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Charged lepton flavor violation on target at GeV scale

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We study the lepton flavor violating process, $e + T \to \tau + T'$, at a few GeV. This process can be studied by experiments directing GeV scale electron or positron beams on internal or fixed targets. We study the effects of some low energy lepton flavor violating interactions on this process. We study the sensitivities of this process on these low energy lepton flavor violating interactions and compare them to the sensitivities of lepton flavor violating τ decay processes. Comparing with τ decay processes, this process provides another way to study the lepton flavor violating effects with $e - \tau$ conversion and it can be searched for in facilities with GeV scale electron or positron beams which are available in a number of laboratories in the world.

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I. INTRODUCTION

As very important possible evidence towards the physics beyond the standard model (SM), processes of charged lepton flavor violation (cLFV) have attracted a lot of attention. This is partly because flavor violation of neutral leptons has been observed in oscillation of neutrinos. It raises the hope that some flavor violating effects of charged leptons, although small but not undetectable, might also appear, if the dynamics underlying the neutrino flavor does not treat the flavor of charged leptons badly unfair. Flavor violating decay processes of muon, such as $\mu \to e\gamma$, $\mu \to 3e$, have been intensively studied by researchers and have been probed to high precision, thanks to the intense muon source available in experiment. They are expected to be further probed to a very high precision by future experiment with muon source improved by orders of magnitude.

Flavor violation related to τ lepton has not been investigated with fever as for muon. Current upper bound to processes $\tau \to e\gamma$, $\tau \to 3e$, $\tau \to \mu\gamma$, $\tau \to \mu e^+ e^-$ etc., are many orders of magnitude weaker than those similar processes of μ decay. The major difference is that τ source is much harder to prepare than the μ source, because of its larger mass and much smaller lifetime. If high luminosity τ-charm factory or Z factory could be built, these τ decay processes can be investigated to much higher precision. Before these possible facilities, we are also allowed to probe lepton flavor violating effect associated with τ lepton by considering inverse processes with electron or positron beam on target. The experiment can be done with electron or positron beam directed on a fixed target or internal target. The typical processes to observe are $e^\pm + T \to \tau^\pm + T'$ where T is the target nucleon or nucleus and T' represent all final particles in the nuclear part of the process. This kind of experiment can be done for electron/positron beam with several GeV. Such kinds of GeV scale electron/positron beams are available at facilities such as BEPCII, SLAC, JLAB etc.

In this article we are going to investigate this kind of cLFV processes with electron or positron on target and study the sensitivity of these kinds of processes on the lepton flavor violating interaction associated with τ lepton. In the next section we first discuss the general kinematics of the process under discussion. Then we concentrate on several processes with typical final states T'. One process to discuss is the elastic scattering with $T' = T$. The other one is the quasielastic scattering in which a nucleon is kicked out of nucleus by elastic scattering with lepton. We discuss the sensitivity of these two typical processes on two types of lepton flavor violating interactions, namely the $e\tau\gamma$ vertex and $e\tau Z$ vertex. Throughout this article we concentrate on $e - \tau$ conversion using electron or positron beam. One can simply apply the discussion in this article to $\mu - \tau$ conversion for appropriate muon beam if it is available.

II. KINEMATICS IN SCATTERING OF $e - \tau$ CONVERSION

We consider production of τ lepton in scattering of electron or positron with target T

$$e^\mp(k) + T(P) \to \tau^\mp(k') + T'(P'), \tag{1}$$

where k, k', P are four-momenta of corresponding particles and P' is the total momentum of the final product, T', which represents all final particles in the target part of the process. The initial target T can be nucleon or some heavy nuclei. In general, T' can be a complicated product. We are not going to discuss very complicated cases in later sections. Instead, we concentrate on some simple processes with $e - \tau$ conversion: (i) elastic scattering (ES) with $T' = T$; (ii) quasielastic (QE) scattering in which a nucleon is kicked out of a nucleus by elastic scattering with $e^\mp$, so that T breaks into several pieces but without a change in the number of nucleon. Two other important processes for scattering at a few GeV scale are (i) inelastic scattering with T' being the

TABLE I. Threshold energy of $e + T \to \tau + T'$ process for a few target nuclei.

Target	proton	deteuron	Helium-3	Helium-4	Lithium-7
Threshold E(GeV)	3.44	2.61	2.33	2.19	2.01

excited state of T; (ii) process with pion production. These two processes are more or less related to the topics in this article but will not be discussed in detail.

Introducing $q = P' - P = k - k'$ and $Q^2 = -q^2$, we can find that

$$Q^2 = -(k - k')^2 = 2k \cdot k' - m_\tau^2, \quad (2)$$

where electron mass has been neglected in comparison with the appearance of m_τ, the mass of τ lepton. Throughout this article, we always neglect the electron mass in our result. For target T at rest, we have $P \cdot q = m_T(E - E') = m_T(E_{T'} - E_T)$. m_T is the mass of the target T. E, E', E_T and $E_{T'}$ are the energies of e, τ, T and T′ separately.

From $P'^2 = (q + P)^2$ we can get

$$\begin{aligned} Q^2 &= -q^2 = 2P \cdot q - (m_{T'}^2 - m_T^2) \\ &= 2m_T(E - E') - (m_{T'}^2 - m_T^2), \end{aligned} \quad (3)$$

where $m_{T'}$ is the invariant mass of T': $m_{T'}^2 = P'^2$.

Using the scattering angle $\theta_{k'}$, the angle between the direction of $\vec{k}'$ and the direction of the incoming electron or positron beam, E' can be solved as illustrated in Appendix A. From (A12) one can see that for $e + T \to \tau + T'$ process to happen the following condition should be satisfied

$$(m_\tau^2 - m_{T'}^2 + m_T^2 + 2m_T E)^2 - 4m_\tau^2(2Em_T + m_T^2) > 0. \quad (4)$$

One can find from (4) that the initial energy should satisfy

$$E > m_\tau \frac{m_{T'}}{m_T} + \frac{1}{2m_T}(m_\tau^2 + m_{T'}^2 - m_T^2). \quad (5)$$

For elastic scattering, we have $T' = T$ and $m_{T'} = m_T$. We can find

$$Q^2 = 2m_T(E - E'), \quad (6)$$

and the threshold condition becomes

$$E > m_\tau + \frac{m_\tau^2}{2m_T}. \quad (7)$$

Apparently condition (7) is weaker than (5), and is the threshold energy condition for this lepton flavor violating process. In Table I we can see numerically, with a few possible targets, this requirement on the beam energy for experiments with this lepton flavor violating process.

Q^2 versus the scattering angle $\theta_{k'}$ in the elastic case can be obtained using (6) and the solution of E' in (A4). In Fig. 1 we show this dependence numerically for $E = 2.5$ GeV and $E = 7$ GeV respectively. One can see that the scattered direction of the τ lepton is limited to a range close to the forward direction due to a kinematic constraint, as discussed in detail in Appendix A, namely (A6). One can see in Fig. 2 that this range of $\theta_{k'}$, although not very large, can still reach $\sin\theta_{k'} \approx 0.2$–$0.3$ for $E = 4$–6 GeV which is fairly large. So there are rooms detecting the final τ lepton even for a detector having problems detecting final particles moving close to the forward direction.

One thing to notice in this process is that due to the unavoidable momentum loss to target in $e \to \tau$ conversion, a minimal amount of energy has to be transferred to the nuclear sector and Q^2 has nonzero minimum in the scattering. In Table II one can see $Q^2_{\rm min}$, the minimum of Q^2, for various nuclei and beam energies. The larger the energy or heavier the target nucleus, the smaller the $Q^2_{\rm min}$ can reach. For a fixed E, $Q^2_{\rm min}$ approaches a constant as the target mass increases, but does not approach zero. For larger energy, the situation would approach the case in which massless limit applies. That is why $Q^2_{\rm min}$ turns to be closer to zero for larger E.

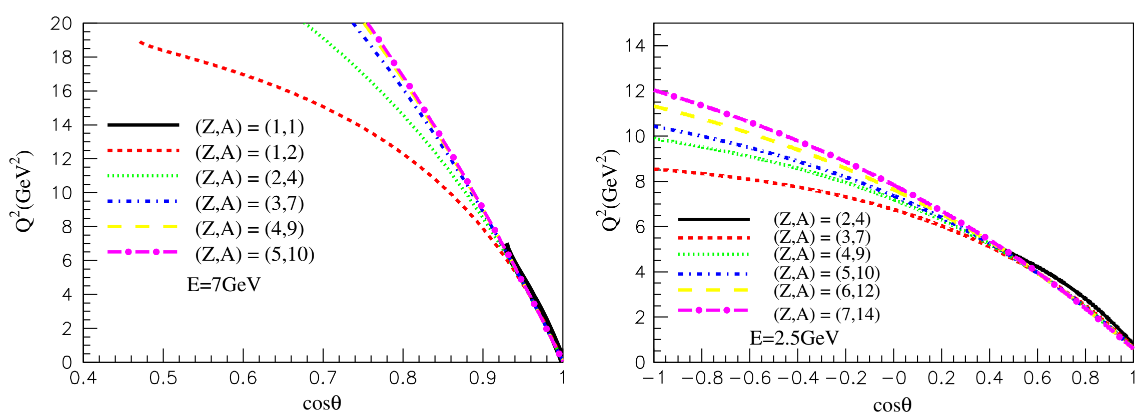

FIG. 1. Q^2 versus $\cos\theta_{k'}$ for elastic scattering $e + T \to \tau + T$.

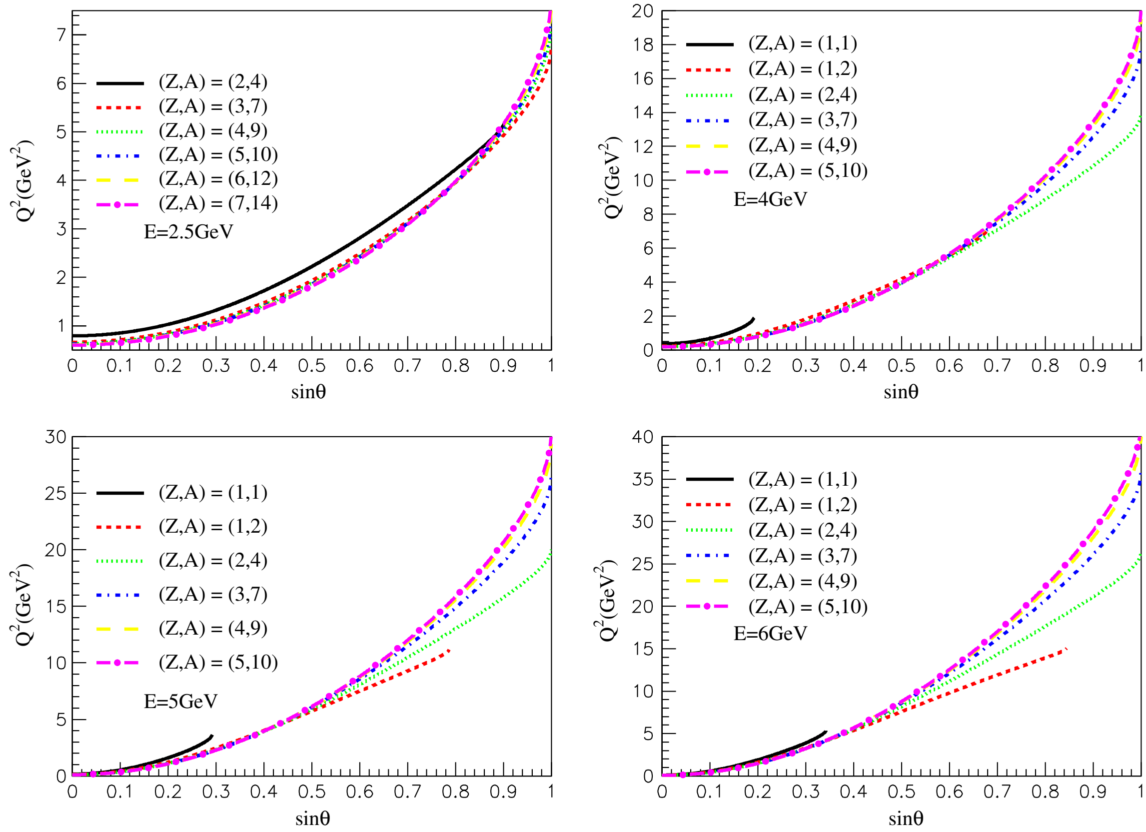

FIG. 2. Q^2 versus $\sin\theta_{k'}$ for elastic scattering $e + T \to \tau + T$.

In quasielastic scattering, the electron/positron is considered to scatter elastically with a nucleon in a nucleus and kicks this nucleon out of the target nucleus. The target nucleus can break into several pieces due to this scattering, but there is no extra nucleon or other hadron produced in this process. If the energy and energy transfer are large enough, the scattering cross section of this QE process can be computed using the impulse approximation in which the initial nucleon is considered at rest inside the nucleus, and outgoing nucleon and the incoming and outgoing leptons are all approximated as plane waves. This approximation greatly simplifies the calculation of this process. In this article we adopt this approximation for QE scattering.

For inelastic scattering, i.e., when T' is an excited state of nucleus, the situation is a bit complicated than ES or QE processes. The discussion would depend on the details of the excited states of various nuclei and results associated with them have to be discussed specifically. Moreover, it depends on the detail of the detector in the experiment, i.e., whether a specific excited state of a nucleus could be detected. More importantly, we expect that using inelastic scattering would not improve very much the sensitivity to $e - \tau$ lepton flavor violating interactions. Therefore, we are not going to elaborate them in the present article.

Production of pion or other heavier particles in the neutral current processes with energy of a few GeV and Q^2 of a few GeV^2 are usually not dominant, comparing with the neutral current elastic scattering with nucleon $e + N \to e + N$. So, considering the processes with production of pion or other hadron with $e - \tau$ lepton flavor conversion are not going to improve a lot the sensitivity on $e - \tau$ lepton flavor violating interactions discussed in this article. Therefore, we are not going to elaborate on these

TABLE II. Minimal Q^2 for elastic scattering $e + T \to \tau + T$.

Target(Z,A)	(1,1)	(1,2)	(2,4)	(3,7)	(4,9)	(5,10)
Q^2_{min}(GeV^2) (E = 2.5 GeV)	×	×	0.79	0.66	0.63	0.62
Q^2_{min}(GeV^2) (E = 4 GeV)	0.37	0.23	0.20	0.19	0.18	0.18
Q^2_{min}(GeV^2) (E = 5 GeV)	0.17	0.13	0.12	0.11	0.11	0.11
Q^2_{min}(GeV^2) (E = 6 GeV)	0.10	0.085	0.078	0.076	0.075	0.075
Q^2_{min}(GeV^2) (E = 7 GeV)	0.071	0.060	0.056	0.055	0.054	0.054

kind processes and leave more detailed researches on them to future works.

III. PROBING $e\tau\gamma$ INTERACTION

As an important example of lepton flavor violating interaction, $e-\tau$ conversion induced by the interaction with a photon should be studied. In this section we study the sensitivity of lepton flavor violating ES and QE scattering processes on the $e\tau\gamma$ interaction vertex.

The effective Lagrangian governing the $e\tau\gamma$ interaction vertex can be written as

$$\Delta L = ec_L\bar{\tau}_R i\sigma^{\mu\nu}e_L F_{\mu\nu} + ec_R\bar{\tau}_L i\sigma^{\mu\nu}e_R F_{\mu\nu} + \text{H.c.}, \quad (8)$$

where $c_{L,R}$ are coupling strengths with dimension -1 for left-handed and right-handed electron field operators, respectively. $F_{\mu\nu}$ is the electromagnetic field strength tensor. From (8) we can get the branching ratio of the $\tau\to e\gamma$ decay

$$Br(\tau\to e\gamma) = \tau_\tau\alpha(|c_L|^2+|c_R|^2)m_\tau^3, \quad (9)$$

where τ_τ is the lifetime of τ and α the fine structure constant. For the present upper bound $\mathrm{Br}(\tau\to e\gamma) < 3.3\times 10^{-8}$ [1] we have

$$|c_L|^2+|c_R|^2 < 1.8\times 10^{-18}\ \text{GeV}^{-2}. \quad (10)$$

For one photon exchange processes, one can write the cross section of $e+T\to\tau+T'$ scattering as follows

$$\frac{d\sigma}{dQ^2} = \frac{\pi Z^2\alpha^2}{Q^4E^2}W_{\mu\nu}L^{\mu\nu} \quad (11)$$

where $W_{\mu\nu}$ is the nuclear(hadronic) tensor and $L^{\mu\nu}$ the leptonic tensor. The nuclear tensor can be parametrized in general as [2–4]

$$W_{\mu\nu} = -(\eta_{\mu\nu}-q_\mu q_\nu/q^2)W_1 + \frac{1}{m_T^2}(P_\mu - q_\mu P\cdot q/q^2)(P_\nu - q_\nu P\cdot q/q^2)W_2. \quad (12)$$

The nuclear tensor obeys $W_{\mu\nu}q^\nu = q^\mu W_{\mu\nu} = 0$ which results from the current conservation. The nuclear tensor is common to one photon exchange processes in which final states in leptonic part can be different but final T' is the same. So $W_{\mu\nu}$ can be measured in electron scattering processes $e+T\to e+T'$.

For $e+T\to e+T'$ scattering, the leptonic tensor is the familiar form

$$L^{\mu\nu} = 2(k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k'\eta^{\mu\nu}). \quad (13)$$

For $e+T\to\tau+T'$ scattering, we can get from (8) the leptonic tensor as

$$L^{\mu\nu} = -2(|c_L|^2+|c_R|^2)[m_\tau^2(m_\tau^2-q^2)(\eta^{\mu\nu}-q^\mu q^\nu/q^2) + 4q^2(k^\mu - q^\mu k\cdot q/q^2)(k^\nu - q^\nu k\cdot q/q^2)], \quad (14)$$

where $q=k-k'$ and $q^2=-Q^2$ as given in the last section. The antisymmetric part of the leptonic tensor does not contribute to the final result and has been neglected in (14).

For elastic scattering in which $T'=T$, the nuclear tensor can be written as

$$\text{spin } 0:\ W_1 = 0,\qquad W_2 = |F(Q^2)|^2, \quad (15)$$

$$\text{spin}\,\frac{1}{2}:\ W_1 = \frac{Q^2}{4m_T^2}(F_1+F_2)^2,$$
$$W_2 = F_1^2 + \frac{Q^2}{4m_T^2}F_2^2. \quad (16)$$

$F(Q^2)$ is the charge form factor. F_1 and F_2 are the Dirac and Pauli form factors respectively. For nucleon, the electromagnetic form factors are familiar to us and are given in Appendix B.

Using (14), (15), and (16), the cross section for scattering $e+T\to\tau+T$ induced by (8) can be found as

$$\frac{d\sigma}{dQ^2} = \frac{2\pi Z^2\alpha^2}{E^2Q^4}(|c_L|^2+|c_R|^2)\frac{|F(Q^2)|^2}{m_T^2}[4Q^2(P\cdot k)^2 + (Q^2+m_\tau^2)(P\cdot q)^2 - (Q^2+m_\tau^2)(4P\cdot qP\cdot k + m_T^2m_\tau^2)] \quad (17)$$

for T of spin 0, and

$$\frac{d\sigma}{dQ^2} = \frac{2\pi Z^2\alpha^2}{E^2Q^4}(|c_L|^2+|c_R|^2)\{W_1(Q^2+m_\tau^2)(2m_\tau^2-Q^2) + \frac{W_2}{m_T^2}[4Q^2(P\cdot k)^2 + (Q^2+m_\tau^2)(P\cdot q)^2 - (Q^2+m_\tau^2)(4P\cdot qP\cdot k + m_T^2m_\tau^2)]\} \quad (18)$$

for T of spin $\frac{1}{2}$. Z in (17) and (18) is the atomic number of nucleus for scattering with nucleus.

For elastic scattering with proton we just need to take $m_T=m_p$ and $Z=1$ in Eq. (18). In Fig. 3 we show numerically the differential cross section versus Q^2. In the plot we take $|c_L|^2+|c_R|^2$ equal to the upper bound given in (10). One can see that the cross section is very small. It is suppressed not only by $|c_L|^2+|c_R|^2$ which is of order 10^{-8} in terms of $G_F^2m_\tau^2$, the square of the Fermi constant times the mass of τ lepton, but also suppressed by the appearance of α^2 which gives a further suppression of order 10^{-4}. The total cross sections for curves of $E=4$ GeV and $E=6$ GeV in Fig. 3 are about 1.5×10^{-10} fb and 5.4×10^{-10} fb respectively. So it is very hard to probe the lepton flavor violating $e\tau\gamma$ interaction vertex to a good sensitivity using $e+N\to\tau+N$ scattering with energy of a few GeV.

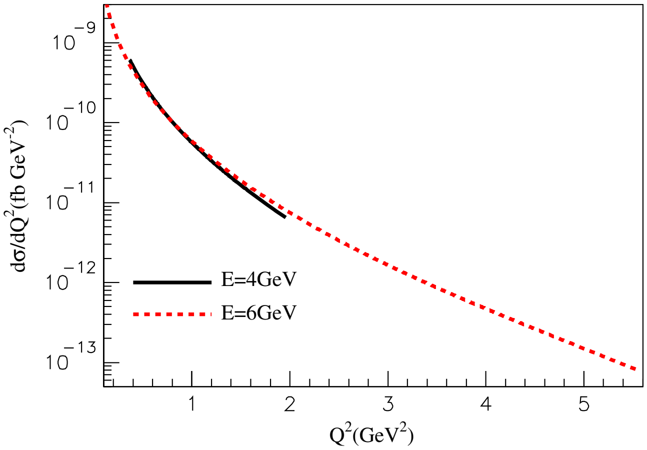

FIG. 3. Differential cross section of $e + p \to \tau + p$ versus Q^2 induced by lepton flavor violating $e\tau\gamma$ interaction.

One way to enhance the sensitivity is to use heavy nuclei so that $Z^2 \sim 10$–100. However, the size of nuclei is of fm scale. As a consequence, the nuclear form factors in elastic scattering, which reflect how much nuclei are like point particles, drop down quickly at the scale of a few fm^{-1}. For example, the form factors of light nuclei drop down by two to three orders of magnitude at scale 3–4 fm^{-1} [5] which corresponds to $Q^2 \approx 0.3$–$0.6\ \mathrm{GeV}^2$. So there is only a very small region of phase space, namely region with $Q^2 \ll 1\ \mathrm{GeV}^2$, that the cross section can potentially be enhanced by Z^2. Moreover, we have learned from Table II that Q^2 is limited by the appearance of the massive τ lepton in the final state and is not allowed to be arbitrarily close to zero for a fixed energy. This is, in particular, for the cases of small energies. So we do not expect that using heavy nuclei would help much in improving the sensitivity on the lepton flavor violating $e\tau\gamma$ interaction.

The quasielastic scattering with nucleus is basically considered as the elastic scattering with nucleons in the nucleus, as explained in the last section. So this process is not going to improve the sensitivity on the lepton flavor violating $e\tau\gamma$ interaction vertex either, for exact the same reason for elastic scattering with free protons discussed above.

IV. PROBING $e\tau Z$ INTERACTION

As another example of lepton flavor violating interaction, $e - \tau$ conversion induced by the interaction with a Z boson is also important. In this section we study the sensitivity of lepton flavor violating ES and QE scattering processes on the $e\tau Z$ interaction vertex.

The lepton flavor violating $e\tau Z$ interaction vertex can be written as

$$\Delta L = \frac{g}{\cos\theta_w}[C_L Z^\mu \bar{\tau}_L \gamma_\mu e_L + C_R Z^\mu \bar{\tau}_R \gamma_\mu e_R + \mathrm{H.c.}] \tag{19}$$

where $C_{L,R}$ are the coupling strength, normalized in unit of $g/\cos\theta_W$, the SM coupling of Z boson with the associated current. The decay rate of the $Z \to e^\pm\tau^\mp$ decay can be calculated using (19)

$$\Gamma(Z \to e^\pm\tau^\mp) = \frac{g^2 m_Z}{12\pi\cos^2\theta_w}(|C_L|^2 + |C_R|^2), \tag{20}$$

where m_Z is the mass of Z boson. Comparing with the SM coupling for ν, i.e., $C_\nu = 1/2$, we can find that

$$\frac{|C_L|^2 + |C_R|^2}{|C_\nu|^2} = \frac{3}{2}\frac{Br(Z \to e^\pm\tau^\mp)}{Br(Z \to \mathrm{invisible})}.$$

For the present upper bound $\mathrm{Br}(Z \to e^\pm\tau^\mp) < 9.8 \times 10^{-6}$ [1,6], we find

$$|C_L|^2 + |C_R|^2 < 7.35 \times 10^{-5}|C_\nu|^2. \tag{21}$$

Another important constraint on (19) comes from the $\tau \to 3e$ decay. One can find that

$$\begin{aligned} Br(\tau \to 3e) &= \tau_\tau \frac{G_F^2 m_\tau^5}{96\pi^3}(|C_L|^2 + |C_R|^2)\left[\left(-\frac{1}{2} + \sin^2\theta_W\right)^2 + \sin^4\theta_W\right] \\ &= 0.125\tau_\tau \frac{G_F^2 m_\tau^5}{96\pi^3}(|C_L|^2 + |C_R|^2), \end{aligned} \tag{22}$$

where G_F is the Fermi constant. For the present upper bound $\mathrm{Br}(\tau \to 3e) < 2.7 \times 10^{-8}$ [1], we get

$$|C_L|^2 + |C_R|^2 < 6 \times 10^{-7} \tag{23}$$

(23) is stronger than (21), the bound from $Z \to e^\pm\tau^\mp$ decay, by about a factor 30. Other constraints, such as those from $\tau \to e + \pi^+ + \pi^-$, $\tau \to e + \mu^+ + \mu^-$ etc., are of the same order of magnitude as the $\tau \to 3e$ constraint and we are not going to elaborate on all of them in this article.

For one Z exchange, one can write the cross section of $e + T \to \tau + T'$ scattering as follows

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2}{32\pi E^2} W_{\mu\nu} L^{\mu\nu} \tag{24}$$

where $W_{\mu\nu}$ is the nuclear(hadronic) tensor and $L^{\mu\nu}$ is the leptonic tensor. In general $W_{\mu\nu}$ can be written as

$$\begin{aligned} W_{\mu\nu} = &-\eta_{\mu\nu} W_1 + \frac{P_\mu P_\nu}{m_T^2} W_2 + \frac{i\epsilon_{\mu\nu\rho\sigma} P^\rho q^\sigma}{2m_T^2} W_3 + \frac{q_\mu q_\nu}{m_T^2} W_4 \\ &+ \frac{P_\mu q_\nu + P_\nu q_\mu}{2m_T^2} W_5 + i\frac{P_\mu q_\nu - P_\nu q_\mu}{2m_T^2} W_6. \end{aligned} \tag{25}$$

This nuclear tensor is common to one Z exchange processes with the same T' irrespective of the leptonic part. In particular, $W_{1,2,3}$ appear in the $\nu + T \to \nu + T'$ scattering and can be measured using the scattering with neutrino.

In our convention, the leptonic tensor $L^{\mu\nu}$ of $e + T \to \tau + T'$ scattering for unpolarized electron or positron beam due to the $e\tau Z$ interaction (19) is

$$L^{\mu\nu} = 16[(k^\mu k'^\nu + k^\nu k'^\mu - k \cdot k' \eta^{\mu\nu})(|C_L|^2 + |C_R|^2) + i\varepsilon^{\mu\nu\rho\sigma} k_\rho q_\sigma (|C_L|^2 - |C_R|^2)]. \qquad (26)$$

As a comparison, the leptonic tensor for $\nu + T \to \nu + T'$ is

$$L^{\mu\nu} = 8(k^\mu k'^\nu + k^\nu k'^\mu - k \cdot k' \eta^{\mu\nu} + i\varepsilon^{\mu\nu\rho\sigma} k_\rho q_\sigma). \qquad (27)$$

Inserting (27) into the expression of cross section gives an expression depending only on $W_{1,2,3}$. Contributions of $W_{4,5}$ vanish in the limit of massless neutrino. For $e + T \to \tau + T'$ scattering, the mass of the τ lepton cannot be neglected in general and terms with $W_{4,5}$ should be included. However, there is no experimental data in principle to extract $W_{4,5}$ as for $W_{1,2,3}$ in the scattering with neutrino. We should rely on the theoretical prediction. Fortunately, for ES scattering with nucleon and the QE scattering, $W_{1,2,3,4,5}$ can all be expressed using the form factors of nucleons which are known quite well. They are discussed in Appendix C.

Now we consider the elastic scattering with nucleon $e + N \to \tau + N$. For unpolarized electron or positron beam, the cross section of $e^\mp + N \to \tau^\mp + N$ elastic scattering is

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2}{2\pi E^2}\left\{(|C_L|^2 + |C_R|^2)\left[(m_\tau^2 + Q^2)\left(W_1 - \frac{1}{2}W_2\right) + \frac{P \cdot k}{m_N^2}(2P \cdot k - Q^2)W_2 + \frac{1}{2}(m_\tau^2 + Q^2)\frac{m_\tau^2}{m_N^2}W_4 - \frac{m_\tau^2}{m_N^2}P \cdot k W_5\right] \pm (|C_L|^2 - |C_R|^2) \times \frac{Q^2}{4m_N^2}(4P \cdot k - Q^2 - m_\tau^2)W_3\right\} \qquad (28)$$

$W_i (i = 1,2,3,4,5)$ are given by (C20), (C21), (C22), (C23) and (C24).

In Fig. 4 we can see numerically the differential cross section of $e^\mp + N \to \tau^\mp + N$ elastic scattering versus Q^2 induced by $e\tau Z$ interaction in (19). In these plots we have set $C_R = 0$ and the upper bound value in (23) has been used for C_L. In this case with $C_R = 0$, the cross section for

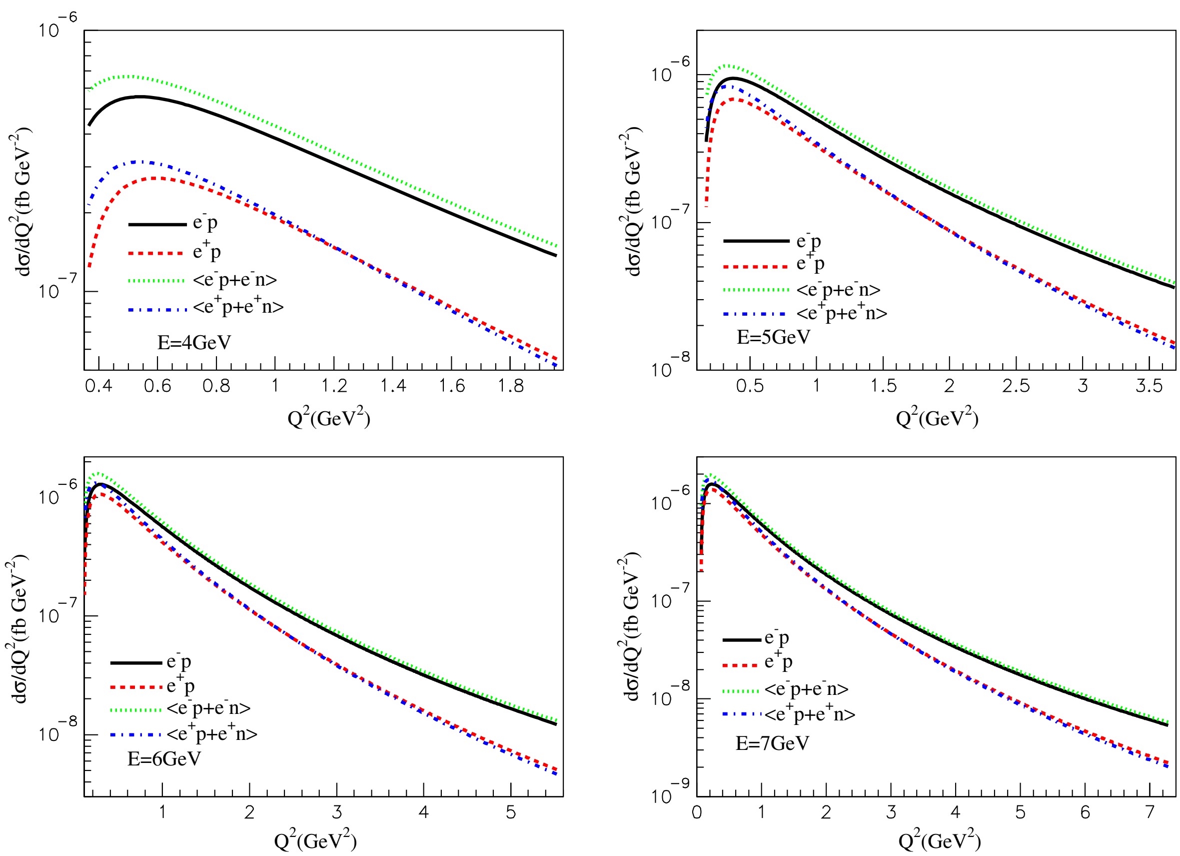

FIG. 4. Differential cross section of $e + N \to \tau + N$ versus Q^2 induced by lepton flavor violating $e\tau Z$ interaction. $e^\pm p$ lines are for scattering with proton and $\langle e^\pm p + e^\pm n \rangle$ lines are the averages of cross sections of the scattering with proton and neutron.

electron beam is larger than the corresponding cross section for positron beam. If $C_L = 0$, the cross section for positron beam is larger. The curves for $\langle ep + en\rangle$ are the averages of cross sections for proton and neutron targets. They are the average cross sections of scattering with nucleon for isoscalar target. We can see that the cross section is not that small as for the $e-\tau$ conversion induced by interaction with photon. For energy of 4–7 GeV as shown in Fig. 4, the total cross sections for e^-p, e^+p, $\langle e^-p + e^-n\rangle$ and $\langle e^+p + e^+n\rangle$ vary in ranges $(0.63-1.6)\times 10^{-6}$ fb, $(0.28-1.2)\times 10^{-6}$ fb, $(0.71-1.8)\times 10^{-6}$ fb, and $(0.30-1.4)\times 10^{-6}$ fb separately. So for this process to reach the sensitivity of $\tau \to 3e$ decay, the luminosity of experiment should reach 1000 ab^{-1}. On the other hand, to reach the sensitivity of $Z \to e^{\pm}\tau^{\mp}$, as shown in (21), the luminosity should reach around 30 ab^{-1}.

For quasielastic scattering with nuclei, nucleons are kicked out of the nuclei by elastic scattering with the initial electron or positron. The cross section can be calculated using that of scattering with free nucleon $e^{\mp} + N \to \tau^{\mp} + N$. Nuclear corrections to the free nucleon approximation have been studied, for example, for the quasielastic scattering of neutrino with nuclei [7], which is called neutral current elastic (NCE) scattering in the neutrino physics community. The results show that for energy of a few GeV, and for Q^2 not very close to zero, the free nucleon approximation works perfectly. So it is fairly good to adopt this approximation when we estimate the sensitivity of probing lepton flavor violating $e\tau Z$ interaction. It is certainly true that we should take these nuclear corrections into account if we want more detailed understanding. But this detailed work should be done for those matter selected to use as targets in experiments. We are not going to elaborate on this topic in this article and leave it to future research.

The signals of the lepton flavor violating processes discussed above are simple and easy to distinguish. In addition to a τ lepton, final particles of the processes include a free nucleon in the case of elastic scattering, or several pieces of the broken nuclei in the case of quasielastic scattering. If the energy of the τ lepton can be reconstructed to a good precision, there should be no missing energy in these final products except that arising from the τ decay.

Processes with production of heavy hadrons by strong interaction, e.g., $e + N \to e + N + D^+ + D^-$ or $e + N \to e + N + D_s^+ + D_s^-$, can potentially give τ lepton in decays of these hadrons. So in principle they can mimic the lepton flavor violating process $e + N \to \tau + N$ discussed in this article if the scattered electron or positron and the decay products other than τ lepton all escape the detection of detector. A great virtue of considering $e + N \to \tau + N$ scattering at a few GeV is that we can avoid these potential backgrounds. One can show that a threshold condition similar to (7) holds for the production of a pair of D mesons or D_s mesons. Consequently, the energy threshold is more than 4 times the threshold for $e + N \to \tau + N$ scattering shown in Table I. By comparing with Table I, we can see that this potential background would disappear if considering experiments with beam energy < 8 GeV. For the same reason, processes with production of a pair of τ lepton by neutral current weak interaction can also be avoided if considering experiments with beam energy < 8 GeV.

A relevant process for background consideration is the process with production of a pair of $(\tau^-, \bar{\nu}_\tau)$ or (τ^+, ν_τ) by charged current weak interaction. The charge in the target part can be balanced by emitting a soft pion. This pion, which should escape the detection by the detector, should carry a small amount of energy, so that this undetected particle does not affect much the energy budget of the scattering process detected in detector. So for $e^- + N \to \tau^- + N$ or $e^+ + N \to \tau^+ + N$ scattering, the relevant background processes are $e^- + N \to e^- + N + \tau^- + \bar{\nu}_\tau + \pi^+$ or $e^+ + N \to e^+ + N + \tau^+ + \nu_\tau + \pi^-$, respectively. If the scattering with the nucleon is through weak interaction, the total cross section with the production of a τ lepton is doubly suppressed by the weak interaction, that is it would be proportional to $G_F^4 E^6$ which is suppressed by $G_F^2 E^4 \sim 10^{-9}$ for energy of a few GeV, compared to the usual weak interaction process. So this kind of contribution to background is negligible. At leading order, the background process of $e^{\mp}$ scattering with nucleon is given by a photon exchange which gives a factor α^2 in the cross section. So this background process is proportional to $\alpha^2 G_F^2 E^2$ which is $\sim 10^{-4}$ fb for energy of a few GeV. Moreover, there are three extra particles in the background process compared to the signal process, namely $e^{\mp}$, $\bar{\nu}_\tau$ or ν_τ, and $\pi^{\pm}$. Their appearance in the final state contributes three phase space factors which altogether should give a suppression factor no less than 10^{-3}. Finally, the scattered $e^{\mp}$ should also escape the detection by the detector and should also carry a small amount of energy, similar to the consideration for soft pion. This means that to mimic the lepton flavor violating process $e^{\mp} + N \to \tau^{\mp} + N$, final $e^{\mp}$ and $\pi^{\pm}$ of the background process are limited to a small region of phase space, e.g., $\lesssim 10\%$ of the momentum region allowed by the kinematics. So the cross sections of the background event production are further suppressed by these phase space considerations. Taking all these into account, we expect that the background process is at most of order $10^{-9} G_F^2 E^2$ which is around 10^{-9} fb for energy of a few GeV. It is several orders of magnitude smaller than the cross section of the signal process presented in Fig. 4.

Another kind of possible background process is through charged current interaction: $e^- + N \to \nu_e + N + \tau^- + \bar{\nu}_\tau$ or $e^+ + N \to \bar{\nu}_e + N + \tau^+ + \nu_\tau$. The cross sections of these processes are either proportional to $G_F^4 E^6$ or $\alpha^2 G_F^2 E^2$. For reasons similar to the above discussions, we expect these processes also give negligible background events. So background processes are negligible and there is quite a lot of room for studying lepton flavor violating effect in the $e + N \to \tau + N$ process. More detailed analysis of the background processes is not the topic of this article and we leave it to future works.

V. SUMMARY

In summary, we have studied the possible lepton flavor violating effects in a simple process of electron or positron scattering with nucleon, $e + N \to \tau + N$, at energy of a few GeV. This lepton flavor violating process $e + N \to \tau + N$ can be searched for by directing electron or positron beam on an internal or fixed target. For scattering with hydrogen target, this process is the elastic scattering of electron or positron with proton. For target of heavier nuclei, such as deuteron, helium, or others, this process can be studied in quasielastic scattering of $e^{\mp}$ with nuclei. The final particles in this quasielastic scattering are some nucleons or nuclei with no extra hadron produced, i.e., just the broken pieces of the initial nuclei, plus a τ lepton. So the final states are quite simple to study in experiments.

We have considered, as two model independent examples, lepton flavor violating $e\tau\gamma$ and $e\tau Z$ interactions which are possible low energy effective interactions arising from physics beyond SM. We have studied the effects of these interactions on the process $e + N \to \tau + N$ and calculated the cross section of this process induced by these two kinds of interactions. We compare the sensitivity of other lepton flavor violating process, such as $\tau \to e\gamma$, $\tau \to 3e$, $Z \to e^{\mp}\tau^{\pm}$ etc, and the sensitivity of the process $e + N \to \tau + N$ on the lepton flavor violating interactions. We have shown that the lepton flavor violating process $e + N \to \tau + N$ is not as sensitive to lepton flavor violating $e\tau\gamma$ interaction, as to $e\tau Z$ interaction. For the latter, a target experiment with luminosity $\gtrsim 30$ ab^{-1} can give a better sensitivity on the $e\tau Z$ interaction vertex than $Z \to e^{\mp}\tau^{\pm}$. To be better than the sensitivity of $\tau \to 3e$ decay, the luminosity is required to reach 1000 ab^{-1}.

For simplicity, we have concentrated on this lepton flavor violating process at GeV scale which can be studied by GeV scale $e^{\mp}$ accelerator facilities at various places in the world. Another reason for considering GeV scale lepton flavor violating effect is that at this energy scale the elastic scattering or quasielastic scattering is the dominant process and possible background from other channels, which could mimic the lepton flavor violating signal, could be significantly suppressed. So, unlike the processes at higher energy, the background study could be significantly simplified at GeV scale. Note that similar process with muon beam if it is available in experiment, i.e. $\mu + N \to \tau + N$, can also be considered and formalism discussed in this article can be similarly applied to elastic or quasielastic scattering of muon with target. Needless to say, this process provides an alternative way to search for lepton flavor violating effect associated with τ lepton. It is particularly interesting because GeV scale electron or positron beams are available in a number of laboratories in the world, while it is hard to prepare the source of τ lepton and study the lepton flavor violating effect in processes with τ lepton in initial state.

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APPENDIX A

Using (2) and (3), we can find for $T' = T$

$$2k \cdot k' - m_\tau^2 = 2m_T(E - E'). \tag{A1}$$

Using $\theta_{k'}$, the angle between $\vec{k}$ and $\vec{k}'$, (A1) can be rewritten as

$$2EE' - 2E|\vec{k}'|\cos\theta_{k'} - m_\tau^2 = 2m_T(E - E'), \tag{A2}$$

or

$$(2E + 2m_T)E' - (2m_T E + m_\tau^2) = 2E|\vec{k}'|\cos\theta_{k'}, \tag{A3}$$

where $E = |\vec{k}|$ has been used and $|\vec{k}'| = \sqrt{E'^2 - m_\tau^2}$. Making a square of (A3) we get an equation for E' which can be solved as

$$E' = \frac{(E + m_T)(m_\tau^2 + 2m_T E) + E\cos\theta_{k'}\sqrt{A}}{2(E^2\sin^2\theta_{k'} + 2Em_T + m_T^2)}, \tag{A4}$$

where

$$A = (m_\tau^2 + 2m_T E)^2 - 4m_\tau^2(E^2\sin^2\theta_k{}' + 2Em_T + m_T^2). \tag{A5}$$

The condition to have a real solution of E' is $A \geq 0$. So we get a condition for the scattering angle $\theta_{k'}$ for a fixed initial energy E:

$$(m_\tau^2 + 2m_T E)^2 - 4m_\tau^2(2Em_T + m_T^2) \geq 4m_\tau^2 E^2\sin^2\theta_{k'}. \tag{A6}$$

Taking the limit $m_\tau = 0$ in (A4) we can find that the solution becomes

$$E' = \frac{m_T E}{E + m_T - E\cos\theta_{k'}} = \frac{E}{1 + \frac{E}{m_T}(1 - \cos\theta_{k'})}, \tag{A7}$$

which is the familiar formula of elastic scattering in massless limit.

For $T' \neq T$ we can find from (2) and (3) that

$$2k \cdot k' - m_S^2 = 2m_T(E - E') \tag{A8}$$

where

$$m_S^2 = m_\tau^2 - (m_{T'}^2 - m_T^2). \tag{A9}$$

The solution of E' can be obtained with a similar procedure

$$E' = \frac{(E + m_T)(m_S^2 + 2m_T E) + E\cos\theta_{k'}\sqrt{A'}}{2(E^2\sin^2\theta_{k'} + 2Em_T + m_T^2)}, \tag{A10}$$

where

$$A' = (m_S^2 + 2m_T E)^2 - 4m_\tau^2(E^2\sin^2\theta_{k'} + 2Em_T + m_T^2). \tag{A11}$$

The condition (A6) becomes

$$(m_S^2 + 2m_T E)^2 - 4m_\tau^2(2Em_T + m_T^2) \geq 4m_\tau^2 E^2 \sin^2\theta_{k'}. \tag{A12}$$

Applying (A12) to $\theta_{k'} = 0$ gives (4), the threshold condition of the energy.

APPENDIX B

For elastic scattering $e + T \to \tau + T$ induced by a photon exchange, the electromagnetic form factors of nucleus or nucleon are introduced as follows. For spin 0 nuclear target, it is given in

$$\langle P + q|J^\mu_{EM}|P\rangle = F(Q^2)\left(P^\mu - \frac{P \cdot q}{q^2}q^\mu\right). \tag{B1}$$

For spin $\frac{1}{2}$ target, it is given in

$$\langle P + q, s'|J^\mu_{EM}|P, s\rangle = \bar{u}_{s'}(P')\left[F_1(Q^2)\gamma^\mu + i\frac{F_2(Q^2)}{2m_T}\sigma^{\mu\rho}q_\rho\right]u_s(P), \tag{B2}$$

where $P' = P + q$, s' and s are spin indices. J^μ_{EM} is the electromagnetic current. F_1 and F_2 are Dirac and Pauli form factors.

For nucleon target, we take $m_T = m_N$ in (B2) where $m_N = m_p \approx m_n$. The corresponding form factors are the nucleon form factors $F_i^N (i = 1, 2)$, and

$$F_i^N = \begin{cases} F_i^p, & \text{N = proton} \\ F_i^n, & \text{N = neutron} \end{cases} \tag{B3}$$

These form factors satisfy:

$$F_1^p(Q^2 = 0) = 1, \qquad F_2^p(Q^2 = 0) = \kappa_p \tag{B4}$$

$$F_1^n(Q^2 = 0) = 0, \qquad F_2^p(Q^2 = 0) = \kappa_p, \tag{B5}$$

where κ_p and κ_n are anomalous magnetic moments in unit of $e/2m_N$ of proton and neutron, respectively.

$F_i^{p,n}$ can be expressed using Sachs form factors G_E and G_M

$$F_1^{p,n}(Q^2) = \frac{G_E^{p,n}(Q^2) + \frac{Q^2}{4m_N^2}G_M^{p,n}(Q^2)}{1 + \frac{Q^2}{4m_N^2}}, \tag{B6}$$

$$F_2^{p,n}(Q^2) = \frac{G_M^{p,n}(Q^2) - G_E^{p,n}(Q^2)}{1 + \frac{Q^2}{4m_N^2}}. \tag{B7}$$

In dipole approximation, G_E and G_M are expressed as

$$G_{E,M}^{p,n}(Q^2) = \frac{G_{E,M}^{p,n}(Q^2 = 0)}{(1 + Q^2/M_V^2)^2}, \tag{B8}$$

where the vector mass is $M_V = 0.843$ GeV, and

$$G_E^p(Q^2 = 0) = 1, \qquad G_M^p(Q^2 = 0) = \mu_p, \tag{B9}$$

$$G_E^n(Q^2 = 0) = 0, \qquad G_M^n(Q^2 = 0) = \mu_n, \tag{B10}$$

where $\mu_N = 2.793$ and $\mu_n = -1.91$ are the total magnetic moment in unit of $e/2m_N$ of proton and neutron respectively. In this article we adopt the dipole approximation of these electromagnetic form factors of nucleons. More sophisticated and complicated approximations can be found in literatures [8,9] which can be used for more accurate calculation. In this article, we do not use them in our calculation and in our estimate of the sensitivities on lepton flavor violating interactions.

APPENDIX C

Using the lepton flavor violating effective coupling (19), a neutral current effective interaction with quark can be induced at low energy by a Z exchange. For the scattering with nucleon, $e + N \to \tau + N$, the relevant interaction is

$$\Delta L = \frac{2G_F}{\sqrt{2}}[C_L\bar{\tau}\gamma_\mu(1 - \gamma_5)e + C_R\bar{\tau}\gamma_\mu(1 + \gamma_5)e] \times \sum_i\left[\bar{q}_i\frac{1}{2}T^3\gamma^\mu(1 - \gamma_5)q_i - 2\sin^2\theta_W J^\mu_{EM}\right], \tag{C1}$$

where θ_W is the weak mixing angle, q = u or d quark, T^3 the isospin operator, J^μ_{EM} the electromagnetic current for nucleon

$$J^\mu_{EM} = \sum_i Q_i \bar{q}_i \gamma^\mu q_i = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d. \quad \text{(C2)}$$

For simplicity we have neglected possible contribution of strange sea quark [10]. The quark part of the current in (C1) can be decomposed into isoscalar and isovector part and (C1) can be rewritten as

$$\Delta L = \frac{2G_F}{\sqrt{2}}[C_L \bar{\tau}\gamma_\mu(1-\gamma_5)e + C_R \bar{\tau}\gamma_\mu(1+\gamma_5)e] \times [xV^\mu_3 + yV^\mu_0 + \gamma A^\mu_3 + \delta A^\mu_0], \quad \text{(C3)}$$

where

$$V^\mu_3 = \frac{1}{2}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d), \qquad V^\mu_0 = \frac{1}{6}(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d), \quad \text{(C4)}$$

$$A^\mu_3 = \frac{1}{2}(\bar{u}\gamma^\mu\gamma_5 u - \bar{d}\gamma^\mu\gamma_5 d), \qquad A^\mu_0 = \frac{1}{2}(\bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d), \quad \text{(C5)}$$

and for this coupling through Z exchange

$$x = 1 - 2\sin^2\theta_W, \qquad y = -2\sin^2\theta_W, \qquad \gamma = -1, \qquad \delta = 0. \quad \text{(C6)}$$

V^μ_0 and A^μ_0 are isoscalar currents, V^μ_3 and A^μ_3 are isovector currents. Using (C3), the matrix element between nucleon state is

$$\langle N(P')|xV^\mu_3 + yV^\mu_0 + \gamma A^\mu_3 + \delta A^\mu_0|N(P)\rangle. \quad \text{(C7)}$$

The vector part of (C7) can be expressed using the electromagnetic form factors $F^N_{1,2}$ of nucleon as follows. Since J^μ_{EM} can be rewritten as a sum of isovector and isoscalar component as

$$J^\mu_{EM} = V^\mu_0 + V^\mu_3, \quad \text{(C8)}$$

(B2) for nucleon can be rewritten as

$$\langle N(P'),s'|J^\mu_{EM}|N(P),s\rangle = \bar{u}_{s'}(P')\left[(F^s_1 + F^v_1\tau^3)\gamma^\mu + \frac{i}{2m_N}\sigma^{\mu\rho}q_\rho(F^s_2 + F^v_2\tau^3)\right]u_s(P), \quad \text{(C9)}$$

where τ^3 takes value +1 for N = proton and takes value −1 for N = neutron. F^s_i and F^v_i are isoscalar and isovector part of the form factors and are given by the matrix elements of V^μ_0 and V^μ_3, respectively. So we have

$$F^p_i = F^s_i + F^v_i, \qquad i = 1,2 \quad \text{(C10)}$$

$$F^n_i = F^s_i - F^v_i \qquad i = 1,2, \quad \text{(C11)}$$

or

$$F^s_i = \frac{1}{2}(F^p_i + F^n_i), \qquad F^v_i = \frac{1}{2}(F^p_i - F^n_i). \quad \text{(C12)}$$

Similar to (C8), the vector current in (C7) is also a linear combination of isoscalar and isovector current V^μ_0 and V^μ_3. So the vector part in (C7) can be written as a linear combination of isoscalar and isovector contributions, similar to (C9). So we get

$$\langle N(P')|xV^\mu_3 + yV^\mu_0|N(P)\rangle = \bar{u}_{s'}(P')\left[F^z_1\gamma^\mu + \frac{i}{2m_N}\sigma^{\mu\rho}q_\rho F^z_2\right]u_s(P), \quad \text{(C13)}$$

where

$$F^z_i = yF^s_i + xF^v_i\tau^3, \qquad i = 1,2. \quad \text{(C14)}$$

τ^3 takes value +1 for proton and −1 for neutron. Inserting (C12) into (C14) we get an expression of matrix element using the electromagnetic form factors $F^{p,n}_i$:

$$F^z_i = \frac{1}{2}(y + x\tau^3)F^p_i + \frac{1}{2}(y - x\tau^3)F^n_i, \quad i = 1,2. \quad \text{(C15)}$$

The axial part in (C7) can be written into two terms [10] and similar to (C13), each term can be written as a linear combination of isoscalar and isovector contributions.

$$\langle N(P')|\gamma A^\mu_3 + \delta A^\mu_0|N(P)\rangle = \bar{u}_{s'}(P')\left[G^z_1\gamma^\mu\gamma_5 + \frac{1}{2m_N}q^\mu\gamma_5 G^z_2\right]u_s(P), \quad \text{(C16)}$$

where G^z_i is a sum of isoscalar and isovector contributions

$$G^z_i = -\frac{\gamma}{2}G^v_i\tau^3 + \frac{\delta}{2}G^s_i = \frac{1}{2}G^v_i\tau^3, \qquad i = 1,2. \quad \text{(C17)}$$

Again, τ^3 takes value +1 for proton and −1 for neutron. Since $\delta = 0$ for the scattering induced by Z exchange, only the isovector part G^v_i give contribution in (C17). G^v_1 at $Q^2 = 0$ is known from the nucleon β decay and for general Q^2 it can be taken as

$$G^v_1 = g_A/(1 + Q^2/M^2_A)^2 \quad \text{(C18)}$$

in dipole approximation where $g_A = 1.267$. For the axial nucleon mass M_A, $M_A = 1.39$ GeV is taken in this article [11]. G^v_2 is related to G^v_1 by the partial conservation of axial current(PCAC)

$$G_2^v = \frac{4m_N^2}{m_\pi^2 + Q^2} G_1^v \tag{C19}$$

Structure functions W_i in (25) are expressed in terms of F_i^z and G_i^z as

$$W_1 = \frac{Q^2}{4m_N^2}(F_1^z + F_2^z)^2 + \left(1 + \frac{Q^2}{4m_N^2}\right)(G_1^z)^2 \tag{C20}$$

$$W_2 = (F_1^z)^2 + \frac{Q^2}{4m_N^2}(F_2^z)^2 + (G_1^z)^2 \tag{C21}$$

$$W_3 = 2G_1^z(F_1^z + F_2^z) \tag{C22}$$

$$W_4 = \frac{1}{4}\left[(F_1^z)^2 + \frac{Q^2}{4m_N^2}(F_2^z)^2 - (F_1^z + F_2^z)^2 + \frac{Q^2}{4m_N^2}(G_2^z)^2 - 2G_1^z G_2^z\right] \tag{C23}$$

$$W_5 = (F_1^z)^2 + \frac{Q^2}{4m_N^2}(F_2^z)^2 + (G_1^z)^2 \tag{C24}$$

$$W_6 = 0. \tag{C25}$$

For the neutral current elastic scattering of neutrino or antineutrino with nucleon, $\nu(\bar{\nu}) + N \to \nu(\bar{\nu}) + N$, the cross section depends only on $W_{1,2,3}$, not on $W_{4,5}$. By setting $m_\tau = 0$ and $C_R = 0$ and normalizing to polarized beam which is appropriate for neutrino and antineutrino, one can recover from (28) the cross section for $\nu(\bar{\nu}) + N \to \nu(\bar{\nu}) + N$ elastic scattering [11–13]

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2 Q^2}{2\pi E^2}\left\{\frac{1}{2}\left[W_1 - \frac{1}{2}\left(1 + \frac{Q^2}{4m_N^2}\right)W_2\right] \pm \frac{1}{8}\left(\frac{4E}{m_N} - \frac{Q^2}{m_N^2}\right)W_3 + \frac{1}{16}\frac{m_N^2}{Q^2}\left(\frac{4E}{m_N} - \frac{Q^2}{m_N^2}\right)^2 W_2\right\}. \tag{C26}$$

(C26) is also available by directly applying (27) into the cross section (24).

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