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Relationship between the CMB, Sunyaev-Zel'dovich cluster counts, and local Hubble parameter measurements in a simple void model

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The discrepancy between the amplitudes of matter fluctuations inferred from Sunyaev-Zel'dovich (SZ) cluster number counts and the measurement of temperature and polarization anisotropies of the cosmic microwave background (CMB) measured by the Planck satellite can be reconciled if the local universe is embedded in an underdense region as shown by Lee, 2014. Here using a simple void model assuming the open Friedmann-Robertson-Walker geometry and a Markov Chain Monte Carlo technique, we investigate how deep the local underdense region needs to be to resolve this discrepancy. Such local void, if it exists, predicts the local Hubble parameter value that is different from the global Hubble constant. We derive the posterior distribution of the local Hubble parameter from a joint fitting of the Planck CMB data and SZ cluster number counts assuming the simple void model. We show that the predicted local Hubble parameter value of $H_{\rm loc} = 70.1 \pm 0.34\ {\rm km\,s^{-1}\,Mpc^{-1}}$ is in better agreement with direct local Hubble parameter measurements, indicating that the local void model may provide provide a consistent solution to the cluster number counts and Hubble parameter discrepancies.

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I. INTRODUCTION

The standard model of cosmology, also referred to the ΛCDM model, is now well established based on a number of cosmological observations. The simplest ΛCDM model contains six free cosmological parameters, each of which has been determined with an accuracy of a few percent through the measurement of cosmic microwave background (CMB) anisotropies by the Planck satellite [1]. The CMB anisotropies mainly manifest the nature of density fluctuations in the universe at the recombination era at $z \approx 1100$. It is therefore of great importance to confront the standard model with observations of the local universe ($z \approx 0$) to confirm that the standard model is correct throughout the history of the universe (see section 5.6 of [2] and references therein).

Some authors, however, have reported cosmological measurements in the local universe that conflict with the CMB result. For example, it has been claimed that the amplitude of density fluctuations measured in the local universe is systematically low compared with the prediction from Planck's CMB data (e.g., [3–9]). In particular, the number of massive clusters derived from the Sunyaev-Zel'dovich (SZ) effect by Planck is about half that expected from the CMB anisotropies [10]. This SZ result from Planck is consistent with the SZ-selected cluster number counts from the Atacama Cosmology Telescope [11] and the South Pole Telescope surveys [12]. Perhaps related to this discrepancy, the local Hubble parameter measured through the traditional distance ladder tends to show a larger value (e.g., $H_0 = 73.8 \pm 2.4\ {\rm km\,s^{-1}\,Mpc^{-1}}$ by [13]) than that inferred from the Planck result ($H_0 \approx 67.3 \pm 1.0\ {\rm km\,s^{-1}\,Mpc^{-1}}$). There have been many proposals to resolve these discrepancies, including beyond-the-standard models, e.g., massive neutrinos [4,5], and decaying dark matter [14], and reconsiderations of astrophysics and calibration issues, e.g., the halo mass function [15–17], and mass calibration [18–22].

Here, we reinvestigate the idea of Lee 2014 [23], where it was shown that the discrepancy can be resolved if we reside in a local underdense region. The idea is particularly interesting because the local underdense region may explain the observed discrepancy in the Hubble parameter as well [24]. In our analysis, we employ a simple Friedmann-Robertson-Walker (FRW) model with an open geometry for the underdense region and investigated how deep the underdense region is in order to resolve the discrepancy using the Markov Chain Monte Carlo (MCMC) technique. We show that the joint fitting of the CMB and SZ data set *predicts* the local Hubble parameter

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value. The posterior distribution of the local Hubble parameter from the MCMC analysis is then confronted with direct local Hubble parameter measurements to check the consistency of the local void picture.

The paper is organized as follows. In the next section we describe our simple void model and our method of analysis. We present our results and discussions in Sec. III. The final section is devoted to our conclusion. The normalized Hubble parameter h is defined by $h \equiv H_0/(100\ \mathrm{km\,s^{-1}\,Mpc^{-1}})$.

II. METHOD

A. Local void model

Suppose that we live in a local (under)dense region in a background flat Friedmann-Robertson-Walker (FRW) universe. If the local region has a constant density profile, i.e., a top-hat profile, the evolution of the region is approximated by a slightly different FRW universe. The local universe and the background universe may be characterized by cosmological parameters:$(H_{\rm loc}, \Omega_{\rm loc})$ and $(H_{\rm bg}, \Omega_{\rm bg})$, respectively. The local cosmological parameters $(H_{\rm loc}, \Omega_{\rm loc})$ can differ in principle from those in the global FRW universe. Here, for simplicity, let us assume that abundance ratios of the energy components in the universe are same in the global and the local underdense regions by demanding that

$$\Omega_{\alpha,{\rm loc}} h_{\rm loc}^2 = \Omega_\alpha h_{\rm bg}^2, \tag{1}$$

where the subscript $\alpha = ({\rm v}, {\rm c}, {\rm b}, \gamma, {\rm n})$ stands for the vacuum energy (v), CDM density (c), baryon density (b), photon density (γ), and neutrino density (n). Note that we have assumed that the curvature parameter in the background FRW model is equal to zero, i.e., $\Omega_{\rm K,bg} = 0$. The local curvature parameter, $\Omega_{\rm K,loc}$ is determined by

$$\Omega_{\rm K,loc} = 1 - \sum_i \Omega_{i,{\rm loc}}, \tag{2}$$

so that the difference between $H_{\rm loc}$ and $H_{\rm bg}$ is compensated. Then, we find $\Omega_{\rm K,loc} > 0$ (< 0) for $H_{\rm loc} > H_{\rm bg}$ ($H_{\rm loc} < H_{\rm bg}$). It should be noted that the present time must be fixed so that the present CMB temperature is 2.725K. Therefore, our assumption (1) implies that, for the present uniform density time slice, the CMB temperature we observe and that in the background universe are equally given by 2.725K. Equation (1) also implies that the local void considered here is in the growing mode (see e.g., Eq. (34) in [25]).

Let us introduce a parameter $f_{\rm loc}$ as follows:

$$H_{\rm loc} = f_{\rm loc} H_{\rm bg}. \tag{3}$$

We are particularly interested in the case $f_{\rm loc} > 1$ to explain the discrepancy in the measured values of the Hubble parameter. In this case, in order to change the present time slice from the uniform density slice to the uniform Hubble slice given by $H = H_{\rm loc} > H_{\rm bg}$, we have to go back in time and take the past slice in the background universe. Consequently, in the background universe, the density on this time slice is larger than that on the uniform density time slice. This means that the density profile has a void structure in the uniform Hubble slice. This observation is consistent with the result reported in Ref. [24], which states that the local underdense region may lead to a larger value of the local Hubble parameter.[1]

In this paper, we calculate the cluster abundance based on the local cosmological parameters: $(H_{\rm loc}, \Omega_{\rm loc})$, which are fixed by $(H_{\rm bg}, \Omega_{\rm bg})$ and $f_{\rm loc}$ through Eqs. (1) and (3). In other words, we assume that the local void covers up the entire region where the Planck clusters were found, namely, $z \lesssim 1$, or in terms of the radial comoving distance, $\lesssim$3000 Mpc.

The effect of the parameter $f_{\rm loc}$ on the cluster abundance is shown in Fig. 1. The decrease in the cluster abundance with increasing $f_{\rm loc}$ mainly comes from a smaller amplitude of density fluctuations at corresponding scales due to a slower growth rate in the open FRW model than that in the flat FRW one. Following the convention, we denote the fluctuation amplitude by $\sigma_{8,{\rm loc}}$, the root-mean-squared of linear fluctuations within a top-hat sphere of $8h^{-1}$ Mpc radius. Assuming that primordial density fluctuations have the same initial power spectrum both in the underdense region and the surrounding background universe, the parameter $\sigma_{8,{\rm loc}}$ is derived from the other cosmological parameters.

For simplicity, we assume that the angular diameter distance to the last scattering surface remains unchanged. This can be justified if the density profile in the high redshift universe, say in $2 \lesssim z \lesssim 1000$, is modified to compensate for the stretch in distance by the local underdense region. In this case, the Hubble parameter $H_{\rm bg}$ is not directly observed but inferred from the CMB measurements. On the other hand, $H_{\rm loc}$ can directly be observed using local distance measurements.

[1]The correlation between the larger value of the local Hubble parameter and the local underdense region can be also understood by considering conventional perturbation theory. Usually, the initial condition for the Hubble flow of the top-hat region is set assuming that the density perturbation grows following linearized perturbation theory; in the radiation dominated era, the density contrast of region δ grows as $\delta \propto a^2$ in the synchronous gauge, where a is the cosmic scale factor. In this case, the relationship between the Hubble parameters of the top-hat region $H_{\rm loc}$ and the background universe $H_{\rm bg}$ is given by [26]

$$H_{{\rm loc},i} = H_{{\rm bg},i} - \frac{2}{3}\delta_i H_{{\rm bg},i},$$

where the subscript i denotes a specific initial time. If the top-hat region is underdense, i.e., $-1 < \delta < 0$, then $H_{{\rm loc},i} > H_{{\rm bg},i}$ and the inequality $H_{\rm loc}(t) > H_{\rm bg}(t)$ always holds true.

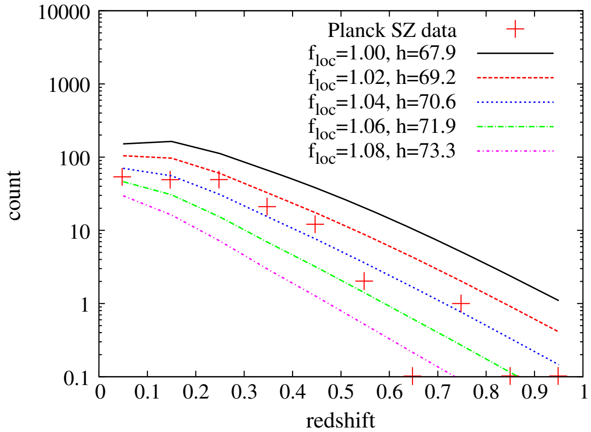

FIG. 1. SZ cluster number counts predicted in the local void model are compared with the observed number counts from the Planck 2013 result [10]. The parameter $f = f_{\rm loc}$ defined in Eq. (3) describes the difference between the local and global Hubble parameters. The parameter h here denotes the normalized local Hubble parameter value corresponding to each $f_{\rm loc}$.

B. Mass function and cluster abundance

To predict the abundance of SZ clusters, we adopt the fitting form of the mass function $dn_{\rm halo}/dM$ presented in [27]. We adopt the parameters $D_{\rm crit} = 500$ and $D_{\rm cluster} = D_{\rm crit}/\Omega_{\rm M}(z)$, where $\Omega_{\rm M}(z)$ is the total matter density over the critical density at redshift z. The cluster abundance of mass M at redshift z is given by

$$\frac{dN}{dz}(z) = f_{\rm sky}\int_0^\infty dM\chi(M)\frac{dN}{dM}(M,z)\frac{dV(z)}{dz}, \qquad (4)$$

where the comoving volume $V(z)$ is given by

$$\frac{dV(z)}{dz} = 4\pi(1+z)^2\frac{d_A(z)^2}{H(z)}, \qquad (5)$$

and the sky fraction that is covered by the Planck SZ cluster counts is fixed to $f_{\rm sky} = 0.65$. The function $\chi(M)$ represents the survey completeness as a function of halo mass M that takes account of the fact that some fraction of clusters falls out from detection due to the survey strategy, detection limit, and so on. We simply write this function as [28]

$$\chi(M) = \int_{M_{\rm min}(z)}^\infty dM' P(M'|M), \qquad (6)$$

where the function $P(M'|M)$ describes the probability that the mass is estimated as M' for a cluster with a true mass M, which for example originates from the scatter in the scaling relation between halo masses and SZ signals. In this analysis, we assumed that it follows a log-normal distribution with variance $\sigma^2_{\ln M}$ as

TABLE I. Mean minimum masses of the Planck 2013 SZ cluster sample as a function of redshift, which are taken from [10]. These masses are defined at 50% completeness on average for the unmasked sky.

z	$M_{\rm min}(z)$ $[M_\odot]$
0.05	3.12e14
0.15	6.15e14
0.25	8.20e14
0.35	9.66e14
0.45	1.06e15
0.55	1.13e15
0.65	1.17e15
0.75	1.20e15
0.85	1.21e15
0.95	1.21e15

$$P(M'|M) = \frac{M'^{-1}}{\sqrt{2\pi\sigma^2_{\ln M}}}\exp\left[-\frac{(\ln M' - \ln M)^2}{2\sigma^2_{\ln M}}\right]. \qquad (7)$$

We fix $\sigma_{\ln M} = 0.2$, which roughly reproduces the theoretical curve for the number counts of Planck SZ clusters presented in [10], as shown by the black curve in Fig. 1. The minimum masses obtained from the Planck SZ survey are listed in Table I.

A caveat is that, while we have used the Tinker mass function to compare our results with those of the Planck fiducial analysis, the fitting function is not very well tested in an open ΛCDM model which is of our main interest in this paper. We expect that the fitting function works fine even in nonflat universes as long as important ingredients of the fitting function such as the matter density, linear power spectrum, and linear growth rate are properly included, but this assumption should be carefully validated by e.g., N-body simulation.

C. MCMC

In our MCMC analysis we use both the Planck CMB data and the SZ cluster number counts. Specifically, we used a flat ΛCDM model when fitting to the CMB data with the six standard parameters, while we used an open ΛCDM model with an extra parameter $f_{\rm loc}$ to fit to the SZ cluster data; the other parameters were derived from the relations in Eqs. (3)–(2). We use a modified version of CosmoMC [29] to explore the likelihoods. For the likelihood of the cluster number counts, we assume that the number of the cluster obeys Poisson statistics [30], which is a good approximation for massive clusters as considered here [31].

III. RESULTS AND DISCUSSION

First, we show the posterior distribution for $f_{\rm loc}$ from the joint fitting of the CMB and SZ number counts in Fig. 2. As shown in the figure, the parameter $f_{\rm loc}$ tends to be $f_{\rm loc} > 1$ indicating that the model with a local underdense region fits

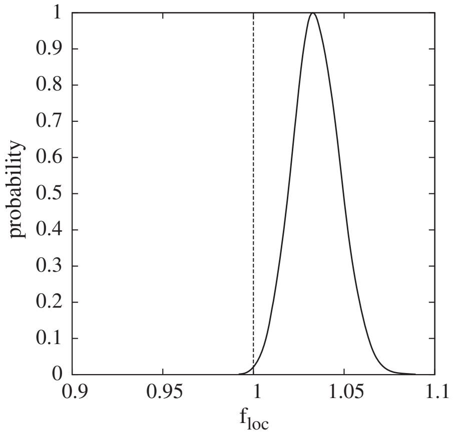

FIG. 2. One-dimensional posterior probability distribution of the parameter f_{loc} (see Eq. (3) from the joint fitting of the CMB and SZ cluster number counts. The dotted line ($f_{\mathrm{loc}} = 1$) denotes the standard cosmological model without any local over- or underdense region.

better with the data set. The result $f_{\mathrm{loc}} = 1.03 \pm 0.01245$ indicates that the standard ΛCDM model is excluded at 2.4σ level. We find the improvement of χ^2 to be $\Delta\chi^2 \approx 8.44$ with one additional parameter, which is, in terms of Akaike's information criterion, AIC ≈ 6.44. This indicates that the void model is preferable to the ΛCDM model for the combined CMB and SZ data set. The two dimensional constraint on the $f_{\mathrm{loc}} - \sigma_{8,\mathrm{loc}}$ plane is shown in the left panel of Fig. 3. It is evident that the trend for $f_{\mathrm{loc}} > 1$ is required in order to realize the smaller σ_8 as inferred from the measurements of the local universe.

An interesting relation is found in the right panel of Fig 3, which shows a positive correlation between $\sigma_{8,\mathrm{loc}}$ and H_{loc}. This correlation is nontrivial because a larger f_{loc} leads a smaller $\sigma_{8,\mathrm{loc}}$ but a larger H_{loc} if the other cosmological parameters are fixed. We find that this correlation is attributed to the well-known σ_8-Ω_m degeneracy in the cluster abundance. In order to produce the same cluster abundance, a larger $\sigma_{8,\mathrm{loc}}$ requires a smaller $\Omega_{m,\mathrm{loc}}$, hence a larger H_{loc} because $\Omega_{m,\mathrm{loc}} h_{\mathrm{loc}}^2 = \Omega_m h_{\mathrm{bg}}^2$ is tightly constrained from the CMB.

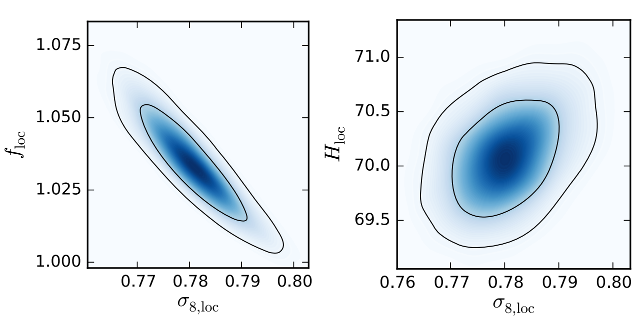

FIG. 3. Posterior probability distribution on the $f_{\mathrm{loc}} - \sigma_{8,\mathrm{loc}}$ (left) and $H_{\mathrm{loc}} - \sigma_{8,\mathrm{loc}}$ (right) planes.

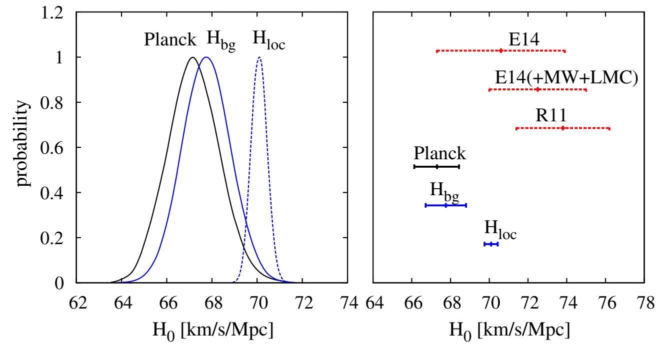

FIG. 4. Left: Probability distribution functions for the Hubble parameter from Planck (black solid line), and H_{bg} (blue solid) and H_{loc} (blue dashed) from the combined CMB and SZ data set. Right: Constraints on the Hubble parameter are compared with the direct local estimates for the Hubble constant by Riess *et al.* (R11), and by Efstathiou using NGC 4258 (E14) and plus other distance anchors (E14 + MW + LMC). The black point with an error bar shows the constraint from Planck CMB. In the local void model, Hubble parameters are separated into local (H_{loc}) and global (H_{bg}) parameters, as shown in the figure.

A key feature of this local void model is that the local Hubble parameter defined in Eq. (3) is automatically adjusted as a result of fitting to the observed SZ cluster number counts. Figure 4 shows the posterior probability distributions of the Hubble parameters with and without the void parameter f_{loc}, together with the local Hubble parameter measurements from Riess *et al.* [13] and Efstathiou [32]. The main results are summarized as follows. By keeping the void parameter at the standard ΛCDM value ($f_{\mathrm{loc}} = 1$), the Planck CMB gives $H_0 = 67.3 \pm 1.2$ km s^{-1} Mpc^{-1} whilst the CMB and SZ combination gives $H_0 = 70.7 \pm 0.3$ km s^{-1} Mpc^{-1}, indicating the disparity between these two data sets. We find that in the current void model, the local Hubble parameter is predicted by the joint fitting of the CMB and SZ cluster number counts as $H_{\mathrm{loc}} = 100 h_{\mathrm{loc}} = 70.1 \pm 0.34$ km s^{-1} Mpc^{-1}. This value is in better agreement with the local Hubble measurements by Riess *et al.* [13], and interestingly, is in excellent agreement with the value $H_0 = 70.6 \pm 3.3$ km s^{-1} Mpc^{-1} recently derived by Efstathiou using the updated geometric maser distance to NGC4258 [32].

There are several other possibilities to resolve the discrepancy. For example it may be possible that the mass bias factor is significantly lower than the inferred value, i.e., masses of clusters observed by Planck SZ is in fact larger than those inferred by the scaling relation, and therefore, they are less abundant in the universe [18]. Since the publishing of their results in 2013, the Planck collaboration has updated their bias calibration using cosmic shear and CMB lensing measurements, and their 2015 estimates give similar mass bias values to that in the baseline model assumed in 2013 [33]. The results of cluster number counts in 2013 and 2015 are consistent with each other if the same mass bias is assumed. Independent analysis also shows that the mass bias cannot fully resolve the discrepancy [34–36]. Recently, an important update has been made in Battaglia *et al.* [22]. They found that an Eddington bias correction, if it is applied to the weak lensing mass calibration analysis of the Planck SZ clusters, brings the mass bias factors measured in von der Linden *et al.* [18] and Hoekstra *et al.* [36] closer to the values that are small enough to explain both the Planck CMB and SZ results. Further analysis of this systematic uncertainty is of primary importance for understanding the origin of the discrepancy.

From a cosmological point of view, while massive neutrinos may be an interesting possibility [4,5], a larger neutrino mass generally demands a smaller Hubble parameter, which makes the discrepancy between the global and local Hubble parameters even worse [37]. Other possibilities include modification of the mass function due to baryonic effects [15–17], and decaying dark matter [14]. Non-Gaussianity may also explain the discrepancy, although the Planck bispectrum already put severe constraints [38], making this explanation difficult as long as the non-Gaussianity is scale-independent [39].

Our current study needs to be advanced in several directions. The large bulk flow associated with the local void generates secondary CMB anisotropies through the kinetic Sunyaev-Zeldovich effect (kSZ). Here we give a rough estimate of the kSZ effect arising from the local void following Moss *et al.* [40]. Suppose that the dipole induced by the radial bulk velocity is the dominant anisotropy seen by the scatterer (i.e., clusters). We estimate the amplitude of this dipole from the difference in redshift between incoming and outgoing CMB photons felt for the same period in conformal time. The dipole anisotropy at clusters is then given by

$$\frac{\Delta T}{T} \approx 1 - \frac{R(\eta_*)}{a(\eta_*)}, \tag{8}$$

where $a(\eta_*)$ and $R(\eta_*)$ denote the scale factors of the background and void regions when the incoming photon enters into the void, and follow the Friedmann equations in flat and open ΛCDM models, respectively. For our void model that gives $H_{\rm loc} = 70$, we obtain $\Delta T/T \approx 7 \times 10^{-4}$ for $a(\eta_*) \approx 0.01$ and $\Delta T/T \approx 9 \times 10^{-3}$ for $a(\eta_*) \approx 0.1$. Therefore, a fairly large void seems necessary to avoid the constraint from peculiar velocities measured by the Planck collaboration, which gives $\Delta T/T \lesssim 6.4 \times 10^{-4}$ (2σ) [41]. These numbers above are order of magnitude estimates; further detailed investigation would be needed for a more quantitative constraint by using concrete models of spherically symmetric inhomogeneous universes, e.g., generalized Λ-Lemaître-Tolman-Bondi dust models [42–44]. The effect on other probes such as baryon acoustic oscillation and cosmic shear measurements should also be investigated in order to see whether the local void is a viable cosmological model.

This study is based on the Planck 2013 results. Interestingly, the discrepancy persists in the Planck 2015 results [33] and therefore our results should still be as valid for the latest data set.

IV. CONCLUSION

In this paper, we have considered a simple FRW model with an open geometry to investigate the magnitude of the effect of the local underdense region on the number of SZ clusters. We have shown that a local underdense region may be responsible for the discrepancy between cosmological parameters from SZ cluster number counts and the CMB. If we attribute the discrepancy to the local void, the local Hubble parameter is predicted as $H_0 = 70.1 \pm 0.34\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, which is in better agreement with direct local Hubble parameter measurements. Our result indicates that the local void model may serve as a new concordance model that explains the CMB, SZ cluster number counts, and local Hubble parameter measurements simultaneously.

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