Surface curvature singularities of polytropic spheres in Palatini $f(R,T)$ gravity

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We consider Palatini $f(R,T)$ gravity models, similar to those introduced by Harko *et al.* (2012), where the gravitational Lagrangian is given by an arbitrary function of the curvature scalar R and of the trace of the energy-momentum tensor T. Interior spherical static solutions are studied considering the model of matter given by a perfect fluid configuration and a polytropic equation of state. We analyze the curvature singularities found previously for Palatini $f(R)$ gravity and discuss the possibility to remove them in some particular $f(R,T)$ models. We show that it is possible to construct a restricted family of models for which these singularities are not present.

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I. INTRODUCTION

Modified theories of gravity have received much attention as an alternative to general relativity (GR). Among the many alternatives to Einstein's gravity, theories including curvature invariants of higher order have been studied in detail; $f(R)$ theories especially have received particular attention. In the Palatini formalism (see [1] for a complete review on modified theories of gravity in the Palatini approach), which considers the metric and the connection as independents fields, these theories have been applied to stellar models, and it has been found that for generic choices of $f(R)$, the theory presents curvature singularities when considering static spherically symmetric solutions and a polytropic equation of state [2–4]. The only exception seems to be the case in which f is a linear function of R, which corresponds to Einstein gravity with a cosmological constant. This feature has also been found in other models, for instance in the Eddington-inspired Born-Infeld gravity, where the presence of higher derivatives of the matter fields in the effective field equations produce curvature singularities; see [5] and references therein. On the other hand, in [6] Kim argues that it is possible that a gravitational backreaction on the matter dynamics can remove the singularities. Furthermore, Harko *et al.* [7] introduced $f(R,T)$ gravity, where the gravitational Lagrangian is given by an arbitrary function of the curvature scalar R and of the trace of the energy-momentum tensor T. The authors present the metric gravitational field equations of some particular models and studied the equations of motion in the Newtonian limit. The features of these models are being studied in different physical situations; see for instance Refs. [8–11].

The purpose of this paper is to present a new family of related models, obtained as a variation of those described above, by considering the Palatini formalism for $f(R,T)$ gravity. In particular, we study the existence of curvature singularities similar to those found in Palatini $f(R)$ gravity for polytropic spheres, and determine if these kind of models are a viable alternative within this context. As a first step, we derive the field equations in the Palatini formalism. We consider the metric $g_{\mu\nu}$ independent of the connection $\Gamma^{\lambda}{}_{\mu\nu}$, and the curvature scalar of this connection is denoted by $\mathcal{R}$. We will follow the procedure described by Kainulainen *et al.* in Ref. [12] to obtain a generalization of the Tolman-Oppenheimer-Volkoff (TOV) equation of hydrostatic equilibrium for $f(R,T)$ gravity in the Palatini formalism and will follow the method described in [2], used to analyze the existence of curvature singularities in Palatini $f(R)$ gravity, and study the possibility to remove them in some particular $f(R,T)$ models.

The content of our paper is organized as follows. In Sec. II we determine the field equations of $f(R,T)$ gravity in the Palatini formalism and reduce them to effective field equations. In Sec. III we consider a static spherically symmetric solution and find a generalized TOV equation for Palatini $f(R,T)$ gravity. In Sec. IV we consider the problem of curvature singularities in polytropic spheres and show that it is possible find a family of functions for which no singularities arise. We conclude in Sec. V with a summary and discussion of some future perspectives.

II. GRAVITATIONAL FIELD EQUATIONS OF PALATINI $f(R,T)$ GRAVITY

In the Palatini formalism we consider that the metric tensor and the connection are independent fields. The scalar constructed with the Christoffel symbols of $g_{\mu\nu}$ will be denoted by R, whereas $\mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu}(\Gamma)$ represents the

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curvature scalar constructed with the independent connection $\Gamma^\lambda_{\mu\nu}$. The scalar $\mathcal{R}$ is invariant under projective transformation of the form $\Gamma^\lambda_{\mu\nu} \to \Gamma^\lambda_{\mu\nu} + \delta^\lambda_\mu \xi_\nu$, where ξ_ν is an arbitrary covariant vector field. Then, the Hilbert-Einstein Lagrangian, or any other Lagrangian constructed from a function of $\mathcal{R}$, is invariant under this kind of transformation. As a consequence, it would not be possible to find a unique solution for the connection, since the field equations determine this field up to projective transformations. A way out of this problem is to assume from the beginning that the connection has no torsion, i.e. that it is symmetric with respect to its lower indices. This assumption breaks projective invariance, since projective transformations do not preserve the torsionless condition. Therefore, we always consider an independent torsionless connection $\Gamma^\lambda_{\mu\nu}$, which on the other hand can in general have nontrivial nonmetricity, i.e. $\nabla_\lambda g_{\mu\nu} \neq 0$.

With these preliminaries, we consider an action given by

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(\mathcal{R},T) + \int d^4x\sqrt{-g}L_m, \tag{2.1}$$

where $\kappa = 8\pi G$; $f(\mathcal{R},T)$ is an arbitrary function of the curvature scalar $\mathcal{R}$; and of the trace T of the energy-momentum tensor of matter $T_{\mu\nu}$, we choose $c = 1$ and follow the notation of Ref. [7]. Finally, L_m is the matter Lagrangian, and we define the energy-momentum tensor of matter as usual as

$$T_{\mu\nu} := -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \tag{2.2}$$

We will assume as in [7] that the Lagrangian L_m of matter depends only on the metric tensor components $g_{\mu\nu}$, not on its derivatives, and thus

$$T_{\mu\nu} = g_{\mu\nu}L_m - 2\frac{\partial L_m}{\partial g^{\mu\nu}}. \tag{2.3}$$

By varying the action (2.1) with respect to the metric tensor $g_{\mu\nu}$ we find

$$\delta S = \frac{1}{2\kappa}\int\left[F(\mathcal{R},T)\delta\mathcal{R} + H(\mathcal{R},T)\frac{\delta T}{\delta g^{\mu\nu}}\delta g^{\mu\nu}\right. \tag{2.4}$$

$$\left.-\frac{1}{2}g_{\mu\nu}f(\mathcal{R},T)\delta g^{\mu\nu} + \kappa\frac{1}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}\right]\sqrt{-g}d^4x, \tag{2.5}$$

where we have denoted

$$F(\mathcal{R},T) := \frac{\partial f(\mathcal{R},T)}{\partial\mathcal{R}}, \qquad H(\mathcal{R},T) := \frac{\partial f(\mathcal{R},T)}{\partial T}. \tag{2.6}$$

Since $\mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu}$ and $\mathcal{R}_{\mu\nu}$ does not depend on the metric tensor, we have that

$$\delta S = \frac{1}{2\kappa}\int\left[F(\mathcal{R},T)\mathcal{R}_{\mu\nu}\delta g^{\mu\nu} + H(\mathcal{R},T)\frac{\delta(g^{\alpha\beta}T_{\alpha\beta})}{\delta g^{\mu\nu}}\delta g^{\mu\nu}\right.$$
$$\left.-\frac{1}{2}g_{\mu\nu}f(\mathcal{R},T)\delta g^{\mu\nu} + \kappa\frac{1}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}\right]\sqrt{-g}d^4x. \tag{2.7}$$

As in Ref. [7] we define the variation of T with respect to the metric tensor as

$$\frac{\delta(g^{\alpha\beta}T_{\alpha\beta})}{\delta g^{\mu\nu}} = T_{\mu\nu} + \Theta_{\mu\nu}, \tag{2.8}$$

where

$$\Theta_{\mu\nu} := g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}. \tag{2.9}$$

Then, we obtain the field equations for the metric of $f(\mathcal{R},T)$ gravity as

$$F(\mathcal{R},T)\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(\mathcal{R},T)$$
$$= \kappa T_{\mu\nu} - H(\mathcal{R},T)T_{\mu\nu} - H(\mathcal{R},T)\Theta_{\mu\nu}. \tag{2.10}$$

Varying now the action (2.1) with respect to the connection $\Gamma^\lambda{}_{\mu\nu}$, and taking into account that the trace of the energy-momentum tensor is independent of the connection and that

$$\delta\mathcal{R}_{\mu\nu} = \bar{\nabla}_\lambda\delta\Gamma^\lambda{}_{\mu\nu} - \bar{\nabla}_\nu\delta\Gamma^\lambda{}_{\mu\lambda}, \tag{2.11}$$

where $\bar{\nabla}_\lambda$ denotes the covariant derivative defined with the independent connection $\Gamma^\lambda{}_{\mu\nu}$, we have that

$$\bar{\nabla}_\lambda(F(\mathcal{R},T)\sqrt{-g}g^{\mu\nu}) - \delta^{(\nu}_\lambda\bar{\nabla}_\rho(F(\mathcal{R},T)\sqrt{-g}g^{\mu)\rho} = 0, \tag{2.12}$$

where $(\mu\nu)$ denotes the symmetrization over μ and ν. Contracting with δ^λ_ν, we can rewrite (2.12) as

$$\bar{\nabla}_\lambda(F(\mathcal{R},T)\sqrt{-g}g^{\mu\nu}) = 0. \tag{2.13}$$

Note that if we consider $f(\mathcal{R},T) = f(\mathcal{R})$, then $H(\mathcal{R},T) = 0$ and we recover the field equations for Palatini $f(R)$ gravity; see [13].

We can eliminate the connection as an independent field if we consider a metric conformal to $g_{\mu\nu}$ as

$$h_{\mu\nu} := F(\mathcal{R},T)g_{\mu\nu}. \tag{2.14}$$

It is easy to show that in terms of $h_{\mu\nu}$ the field equation (2.13) reduces to $\bar{\nabla}_\lambda(\sqrt{-h}h^{\mu\nu}) = 0$, so that the solutions are the Christoffel symbols of $h_{\mu\nu}$,

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} h^{\lambda\sigma}(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma} - \partial_\sigma h_{\mu\nu}), \qquad (2.15)$$

and therefore,

$$\begin{aligned}\Gamma^{\lambda}_{\mu\nu} &= \frac{1}{2}\frac{1}{F(\mathcal{R},T)} g^{\lambda\sigma}[\partial_\mu(F(\mathcal{R},T)g_{\nu\sigma}) + \partial_\nu(F(\mathcal{R},T)g_{\mu\sigma}) \\ &\quad - \partial_\sigma(F(\mathcal{R},T)g_{\mu\nu})] \\ &= \left\{ {\lambda \atop \mu\nu} \right\} + \frac{1}{2}\frac{1}{F(\mathcal{R},T)}[\delta^\lambda_\nu \partial_\mu(F(\mathcal{R},T)) \\ &\quad + \delta^\lambda_\mu \partial_\nu(F(\mathcal{R},T)) - g_{\mu\nu}\partial^\lambda(F(\mathcal{R},T))], \end{aligned} \qquad (2.16)$$

where $\left\{ {\lambda \atop \mu\nu} \right\}$ denotes the Christoffel symbols of the metric tensor $g_{\mu\nu}$. As a result, we have that

$$\begin{aligned}\mathcal{R}_{\mu\nu} &= R_{\mu\nu} + \frac{3}{2}\frac{1}{(F(\mathcal{R},T))^2}(\nabla_\mu F(\mathcal{R},T))(\nabla_\nu F(\mathcal{R},T)) \\ &\quad - \frac{1}{F(\mathcal{R},T)}\left(\nabla_\mu\nabla_\nu + \frac{1}{2}g_{\mu\nu}\Box\right)F(\mathcal{R},T), \end{aligned} \qquad (2.17)$$

and

$$\mathcal{R} = R + \frac{3}{2}\frac{1}{(F(\mathcal{R},T))^2}(\nabla F(\mathcal{R},T))^2 - \frac{3}{F(\mathcal{R},T)}\Box F(\mathcal{R},T), \qquad (2.18)$$

where $R_{\mu\nu}$ and R are the Ricci tensor and the curvature scalar defined by the Christoffel symbols of the metric tensor $g_{\mu\nu}$, respectively. Replacing Eqs. (2.17) and (2.18) in Eq. (2.10), and after some manipulations we can write the field equations as effective Einstein equations, namely

$$\begin{aligned}G_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \\ &= \frac{\kappa - H(\mathcal{R},T)}{F(\mathcal{R},T)}T_{\mu\nu} - \frac{H(\mathcal{R},T)}{F(\mathcal{R},T)}\Theta_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\left(\mathcal{R} - \frac{f(\mathcal{R},T)}{F(\mathcal{R},T)}\right) \\ &\quad - \frac{3}{2F^2(\mathcal{R},T)}((\nabla_\mu F(\mathcal{R},T))(\nabla_\nu F(\mathcal{R},T)) \\ &\quad - \frac{1}{2}g_{\mu\nu}(\nabla F(\mathcal{R},T))^2) + \frac{1}{F(\mathcal{R},T)}(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)F(\mathcal{R},T). \end{aligned} \qquad (2.19)$$

III. MODIFIED TOLMAN-OPPENHEIMER-VOLKOFF EQUATION

We now take the covariant derivative ∇^μ of Eq. (2.10) and obtain for the divergence of the energy-momentum tensor $T_{\mu\nu}$

$$\begin{aligned}\nabla^\mu T_{\mu\nu} &= \frac{H(\mathcal{R},T)}{\kappa - H(\mathcal{R},T)}\bigg[(T_{\mu\nu} + \Theta_{\mu\nu})\nabla^\mu \ln H(\mathcal{R},T) \\ &\quad + \nabla^\mu\Theta_{\mu\nu} - \frac{1}{2}\nabla_\nu T\bigg], \end{aligned} \qquad (3.1)$$

where ∇^μ denotes the covariant derivative with respect to the connection of the metric tensor $g_{\mu\nu}$. Note that Eq. (3.1) corresponds to the conservation law found in Ref. [14] in the metric formalism. If $H(\mathcal{R},T) = 0$, we obtain $\nabla^\mu T_{\mu\nu} = 0$ as in GR and $f(R)$ gravity.

Now, in order to obtain a modified TOV equation of hydrostatic equilibrium for Palatini $f(R,T)$ gravity we assume that the metric is static and spherically symmetric, and we write it in the form

$$ds^2 \equiv e^{A(r)}dt^2 - e^{B(r)}dr^2 - r^2 d\Omega^2. \qquad (3.2)$$

We also assume a perfect fluid description for matter, so that

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu - P g_{\mu\nu}, \qquad (3.3)$$

where ρ is the mass/energy density, P is the pressure, and u^μ is the fluid 4-velocity. As in Ref. [7] we consider the matter Lagrangian to be given by $L_m = -P$; see also [15]. Then, we find that $\Theta_{\mu\nu} = -2T_{\mu\nu} - g_{\mu\nu}P$. Now we can replace the metric (3.2) and the energy-momentum tensor (3.3) in (2.19) and consider the components G_{00} and G_{11}. Denoting d/dr with a prime $'$, we arrive at

$$A' = -\frac{1}{1+\gamma}\left(\frac{1 - e^B}{r} - \frac{\kappa}{F}e^B P r + \frac{\alpha}{r}\right), \qquad (3.4)$$

$$B' = \frac{1}{1+\gamma}\left(\frac{1 - e^B}{r} + \frac{\kappa}{F}e^B \rho r + \frac{H}{F}e^B(\rho + P)r + \frac{\alpha + \beta}{r}\right), \qquad (3.5)$$

where we define

$$\alpha \coloneqq r^2\left[\frac{3}{4}\left(\frac{F'}{F}\right)^2 + \frac{2F'}{Fr} - \frac{e^B}{2}\left(\mathcal{R} - \frac{f}{F}\right)\right], \qquad (3.6)$$

$$\beta \coloneqq r^2\left[\frac{F''}{F} - \frac{3}{2}\left(\frac{F'}{F}\right)^2\right], \qquad (3.7)$$

$$\gamma \coloneqq \frac{rF'}{2F}. \qquad (3.8)$$

Note that Eq. (3.4) for A' is not modified by the new coupling between geometry and matter as compared to $f(R)$ gravity, while B' is modified, due to the term proportional to H. We consider the conservation of the energy-momentum tensor, Eq. (3.1); take into account that $\nabla_\mu T^{\mu 1} = P' + A'(\rho + P)/2$; and assume an equation of state of the form $\rho = \rho(P)$, so that $\rho' = P'(d\rho/dP)$. Therefore, we can use Eqs. (3.4) and (3.5) to arrive at the modified TOV equation for Palatini $f(R,T)$ gravity:

$$\frac{dP}{dr} = -\frac{\kappa + H}{\kappa + H(1 + \frac{e^{-B}}{2}(1 - \frac{d\rho}{dP}))}\left(\frac{1}{1+\gamma}\right)\frac{(\rho + P)}{r(r - 2m_{\rm tot})} \times \left(m_{\rm tot} + \frac{4\pi G P r^3}{F} - \frac{\alpha}{2}(r - 2m_{\rm tot})\right). \tag{3.9}$$

Here we have defined $m_{\rm tot}(r) \coloneqq r(1 - e^{-B})/2$. Note that if $H = 0$ we obtain the corresponding modified TOV equation for Palatini $f(R)$ gravity [12].

IV. CURVATURE SINGULARITIES

Our purpose is to determine if Palatini $f(R,T)$ gravity presents the same kind of surface curvature singularities which appear in $f(R)$ theory. The fact that the function f depends, in addition to the curvature scalar $\mathcal{R}$, on the trace of the energy-momentum tensor T, could provide a possibility that these singularities are not present in some particular model. As stated above, our analysis follows Ref. [2]; therefore we will use the modified TOV equation (3.9), and the general equations for the metric components (3.4) and (3.5). Indeed, multiplying (3.9) by dF/dP and after some algebra, we find a quadratic equation for F', $aF'^2 + bF' + c = 0$, where

$$\begin{aligned} a &= 2r^2(r - 2m_{\rm tot})(\kappa + H)(3\mathcal{C} - 4F) \\ &\quad - 4r(r - 2m_{\rm tot})^2 HF\left(1 - \frac{d\rho}{dP}\right), \\ b &= 16rF(r - 2m_{\rm tot})(\kappa + H)(\mathcal{C} - F) \\ &\quad - 8HF^2(r - 2m_{\rm tot})^2\left(1 - \frac{d\rho}{dP}\right), \\ c &= -8\mathcal{C}F(\kappa + H)\left(2m_{\rm tot}F + 8\pi G P r^3 + \frac{r^3}{2}(RF - f)\right), \end{aligned} \tag{4.1}$$

whose solution is then given by

$$F' = \frac{1}{2}\left(\frac{-b}{a} \pm \sqrt{\left(\frac{b}{a}\right)^2 - 4\left(\frac{c}{|a|}\right)}\right), \tag{4.2}$$

and where we have defined

$$\mathcal{C} \coloneqq (\rho + P)\frac{dF}{dP} = (\rho + P)\frac{dF}{d\rho}\frac{d\rho}{dP}. \tag{4.3}$$

We focus on a polytropic equation of state, which can be written as

$$\rho(P) = \left(\frac{P}{K}\right)^{1/\Gamma} + \frac{P}{\Gamma - 1}, \tag{4.4}$$

where $K > 0$ and $\Gamma > 1$ are constants, the latter being the polytropic index. Note that, even when $d\rho/dP$ diverges for $P \to 0$, the product $(\rho + P)d\rho/dP \to 0$ provided $\Gamma < 2$ [2]. Notice also that taking the trace of Eq. (2.10) we can find an algebraic relation among $\mathcal{R}$, T and P,

$$F(\mathcal{R},T)\mathcal{R} - 2f(\mathcal{R},T) = \kappa T + H(\mathcal{R},T)T + 4H(\mathcal{R},T)P, \tag{4.5}$$

so that after solving for $\mathcal{R}$ we find a relation of the form $\mathcal{R} = \mathcal{R}(T,P)$. Taking this into account, as well as the dependency of the trace of energy-momentum tensor on ρ and P and the equation of state, we can evaluate the total variation of F under changes of the density ρ, i.e. the derivative $dF/d\rho$ appearing in (4.3), and find that

$$\frac{dF}{d\rho} = \frac{\partial F}{\partial \mathcal{R}}\frac{\partial \mathcal{R}}{\partial T}\left(1 - 3\frac{dP}{d\rho}\right) + \frac{\partial F}{\partial \mathcal{R}}\frac{\partial \mathcal{R}}{\partial P}\frac{dP}{d\rho} + \frac{\partial F}{\partial T}\left(1 - 3\frac{dP}{d\rho}\right), \tag{4.6}$$

where $\partial\mathcal{R}/\partial T$ and $\partial\mathcal{R}/\partial P$ can be calculated from (4.5), obtaining that

$$\frac{\partial \mathcal{R}}{\partial T} = \frac{\kappa + 3H + \frac{\partial H}{\partial T}(T + 4P)}{\frac{\partial F}{\partial \mathcal{R}}\mathcal{R} - F - \frac{\partial H}{\partial \mathcal{R}}(T + 4P)} \tag{4.7}$$

and

$$\frac{\partial \mathcal{R}}{\partial P} = \frac{4H}{\frac{\partial F}{\partial \mathcal{R}}\mathcal{R} - F - \frac{\partial H}{\partial \mathcal{R}}(T + 4P)}. \tag{4.8}$$

For the study of surface singularities, we will consider, for simplicity, the case in which the function f takes the form $f(\mathcal{R},T) = f_1(\mathcal{R}) + f_2(T)$; then we have that $F = F(\mathcal{R})$ and $H = H(T)$. In addition, we consider $f_2(T)$ to be a polynomial of T. Then, $\partial F/\partial\mathcal{R}$, $\partial\mathcal{R}/\partial T$, $\partial\mathcal{R}/\partial P$ and $\partial F/\partial T$ are in general finite [check for instance, $f(\mathcal{R},T) = \mathcal{R} + \mathcal{R}^2 + T^2$], even when $T \to 0$ and $dP/d\rho \to 0$ when $P = 0$ at the surface, which is located at $r = r_{\rm out}$. Moreover, f, F, H and $\mathcal{R}$ take finite values f_0, F_0, H_0 and $\mathcal{R}_0$ at the surface, respectively. Additionally, it can be easily shown from Eqs. (2.19) and (4.5) that the vacuum static spherically symmetric solution for these cases is the Schwarzschild-(anti–)de Sitter solution. Therefore, when $\Gamma < 2$, $\mathcal{C} = 0$ at $r = r_{\rm out}$. Furthermore, in contrast to the $f(R)$ case [2], the solutions of F', Eq. (4.2), contain terms proportional to $H d\rho/dP$. While $d\rho/dP$ diverges when $P \to 0$, the product $H(d\rho/dP)$ could converge in a particular model. Thus, for simplicity we consider two cases: (1) $H(T = 0) = H_0 = \text{const.}$ and (2) $H(T) \propto T^n, n \geq 1$. We have to calculate the solutions for F' at the surface considering Eq. (4.2). For the first case, $H(T = 0) = H_0 = \text{const}$, we have that

$$\lim_{P\to 0}\left(\frac{b}{a}\right) \to \frac{-8H_0F_0^2(r_{\rm out} - 2m_{\rm tot})^2(1 - \frac{d\rho}{dP})}{-4r_{\rm out}(r_{\rm out} - 2m_{\rm tot})^2 H_0 F_0(1 - \frac{d\rho}{dP})} \to \frac{2F_0}{r_{\rm out}}. \tag{4.9}$$

Similarly, and considering $\mathcal{C} = 0$ at the surface if $\Gamma < 2$, we find

$$\lim_{P\to 0}\left(\frac{c}{a}\right) \to 0. \quad (4.10)$$

Then, replacing the previous values in Eq. (4.2) we find the solutions for F' at the surface:

$$F'(r_{\rm out}) = -\frac{2F_0}{r_{\rm out}} \quad \text{and} \quad F'(r_{\rm out}) = 0. \quad (4.11)$$

For the second case $H(T) \propto T^n, n \geq 1$, the calculation is analogous to the previous one and it can be shown that we have the same solutions for F' at the surface, namely Eq. (4.11).

In both cases, considering the first solution, $F'(r_{\rm out}) = -2F_0/r_{\rm out}$, we have that $\gamma = -1$ and then A' diverges for all r [see Eq. (3.4)]. Therefore, we focus on the second solution, $F'(r_{\rm out}) = 0$. Then, $\gamma = 0$ and Eq. (3.4) implies that A' is finite at the surface. We also need B' to be finite when we approach the surface. Therefore, we also need to determine the behavior of F'', since B' depends on this latter quantity through β; see Eqs. (3.5) and (3.7). Deriving the quadratic equation of F' with respect to r, and taking into account that $F' = \mathcal{C} = 0$ at $r = r_{\rm out}$ we find

$$F''(r_{\rm out}) = -\frac{(\kappa + H_0)(8m_{\rm tot} + r_{\rm out}^3\mathcal{R}_0)\mathcal{C}'}{4(r_{\rm out} - 2m_{\rm tot})[2r_{\rm out}(\kappa + H_0) + (r_{\rm out} - 2m_{\rm tot})H_0(1 - \frac{d\rho}{dP})]}. \quad (4.12)$$

To completely determine F'' we must compute $\mathcal{C}'$ and, in contrast to $f(R)$ theory, the second term in the denominator, proportional to $H_0(1 - d\rho/dP)$, could provide a possibility that even when $\mathcal{C}'$ diverges, F'' converges.

Deriving Eq. (4.5) with respect to r, we find that

$$F'\mathcal{R} - F\mathcal{R}' = \kappa T' + 3HT' + \frac{dH}{dT}T'(T + 4P) + 4HP'. \quad (4.13)$$

We will now proceed to determine the behavior of $\mathcal{C}'$ at the surface, for which we have to consider the two cases mentioned above.

A. Case 1: $H(T = 0) = H_0 = \mathrm{const}$

As we are interested in the behavior at the surface, $F' = 0$ and we also have that $T' = (d\rho/dP - 3)P'$. From Eq. (3.9) we note that if $H_0 = \mathrm{const}$ then $P' \propto (\rho + P)/(1 - d\rho/dP)$, so that $T' \propto (d\rho/dP - 3)(\rho + P)/(1 - d\rho/dP)$ which goes to zero at the surface. Thus, considering Eq. (4.13) we find that $\mathcal{R}' = T' = 0$ at the surface. Furthermore, deriving Eq. (4.3) with respect to r we find that

$$\mathcal{C}' \propto \lambda\frac{d^2\rho}{dP^2}\frac{(\rho + P)^2}{(1 - d\rho/dP)} + \mu\left[\left(\frac{d\rho}{dP}\right)^2 - 2\frac{d\rho}{dP} - 3\right] \times \frac{(\rho + P)}{(1 - d\rho/dP)} + \nu\left(1 + \frac{d\rho}{dP}\right)\frac{(\rho + P)}{(1 - d\rho/dP)}, \quad (4.14)$$

where λ, μ and ν are the constant values of the derivatives $\lambda = (dF/d\mathcal{R})(\partial\mathcal{R}/\partial T)$, $\mu = (dF/d\mathcal{R})(\partial\mathcal{R}/\partial T)$ and $\nu = (dF/d\mathcal{R})(\partial\mathcal{R}/\partial P)$ at the surface, respectively. Using the polytropic equation of state (4.4) and their respective derivatives, $d\rho/dP$ and $d^2\rho/dP^2$, we find after some algebra that $\mathcal{C}'$ converges at the surface provided $\Gamma < 2$, since the leading term turns out to be proportional to $P^{2/\Gamma - 1}$. This is different compared to the case of $f(R)$ gravity where $\mathcal{C}'$ goes to zero as $P^{3/\Gamma - 2}$ so that it converges only for $\Gamma < 3/2$; see [2] for further details. Note that in the Palatini $f(R,T)$ model F'', cf. Eq. (4.12), includes the term $H_0(1 - d\rho/dP)$ in the denominator, and since $H_0 = \mathrm{const}$ and $d\rho/dP$ diverges at the surface we have that

$$\lim_{P\to 0} F'' \propto \lim_{P\to 0}\frac{\mathcal{C}'}{(1 - \frac{d\rho}{dP})} \to 0. \quad (4.15)$$

Therefore, we find that F'' converges at the surface if $1 < \Gamma < 2$. Thus, B' (or $m'_{\rm tot}$) converges at $r = r_{\rm out}$, making it possible to find solutions where the metric and its first derivative are continuous across the surface.

B. Case 2: $H(T) \propto T^n, n \geq 1$

In this case we have that $H(T)(1 - d\rho/dP) \propto P^{(n+1)/\Gamma - 1} \to 0$ when $P \to 0$ if $\Gamma < 2$, such that from Eq. (3.9) we find that $P' \propto (\rho + P) \to 0$ and from Eq. (4.13) we have that $\mathcal{R}' = T' = 0$ at the surface. Thus, we have that $\mathcal{C}'$ is given by

$$\mathcal{C}' \propto \lambda\frac{d^2\rho}{dP^2}(\rho + P)^2 + \mu\left[\left(\frac{d\rho}{dP}\right)^2 - 2\frac{d\rho}{dP} - 3\right](\rho + P) + \nu\left(1 + \frac{d\rho}{dP}\right)(\rho + P), \quad (4.16)$$

which differs from Eq. (4.14) since now the new terms proportional to H in the denominator vanish at the surface.

Replacing the equation of state and its derivatives we find that $\mathcal{C}'$ goes to zero if $\Gamma < 3/2$ when $P \to 0$ at the surface. Finally, considering the expression for F'' given by (4.12), and since $H(T)(1 - d\rho/dP) \to 0$ we have that

$$F''(r_{\rm out}) = -\frac{(8m_{\rm tot} + r_{\rm out}^3\mathcal{R}_0)\mathcal{C}'}{8r_{\rm out}(r_{\rm out} - 2m_{\rm tot})}. \quad (4.17)$$

Note that the above expression is identical to that obtained by Barausse *et al.* for $f(R)$ models [2], a fact which is not surprising since in the present case we have found that the new terms introduced by $f(R,T)$ gravity vanish at the surface. Therefore, F'' goes to zero at the surface if $\Gamma < 3/2$ and diverges if $3/2 < \Gamma < 2$. See Ref. [2] for further details.

V. CONCLUSIONS AND FINAL REMARKS

Considering the case 1 discussed above it is possible to build a family of functions of the form

$$f(\mathcal{R},T) = f_1(\mathcal{R}) + f_2(T), \tag{5.1}$$

that allows nonsingular interior solutions with a polytropic equation of state and $\Gamma < 2$, so that they can be considered as a viable alternative to GR. As we mentioned before, in the Palatini $f(R)$ theory it was not possible to have physically acceptable solutions with a polytropic equation of state in the range $3/2 < \Gamma < 2$, leading to curvature singularities at the stellar surface, which would in turn induce infinite tidal forces. This excluded $\Gamma = 5/3$, which is the case of a degenerate nonrelativistic particle gas, a physical system which is well described even in Newtonian gravity [2]. We can construct a family of $f(\mathcal{R},T)$ models, of the form (5.1), which allow interior solutions provided $f_2(T)$ can be written, when $T \to 0$, as

$$f_2(T) \to H_0 T + \mathcal{O}(T^2), \tag{5.2}$$

where $H_0 = (df_2(T)/dT)|_{T=0} \neq 0$. For instance, we have the case in which $f_2(T)$ is a polynomial of degree n in T, $\mathcal{P}_n(T)$, i.e.,

$$\begin{aligned} f_2(T) &= \mathcal{P}_n(T) \\ &= a_n T^n + a_{n-1}T^{n-1} + \cdots + a_1 T^1 + a_0, \end{aligned} \tag{5.3}$$

where a_n are constants and $n \geq 1$, and $H_0 = a_1 \neq 0$. On the other hand, if $H_0 = 0$ the model behaves as in case 2 above, where surface singularities arise for $3/2 < \Gamma < 2$, just like in $f(R)$ gravity.

For example, if we consider the simplest case within the family of models defined by Eq. (5.3), i.e., $f(\mathcal{R},T) = f_1(\mathcal{R}) + \lambda T$, where λ is a constant, then $H(T) = \lambda$ and Eqs. (2.19) and (3.1) reduce, respectively, to

$$\begin{aligned} G_{\mu\nu} = &\frac{\kappa+\lambda}{F(\mathcal{R})} T_{\mu\nu} + \frac{\lambda}{2F(\mathcal{R})} g_{\mu\nu}(\rho - P) - \frac{1}{2} g_{\mu\nu}\left(\mathcal{R} - \frac{f_1(\mathcal{R})}{F(\mathcal{R})}\right) \\ &- \frac{3}{2F^2(\mathcal{R})}\left((\nabla_\mu F(\mathcal{R}))(\nabla_\nu F(\mathcal{R})) - \frac{1}{2} g_{\mu\nu}(\nabla F(\mathcal{R}))^2\right) \\ &+ \frac{1}{F(\mathcal{R})}(\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box)F(\mathcal{R}) \end{aligned} \tag{5.4}$$

and

$$\nabla^\mu T_{\mu\nu} = \frac{\lambda}{2(\kappa+\lambda)} \nabla_\nu (P - \rho). \tag{5.5}$$

As expected, if $\lambda = 0$ then Eqs. (5.4) and (5.5) reduce to the corresponding equations of Palatini $f(R)$ gravity; see Ref. [12]. The model with $\lambda \neq 0$, although the simplest nonsingular one, is physically nontrivial and may then deserve further investigation. This particular case has been recently studied within the metric formalism in Ref. [16].

In summary, we have analyzed $f(R,T)$ gravity in the Palatini formalism and studied the simplest stellar model defined by a static spherically symmetric solution with a polytropic equation of state. We have determined the modified TOV equation for this theory and compared it with the one obtained for $f(R)$ gravity [12]. We have also shown that, as expected, most of these models present curvature singularities. This happens when a constant term H_0 is not present and the theory behaves similarly as Palatini $f(R)$ gravity; see Eq. (4.17) and Ref. [2]. Rather surprisingly, however, we have found that it is possible to construct a family of $f(R,T)$ models allowing finite solutions in a range in which $f(R)$ gravity does not allow them, i.e., when $3/2 < \Gamma < 2$.

We have studied the simplest case in which the action is a function of separable variables, i.e., $f(\mathcal{R},T) = f_1(\mathcal{R}) + f_2(T)$, and we considered the two simplest cases in Secs. IV A and IV B. For future work it will be interesting to analyze the cases in which the function $f_2(T)$ or its derivative $H(T)$ diverges. This occurs, for instance, in a model recently considered in the literature[1]: $f_2(T) = \sqrt{T}$ [9,17] for which $H(T) = 1/(2\sqrt{T})$. In this case the solutions of F' at the surface are the same found in the present paper, Eq. (4.11), but the value of $\mathcal{C}$ at the surface depends on the polytropic index, which converges for $1 < \Gamma < 3/2$ and diverges for $3/2 < \Gamma < 2$. Then, the form of $\mathcal{C}'$ and F'' are more complicated to analyze. Furthermore, for models in which $f_2(T)$ diverges in vacuum, we note that the vacuum solution to (2.19) exists only if we limit the value of the polytropic index depending on the particular $f(\mathcal{R},T)$ model.

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[1]In [9,17] the authors use $f_2(T) = \sqrt{-T}$. We write $f_2(T) = \sqrt{T}$ due to our different choice of signature.

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