2}}\varphi_N\varphi_S(\bar{\Psi}_N\gamma^5\Psi_{N*}-\bar{\Psi}_{N*}\gamma^5\Psi_N)-\frac{\kappa_2'-\kappa_2}{4\sqrt{2}}\varphi_N\varphi_S(\bar{\Psi}_M\gamma^5\Psi_{M*}-\bar{\Psi}_{M*}\gamma^5\Psi_M)\\
&-\frac{m_{0,1}+m_{0,2}}{2}(\bar{\Psi}_N\Psi_M+\bar{\Psi}_{N*}\Psi_{M*}+\bar{\Psi}_M\Psi_N+\bar{\Psi}_{M*}\Psi_{N*})\\
&-\frac{m_{0,2}-m_{0,1}}{2}(\bar{\Psi}_N\gamma^5\Psi_{M*}+\bar{\Psi}_{N*}\gamma^5\Psi_M-\bar{\Psi}_M\gamma^5\Psi_{N*}-\bar{\Psi}_{M*}\gamma^5\Psi_N),
\end{aligned}
\tag{18}
$$

where φ_N and φ_S are the VEVs of the σ_N and σ_S mesons, respectively. In order to determine the physical fields N_{939}, N_{1535}, N_{1440}, and N_{1650} corresponding to the resonances $N(939)$, $N(1525)$, $N(1535)$, and $N(1640)$, we have to diagonalize the Lagrangian. To this end, we define the vector

$$
\Psi = (\Psi_N, \gamma^5\Psi_{N*}, \Psi_M, \gamma^5\Psi_{M*})^T \Rightarrow \bar{\Psi} = (\bar{\Psi}_N, -\bar{\Psi}_{N*}\gamma^5, \bar{\Psi}_M, -\bar{\Psi}_{M*}\gamma^5). \tag{19}
$$

The additional γ^5 matrices are introduced in order to avoid such matrices in the mass matrix (20). As a consequence, all four components of the vector Ψ have the same parity.

Rewriting Eq. (18) in matrix form, $\mathcal{L}_{\text{mass}} = -\bar{\Psi}M\Psi$, we obtain the mass matrix

$$M \equiv \frac{1}{2}\begin{pmatrix} g_N\varphi_N - \frac{\kappa_1'+\kappa_1}{2\sqrt{2}}\varphi_N\varphi_S & \frac{\kappa_1'-\kappa_1}{2\sqrt{2}}\varphi_N\varphi_S & m_{0,1}+m_{0,2} & m_{0,1}-m_{0,2} \\ \frac{\kappa_1'-\kappa_1}{2\sqrt{2}}\varphi_N\varphi_S & -g_N\varphi_N - \frac{\kappa_1'+\kappa_1}{2\sqrt{2}}\varphi_N\varphi_S & m_{0,2}-m_{0,1} & -m_{0,1}-m_{0,2} \\ m_{0,1}+m_{0,2} & m_{0,2}-m_{0,1} & g_M\varphi_N - \frac{\kappa_2'+\kappa_2}{2\sqrt{2}}\varphi_N\varphi_S & \frac{\kappa_2'-\kappa_2}{2\sqrt{2}}\varphi_N\varphi_S \\ m_{0,1}-m_{0,2} & -m_{0,1}-m_{0,2} & \frac{\kappa_2'-\kappa_2}{2\sqrt{2}}\varphi_N\varphi_S & -g_M\varphi_N - \frac{\kappa_2'+\kappa_2}{2\sqrt{2}}\varphi_N\varphi_S \end{pmatrix}. \tag{20}$$

At this point it is possible to compare to the Lagrangian of Ref. [9], which only describes the nucleon and its chiral partner. If $m_{0,1} = -m_{0,2}$ and $\kappa_{1(2)} = \kappa'_{1(2)}$, the mass matrix is of the form

$$M_{\mathrm{decoupled}} = \frac{1}{2}\begin{pmatrix} g_N\varphi_N - \frac{\kappa_1}{\sqrt{2}}\varphi_N\varphi_S & 0 & 0 & 2m_{0,1} \\ 0 & -g_N\varphi_N - \frac{\kappa_1}{\sqrt{2}}\varphi_N\varphi_S & -2m_{0,1} & 0 \\ 0 & -2m_{0,1} & g_M\varphi_N - \frac{\kappa_2}{\sqrt{2}}\varphi_N\varphi_S & 0 \\ 2m_{0,1} & 0 & 0 & -g_M\varphi_N - \frac{\kappa_2}{\sqrt{2}}\varphi_N\varphi_S \end{pmatrix}.$$

Obviously, the fields Ψ_N and Ψ_{M*} completely decouple from the fields Ψ_{N*} and Ψ_M, and the diagonalization of the two sets can be performed independently. However, it is not clear which of the two states Ψ_{N*} and Ψ_{M*} should be identified with the chiral partner of Ψ_N (the putative nucleon field), because all states become degenerate in mass (all masses are equal to $m_{0,1}$) when chiral symmetry is restored ($\varphi_N, \varphi_S \to 0$).

In order to diagonalize the mass matrix (20), we have to solve the eigenvalue problem

$$\begin{aligned} M\boldsymbol{u}_k &= m_k\boldsymbol{u}_k, \\ M^{ij}u_k^j &= m_k u_k^i, \end{aligned} \tag{21}$$

where $\boldsymbol{u}_k$ $[k \in \{1, \ldots, \dim(M) = 4\}]$ are the eigenvectors and m_k are the four eigenvalues of the mass matrix M. Note that a sum over j (but not over k) is understood. By multiplying Eq. (21) with $\boldsymbol{u}_l$ from the left-hand side, we find

$$u_l^i M_{ij} u_k^j = m_k u_l^i u_k^i \equiv m_k \delta_{kl}$$

for orthogonal eigenvectors, $\boldsymbol{u}_l \cdot \boldsymbol{u}_k = \delta_{lk}$. Hence, the matrix

$$U_{ij} = u_j^i \tag{22}$$

diagonalizes M:

$$\begin{aligned} U^\dagger M U &= \mathrm{diag}(m_1, m_2, m_3, m_4) \\ &\equiv \mathrm{diag}(m_{939}, -m_{1535}, m_{1440}, -m_{1650}). \end{aligned}$$

In the second equality, we take into account that, due to the definitions (19) and (23), the masses of the negative-parity states correspond to negative eigenvalues of M. Returning to the Lagrangian (18), we now realize that it is diagonalized by

$$\begin{aligned} \mathcal{L}_{\mathrm{mass}} &= -\bar{\Psi} U U^\dagger M U U^\dagger \Psi \\ &= -\bar{\Psi}^{\mathrm{phys}} \mathrm{diag}(m_1, m_2, m_3, m_4) \Psi^{\mathrm{phys}}, \end{aligned}$$

with the physical fields

$$\Psi^{\mathrm{phys}} = U^\dagger \Psi \equiv (N_{939}, \gamma^5 N_{1535}, N_{1440}, \gamma^5 N_{1650})^T. \tag{23}$$

The eigenvalues of M, which (up to a sign) correspond to the masses of the physical fields $N_{939}, N_{1535}, N_{1440}$, and N_{1650}, are determined by the roots of the equation

$$\det\Big[M - m_i \mathbb{1}_{4\times 4}\Big] = 0.$$

In the general case, this will be done numerically; see Sec. III. However, in the chiral limit, i.e., $\varphi_N, \varphi_S \to 0$, one can easily do this analytically. In this case, denoting $\bar{M} \equiv (m_{0,1} + m_{0,2})/2$ and $\mu \equiv (m_{0,1} - m_{0,2})/2$, the mass matrix reads

$$M_{\mathrm{chiral\ limit}} \equiv \begin{pmatrix} 0 & 0 & \bar{M} & \mu \\ 0 & 0 & -\mu & -\bar{M} \\ \bar{M} & -\mu & 0 & 0 \\ \mu & -\bar{M} & 0 & 0 \end{pmatrix}.$$

The eigenvalues of this matrix are $\lambda_{1,2} = \pm(\bar{M} + \mu) = \pm m_{0,1}$ and $\lambda_{3,4} = \pm(\bar{M} - \mu) = \pm m_{0,2}$. As expected, we have two distinct sets of chiral partners. One set has the mass $m_{0,1}$ and the other the mass $m_{0,2}$, which is in general different from $m_{0,1}$. In order to decide which mass eigenstates are chiral partners, we need to compute the transformation matrix U. Somewhat surprisingly,

$$U = \frac{1}{2}\begin{pmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \end{pmatrix} \equiv U^\dagger,$$

which means that the mass eigenstates are uniform mixtures of the fields $\Psi_N, \gamma_5\Psi_{N*}, \Psi_M$, and $\gamma_5\Psi_{M*}$. The chiral partners with mass $m_{0,1}$ are given by the linear combinations $\Psi_N - \gamma_5\Psi_{N*} + (\Psi_M + \gamma_5\Psi_{M*})$ and $-\Psi_N + \gamma_5\Psi_{N*} + (\Psi_M + \gamma_5\Psi_{M*})$, while the chiral partners with mass $m_{0,2}$ are given by $\Psi_N + \gamma_5\Psi_{N*} + (\Psi_M - \gamma_5\Psi_{M*})$ and $\Psi_N + \gamma_5\Psi_{N*} - (\Psi_M - \gamma_5\Psi_{M*})$, respectively. Therefore, it is impossible to decide whether $N(1535)$ or $N(1650)$ is the chiral partner of the nucleon. The solution to this problem will be presented in the next section, where we compute the eigenvalues as a function of φ_N to trace whether the mass of $N(1535)$ or that of $N(1650)$ approaches the nucleon mass in the chiral limit.

III. RESULTS

The Lagrangian of the model in the $N_f = 2$ case [cf. Eqs. (17) and (C1)] contains the following 12 parameters in the baryonic sector: the mass parameters $m_{0,1}$ and $m_{0,2}$, and the coupling constants c_N, c_M, c_{A_N}, c_{A_M}, g_N, g_M, κ_1, κ_2, κ_1', and κ_2'. To determine these parameters, we use the experimental values of the masses of the four baryonic states; the partial decay widths of the baryonic resonances into a nucleon and a pseudoscalar meson, $\Gamma_{N(1535)\to N\pi}$, $\Gamma_{N(1535)\to N\eta}$, $\Gamma_{N(1650)\to N\pi}$, $\Gamma_{N(1650)\to N\eta}$, and $\Gamma_{N(1440)\to N(939)\pi}$; and the axial coupling constant $g_A^{N(939)}$; as well as lattice results [37] for $g_A^{N(1440)}$, $g_A^{N(1535)}$, and $g_A^{N(1650)}$. In total, there are 13 experimental values, which are fitted to 12 parameters. The parameters Z, w, φ_N, and φ_S are already determined by meson physics [11].

For the baryon masses, we use the values given by the PDG [34]. Since our model does not contain isospin-breaking effects, it is not expected to describe the baryon masses to (in some cases very high) experimental precision. Therefore, we assume a 5% uncertainty of the masses (a strategy that was already followed in the fit of Ref. [12]).

The expressions for the decay widths into pseudoscalar mesons and the axial coupling constants are given in Appendixes D and E. The experimental values of the decay widths are obtained from the total width and the branching ratios given by the PDG [34]. The nucleon axial coupling constant is also quoted by the PDG [34], while all other axial coupling constants result from lattice-QCD calculations [37].

Using a standard χ^2 procedure, we find that three acceptable and almost equally deep minima exist. Their corresponding parameter values are given in Table I. It is interesting to note that the first two minima lead to small values of $m_{0,1}$ and $m_{0,2}$, while the third one features values of these constants which are close to the vacuum mass of

TABLE I. The parameter values of the three χ^2 minima and the comparison to experimental quantities.

	Minimum 1		Minimum 2		Minimum 3		Experiment/lattice	
$m_{0,1}$[GeV]	0.1393	±0.0026	0.14	±0.11	−1.078	±0.017	⋯	
$m_{0,2}$[GeV]	−0.2069	±0.0027	−0.18	±0.12	0.894	±0.019	⋯	
c_N	−2.071	±0.023	−2.83	±0.39	−33.6	±2.2	⋯	
c_M	12.4	±1.3	11.7	±1.8	−19.1	±3.1	⋯	
c_{A_N}	−1.00	±0.23	0.03	±0.40	−2.68	±0.80	⋯	
c_{A_M}	−51.0	±2.8	80	±41	−71.7	±6.5	⋯	
g_N	15.485	±0.012	15.24	±0.36	10.58	±0.24	⋯	
g_M	17.96	±0.17	18.26	±0.52	13.07	±0.33	⋯	
κ_1[GeV^{-1}]	37.80	±0.26	59.9	±8.5	32.4	±4.2	⋯	
κ_1'[GeV^{-1}]	57.12	±0.29	29.8	±6.6	55.2	±4.0	⋯	
κ_2[GeV^{-1}]	−20.7	±2.5	32	±13	−20	±13	⋯	
κ_2'[GeV^{-1}]	41.5	±3.2	−8	±13	48.9	±4.5	⋯	
m_N[GeV]	0.9389	±0.0010	0.9389	±0.0010	0.9389	±0.0010	0.9389	±0.001
$m_{N(1440)}$[GeV]	1.430	±0.071	1.432	±0.073	1.429	±0.074	1.43	±0.07
$m_{N(1535)}$[GeV]	1.561	±0.065	1.585	±0.069	1.559	±0.069	1.53	±0.08
$m_{N(1650)}$[GeV]	1.658	±0.076	1.619	±0.071	1.663	±0.081	1.65	±0.08
$\Gamma_{N(1440)\to N\pi}$[GeV]	0.195	±0.087	0.195	±0.088	0.196	±0.087	0.195	±0.087
$\Gamma_{N(1535)\to N\pi}$[GeV]	0.072	±0.019	0.073	±0.019	0.072	±0.019	0.068	±0.019
$\Gamma_{N(1535)\to N\eta}$[GeV]	0.0055	±0.0025	0.0062	±0.0024	0.0055	±0.0027	0.063	±0.018
$\Gamma_{N(1650)\to N\pi}$[GeV]	0.112	±0.033	0.114	±0.033	0.112	±0.033	0.105	±0.037
$\Gamma_{N(1650)\to N\eta}$[GeV]	0.0117	±0.0038	0.0109	±0.0038	0.0119	±0.0038	0.015	±0.008
g_A^N	1.2670	±0.0025	1.2670	±0.0025	1.2670	±0.0025	1.267	±0.003
$g_A^{N(1440)}$	1.20	±0.20	1.19	±0.20	1.21	±0.21	1.2	±0.2
$g_A^{N(1535)}$	0.20	±0.30	0.21	±0.30	0.20	±0.31	0.2	±0.3
$g_A^{N(1650)}$	0.55	±0.20	0.55	±0.20	0.55	±0.20	0.55	±0.2
χ^2	10.3		10.7		10.3		⋯	

the nucleon. Thus, for the first two minima, the main contribution to all masses arises from chiral symmetry breaking, while in the third minimum, most of the mass is generated by another source, e.g. a gluon condensate.

The numerical results for the experimental quantities obtained using the above parameters are also given in Table I. Most of these quantities are described by all solutions of the model within one standard deviation. The most important exception is the $N(1535) \to N\eta$ decay width, which deviates by about an order of magnitude from the experimental value for all scenarios explored (in fact, this deviation completely dominates the value of χ^2). Note that this quantity was also not well described in the study of Ref. [9]. Thus, including more multiplets does not solve this problem, as was erroneously speculated in that reference. Other ideas towards a solution are described in the next section.

It is interesting to discuss the numerical results for the mass matrix M and the mixing matrix U:

Minimum 1: Using the parameters corresponding to minimum 1, the mass matrix (20) reads

$$M_{\min 1} = \begin{pmatrix} 0.926 & 0.071 & -0.034 & 0.173 \\ 0.071 & -1.623 & -0.173 & 0.034 \\ -0.034 & -0.173 & 1.402 & 0.228 \\ 0.173 & 0.034 & 0.228 & -1.555 \end{pmatrix} \text{GeV}.$$

Furthermore, with the numerical value for the transformation matrix U_{ij} defined in Eq. (22) and composed of the eigenvectors of the mass matrix, Eq. (23) can be written as

$$\begin{pmatrix} N_{939} \\ \gamma^5 N_{1535} \\ N_{1440} \\ \gamma^5 N_{1650} \end{pmatrix} = \begin{pmatrix} \mathbf{-0.996} & -0.025 & -0.046 & -0.074 \\ 0.075 & \mathbf{-0.492} & 0.039 & \mathbf{-0.867} \\ -0.050 & -0.057 & \mathbf{0.995} & 0.073 \\ 0.010 & \mathbf{0.869} & 0.086 & \mathbf{-0.488} \end{pmatrix} \begin{pmatrix} \Psi_N \\ \gamma^5 \Psi_{N*} \\ \Psi_M \\ \gamma^5 \Psi_{M*} \end{pmatrix}. \quad (24)$$

Here one can see that, to a first approximation, $N_{939} \approx \Psi_N$, $N_{1440} \approx \Psi_M$, $N_{1535} \approx \Psi_{M*}$, and $N_{1650} \approx \Psi_{N*}$. Furthermore, the two negative-parity states N_{1535} and N_{1650} mix appreciably with each other; the mixing angle is $\sim 30°$.

In order to decide which states form chiral partners, we also compute the masses as a function of φ_N, keeping φ_S at its vacuum value. This allows us to trace the masses when chiral symmetry is restored, $\varphi_N \to 0$. [Note that φ_S only appears together with a factor φ_N in the mass matrix (20)]. The result is shown in Fig. 1, from which we unanimously conclude that $N(939)$ and $N(1535)$ are chiral partners, with

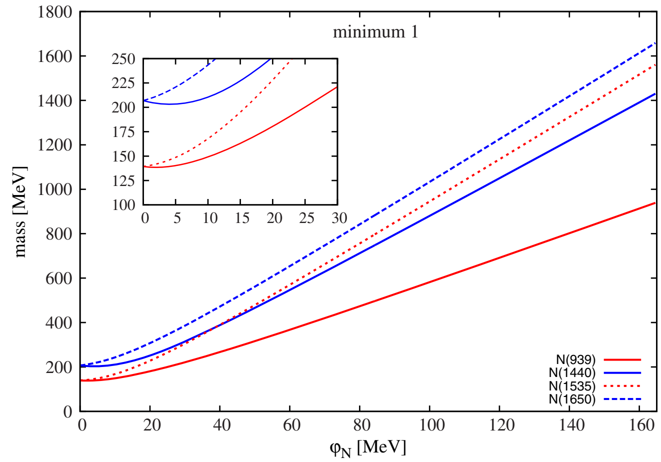

FIG. 1. Masses as a function of φ_N for minimum 1.

a common mass $m_{0,1} = 139$ MeV when chiral symmetry is restored. Consequently, $N(1440)$ and $N(1650)$ are chiral partners with a mass $|m_{0,2}| = 207$ MeV as $\varphi_N \to 0$.

Minimum 2: In this case, the mass matrix reads

$$M_{\min 2} = \begin{pmatrix} 0.925 & -0.111 & -0.017 & 0.161 \\ -0.111 & -1.583 & -0.161 & 0.017 \\ -0.017 & -0.161 & 1.415 & -0.146 \\ 0.161 & 0.017 & -0.146 & -1.590 \end{pmatrix} \text{GeV}.$$

Furthermore, the second minimum has the following transformation matrix:

$$\begin{pmatrix} N_{939} \\ \gamma^5 N_{1535} \\ N_{1440} \\ \gamma^5 N_{1650} \end{pmatrix} = \begin{pmatrix} \mathbf{-0.996} & 0.046 & -0.039 & -0.061 \\ -0.002 & \mathbf{0.806} & 0.072 & \mathbf{0.587} \\ -0.038 & -0.052 & \mathbf{0.997} & -0.051 \\ 0.076 & \mathbf{0.588} & -0.007 & \mathbf{-0.805} \end{pmatrix} \begin{pmatrix} \Psi_N \\ \gamma^5 \Psi_{N*} \\ \Psi_M \\ \gamma^5 \Psi_{M*} \end{pmatrix}. \quad (25)$$

As with minimum 1, the negative-parity states mix strongly, but the mixing matrix is different. Here, we may conclude that $N(1650)$ can be predominantly assigned to Ψ_{M*}.

In order to decide which states form chiral partners, we again compute the masses as a function of φ_N, keeping φ_S at its vacuum value. The result is shown in Fig. 2, from which we again unanimously conclude that $N(939)$ and $N(1535)$ are chiral partners, with a common mass $m_{0,1} = 144$ MeV when chiral symmetry is restored. Consequently, $N(1440)$ and $N(1650)$ are chiral partners with a mass $|m_{0,2}| = 178$ MeV as $\varphi_N \to 0$.

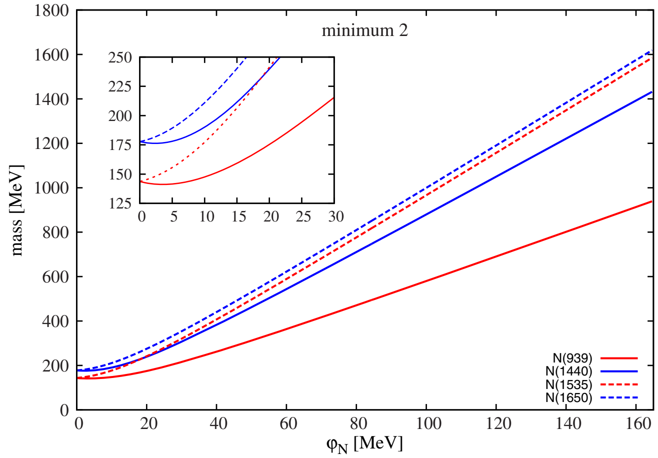

FIG. 2. Masses as a function of φ_N for minimum 2.

Minimum 3: In this case, the mass matrix reads

$$M_{\mathrm{min}\,3} = \begin{pmatrix} 0.549 & 0.084 & -0.092 & -0.986 \\ 0.084 & -1.192 & 0.986 & 0.092 \\ -0.092 & 0.986 & 0.970 & 0.253 \\ -0.986 & 0.092 & 0.253 & -1.181 \end{pmatrix} \mathrm{GeV}.$$

The transformation matrix of the third minimum has a form that is completely different from those of the other two minima:

$$\begin{pmatrix} N_{939} \\ \gamma^5 N_{1535} \\ N_{1440} \\ \gamma^5 N_{1650} \end{pmatrix} = \begin{pmatrix} \mathbf{-0.865} & -0.163 & \mathbf{-0.312} & \mathbf{0.358} \\ 0.140 & \mathbf{0.830} & \mathbf{-0.359} & \mathbf{0.404} \\ \mathbf{-0.292} & \mathbf{0.327} & \mathbf{0.875} & 0.207 \\ \mathbf{-0.384} & \mathbf{0.422} & -0.093 & \mathbf{-0.816} \end{pmatrix} \begin{pmatrix} \Psi_N \\ \gamma^5 \Psi_{N*} \\ \Psi_M \\ \gamma^5 \Psi_{M*} \end{pmatrix}. \tag{26}$$

In this case, all states mix strongly with each other.

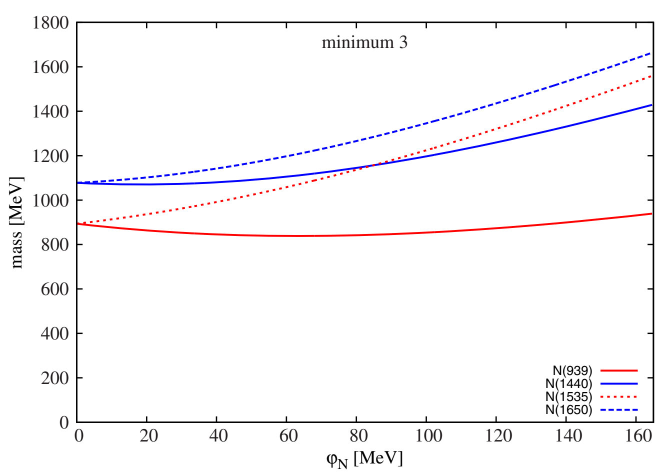

FIG. 3. Masses as a function of φ_N for minimum 3.

In order to decide which states form chiral partners, we again compute the masses as a function of φ_N, keeping φ_S at its vacuum value. The result is shown in Fig. 3, from which we again unanimously conclude that $N(939)$ and $N(1535)$ are chiral partners, with a common mass $m_{0,2} =$ 894 MeV when chiral symmetry is restored. Consequently, $N(1440)$ and $N(1650)$ are chiral partners with a mass $|m_{0,1}| = 1078$ MeV as $\varphi_N \to 0$.

IV. CONCLUSIONS AND OUTLOOK

In this work we have studied the generalization of the eLSM to the three-flavor case, thus including baryons with strangeness ($N_f = 3$). We have found that, in a chiral quark-diquark model for the baryons, we naturally need to consider four baryonic multiplets, if we require the presence of chirally invariant mass terms like in the mirror assignment. Subsequently, we have reduced the model to the case $N_f = 2$ and performed a fit of the parameters of the model to the masses and decay widths, as well as the axial coupling constants of the nucleonic resonances $N(939)$, $N(1440)$, $N(1535)$, and $N(1650)$. Masses and decay widths as well as the axial coupling constant of the nucleon are experimentally known [34]; for the axial coupling constants of the other resonances we used lattice-QCD data [37].

From this fit, we found three minima which, with the exception of the decay $N(1535) \to N\eta$, yield results for the masses, for the decay widths, and for the axial coupling constants that are in very good agreement with data; see Table I. Studying the approach to chiral symmetry restoration $\varphi_N \to 0$, we were able to unanimously identify which of the four nucleonic resonances form chiral partners. For all three minima, these are the pairs $N(939)$, $N(1535)$, as well as $N(1440)$, $N(1650)$.

Finally, let us discuss the issue with the decay width $N(1535) \to N\eta$. Our result that the theoretical value turns out to be too small when compared to the experimental value is stable under parameter variations. This implies that further studies are needed to understand the resonance $N(1535)$. Some authors have argued that $N(1535)$ may contain a sizable amount of $s\bar{s}$ [38–40]. Another interesting possibility is to investigate the role of the chiral anomaly in the baryonic sector [41], which can lead to an enhanced coupling to the resonances η and η'.

In the very recent study of Ref. [42] a chiral baryonic model with three flavors was constructed by making use of parity doublets. There, a large variety of baryonic fields was included (also, the decuplet is present), but no (axial-) vector degrees of freedom were considered in the mesonic sector. The chirally invariant contribution to the nucleon mass is in the range 500–800 MeV, in agreement with our result for minimum 3. Interestingly, in Ref. [42] upper bounds for the axial coupling constants were derived which fit well to our results.

Finally, in order to decide which of the three minima that resulted from our fit is preferable, we plan to investigate the complete three-flavor case. Note that most of the parameters of the Lagrangian (11) are already determined from the $N_f = 2$ fit, but many more experimental data, such as the masses of hyperons and their decay widths, are available to discriminate between the three minima. The obtained values for the coupling constants of hyperons to (pseudo)scalar and (axial-)vector mesons will be relevant for studies of scattering processes in the vacuum [28–31], as well as for neutron stars [32,33]. In connection to the latter topic, one can study nuclear matter at nonzero density and inhomogeneous chiral condensation, thus extending previous investigations on the subjects [27,43] in a more complete framework.

ACKNOWLEDGMENTS

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APPENDIX A: MASS DEGENERACY IN THE CASE OF AN $N_f = 2$ LAGRANGIAN WITHOUT κ TERMS

In this appendix, we want to clarify why it is mandatory to include the κ and ϵ terms in the Lagrangian (11), although they are not dilatation invariant. Therefore, we consider the two-flavor case, given in Eq. (17) and in Appendix C. Setting the constants $\kappa_1, \kappa_2, \kappa_1', \kappa_2'$ (and ϵ_i with $i = 1, 2, 3, 4$) to zero, the part of the Lagrangian which contains the terms contributing to the mass matrix of the four fields Ψ_N, Ψ_{N*}, Ψ_M, and Ψ_{M*} reads $\mathcal{L}_{\mathrm{mass}} = -\bar{\Psi} M' \Psi$. The definition of the vector Ψ is given in Eq. (19), and the mass matrix is given by

$$M' = \frac{1}{2}\begin{pmatrix} g_N\varphi_N & 0 & m_{0,1}+m_{0,2} & m_{0,1}-m_{0,2} \\ 0 & -g_N\varphi_N & m_{0,2}-m_{0,1} & -m_{0,1}-m_{0,2} \\ m_{0,1}+m_{0,2} & m_{0,2}-m_{0,1} & g_M\varphi_N & 0 \\ m_{0,1}-m_{0,2} & -m_{0,1}-m_{0,2} & 0 & -g_M\varphi_N \end{pmatrix}.$$

Evaluating $\det(M' - m_i \mathbb{1}_{4\times4}) = 0$ and denoting $\Omega_{1/2} = \sqrt{\frac{1}{4}(m_{0,1} \pm m_{0,2})^2 + \frac{1}{16}(g_N \mp g_M)^2 \varphi_N^2}$, we find the eigenvalues

$$\begin{aligned} m_1 &= \Omega_1 + \Omega_2 = -m_2, \\ m_3 &= \Omega_1 - \Omega_2 = -m_4. \end{aligned} \tag{A1}$$

Due to the definition of the vectors Ψ given in Eq. (19) and the definition of physical states in Eq. (23), the physical masses correspond to the eigenvalues as follows: $m_{939} = m_1$ and $m_{1535} = -m_2$, as well as $m_{1440} = m_3$ and $m_{1650} = -m_4$. Having this in mind and considering the results for the eigenvalues given in Eq. (A1), it is clear that the masses of $N(939)$ and $N(1535)$ as well as the masses of $N(1440)$ and $N(1650)$ would be degenerate. The only possibility to avoid a mass degeneracy but still keep chiral symmetry is to introduce the κ (and ϵ) terms as shown in Eq. (11).

APPENDIX B: EXPLICIT LAGRANGIAN FOR $N_f = 3$ IN TERMS OF PARITY EIGENVALUES

From Eq. (8) and the Lagrangian (11), one obtains the following baryonic Lagrangian for $N_f = 3$ flavors as a function of parity eigenstates:

$$\begin{aligned} \mathcal{L}_{N_f=3} = {} & \mathrm{Tr}\{\bar{B}_{\mathrm{NR}} i\gamma_\mu D^\mu_{\mathrm{NR}} B_{\mathrm{NR}} + \bar{B}_{NL} i\gamma_\mu D^\mu_{NL} B_{NL} + \bar{B}_{N*R} i\gamma_\mu D^\mu_{\mathrm{NR}} B_{N*R} + \bar{B}_{N*L} i\gamma_\mu D^\mu_{NL} B_{N*L} \\ & + \bar{B}_{MR} i\gamma_\mu D^\mu_{ML} B_{MR} + \bar{B}_{ML} i\gamma_\mu D^\mu_{MR} B_{ML} + \bar{B}_{M*R} i\gamma_\mu D^\mu_{ML} B_{M*R} + \bar{B}_{M*L} i\gamma_\mu D^\mu_{MR} B_{M*L}\} \\ & + c_{A_N} \mathrm{Tr}\{\bar{B}_{\mathrm{NR}} i\gamma_\mu R^\mu B_{N*R} + \bar{B}_{N*R} i\gamma_\mu R^\mu B_{\mathrm{NR}} - \bar{B}_{NL} i\gamma_\mu L^\mu B_{N*L} - \bar{B}_{N*L} i\gamma_\mu L^\mu B_{NL}\} \\ & + c_{A_M} \mathrm{Tr}\{\bar{B}_{MR} i\gamma_\mu L^\mu B_{M*R} + \bar{B}_{M*R} i\gamma_\mu L^\mu B_{MR} - \bar{B}_{ML} i\gamma_\mu R^\mu B_{M*L} - \bar{B}_{M*L} i\gamma_\mu R^\mu B_{ML}\} \\ & - g_N \mathrm{Tr}\{\bar{B}_{NL} \Phi B_{\mathrm{NR}} + \bar{B}_{\mathrm{NR}} \Phi^\dagger B_{NL} + \bar{B}_{N*L} \Phi B_{N*R} + \bar{B}_{N*L} \Phi^\dagger B_{N*R}\} \\ & - g_M \mathrm{Tr}\{\bar{B}_{ML} \Phi^\dagger B_{MR} + \bar{B}_{MR} \Phi B_{ML} + \bar{B}_{M*L} \Phi^\dagger B_{M*R} + \bar{B}_{M*L} \Phi B_{M*R}\} \\ & - \frac{\kappa_1}{2} \mathrm{Tr}\{-\bar{B}_{NL} \Phi B_{\mathrm{NR}} \Phi^\dagger - \bar{B}_{\mathrm{NR}} \Phi^\dagger B_{NL} \Phi + \bar{B}_{N*L} \Phi B_{N*R} \Phi^\dagger + \bar{B}_{N*R} \Phi^\dagger B_{N*L} \Phi \\ & - \bar{B}_{NL} \Phi B_{N*R} \Phi^\dagger + \bar{B}_{\mathrm{NR}} \Phi^\dagger B_{N*L} \Phi + \bar{B}_{N*L} \Phi B_{\mathrm{NR}} \Phi^\dagger - \bar{B}_{N*R} \Phi^\dagger B_{NL} \Phi\} \end{aligned}$$

$$
\begin{aligned}
&-\frac{\kappa_1'}{2}\mathrm{Tr}\{-\bar{B}_{NL}\Phi B_{\mathrm{NR}}\Phi-\bar{B}_{\mathrm{NR}}\Phi^\dagger B_{NL}\Phi^\dagger+\bar{B}_{N*L}\Phi B_{N*R}\Phi+\bar{B}_{N*R}\Phi^\dagger B_{N*L}\Phi^\dagger\\
&+\bar{B}_{NL}\Phi B_{N*R}\Phi-\bar{B}_{\mathrm{NR}}\Phi^\dagger B_{N*L}\Phi^\dagger-\bar{B}_{N*L}\Phi B_{\mathrm{NR}}\Phi+\bar{B}_{N*R}\Phi^\dagger B_{NL}\Phi^\dagger\}\\
&-\frac{\kappa_2}{2}\mathrm{Tr}\{-\bar{B}_{ML}\Phi^\dagger B_{MR}\Phi^\dagger-\bar{B}_{MR}\Phi B_{ML}\Phi+\bar{B}_{M*L}\Phi^\dagger B_{M*R}\Phi^\dagger+\bar{B}_{M*R}\Phi B_{M*L}\Phi\\
&-\bar{B}_{ML}\Phi^\dagger B_{M*R}\Phi^\dagger+\bar{B}_{MR}\Phi B_{M*L}\Phi+\bar{B}_{M*L}\Phi^\dagger B_{MR}\Phi^\dagger-\bar{B}_{M*R}\Phi B_{ML}\Phi\}\\
&-\frac{\kappa_2'}{2}\mathrm{Tr}\{-\bar{B}_{ML}\Phi^\dagger B_{MR}\Phi-\bar{B}_{MR}\Phi B_{ML}\Phi^\dagger+\bar{B}_{M*L}\Phi^\dagger B_{M*R}\Phi+\bar{B}_{M*R}\Phi B_{M*L}\Phi^\dagger\\
&+\bar{B}_{ML}\Phi^\dagger B_{M*R}\Phi-\bar{B}_{MR}\Phi B_{M*L}\Phi^\dagger-\bar{B}_{M*L}\Phi^\dagger B_{MR}\Phi+\bar{B}_{M*R}\Phi B_{ML}\Phi^\dagger\}\\
&-\frac{m_{0,1}+m_{0,2}}{2}\mathrm{Tr}\{\bar{B}_{NL}B_{MR}+\bar{B}_{\mathrm{NR}}B_{ML}+\bar{B}_{N*L}B_{M*R}+\bar{B}_{N*R}B_{M*L}\\
&+\bar{B}_{ML}B_{\mathrm{NR}}+\bar{B}_{MR}B_{NL}+\bar{B}_{M*L}B_{N*R}+\bar{B}_{M*R}B_{N*L}\}\\
&-\frac{m_{0,1}-m_{0,2}}{2}\mathrm{Tr}\{\bar{B}_{NL}B_{M*R}-\bar{B}_{\mathrm{NR}}B_{M*L}-\bar{B}_{ML}B_{N*R}+\bar{B}_{MR}B_{N*L}\\
&-\bar{B}_{N*L}B_{MR}+\bar{B}_{N*R}B_{ML}+\bar{B}_{M*L}B_{\mathrm{NR}}-\bar{B}_{M*R}B_{NL}\},
\end{aligned}
$$

where the covariant derivatives are

$$
\begin{aligned}
D^\mu_{NR}&=\partial^\mu-ic_NR^\mu, & D^\mu_{NL}&=\partial^\mu-ic_NL^\mu,\\
D^\mu_{MR}&=\partial^\mu-ic_MR^\mu, & D^\mu_{ML}&=\partial^\mu-ic_ML^\mu,
\end{aligned}
$$

with

$$
c_N=\frac{c_1+c_2}{2}\quad\text{and}\quad c_M=\frac{c_3+c_4}{2}.
$$

These two constants parametrize the coupling between baryons of equal parity. The constants

$$
c_{A_N}=\frac{c_1-c_2}{2}\quad\text{and}\quad c_{A_M}=\frac{c_3-c_4}{2}
$$

describe the coupling of two baryons with different parity to (axial-)vector mesons. The interaction of the baryonic fields with the scalar and pseudoscalar mesonic fields are parametrized by g_N and g_M. The chirally invariant mass terms are characterized by $m_{0,1}$ and $m_{0,2}$. The terms proportional to $\kappa^{(\prime)}_{1(2)}$ are introduced to avoid mass degeneracy (see Appendix A). In total, the Lagrangian has 12 free parameters.

APPENDIX C: EXPLICIT LAGRANGIAN FOR $N_f=2$ AFTER SSB

After SSB in the meson sector ($\sigma_N\to\sigma_N+\varphi_N$ and $\sigma_S\to\sigma_S+\varphi_S$), the full Lagrangian with two flavors describing the nucleon $N(1440)$, and their chiral partners, as well as their interaction with scalar, pseudoscalar, vector, and axial-vector mesons reads

$$
\begin{aligned}
\mathcal{L}=&\bar{\Psi}_Ni\gamma^\mu\partial_\mu\Psi_N+\bar{\Psi}_{N*}i\gamma^\mu\partial_\mu\Psi_{N*}+\bar{\Psi}_Mi\gamma^\mu\partial_\mu\Psi_M+\bar{\Psi}_{M*}i\gamma^\mu\partial_\mu\Psi_{M*}\\
&+c_N(\bar{\Psi}_N\gamma_\mu\{[\omega^\mu-\gamma^5(f_1^\mu+Zw\partial^\mu\eta_N)]T^0+[\boldsymbol{\rho}^\mu-\gamma^5(\boldsymbol{a}_1^\mu+Zw\partial^\mu\boldsymbol{\pi})]\cdot\boldsymbol{T}\}\Psi_N\\
&+\bar{\Psi}_{N*}\gamma_\mu\{[\omega^\mu-\gamma^5(f_1^\mu+Zw\partial^\mu\eta_N)]T^0+[\boldsymbol{\rho}^\mu-\gamma^5(\boldsymbol{a}_1^\mu+Zw\partial^\mu\boldsymbol{\pi})]\cdot\boldsymbol{T}\}\Psi_{N*})\\
&+c_M(\bar{\Psi}_M\gamma_\mu\{[\omega^\mu+\gamma^5(f_1^\mu+Zw\partial^\mu\eta_N)]T^0+[\boldsymbol{\rho}^\mu+\gamma^5(\boldsymbol{a}_1^\mu+Zw\partial^\mu\boldsymbol{\pi})]\cdot\boldsymbol{T}\}\Psi_M\\
&+\bar{\Psi}_{M*}\gamma_\mu\{[\omega^\mu+\gamma^5(f_1^\mu+Zw\partial^\mu\eta_N)]T^0+[\boldsymbol{\rho}^\mu+\gamma^5(\boldsymbol{a}_1^\mu+Zw\partial^\mu\boldsymbol{\pi})]\cdot\boldsymbol{T}\}\Psi_{M*})\\
&+c_{A_N}\{\bar{\Psi}_N\gamma_\mu[(-f_1^\mu-Zw\partial^\mu\eta_N+\gamma^5\omega^\mu)T^0+(-\boldsymbol{a}_1^\mu-Zw\partial^\mu\boldsymbol{\pi}+\gamma^5\boldsymbol{\rho}^\mu)\cdot\boldsymbol{T}]\Psi_{N*}\\
&+\bar{\Psi}_{N*}\gamma_\mu[(-f_1^\mu-Zw\partial^\mu\eta_N+\gamma^5\omega^\mu)T^0+(-\boldsymbol{a}_1^\mu-Zw\partial^\mu\boldsymbol{\pi}+\gamma^5\boldsymbol{\rho}^\mu)\cdot\boldsymbol{T}]\Psi_N\}\\
&+c_{A_M}\{\bar{\Psi}_M\gamma_\mu[(f_1^\mu+Zw\partial^\mu\eta_N+\gamma^5\omega^\mu)T^0+(\boldsymbol{a}_1^\mu+Zw\partial^\mu\boldsymbol{\pi}+\gamma^5\boldsymbol{\rho}^\mu)\cdot\boldsymbol{T}]\Psi_{M*}\\
&+\bar{\Psi}_{M*}\gamma_\mu[(f_1^\mu+Zw\partial^\mu\eta_N+\gamma^5\omega^\mu)T^0+(\boldsymbol{a}_1^\mu+Zw\partial^\mu\boldsymbol{\pi}+\gamma^5\boldsymbol{\rho}^\mu)\cdot\boldsymbol{T}]\Psi_M\}
\end{aligned}
$$

$$
\begin{aligned}
&- g_N\{\bar{\Psi}_N[(\sigma + \varphi_N + i\gamma^5 Z\eta_N)T^0 + (\boldsymbol{a}_0 + i\gamma^5 Z\boldsymbol{\pi})\cdot\boldsymbol{T}]\Psi_N \\
&+ \bar{\Psi}_{N*}[(\sigma + \varphi_N + i\gamma^5 Z\eta_N)T^0 + (\boldsymbol{a}_0 + i\gamma^5 Z\boldsymbol{\pi})\cdot\boldsymbol{T}]\Psi_{N*}\} \\
&- g_M\{\bar{\Psi}_M[(\sigma + \varphi_N - i\gamma^5 Z\eta_N)T^0 + (\boldsymbol{a}_0 - i\gamma^5 Z\boldsymbol{\pi})\cdot\boldsymbol{T}]\Psi_M \\
&+ \bar{\Psi}_{M*}[(\sigma + \varphi_N - i\gamma^5 Z\eta_N)T^0 + (\boldsymbol{a}_0 - i\gamma^5 Z\boldsymbol{\pi})\cdot\boldsymbol{T}]\Psi_{M*}\} \\
&- \frac{\kappa_1' + \kappa_1}{2\sqrt{2}}\varphi_S\{-\bar{\Psi}_N[(\sigma + \varphi_N + i\gamma^5 Z\eta_N)T^0 + (\boldsymbol{a}_0 + i\gamma^5 Z\boldsymbol{\pi})\cdot\boldsymbol{T}]\Psi_N \\
&+ \bar{\Psi}_{N*}[(\sigma + \varphi_N + i\gamma^5 Z\eta_N)T^0 + (\boldsymbol{a}_0 + i\gamma^5 Z\boldsymbol{\pi})\cdot\boldsymbol{T}]\Psi_{N*}\} \\
&- \frac{\kappa_1' - \kappa_1}{2\sqrt{2}}\varphi_S\{\bar{\Psi}_N[(iZ\eta_N + \gamma^5(\sigma + \varphi_N))T^0 + (iZ\boldsymbol{\pi} + \gamma^5\boldsymbol{a}_0)\cdot\boldsymbol{T}]\Psi_{N*} \\
&- \bar{\Psi}_{N*}[(iZ\eta_N + \gamma^5(\sigma + \varphi_N))T^0 + (iZ\boldsymbol{\pi} + \gamma^5\boldsymbol{a}_0)\cdot\boldsymbol{T}]\Psi_N\} \\
&- \frac{\kappa_2' + \kappa_2}{2\sqrt{2}}\varphi_S\{-\bar{\Psi}_M[(\sigma + \varphi_N - i\gamma^5 Z\eta_N)T^0 + (\boldsymbol{a}_0 - i\gamma^5 Z\boldsymbol{\pi})\cdot\boldsymbol{T}]\Psi_M \\
&+ \bar{\Psi}_{M*}[(\sigma + \varphi_N - i\gamma^5 Z\eta_N)T^0 + (\boldsymbol{a}_0 - i\gamma^5 Z\boldsymbol{\pi})\cdot\boldsymbol{T}]\Psi_{M*}\} \\
&- \frac{\kappa_2' - \kappa_2}{2\sqrt{2}}\varphi_S\{-\bar{\Psi}_M[(iZ\eta_N - \gamma^5(\sigma + \varphi_N))T^0 + (iZ\boldsymbol{\pi} - \gamma^5\boldsymbol{a}_0)\cdot\boldsymbol{T}]\Psi_{M*} \\
&+ \bar{\Psi}_{M*}[(iZ\eta_N - \gamma^5(\sigma + \varphi_N))T^0 + (iZ\boldsymbol{\pi} - \gamma^5\boldsymbol{a}_0)\cdot\boldsymbol{T}]\Psi_M\} \\
&- \frac{m_{0,1} + m_{0,2}}{2}(\bar{\Psi}_N\Psi_M + \bar{\Psi}_{N*}\Psi_{M*} + \bar{\Psi}_M\Psi_N + \bar{\Psi}_{M*}\Psi_{N*}) \\
&- \frac{m_{0,2} - m_{0,1}}{2}(\bar{\Psi}_N\gamma^5\Psi_{M*} + \bar{\Psi}_{N*}\gamma^5\Psi_M - \bar{\Psi}_M\gamma^5\Psi_{N*} - \bar{\Psi}_{M*}\gamma^5\Psi_N), \qquad \text{(C1)}
\end{aligned}
$$

where the coupling to (axial-)vector mesons of two baryons with equal parity and a vector meson is parametrized by $c_N = (c_1 + c_2)/2$ and $c_M = (c_3 + c_4)/2$, and that of two baryons with opposite parity by $c_{A_N} = (c_1 - c_2)/2$ and $c_{A_M} = (c_3 - c_4)/2$. All other constants are the same as in the Lagrangian (17). The factor w is introduced due to the shift of the axial-vector fields in order to eliminate the mixing with the pseudoscalar fields, which occurs after SSB, and Z is the so-called wave-function renormalization factor that takes care of the normalization of the kinetic terms of the pseudoscalar mesonic fields after the shift; see Ref. [11] for more details.

APPENDIX D: DECAY WIDTHS

Because of the existing experimental data [34], we are especially interested in the decays of nucleon resonances into the pseudoscalar mesons π and η. The Lagrangian describing the decay of a resonance N^* into a nucleon N and a pseudoscalar meson $P = \pi, \eta$ has the general structure

$$\mathcal{L} = g^{N_* \to N\partial P}\bar{N}\Gamma\gamma_\mu N_*\partial^\mu P - ig^{N_* \to NP}\bar{N}\Gamma\gamma_\mu N_* P, \qquad \text{(D1)}$$

where $\Gamma = \gamma_5$ (1) for a positive-(negative-)parity N_*. The explicit expressions for the coupling constants $g^{N_* \to N\partial P}$ and $g^{N_* \to NP}$ can be obtained from the relevant terms of the Lagrangian (C1), carrying out the transformation (23). Using this, the tree-level decay width can be calculated to be

$$
\begin{aligned}
\Gamma_{N_* \to NP} &= \lambda_P \frac{p_f}{8\pi m_{N_*}^2}\overline{|i\mathcal{M}|}^2 \\
&= \kappa_P \frac{p_f}{4\pi m_{N_*}}[g^{N_* \to NP} - (m_{N_*} \pm m_N)g^{N_* \to N\partial P}]^2 \\
&\quad \times (E_N \mp m_N), \qquad \text{(D2)}
\end{aligned}
$$

where the upper (lower) sign is valid for a positive-(negative-)parity N_*, and E_N is the nucleon energy in the rest frame of the decaying N_*, while the magnitude of the three-momenta of the decay products is

$$p_f = \frac{1}{2m_{N_*}}\sqrt{(m_{N_*}^2 - m_N^2 - m_P^2)^2 - 4m_N^2 m_P^2}. \qquad \text{(D3)}$$

Furthermore, the factor λ_P is added by hand and should

(1) For $P = \pi$, pay attention to the three possible isospin states of the pion, i.e.,

$$\lambda_\pi = 3.$$

(2) For $P = \eta$, take into account that

$$\eta = \eta_N \cos\phi_P + \eta_S \sin\phi_P,$$

where $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $\eta_S \equiv \bar{s}s$ and ϕ_P is the mixing angle. Its value lies between $-32°$ and $-45°$ [44]. In this paper we have chosen $\phi_P = -44.6°$ obtained from Ref. [12]. It is assumed that the amplitude of the decay $N_* \to N\eta_S$ is massively suppressed. This means that to good approximation,

$$\Gamma_{N_*\to N\eta} \simeq \cos^2\phi_P \Gamma_{N_*\to N\eta_N}.$$

Thus,

$$\lambda_\eta = \cos^2\phi_P.$$

APPENDIX E: AXIAL COUPLING CONSTANTS

The Lagrangians in Appendixes B and C are invariant under $U_A = \exp(-i\theta^a\gamma^5\tau^a/2) \in U(N_f)_A$ axial transformations (θ^a are the parameters and $\tau^a/2$ the generators). Due to Noether's theorem [45], one gets the following axial current:

$$\begin{aligned} A^{a,\mu} = {}& g_A^{(1)}\bar{\Psi}_N\gamma^\mu\gamma^5\frac{\tau^a}{2}\Psi_N + g_A^{(1)}\bar{\Psi}_{N*}\gamma^\mu\gamma^5\frac{\tau^a}{2}\Psi_{N*} \\ &+ g_A^{(2)}\bar{\Psi}_M\gamma^\mu\gamma^5\frac{\tau^a}{2}\Psi_M + g_A^{(2)}\bar{\Psi}_{M*}\gamma^\mu\gamma^5\frac{\tau^a}{2}\Psi_{M*} \\ &+ g_A^{(12)}\bar{\Psi}_N\gamma^\mu\frac{\tau^a}{2}\Psi_{N*} + g_A^{(12)}\bar{\Psi}_{N*}\gamma^\mu\frac{\tau^a}{2}\Psi_N \\ &+ g_A^{(34)}\bar{\Psi}_M\gamma^\mu\frac{\tau^a}{2}\Psi_{M*} + g_A^{(34)}\bar{\Psi}_{M*}\gamma^\mu\frac{\tau^a}{2}\Psi_M, \end{aligned} \tag{E1}$$

where

$$g_A^{(1)} = 1 - \frac{c_N}{g_1}\left(1 - \frac{1}{Z^2}\right), \qquad g_A^{(2)} = -1 + \frac{c_M}{g_1}\left(1 - \frac{1}{Z^2}\right)$$

are the axial coupling constants of the bare fields Ψ_N, Ψ_{N*}, Ψ_M, and Ψ_{M*}, and

$$g_A^{(12)} = -\frac{c_{A_N}}{g_1}\left(1 - \frac{1}{Z^2}\right), \qquad g_A^{(34)} = \frac{c_{A_M}}{g_1}\left(1 - \frac{1}{Z^2}\right)$$

are the "mixed" axial coupling constants of the bare fields Ψ_N with Ψ_{N*} and Ψ_M with Ψ_{M*}.

The expressions for the axial coupling constants of the physical fields can be obtained from the relevant terms of the axial current (E1) after the transformation to parity eigenstates (23) has been carried out.

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