An ultralight pseudoscalar boson

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Using a fundamental discrete symmetry, $\mathbf{Z}_N$, we construct a two-axion model with the QCD axion solving the strong-CP problem, and an ultralight axion (ULA) with $m_{\mathrm{ULA}} \approx 10^{-22}$ eV providing the dominant form of dark matter (DM). The ULA is light enough to be detectable in cosmology from its imprints on structure formation, and may resolve the small-scale problems of cold DM. The necessary relative DM abundances occur without fine-tuning in constructions with decay constants $f_{\mathrm{ULA}} \sim 10^{17}$ GeV, and $f_{\mathrm{QCD}} \sim 10^{11}$ GeV. An example model achieving this has $N = 24$, and we construct a range of other possibilities. We compute the ULA couplings to the standard model, and discuss prospects for direct detection. The QCD axion may be detectable in standard experiments through the $\vec{E} \cdot \vec{B}$ and $G\tilde{G}$ couplings. In the simplest models, however, the ULA has identically zero coupling to both $G\tilde{G}$ of QCD and $\vec{E} \cdot \vec{B}$ of electromagnetism due to vanishing electromagnetic and color anomalies. The ULA couples to fermions with strength $g \propto 1/f_{\mathrm{ULA}}$. This coupling causes spin precession of nucleons and electrons with respect to the DM wind with period $t \sim$ months. Current limits do not exclude the predicted coupling strength, and our model is within reach of the CASPEr-Wind experiment, using nuclear magnetic resonance.

I. INTRODUCTION

Dark matter (DM) is known to comprise around 30% of the energy density of the Universe [1], yet its origin within a fundamental theory remains unknown. A classic candidate embedded in a minimal extension of the standard model (SM) is the QCD axion [2–5] based on the Peccei-Quinn (PQ) symmetry [6–9]. In addition to being a good DM model, the QCD axion also solves the strong-CP problem, by setting the (static) neutron electric dipole moment to zero, consistent with observations [10]. The QCD axion behaves, for all intents and purposes, like cold dark matter (CDM): its background energy density scales like a^{-3} (where a is the cosmic scale factor) on all post-big-bang-nucleosynthesis time scales, and the Jeans scale associated to the gradient energy is vanishingly small on astrophysical length scales [11]. However, CDM, and thus the QCD axion, suffers from some possible problems stemming from exactly these properties: the scale invariance of CDM structure formation leads to the "small-scale crises" [12].

The small-scale crises may be alleviated by proper inclusion of the effects of star formation and feedback (see e.g. Ref. [13] for recent simulations, and Ref. [14] for a review). The small-scale crises can also be addressed by allowing more freedom in the model of DM, deviating from exact CDM. One possibility is to introduce thermal velocities, so-called warm dark matter (WDM) [15,16], and particle physics candidates include the gravitino and

sterile neutrinos. WDM, however, cannot give large cores to dwarf spheroidal (dSph) galaxies while also being consistent with cosmological structure formation [17], i.e. WDM cannot completely solve the small-scale crises.

In the context of axion theories, an ultralight axion (ULA) offers an elegant solution [18,19]. ULAs differ from CDM essentially because of the large de Broglie wavelength, which imprints a scale on structure formation, suppressing linear density perturbations (e.g. Refs. [20,21]) and allowing for the formation of cored pseudosolitons on nonlinear scales [22–25]. An ULA with mass $m_{\mathrm{ULA}} \approx 10^{-22}$ eV making up a dominant ($\gtrsim 90\%$) component of the DM can provide large cores to dSph galaxies [26] (solving the "cusp-core" problem) and is consistent with both the CMB [27] and high-z galaxy formation [28,29], thus avoiding the *Catch 22* of WDM. If the DM is dominantly composed of an ULA with 10^{-22} eV $\lesssim m_{\mathrm{ULA}} \lesssim 10^{-18}$ eV, then irrespective of its role in the small-scale crises its effects may be detectable in the epoch of reionization [28,29] and the 21 cm power spectrum [30]. The origin of such a small mass scale for the ULA, which is at the same time able to give a dominant contribution to the DM, is the subject of this paper.

In this paper, we aim to construct a model based on principles dictated by quantum gravity in which such a DM model is somewhat natural. We use a fundamental discrete symmetry $\mathbf{Z}_N$ giving rise to approximate global symmetries.

We demand the existence of the QCD axion and a successful resolution of the strong-*CP* problem: by symmetry arguments this dictates the couplings of the two-axion model. The QCD axion should, however, be only a fraction of the DM. This occurs with minimal fine-tuning and model uncertainty if the axion decay constant $f_{\rm QCD} \lesssim 10^{14}$ GeV, and the PQ symmetry, $U(1)_{\rm PQ}$, is broken before or during inflation. The rest of the DM is composed of an ULA with $m_{\rm ULA} \approx 10^{-22}$ eV, whose mass is protected from large quantum corrections by the discrete symmetry. In order for this ULA to contribute a large amount to the DM density its decay constant must be $f_{\rm ULA} \sim 10^{17}$ GeV. The corresponding PQ symmetry, $U(1)_{\rm ULA}$, must also be broken before or during inflation. Constraints from isocurvature perturbations require that the Hubble scale during inflation be $H_I < 10^{10}$ GeV, and so this model predicts vanishingly small tensor perturbations, $r_T \ll 10^{-9}$ [31,32]. The demands on $m_{\rm ULA}$ and $f_{\rm ULA}$ place restrictions on the possible symmetry groups $\mathbf{Z}_N$.

The benefit of constructing an explicit QCD-ULA model is that we can compute all of the couplings of the ULA to the SM, which has not been possible before. Demanding a successful solution to the strong-*CP* problem puts constraints on these couplings, and in the simplest constructions forbids an ULA coupling to $\vec{E} \cdot \vec{B}$ of electromagnetism, thus making it invisible to many standard searches for axionlike particles (ALPs, e.g. Ref. [33], which also discusses the role of discrete symmetries). The demand of vanishing ULA color anomaly does, however, predict tree-level couplings between the ULA and SM fermions, which may be detectable via searches for spin precession of nucleons [34,35]. These are key results of this paper.

The existence of ULAs is often discussed within a string theory context [36–39]. Our construction is based purely on field theory,[1] though the use of discrete symmetries is consistent with expectations about global symmetries in quantum gravity (e.g. Refs. [41–43]). Discrete symmetries may arise in phenomenologically consistent orbifold compactifications of string theory [44–46].

The rest of the paper is organized as follows. In Sec. II we describe our criteria for the DM model. In Sec. III we build the desired model, and a number of variants. We compute the axion couplings to the SM in Sec. IV, and analyze the direct and indirect detection possibilities in Sec. V. We conclude in Sec. VI. In the Appendix we present another model variant with additional heavy quarks and a nonvanishing ULA coupling to electromagnetism.

II. TWO-COMPONENT AXION DARK MATTER

Our model contains two axions: the QCD axion with decay constant $f_{\rm QCD}$ arising from a spontaneously broken symmetry $U(1)_{\rm PQ}$, and an ULA with decay constant $f_{\rm ULA}$

TABLE I. Relevant cosmological parameters, as defined in the text. Errors are 68% C.L., upper limits are 95% C.L.

Parameter	Constraint	Ref.
$\Omega_c h^2$	0.1193 ± 0.0014	[1]
$z_{\rm eq}$	3382 ± 32	[1]
h	0.6751 ± 0.0064	[1]
Ω_m	0.3121 ± 0.0087	[1]
$\ln(10^{10} A_s)$	3.059 ± 0.025	[1]
A_I/A_s	<0.038	[50]
r_T	<0.12	[51]

and mass $m_{\rm ULA}$ arising from a spontaneously broken symmetry $U(1)_{\rm ULA}$. We impose a constraint on the QCD axion decay constant 10^9 GeV $< f_{\rm QCD} < 10^{17}$ GeV, where the lower bound comes from e.g. supernova cooling [47,48], and the upper bound from black hole superradiance [49]. We are interested in ULA models with observable cosmological consequences, and limit the ULA mass to be in the range 10^{-22} eV $< m_{\rm ULA} < 10^{-18}$ eV where the lower limit is allowed by cosmic structure formation [27–29] and the upper limit is the possible reach of 21 cm surveys [30]. Other relevant cosmological parameters are listed in Table I.

When $U(1)_{\rm PQ}$ and $U(1)_{\rm ULA}$ are both broken before or during inflation, $f_X > H_I/2\pi$ (H_I is the Hubble scale during inflation; we discuss the reason for this assumption shortly), the dominant axion production is nonthermal, occurring via vacuum realignment with the axions at essentially zero temperature. The relic density in each axion, X, is determined by the spatially averaged misalignment angle:

$$\langle \theta_{i,\rm X}^2 \rangle = \bar{\theta}_{i,\rm X}^2 + (H_I/2\pi f_{\rm X})^2. \tag{1}$$

The first term in Eq. (1) is the initial random value from spontaneous symmetry breaking smoothed by inflation, and the second term arises from backreaction of the inflationary isocurvature perturbations [49]. The relic densities from vacuum realignment are[2]

$$\Omega_{\rm QCD} h^2 = 2 \times 10^4 \left(\frac{f_{\rm QCD}}{10^{16}~{\rm GeV}} \right)^{7/6} \langle \theta_{i,\rm QCD}^2 \rangle \mathcal{F}_{\rm anh}(\bar{\theta}_{i,\rm QCD}), \tag{2}$$

$$\Omega_{\rm ULA} h^2 = \frac{h^2}{6} (9\Omega_r)^{3/4} \left(\frac{m_{\rm ULA}}{H_0} \right)^{1/2} \left(\frac{f_{\rm ULA}}{M_{pl}} \right)^2 \langle (\theta_{i,\rm ULA})^2 \rangle, \tag{3}$$

where h is the reduced Hubble rate today, $H_0 = 100h$ km s^{-1} Mpc^{-1}, and Ω_r is the energy density in radiation, determined by the redshift of equality, $z_{\rm eq}$, and the total matter density, both of which we hold fixed. The

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[1]An alternative field theory model is discussed in Ref. [40].

[2]These are approximate expressions, but are accurate enough for the purposes of the present study. For more details, see e.g. Refs. [27,52–54].
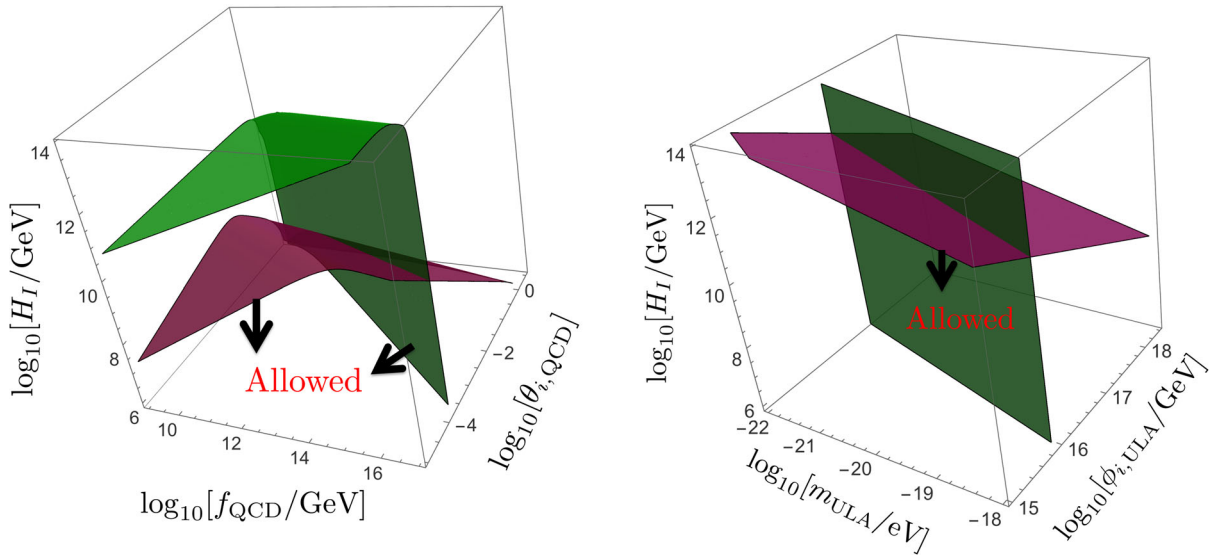
FIG. 1. Constraints on the QCD axion and ULA parameter space imposed by DM relic density (green) and isocurvature (purple) in the scenario where $U(1)_{PQ}$ and $U(1)_{ULA}$ are both broken during inflation. Left panel: QCD axion. We demand this is a subdominant DM component $\Omega_{QCD}/\Omega_c < 0.1$. The maximum allowed inflationary scale is $H_I \sim 10^{10}$ GeV, with the full parameter space consistent with $H_I \lesssim 10^8$ GeV. All $f_{QCD} < 10^{17}$ GeV can be accommodated with $\bar{\theta}_{i,QCD} \gtrsim 10^{-4}$ fine tuning. Restricting to $\bar{\theta}_{i,QCD} \geq 10^{-2}$ imposes $f_{QCD} \lesssim 10^{14}$ GeV. Right panel: ULA. For the range of masses and relic densities considered, backreaction of isocurvature perturbations is negligible and the relic density is independent of H_I for $r_T < 0.12$. Isocurvature perturbations constrain $H_I \lesssim 10^{12}$ GeV. On the scale shown, obtaining relic density $0.9 < \Omega_{ULA}/\Omega_c < 1$ constrains the model to live in the green plane, implying $f_{ULA} > 10^{16}$ GeV if $\bar{\theta}_{i,ULA} < 1$.

factor $\mathcal{F}_{anh}(x)$ accounts for anharmonic corrections to the QCD axion relic density, assuming a cosine potential, $V(\theta_{QCD}) \propto (1 - \cos\theta_{QCD})$, for which we use the analytic fit of Ref. [52]. We only assume a quadratic ULA potential, $V(\theta_{ULA}) \propto \theta_{ULA}^2$, which is valid for $\theta_{ULA} \lesssim 1$. The ULA relic density depends on the combination of parameters $\bar{\theta}_{i,ULA} f_{ULA} \equiv \phi_i$, and thus the relic density in each axion is a function of three free parameters: $\Omega_{QCD} = \Omega_{QCD}(f_{QCD}, \bar{\theta}_{i,QCD}, H_I)$, $\Omega_{ULA} = \Omega_{ULA}(m_{ULA}, \phi_i, H_I)$.

The QCD relic density, Eq. (2), is strictly valid only for $f_{QCD} \lesssim 2 \times 10^{15}$ GeV [53,54]. The differences at large f_{QCD} are fairly small on the scale of the present analysis, and in most cases we restrict ourselves to smaller f_{QCD} anyway. The ULA relic density, Eq. (3), is valid as long as ULA coherent oscillations begin in the radiation dominated era. Oscillations begin when $H \sim m_{ULA}$, and the Hubble rate at z_{eq} is $H_{eq} \sim 10^{-28}$ eV. We are concerned only with ULAs that are allowed to be the dominant form of DM, which requires at the very least $m > 10^{-24}$ eV [27]. Equation (3) is thus an excellent approximation. We assume that temperature dependence of the ULA mass can be neglected, or more accurately that the mass has reached its zero-temperature value by the time coherent oscillations begin.[3]

When the PQ symmetry is broken during inflation the axion fields obtain (uncorrelated) isocurvature perturbations with amplitude

$$A_{I,X} = \left(\frac{\Omega_X h^2}{\Omega_c h^2}\right)^2 \left(\frac{H_I}{\pi \bar{\theta}_{i,X} f_X}\right)^2. \quad (4)$$

The total DM density is $\Omega_c h^2 = \Omega_{QCD} h^2 + \Omega_{ULA} h^2$ and the total isocurvature amplitude is $A_I = A_{I,QCD} + A_{I,ULA}$. For the ULA masses we consider, both the QCD axion and the ULA are indistinguishable from CDM in terms of CMB data [27,31,32], and so only these total combinations, $\{\Omega_c h^2, A_I\}$, are constrained. The value of H_I is bounded from above by the combined *Planck*/BICEP2 constraint [51] on the tensor-to-scalar ratio, $r_T = (8/A_s)(H_I/2\pi M_{pl})^2$, where $M_{pl} = 1/\sqrt{8\pi G_N}$ is the reduced Planck mass.

We are interested in models where the DM is dominantly composed of ULAs, with a small component of the QCD axion, and in which this can be obtained "naturally," i.e. without too much fine-tuning. We demand that $\Omega_{QCD}/\Omega_c < 0.1$ with the rest of the DM in ULAs. We impose naturalness by limiting the misalignment angles $\bar{\theta}_{i,X} > \bar{\theta}_{i,min}$, with $\bar{\theta}_{i,min}$ depending on the severity of fine-tuning we are prepared to allow (recall that the strong-*CP* problem itself, which we are trying to alleviate by introducing the QCD axion, is a tuning of $\theta_{QCD} \sim 10^{-10}$).

Figure 1 sketches the constraints on our model imposed by the DM relic density and isocurvature

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[3]See Ref. [55] for an interesting discussion of the role of mixing and mass evolution in multi-axion systems.

bounds.[4] The dominant ULA component of DM requires $\phi_i > 10^{16}$ GeV and thus $f_{ULA} > 10^{16}$ GeV for $\theta_{ULA} < 1$. The constraint $r_T < 0.12 \Rightarrow H_I < 8.8 \times 10^{13}$ GeV implies that ULAs must have $U(1)_{ULA}$ broken during inflation. Isocurvature constraints on the ULAs then further limit $H_I \lesssim 10^{12}$ GeV, implying all $f_{QCD} \gtrsim 10^{11}$ GeV in our model have $U(1)_{PQ}$ broken during inflation. The strongest constraint on H_I comes from the QCD axion, which even as a subdominant DM component demands $H_I \lesssim 10^{10}$ GeV, falling to $H_I \lesssim 10^9$ GeV if we restrict $\bar{\theta}_{i,\text{min}} = 10^{-2}$. We note that such low values of H_I can be achieved in, e.g., Kachru-Kallosh-Linde-Trivedi (KKLT) or brane inflation [56,57]. $H_I < 10^9$ GeV implies unobservably small [58] tensor modes: $r_T < 1.6 \times 10^{-11}$.

For $f_{QCD} \lesssim 10^{11}$ GeV it is possible to have $U(1)_{PQ}$ broken after inflation while satisfying the ULA isocurvature constraint. In this case, the QCD axion suffers from constraints from domain walls [59–61], and uncertainty on the relic density due to axion string decay. It does, however, avoid all isocurvature constraints. The maximum value of $H_{I,\text{max}} \approx 10^{12}$ GeV imposed by ULA isocurvature constraints thus sets a maximum value of $r_{T,\text{max}} \approx 10^{-5}$ in our model. Due to the constraints from domain walls and uncertainty due to string decay, we will not discuss the scenario in which $U(1)_{PQ}$ is broken after inflation any further.

Our preferred cosmological model is the following:

(i) $U(1)_{PQ}$ and $U(1)_{ULA}$ broken during inflation. Necessary for ULAs. Avoids phase transition relics for QCD axion.

(ii) $m_{ULA} \sim 10^{-22}$ eV: solution to dSph cusp-core problem if this is the dominant DM component.

(iii) $f_{ULA} \sim 10^{17}$ GeV: allows for dominant ULA DM.

(iv) 10^9 GeV $< f_{QCD} < 10^{14}$ GeV: allows for the QCD axion to be up to 10% of the DM with fine-tuning no worse than $\bar{\theta}_i \gtrsim 10^{-2}$.

(v) $H_I < 10^9$ GeV: maximum inflationary Hubble scale allowed by isocurvature if the QCD axion is not fine-tuned.

III. SYMMETRIES AND LIGHT PARTICLES

A. Global symmetries and anomalies

Two axions, the QCD axion, a, and the ULA, ϕ, are introduced by spontaneously breaking the approximate global symmetries $U(1)_{PQ}$ and $U(1)_{ULA}$. The spontaneous breaking is achieved with two SM singlet complex scalar fields, X_1 and X_2, of which the axions are the angular degrees of freedom. The symmetries $U(1)_{PQ}$ and $U(1)_{ULA}$

TABLE II. Global charge assignments required for vanishing ULA color anomaly in the model without heavy quarks.

	q_L	u_L^c	d_L^c	H_u	H_d	X_1	X_2
$U(1)_{PQ}$	1	1	1	-2	-2	2	0
$U(1)_{ULA}$	1	-3	1	2	-3	0	1

are axial symmetries acting as rotations on the angular parts of X_1 and X_2: hence the axions are pseudoscalars. We work in a supersymmetry (SUSY) setting where there are two Higgs fields, H_u and H_d, which give masses to the up-type, u, and down-type, d, quarks, q, respectively.

If we do not want to consider the QCD anomaly, one may consider only the SM singlet scalars and the global symmetries. But, as soon as the quarks and their PQ charges are taken into account, consideration of the QCD anomaly is inevitable. At a more fundamental level, such as in string theory, it is very hard to make the underlying discrete symmetries blind to quarks. If we construct approximate global symmetries starting from exact discrete symmetries, the SM quarks or some heavy quarks are supposed to carry the discrete charges. So, consideration of the QCD anomaly, and thus the QCD axion as well as an ULA, seems inevitable.

A successful resolution of the strong-CP problem can be achieved if the ULA carries zero QCD anomaly, and we use this to fix the quark charges under the approximate global symmetries. If there are no additional heavy quarks, one may consider the charges shown in Table II. These charge assignments ensure that the allowed dimension-4 operators in the Lagrangian respect the approximate $U(1)_{PQ}$ and $U(1)_{ULA}$ symmetries.

The $U(1)_{ULA}$ charges are assigned such that there is no $U(1)_{ULA}$-SU(3)-SU(3) anomaly. There is, however, a $U(1)_{PQ}$-SU(3)-SU(3) anomaly, and thus the QCD axion acquires its mass as usual through QCD instantons after the confinement transition. The ULA has no QCD anomaly, but can acquire a mass from higher-order operators in the Lagrangian. We now turn to these.

B. Discrete symmetries

Quantum gravity, thanks to the black hole no-hair theorems, is expected to violate all continuous global symmetries, with the exchange of Planck scale black holes allowing, for example, baryon number violating processes.[5] Our axion model involves the introduction of two global symmetries, $U(1)_{PQ}$ and $U(1)_{ULA}$, and both of these symmetries will be broken by Planck-suppressed higher-dimensional operators, inducing effective masses for the axions at low energy. Discrete global symmetries, on the other hand, are allowed by quantum gravity (consider

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[4]Since the chosen ULA masses behave like CDM in the CMB, it is a trivial matter to perform a full analysis from the public *Planck* chains, using the given formulas and imposing our priors. At the present level of model building, this level of detail is unnecessary and the sketched constraints are a good enough guide on the logarithmic scale of parameters.

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[5]For discussion relating specifically to the QCD axion, see Refs. [42,43].

TABLE III. $\mathbf{Z}_4$ example, allowing the approximate global charges of Table II.

	q_L	u_L^c	d_L^c	H_u	H_d	X_1	X_2
$\mathbf{Z}_4$	1	−3	1	2	−3	0	−3

e.g. a $\mathbf{Z}_N$ orbifold compactification of string theory). By imposing an exact discrete symmetry we can control the order of higher-dimensional operators appearing in the effective theory breaking that global symmetry, and thus gain some control over the induced axion masses.

To have the global charges of Table II, we may consider a discrete symmetry $\mathbf{Z}_4$ with the quantum numbers given in Table III. The discrete charges of Table III allow the following terms at lowest order in the low-energy action:

$$q_L u_L^c H_u, \tag{5}$$

$$q_L d_L^c H_d \frac{X_2}{M_{\mathrm{UV}}}, \tag{6}$$

$$\frac{H_u H_d X_1^2 X_2}{M_{\mathrm{UV}}^2}, \tag{7}$$

where M_{UV} is a UV mass scale (e.g. the string or the Planck scale). The term (5) gives t quark an electroweak scale mass, and the term (6) gives mass to b quark. For the b quark mass near the electroweak scale the VEV of X_2 must be near M_{UV}. The term (7) is the one giving the μ-term in SUSY models [62,63], and defines the $\mathrm{U}(1)_{\mathrm{ULA}}$ symmetry.

The additional term $H_u H_d X_2$ is also allowed by the $\mathbf{Z}_4$ discrete symmetry. However, this spoils a solution of the μ problem, and must be forbidden. By introducing an additional discrete group $\mathbf{Z}_3$ as in Table IV, we can forbid the unwanted term $H_u H_d X_2$ while keeping (5), (6), (7). In the last row of Table IV, the combined effect, i.e. the $\mathbf{Z}_{12}$ charges are shown.

C. Ultralight axions

1. The simplest model

For simplicity of discussion, we work in a SUSY framework. The superpotential W gives the information on discrete and global symmetries. As an illustration, consider the global and discrete charges presented in

TABLE IV. $\mathbf{Z}_4 \times \mathbf{Z}_3$ example, where $\mathbf{Z}_3$ is used to ensure a solution to the μ problem.

	q_L	u_L^c	d_L^c	H_u	H_d	X_1	X_2
$\mathbf{Z}_4$	1	−3	1	2	−3	0	−3
$\mathbf{Z}_3$	2	0	0	1	1	2	0
$\mathbf{Z}_{12}$	5	9	9	10	1	8	9

Tables II and IV. Obviously, the approximate global symmetries of Table II are broken if we consider all the terms in W allowed by the discrete symmetries. $\mathrm{U}(1)_{\mathrm{PQ}}$ is in addition broken by the nonvanishing color anomaly.

We take the VEV of X_1 to fix the decay constant of the QCD axion: $f_{\mathrm{QCD}}/\sqrt{2} = \langle X_1 \rangle$. This is a free parameter, which we take to be around 10^{11} GeV. $\mathrm{U}(1)_{\mathrm{ULA}}$ is broken by

$$\frac{H_u H_d X_1^2 X_2^5}{M_{\mathrm{UV}}^6}. \tag{8}$$

Equation (8) respects the discrete symmetries of Table IV and the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry, but breaks the $\mathrm{U}(1)_{\mathrm{ULA}}$ symmetry of Table II. When SUSY is broken, this term generates the scale, $V = f_{\mathrm{ULA}}^2 m_{\mathrm{ULA}}^2$, of the ULA potential. Taking $\langle X_2 \rangle = f_{\mathrm{ULA}}/\sqrt{2} = \alpha M_{\mathrm{UV}}$ for some constant α we have

$$V = \alpha^5 s_\beta c_\beta \frac{m_{3/2} v_{ew}^2 f_{\mathrm{QCD}}^2}{M_{\mathrm{UV}}}, \tag{9}$$

where $s_\beta \equiv \sin\beta = v_u/v_{ew}$ and $c_\beta \equiv \cos\beta = v_d/v_{ew}$ (as is standard in two-Higgs SUSY models). The ULA mass is given by

$$m_{\mathrm{ULA}} = (2 \times 10^{-4}\ \mathrm{eV})\alpha^{3/2}\sqrt{s_\beta c_\beta}\left(\frac{m_{3/2}}{\mathrm{TeV}}\right)^{1/2}$$
$$\times \left(\frac{M_{pl}}{M_{\mathrm{UV}}}\right)^{3/2}\left(\frac{f_{\mathrm{QCD}}}{10^{11}\ \mathrm{GeV}}\right)\left(\frac{v_{ew}}{246\ \mathrm{GeV}}\right). \tag{10}$$

Setting $\sqrt{s_\beta c_\beta} \approx 1$ and taking all nonaxion parameters at the fiducial values given in Eq. (10) gives

$$\frac{m_{\mathrm{ULA}}}{10^{-22}\ \mathrm{eV}} \approx 4.25 \times 10^{15}\left(\frac{f_{\mathrm{ULA}}}{10^{17}\ \mathrm{GeV}}\right)^{3/2}\left(\frac{f_{\mathrm{QCD}}}{10^{11}\ \mathrm{GeV}}\right). \tag{11}$$

It is clear that this simplest model cannot produce a cosmology matching our needs: either f_{ULA} is too low, or m_{ULA} is too large.

2. More general discrete charges

Consider a variation of our simplest model, generalizing the discrete charges we allow our fields to carry, while keeping $\langle X_2 \rangle = f_{\mathrm{ULA}}/\sqrt{2}$, and the global charges of Table II. Take $\mathbf{Z}_N$ with a slight modification of Table III

TABLE V. More general $\mathbf{Z}_N$ charges.

	q_L	u_L^c	d_L^c	H_u	H_d	X_1	X_2
$\mathbf{Z}_N$	1	−3	1	2	n	0	m

using arbitrary charges m and n for X_2 and H_d respectively, which is shown in Table V.

We must allow the t quark mass by

$$q_L t_L^c H_u, \qquad (12)$$

and b quark mass by

$$q_L d_L^c H_d \frac{X_2}{M_{UV}}. \qquad (13)$$

$\mathbf{Z}_N$ invariance then constrains

$$2 + n + m = 0 \mod N, \qquad (14)$$

$U(1)_{ULA}$ breaking occurs via the operator,

$$\frac{1}{M_{UV}^{p+1}} H_u H_d X_1^2 X_2^p, \qquad (15)$$

for which $\mathbf{Z}_N$ invariance implies

$$2 + n + mp = 0 \mod N. \qquad (16)$$

After SUSY breaking the scale of the ULA potential is given as before by

$$V = f_{ULA}^2 m_{ULA}^2 = \alpha^p s_\beta c_\beta \frac{m_{3/2} v_{ew}^2 f_{QCD}^2}{M_{UV}}, \qquad (17)$$

and the mass is

$$m_{ULA} = (2 \times 10^{-4} \text{ eV}) \alpha^{p/2-1} \sqrt{s_\beta c_\beta} \left(\frac{m_{3/2}}{\text{TeV}}\right)^{1/2}$$
$$\times \left(\frac{M_{pl}}{M_{UV}}\right)^{3/2} \left(\frac{f_{QCD}}{10^{11} \text{ GeV}}\right) \left(\frac{v_{ew}}{246 \text{ GeV}}\right). \qquad (18)$$

Fixing the fiducial values of the nonaxion parameters this gives

$$\frac{m_{ULA}}{10^{-22} \text{ eV}} \approx 10^{19.69-0.77p} \left(\frac{f_{ULA}}{10^{17} \text{ GeV}}\right)^{p/2-1} \left(\frac{f_{QCD}}{10^{11} \text{ GeV}}\right). \qquad (19)$$

We now have a class of models defined by the value of p.

The system of Eqs. (14),(16) is simple to solve for the integers $\{m, n, p\}$. Each solution is characterized by three free integers, $\{\nu_1, \nu_2, \nu_3\}$, since each of $\{m, n, p\}$ is fixed up to addition of some multiple of N. We require nontrivial solutions with $m \neq 0$. The most important part of the solution for our purposes is the value of $p = q + N\nu_3$. The term with $p = 1$ is $U(1)_{ULA}$ invariant. This is the SUSY μ-term [62], and we must allow it. Therefore we require $q = 1$, and the lowest order $U(1)_{ULA}$ breaking term
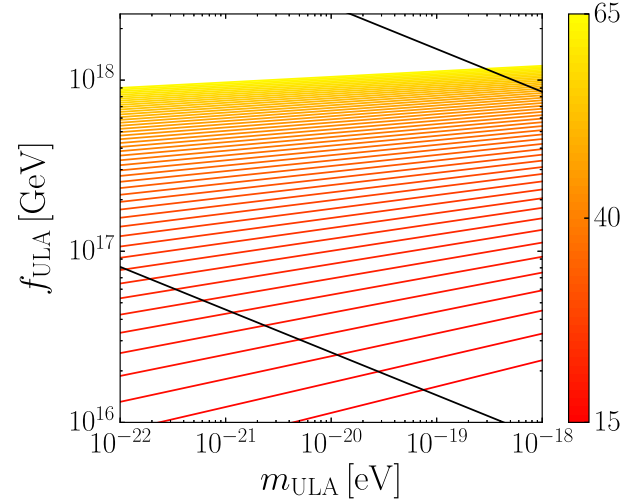

FIG. 2. ULA models generated by the generalized discrete symmetry model presented in Table V, labeled by the integer parameter p of Eq. (15). The given values of p are most easily achieved for a discrete symmetry $\mathbf{Z}_N$ with $N = p - 1$. The demand that the ULA make up $> 90\%$ of the DM with fine-tuning no greater than 10^{-2} constrains the desired models to live between the solid black lines.

occurs when $\nu_3 = 1$. Equation (14) implies there is always a nontrivial solution with $q = 1$, and therefore by fixing N we can easily obtain any $p = 1 + N$.

Let us consider one explicit example. Take the model with

$$(N = 24, m = 8, n = 14) \Rightarrow p = 25. \qquad (20)$$

This gives $m_{ULA} = 2.8 \times 10^{-22}$ eV with $f_{ULA} = 10^{17}$ GeV and $f_{QCD} = 10^{11}$ GeV.

Figure 2 shows the space of models generated in this manner, for $p \in [15, 65]$, along with the constraints from the DM abundance. We fix all the nonaxion parameters to their fiducial values. In addition we fix $f_{QCD} = 10^{11}$ GeV, which allows the QCD axion to make up anywhere between $\ll 1\%$ and 10% of the DM with $10^{-2} \lesssim \theta_{i,QCD} \lesssim 0.6$. We impose the constraint that the ULA make up $> 90\%$ of the DM with $10^{-2} < \theta_{i,ULA} < 1$, which limits the range of allowed f_{ULA} for a given m_{ULA}. We are able to generate acceptable models across the entire desired range of ULA masses. Interestingly, in this mass range our models seem to display a maximum value of $f_{ULA} \sim 10^{18}$ GeV.

IV. COUPLINGS TO THE STANDARD MODEL

Having built our desired two-axion models, we now compute the couplings of a and ϕ to the SM. For the axion and ULA, we define the couplings as

$$\mathcal{L} \ni e^2 c_{a\gamma\gamma} \frac{a}{32\pi^2 f_a} F_{\text{em},\mu\nu} \tilde{F}_{\text{em}}^{\mu\nu} + \bar{c}_{1,ae} \frac{\partial^\mu a}{f_a} \bar{e}\gamma_\mu\gamma_5 e + \frac{\partial^\mu a}{f_a}(\bar{c}_{1,au} \bar{u}\gamma_\mu\gamma_5 u + \bar{c}_{1,ad} \bar{d}\gamma_\mu\gamma_5 d)$$

$$+ e^2 c_{\phi\gamma\gamma} \frac{\phi}{32\pi^2 f_{\text{ULA}}} F_{\text{em},\mu\nu} \tilde{F}_{\text{em}}^{\mu\nu} + \bar{c}_{1,\phi e} \frac{\partial^\mu \phi}{f_{\text{ULA}}} \bar{e}\gamma_\mu\gamma_5 e + \frac{\partial^\mu \phi}{f_{\text{ULA}}}(\bar{c}_{1,\phi u} \bar{u}\gamma_\mu\gamma_5 u + \bar{c}_{1,\phi d} \bar{d}\gamma_\mu\gamma_5 d) \quad (21)$$

where the derivative couplings preserve the PQ and ULA symmetries. Using equations of motion, we consider the following couplings:

$$\mathcal{L} \to e^2 c_{a\gamma\gamma} \frac{a}{32\pi^2 f_a} F_{\text{em},\mu\nu} \tilde{F}_{\text{em}}^{\mu\nu} + c_{aee} \frac{m_e}{f_a} \bar{e} i\gamma_5 e a + \frac{m_N}{f_a}(C_{app} \bar{p} i\gamma_5 p + C_{ann} \bar{n} i\gamma_5 n)a$$

$$+ e^2 c_{\phi\gamma\gamma} \frac{\phi}{32\pi^2 f_{\text{ULA}}} F_{\text{em},\mu\nu} \tilde{F}_{\text{em}}^{\mu\nu} + c_{\phi ee} \frac{m_e}{f_{\text{ULA}}} \bar{e} i\gamma_5 e a + \frac{m_N}{f_{\text{ULA}}}(C_{\phi pp} \bar{p} i\gamma_5 p + C_{\phi nn} \bar{n} i\gamma_5 n)\phi \quad (22)$$

where m_N is the nucleon mass, $m_p \approx m_n \approx m_N$.

For example, from the action $\int d^4 x \mathcal{L}$ we may consider $\mathcal{L} \ni \frac{\partial^\mu a}{f_a} \bar{e}\gamma_\mu\gamma_5 e$, neglecting the surface term

$$-\frac{a}{f_a} \bar{e}[\gamma^\mu \partial_\mu e] + \bar{e}(i\gamma^\mu \partial_\mu)e - m_e \bar{e}e = -\frac{a}{f_a} \bar{e}[\gamma^\mu \partial_\mu (e_L + e_R)] \quad (\text{with} \quad \gamma^\mu \partial_\mu = -im_e)$$

$$\to a \frac{im_e}{f_a} \bar{e}(\Gamma_L e_L - \Gamma_R e_R) = (\Gamma_L - \Gamma_R)\frac{m_e}{f_a} \bar{e} i\gamma_5 e a \quad (23)$$

where $e_{L,R} = \frac{1 \pm \gamma_5}{2} e$. In the last part of (23), we choose the L-chiral representation. From the $U(1)_{\text{ULA}}$ transformation of e, $e_{L,R} \to e^{i\theta\Gamma_{L,R}} e_{L,R}$ with $\theta = \phi/f_{\text{ULA}}$, thus we obtain

$$\mathcal{L} \to (\Gamma_L - \Gamma_R)\frac{m_e}{f_{\text{ULA}}} \bar{e} i\gamma_5 e \phi, \quad (24)$$

where Γ_L (Γ_R) is the $U(1)_{\text{ULA}}$ charge of $\ell_L(e_R)$ (i.e. for e_R it is minus that of e_L^c) in the tables with $U(1)_{\text{ULA}}$ quantum number +1. Thus, whether H_d or H_u is responsible for the electron mass, we obtain in both cases $\Gamma_L - \Gamma_R = -2$ from Table II. On the other hand, if we use the model with heavy quarks, i.e. for Table VI in the Appendix, we obtain $\Gamma_L - \Gamma_R = -3$. There is a freedom in assigning the electron-ULA coupling at this bottom-up approach.

For the axion and ULA couplings to neutrons and protons, we borrow the results from [64–66]. We use the global quantum numbers presented in Table II. Note that the phase of X_1 is the QCD axion and the phase of X_2 is the ULA. Since the ULA charge of X_2 is 1, we use the quark ULA charges those of Table II. On the other hand, the PQ charge of X_1 is 2 and hence we use the quark PQ charges the halves of those given in Table II.

TABLE VI. Global symmetry charge assignments in the model with a single heavy quark.

	q_L	u_L^c	d_L^c	H_u	H_d	X_1	X_2	Q_L	Q_L^c
$U(1)_{\text{PQ}}$	1	1	1	-2	-2	2	0	-1	-1
$U(1)_{\text{ULA}}$	1	-3	2	2	-3	0	1	$-\frac{3}{2}$	$-\frac{3}{2}$

A. QCD axion couplings

From Table II, the electron quantum numbers are read and we obtain $-m_e/f_a$ for the aee coupling. For Table II, we choose $\delta_{H_u} = -1$ and $\delta_{H_d} = -1$ in Eqs. (61), (62) of [66],

$$\bar{c}_1^u = \frac{1}{2(1+Z)} + \frac{v_d^2}{2v_{ew}^2} \simeq -\frac{1}{6} - \frac{1}{2}s_\beta^2,$$

$$\bar{c}_1^d = \frac{Z}{2(1+Z)} + \frac{v_u^2}{2v_{ew}^2} \simeq -\frac{1}{3} - \frac{1}{2}s_\beta^2 \quad (25)$$

where $v_{ew}^2 = v_u^2 + v_d^2, s_\beta^2 = v_u^2/v_{ew}^2, c_\beta^2 = v_d^2/v_{ew}^2$, and $Z = m_u/m_d \simeq 0.5$. Now, the nucleon couplings are given by

$$C_{app} = \bar{c}_1^u F + \frac{\bar{c}_1^u - 2\bar{c}_1^d}{3}D + \frac{\bar{c}_1^u + \bar{c}_1^d}{6}S$$

$$= \left(-\frac{1}{6} - \frac{1}{2}s_\beta^2\right)F + \left(\frac{1}{2} + \frac{1}{2}s_\beta^2\right)D + \left(-\frac{1}{12} - \frac{1}{6}s_\beta^2\right)S,$$

$$C_{ann} = \bar{c}_1^d F + \frac{\bar{c}_1^d - 2\bar{c}_1^u}{3}D + \frac{\bar{c}_1^u + \bar{c}_1^d}{6}S$$

$$= \left(-\frac{1}{3} - \frac{1}{2}s_\beta^2\right)F + \frac{1}{6}s_\beta^2 D + \left(-\frac{1}{12} - \frac{1}{6}s_\beta^2\right)S. \quad (26)$$

Using the nucleon parameters $F = 0.47, D = 0.81$, and $S \simeq 0.13(\pm 0.2)$:

$$C_{app} = 0.316 + 0.148 s_\beta^2, \quad (27)$$

$$C_{\text{ann}} = -0.168 - 0.122 s_\beta^2. \qquad (28)$$

B. ULA couplings

The value of the coupling $c_{\phi\gamma\gamma}$ depends on the $\mathrm{U}(1)_{\text{ULA}}$ charges of the leptons, which we have not yet discussed, as they do not contribute to the color anomaly. For example, if H_d couples to leptons as, $\ell_L e_L^c H_d$, the $\mathrm{U}(1)_{\text{ULA}}$ charges of the lepton doublet ℓ_L and e_L^c are 1 and 1, respectively, as for q_L and d_L^c of Table II. Then,

$$
\begin{aligned}
c_{\phi\gamma\gamma} \propto\ & 1_{\text{from}q_L} \times 3\left(\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}a^2\right)\right) - 3_{\text{from}u_L^c} \\
& \times 3\left(\left(\frac{2}{3}\right)^2\right) + 1_{\text{from}d_L^c} \times 3\left(\left(\frac{-1}{3}\right)^2\right) \\
& + 1_{\text{from}\ell_L} \times ((-1)^2) + 1_{\text{from}e_L^c} \times ((-1)^2) = 0.
\end{aligned}
\qquad (29)
$$

If H_u couples to leptons as, $\ell_L e_L^c \tilde{H}_u$ where $\tilde{H}_u = i\sigma_2 H_u^*$, the $\mathrm{U}(1)_{\text{ULA}}$ charges of the lepton doublet ℓ_L and e_L^c are again 1 and 1, and we again obtain $c_{\phi\gamma\gamma} = 0$.

The above is a key result of this paper: *in the simplest models, ULAs do not have the usual two-photon coupling.* In nonminimal models, it is possible to avoid this conclusion by introducing additional heavy quarks charged under $\mathrm{U}(1)_{\text{ULA}}$. We discuss such a possibility in the Appendix.

The electron quantum numbers are read off and we obtain $-2m_e/f_{\text{ULA}}$ for the ϕee coupling. For the ϕ coupling to neutrons and protons, we borrow the results from [64,65] for the case of $c_3 = 0$ [66]. For Table II, we choose $\delta_{H_u} = 2$ and $\delta_{H_d} = -3$ in Eqs. (61), (62) of [66],

$$
\begin{aligned}
\bar{c}_1^u &= \frac{1}{2(1+Z)} - \frac{v_d^2}{v_{ew}^2} \simeq -\frac{2}{3} + s_\beta^2, \\
\bar{c}_1^d &= \frac{Z}{2(1+Z)} + \frac{3v_u^2}{2v_{ew}^2} \simeq \frac{1}{6} + 1.5 s_\beta^2.
\end{aligned}
\qquad (30)
$$

Now, the nucleon couplings are given by

$$
\begin{aligned}
C_{\phi\text{pp}} &= \bar{c}_1^u F + \frac{\bar{c}_1^u - 2\bar{c}_1^d}{3}D + \frac{\bar{c}_1^u + \bar{c}_1^d}{6}S \\
&= \left(-\frac{2}{3} + s_\beta^2\right)F + \left(-\frac{1}{3} - \frac{2}{3}s_\beta^2\right)D + \left(-\frac{1}{12} + \frac{5}{12}s_\beta^2\right)S, \\
C_{\phi\text{nn}} &= \bar{c}_1^d F + \frac{\bar{c}_1^d - 2\bar{c}_1^u}{3}D + \frac{\bar{c}_1^u + \bar{c}_1^d}{6}S \\
&= \left(\frac{1}{6} + \frac{3}{2}s_\beta^2\right)F + \left(\frac{1}{2} - \frac{1}{6}s_\beta^2\right)D + \left(-\frac{1}{12} + \frac{5}{12}s_\beta^2\right)S.
\end{aligned}
\qquad (31)
$$

Using again $F = 0.47$, $D = 0.81$, and $S \simeq 0.13(\pm 0.2)$:

$$C_{\phi\text{pp}} = -0.594 - 0.016 s_\beta^2, \qquad (32)$$

$$C_{\phi\text{nn}} = 0.473 + 0.624 s_\beta^2. \qquad (33)$$

V. DETECTION IN THE LAB AND IN ASTROPHYSICS

Both the QCD axion and the ULA in our model can be detected directly in the lab by their interactions with the SM. The standard limits from the $\vec{E}\cdot\vec{B}$ coupling (e.g. Ref. [67]) apply only to the QCD axion, as the ULA has no such coupling. Furthermore, constraints relying on the QCD axion contributing a large fraction of the DM (e.g. those of ADMX [68]) must be scaled appropriately to account for it contributing a subdominant DM component in our model. Recently, a whole range of new ideas have emerged for direct detection of axions and ALPs (see e.g. the review in Ref. [69]). We focus here on those aspects novel to the ULA case, where cosmological and astrophysical information is also important.

The most important coupling between ULAs and the SM is the nucleon coupling, and so we consider our models in the plane (m, g_N), shown in Fig. 3 (we will discuss the electron coupling briefly later). We display our models as curves in this plane derived from the relationship between f_{ULA} and m_{ULA}, Eq. (19), with $12 < p < 65$, and fixing the reference values of all other parameters, including $f_{\text{QCD}} = 10^{11}$ GeV. For simplicity of presentation, we restrict the ULA mass to the cosmologically relevant regime, but note that our model also works outside this limit. The nucleon coupling is

$$g_{NX} = \frac{rC_p}{f_X} + \frac{(1-r)C_n}{f_X}, \qquad (34)$$

where r is the proton-to-neutron ratio in the given medium. As a reference value, we assume a proton-to-neutron ratio taken from the Ref. [70] SN1987A bounds. Our conclusions are not strongly sensitive to this assumption.

The region where ULAs are allowed as a dominant form of DM, yet remain detectable by their imprint on cosmic structure formation is 10^{-22} eV $\lesssim m \lesssim 10^{-18}$ eV. In general, ULAs need have no coupling to nucleons, and the region accessible to studies of structure formation are independent of g_N. In our model, however, we have $g_N = g_N(f_{\text{ULA}})$, and therefore we also show the region where ULAs based on the $\mathbf{Z}_N$ symmetry are able to contribute the dominant form of DM for the given value of g_N (cf. the region shown in Fig. 2). The overlap of the model curves, the desired region for our DM model, and the region accessible to studies of structure formation, defines a small window in the (m, g_N) plane.

Black hole superradiance (BHSR) [71–73] provides a constraint on light bosons, independent of coupling and DM abundance. Spinning stellar mass BHs exclude 6×10^{-13} eV $< m < 2 \times 10^{-11}$ eV at 2σ [71]. These BHSR

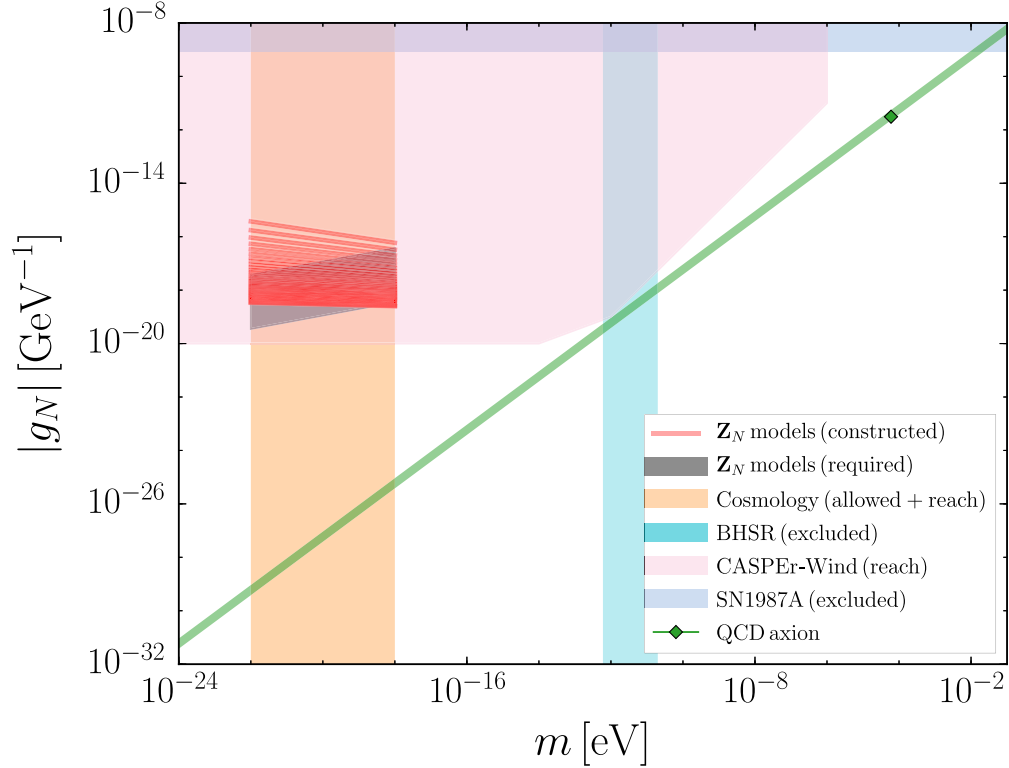FIG. 3.   Axion parameter space, (m, g_N). The QCD axion paired with the ULA is shown for reference, along with the specific point $f_{QCD} = 10^{11}$ GeV, which is our reference value. BHSR excludes a range of masses at 2σ independent of DM abundance and coupling strength [71]. SN1987A excludes the shaded region with $g_N \gtrsim 8.2 \times 10^{-10}$, independent of DM abundance and axion mass [70]. The region both allowed and detectable using cosmology, and relevant to the small-scale crises of CDM is 10^{-22} eV $\lesssim m \lesssim 10^{-18}$ eV [24–26,28–30]. We show the $\mathbf{Z}_N$ models in this regime only, and also show the target region where f_{ULA} allows for the ULAs to be the dominant form of DM without fine-tuning. The region accessible to direct detection using the spin precession technique of CASPEr-Wind [34,35] is also shown. The cosmologically relevant regime of the $\mathbf{Z}_N$ models lies well within the projected sensitivity of CASPEr-Wind, and is not excluded by any other probes.

limits are shown on Fig. 3. Supermassive BHs provide weak constraints on $m \sim 10^{-17}$ eV, and future studies will improve sensitivity in this regime. The range of masses probed by supermassive BHs is highly complementary to cosmological studies.

The first limits on g_N that we consider are those of Ref. [70] from the cooling of SN1987A, which provides the strongest constraints on the absolute value of the axion-nucleon coupling. The model quoted in Ref. [70] has helium mass fraction $Y_p = 0.3$ and constrains a hadronic axion with $C_p = 0.4$, $C_n = 0$ to have $f_a \gtrsim 4 \times 10^8$ GeV. This translates into a bound $|g_N| \lesssim 8 \times 10^{-10}$ GeV, which is applicable to all ALPs, regardless of their DM contribution. Our models easily avoid the SN1987A constraint, due to the large values of f_{ULA}, as shown in Fig. 3.

A direct coupling of axion DM to nucleons induces nuclear spin precession with respect to the DM wind, and this could be detected using nuclear magnetic resonance [34]. An experiment to detect this effect works on similar principles to the Cosmic Axion Spin Precession Experiment (CASPEr), and has been dubbed "CASPEr-Wind."

In contrast, the original CASPEr experiment of Ref. [35] has been dubbed "CASPEr-Electric." CASPEr-Electric uses the axion coupling to the neutron electric dipole moment (EDM), and requires an applied electric field in order to detect axions. In our model, the QCD axion has an EDM coupling via $G\tilde{G}$, but the ULA has no such coupling due to the vanishing color anomaly. As the ULA contributes the dominant form of DM, this renders our model largely invisible to CASPEr-Electric.

CASPEr-Wind searches for axions using the nucleon coupling, and thus requires no applied electric field. Approximate sensitivity curves for CASPEr-Wind are shown in Fig. 3. ULAs oscillate with a very low frequency, and for $m = 10^{-22}$ eV the spin precession frequency $\nu = m^{-1} \sim 10^{-7}$ Hz. The corresponding period is $t \sim$ months. No detailed experimental study has been made of the sensitivity of CASPEr-Wind to such low frequencies, and as such the backgrounds and other issues are unknown. We extrapolate the sensitivity from the lowest mass considered in [34], $m = 10^{-14}$ eV, as a constant. This is likely a conservative extrapolation, assuming that all low

frequencies can be constrained simultaneously with a broadband search. Assuming a large ^{3}He sample limited by magnetometer noise at low frequencies gives $g_N = 10^{-20}$ GeV^{-1}. This extrapolation easily covers all of our model space. Even the more conservative extrapolation assuming a Xe sample limited by magnetometer and magnetization noise, with $g_N = 10^{-18}$ GeV^{-1}, covers the vast majority of the cosmologically and astrophysically relevant region of the $\mathbf{Z}_N$ ULA models.

VI. DISCUSSION AND CONCLUSIONS

We considered models of DM that can simultaneously solve the strong-CP problem and the small-scale crises of CDM, by using two axions with associated approximate global symmetries. In order to solve the small-scale crises of CDM, one axion must be ultralight, with $m_{\mathrm{ULA}} \sim 10^{-22}$ eV, and be the dominant form of DM, requiring a large decay constant, $f_{\mathrm{ULA}} \sim 10^{17}$ GeV. The QCD axion must be subdominant in the DM and therefore have a relatively small decay constant: our benchmark that requires minimal fine-tuning has $f_{\mathrm{QCD}} \sim 10^{11}$ GeV. Minimal model uncertainty occurs if both axions are formed by symmetry breaking before inflation. In the simplest case, our model requires low scale inflation, with $H_I \lesssim 10^{12}$ GeV.

Axions in general have their masses protected by global symmetries, however these symmetries are broken by quantum gravitational effects. In order to control the quantum corrections to the ULA mass, one can make the global symmetries approximate, descending from some exact, and fundamental (i.e. respected by quantum gravity), discrete symmetry $\mathbf{Z}_N$. Achieving low masses and large decay constants requires the approximate global symmetries to be of high quality, which requires fixing the order, N, of the discrete symmetry. We were able to construct a large family of such models. A benchmark model has $N = 24$, and we noted asymptotic behavior of the models at large N. Varying f_{QCD} shifts the benchmark value of N by approximately one, up or down, per order of magnitude in energy.

In order to maintain a satisfactory solution to the strong-CP problem, we demanded that the QCD axion carry all of the color anomaly. This fixed the charges of the SM fermions under the PQ symmetries and we computed the couplings of the QCD axion and the ULA. We found that, in a minimal model with no additional heavy quarks, the ULA has identically zero coupling to $\vec{E} \cdot \vec{B}$ of electromagnetism, and so is invisible to the standard searches for axionlike particles. The axion-nucleon coupling, however, is nonvanishing, and we explored the astrophysical constraints and sensitivity of direct-detection experiments.

We found that the cosmologically relevant region of the $\mathbf{Z}_N$ models is well within the direct-detection sensitivity of the proposed CASPEr-Wind experiment. This is the main finding of our work. We have constructed a class of models where one can obtain ULAs with their masses protected from quantum corrections by a symmetry. The ULAs will be detectable in the near future by their distinctive imprints on cosmic structure formation compared to CDM. The ULAs are also sufficiently strongly coupled to nucleons to be detectable by nuclear spin precession relative to the DM wind using nuclear magnetic resonance.

We have so far not considered the fifth force mediated by the axion-fermion couplings in our model [74,75]. The force is a Yukawa-type force, which for the ULA is extremely long range, $\sim 10^3$ AU. However, because axions are pseudoscalars, the force is also spin dependent. Astrophysical objects are not spin polarized, and so axions do not mediate an astrophysical fifth force. Our model is not excluded by standard searches for additional forces, which tend to assume the forces are scalar mediated.

Current constraints on pseudoscalar forces are reviewed in e.g. Refs. [34,70,76]. For the nucleon coupling, the constraint from fifth forces is weaker than the SN1987A constraint we have already considered. For the electron coupling, fifth force constraints are weaker than those from white dwarf cooling. The new technique proposed by Ref. [76] using NMR to search for spin-dependent forces will be able to detect the QCD axion in our model, via the monopole-dipole interaction. However, even in an optimistic case where the ULA has scalar couplings as large as the QCD axion, the ULA in our model will be invisible to this technique due to the large value of f_{ULA} necessary for the DM abundance.

Since astrophysics and cosmology do not care about the scalar or pseudoscalar nature of the DM, evidence in structure formation for an ultralight particle might be due to a scalar and not an axion. Scalar masses cannot be so easily protected from quantum corrections using the methods we have presented here. However, just like the case of no coupling to the visible sector whatsoever, it is a logical possibility to have an ultralight boson with scalar and dilatonlike couplings. Therefore, consider the constraints of Ref. [77], which searches for ultralight DM using atomic spectroscopy via a dilatonlike coupling [78]. The sensitivity of such a search peaks at $m \sim 10^{-22}$ eV, and is stronger than equivalence principle tests. If astrophysical evidence for $m \sim 10^{-22}$ eV were to build, while detection with CASPEr-Wind fails, then an experiment such as Ref. [77] may still find direct evidence for ultralight particles. Building a satisfactory model with ultralight scalars poses an interesting topic for future work.

APPENDIX: HEAVY QUARKS AND THE ULA PHOTON COUPLING

In the minimal QCD-ULA models considered in the main text, the ULA has no coupling to photons, $c_{\phi\gamma\gamma} = 0$. This is because the charge assignments required for vanishing color anomaly in turn impose vanishing electromagnetic (EM) anomaly. This conclusion can be avoided in a nonminimal model, which we now discuss.

To have a nontrivial coupling, $c_{\phi\gamma\gamma} \neq 0$, we introduce $U(1)_{\mathrm{ULA}}$ charge carrying heavy quarks. In Table VI, we list the charges for an example model with a single heavy quark, Q. For the photon coupling, we calculate the SM fermion contribution and also the heavy quark contribution,

$$
\begin{aligned}
c_{\phi\gamma\gamma} \propto\ & -1_{\mathrm{from}\ u_{\mathrm{L}}} \times 3\left(\left(\frac{2}{3}\right)^2\right) + 3_{\mathrm{from}\ u_{\mathrm{L}}^c} \times 3\left(\left(\frac{2}{3}\right)^2\right) \\
& -1_{\mathrm{from}\ d_{\mathrm{L}}} \times 3\left(\left(\frac{-1}{3}\right)^2\right) - 2_{\mathrm{from}\ d_{\mathrm{L}}^c} \times 3\left(\left(\frac{-1}{3}\right)^2\right) \\
& -1_{\mathrm{from}\ \ell_{\mathrm{L}}} \times ((-1)^2) - 2_{\mathrm{from}\ e_{\mathrm{L}}^c} \times ((-1)^2) \\
& +\frac{3}{2}_{\mathrm{from}\ Q_{\mathrm{L}}} \times 3((e_Q)^2) + \frac{3}{2}_{\mathrm{from}\ Q_{\mathrm{L}}^c} \times 3((e_Q)^2) \\
=\ & 9(e_Q)^2 - \frac{4}{3}.
\end{aligned}
$$

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