Charm degrees of freedom in the quark gluon plasma

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Lattice QCD studies on fluctuations and correlations of charm quantum number have established that deconfinement of charm degrees of freedom sets in around the chiral crossover temperature, T_c ; i.e., charm degrees of freedom carrying fractional baryonic charge start to appear. By reexamining those same lattice QCD data we show that, in addition to the contributions from quarklike excitations, the partial pressure of charm degrees of freedom may still contain significant contributions from open-charm-meson- and baryonlike excitations associated with integral baryonic charges for temperatures up to $1.2T_c$. Charm-quark quasiparticles become the dominant degrees of freedom for temperatures $T > 1.2T_c$.

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Nuclear modification factor and elliptic flow of opencharm hadrons in heavy-ion collision experiments are important observables that provide us with detailed knowledge of the strongly coupled quark gluon plasma (QGP) [\[1\]](#page-3-0). Most of the theoretical models that try to describe these quantities rely on the energy loss of heavy quarks via Langevin dynamics [2–[4\].](#page-3-1) However, the importance of possible heavy-light (strange) bound states inside the QGP has been pointed out in Refs. [5–[8\].](#page-3-2) In particular, the presence of such heavy-light bound states above the QCD transition temperature seems to be necessary for the simultaneous description of elliptic flow and nuclear modification factor of D_s mesons [\[5\]](#page-3-2). The presence of various hadronic bound states [\[9\]](#page-3-3) as well as colored ones [\[10,11\]](#page-3-4) in strongly coupled QGP created in heavy-ion collisions has also been speculated in other contexts.

By utilizing various novel combinations of up-to-fourthorder cumulants of fluctuations of charm quantum number (C) and its correlations with baryon number (B) , electric charge, and strangeness (S), lattice QCD studies [\[12\]](#page-3-5) have established that charm degrees of freedom associated with fractional baryonic and electric charge start appearing at the chiral crossover temperature, $T_c = 154 \pm 9$ MeV [\[13](#page-3-6)–15]. Below T_c the charm degrees of freedom are well described by an uncorrelated gas of charm hadrons having vacuum masses [\[12\]](#page-3-5), i.e., by the hadron resonance gas (HRG) model. Similar conclusions were also obtained from lattice QCD studies involving the light up, down and strange quarks [\[16,17\].](#page-3-7)

On the other hand, lattice QCD calculations have also shown that weakly interacting quasiquarks are good descriptions for the light quark degrees only for temperatures $T \gtrsim 2T_c$ [\[16,18](#page-3-7)–20]. The situation for the heavier charm quarks is also analogous. By reexpressing the lattice QCD results for charm fluctuations and correlations up to fourth order from Ref. [\[12\]](#page-3-5) in the charm- (c) and up- (u) quark flavor basis, we show the $u-c$ flavor correlations, defined as $\chi_{mn}^{uc} = (\partial^{m+n} p / \partial \hat{\mu}_u^m \partial \hat{\mu}_c^n)$ at $\mu_u = \mu_c = 0$, in Fig. [1.](#page-0-0) Here, p denotes the total pressure in QCD and μ_u and μ_c indicate the up- and charm-quark chemical potentials with $\hat{\mu}_X \equiv \mu_X/T$. In order to compare these lattice QCD data with resummed perturbation theory results, which are available only for zero quark masses, we normalize the off-diagonal flavor susceptibilities with the second-order charm-quark susceptibility χ_2^c = $\left(\frac{\partial^2 p}{\partial \hat{\mu}_c^2}\right)$ calculated at $\mu_X = \mu_c = 0$. Such a normalization largely cancels the explicit charm-quark mass dependence of the off-diagonal susceptibilities and enables us to probe whether the $u-c$ flavor correlations can be described by the weak coupling calculations. In the weak coupling limit χ_{11}^{uc} , χ_{13}^{uc} , and χ_{31}^{uc} are expected to have leading-order contributions at $\mathcal{O}(\alpha_s^3)$ [\[21\]](#page-4-0), where α_s is the QCD strong coupling constant. This contribution is, strictly speaking, nonperturbative, but can be calculated on the lattice using

FIG. 1. Off-diagonal quark number susceptibilities χ_{nm}^{uc} normalized by the second-order diagonal charm susceptibility χ_2^c as a function of temperature [\[12\].](#page-3-5) The shaded band shows the threeloop hard-thermal-loop perturbation theory calculation for χ_{22}/χ_2 ; the width of the band corresponds to a variation of the renormalization scale from πT to $4\pi T$ [\[23\].](#page-4-1) Also shown, as dashed lines, are the results of dimensionally reduced electrostatic QCD (EQCD) calculations for χ_{11} corresponding to temperatures $1.32T_c$ and $2.30T_c$ from [\[22\].](#page-4-2)

dimensionally reduced effective theory for high-temperature QCD, the so-called EQCD [\[22\].](#page-4-2) Similarly, in the weak coupling picture, the leading contribution to χ_{22}^{uc} arises from the so-called plasmon term and starts at $\mathcal{O}(\alpha_s^{3/2})$ [\[23\].](#page-4-1) Thus, it is generically expected that $\chi_{22}^{uc} \gg \chi_{13}^{uc} \sim \chi_{31}^{uc} \sim \chi_{11}^{uc}$ in the weak coupling limit. As shown in Fig. [1,](#page-0-0) such an obvious hierarchy in magnitude of the off-diagonal susceptibilities is clearly absent in the lattice data for $T < 200$ MeV. However, for $T \ge 200$ MeV these lattice results are largely consistent with the weak coupling calculations, indicating that the weakly coupled quasiquarks can be considered as the dominant charm degrees of freedom only above this temperature. The fact that for $T_c \lesssim T \lesssim 200$ MeV the charm degrees of freedom are far from weakly interacting quasiquarks is also supported by lattice QCD studies of the screening properties of the open-charm mesons. In this temperature range the screening masses of open-charm mesons also turn out to be quite different from the expectation based on an uncorrelated charm and a light quark degrees of freedom [\[24\].](#page-4-3)

From the preceding discussion it is clear that the weakly interacting charm quasiquarks cannot be the only carriers of charm quantum number for $T \le 200$ MeV. Such an observation naturally raises the question of whether charm excitations associated with baryon number zero and one exist in the QGP for $T_c \lesssim T \lesssim 200$, along with the charm quasiquark excitations carrying $1/3$ baryonic charge. In the present work, we address this question by postulating that such open-charm-meson- and baryonlike excitations exist alongside the charm quasiquarks in the QGP; we then investigate whether such an assumption is compatible with the exact lattice QCD results on charm fluctuations and correlations.

Fluctuations of charm quantum number and its correlation with other conserved quantum numbers can be measured on the lattice through the generalized charm susceptibilities

$$
\chi_{ijk}^{\text{XYC}} = \frac{\partial^{i+j+k} p(T, \mu_X, \mu_Y, \mu_C)}{\partial \hat{\mu}_X^i \partial \hat{\mu}_Y^j \partial \hat{\mu}_C^k} \bigg|_{\mu_X = \mu_Y = \mu_C = 0}, \qquad (1)
$$

where $\hat{\mu}_X = \mu_X/T$. For notational brevity we will suppress the superscripts of χ whenever the corresponding subscript is zero. To check our postulates against the lattice QCD results, we will use throughout this study the lattice QCD data of Ref. [\[12\]](#page-3-5) on up-to-fourth-order generalized charm susceptibilities, i.e., for $i + j + k \leq 4$.

To avoid the introduction of unknown tunable parameters we simply postulate an uncorrelated, i.e., noninteracting, gas of charm-meson-, baryon-, and quarklike excitations for $T \gtrsim T_c$. Owing to the large mass of the charm quark itself, compared to $T \sim 2T_c$, it is safe to treat all the quark-, meson-, and baryonlike excitations as classical quasiparticles, i.e., within the Boltzmann approximations. Furthermore, as discussed in Ref. [\[12\]](#page-3-5), the

FIG. 2. (Top) Fractional contributions of partial pressures of charm quarklike (p_q^C) , mesonlike (p_M^C) , and baryonlike (p_B^C) excitations to the total charm partial pressure (p^C) . (Bottom) Fractional contributions of partial pressures of charm-strange mesonlike $(p_M^{C,S=1})$, charm-singly-strange baryonlike $(p_B^{C,S=1})$, and charm-doubly-strange baryonlike $(p_B^{C,S=2})$ excitations to the total charm partial pressure (p^C) . The solid lines show the corresponding partial pressures obtained from the HRG model including additional quark-model-predicted charm hadrons (see text).

doubly and triply charmed baryons are too heavy to have any significant contributions to QCD thermodynamics in the temperature range of interest and we thus neglect their contributions. With these simplifications the partial pressure of the open-charm sector, p^C , can be written as

$$
p^{C}(T, \mu_C, \mu_B)
$$

= $p_q^{C}(T) \cosh(\hat{\mu}_C + \hat{\mu}_B/3)$
+ $p_B^{C}(T) \cosh(\hat{\mu}_C + \hat{\mu}_B) + p_M^{C}(T) \cosh(\hat{\mu}_C),$ (2)

where p_q^C , p_B^C , and p_M^C denote the partial pressure of the quarklike, mesonlike, and baryonlike excitations, respectively, and μ_B and $\mu_C = \mu_c$ represents the baryon and charm chemical potentials.

Using combinations of up-to-fourth-order baryon-charm susceptibilities, it is easy to isolate the partial pressures of p_q^C , p_M^C , and p_B^C appearing in Eq. [\(2\).](#page-1-0) For example, $p_q^C = 9(\chi_{13}^{BC} - \chi_{22}^{BC})/2$, $p_B^C = (3\chi_{22}^{BC} - \chi_{13}^{BC})/2$, and $p_M^C =$ $\chi_2^C + 3\chi_{22}^{BC} - 4\chi_{13}^{BC}$. The contributions of these partial pressures compared to total charm pressure $p^{C}(T, 0, 0) = \chi_2^{C}$ is shown in Fig. [2](#page-1-1) (top). For $T \lesssim T_c$ the partial pressure of mesons, p_M^C , and the partial pressure of baryons, p_B^C , agree with the corresponding partial pressures from the HRG model including all the experimentally observed as well as additional quark-model-predicted, but yet unobserved, open-charm hadrons with vacuum masses [\[12\].](#page-3-5) The contributions of p_M^C and p_B^C remain significant until $T \lesssim 200$ MeV. In fact, for $T \lesssim 180$ MeV the combined contributions of p_M^C and p_B^C exceeds the contribution from p_q^C . With increasing temperatures p_M^C and p_B^C deviate from the HRG model predictions. This indicate that these charm-meson- and baryonlike excitations can no longer be considered as vacuum charm mesons and baryons. This is in line with the lattice QCD studies on spatial correlation functions of open-charm mesons [\[24\],](#page-4-3) which show significant in-medium modifications of open-charm mesons already in the vicinity of T_c . The partial pressure of quarklike excitations is quite small for $T \sim T_c$ and becomes the dominant contribution to p^C only for $T > 200$ MeV.

Since a charm-quark-like excitation does not carry a strangeness quantum number, the excitations carrying both strangeness and charm quantum numbers are a much cleaner probe of the postulated existence of the charmhadron-like excitations. In this subsector, the pressure can be partitioned into partial pressures of $|C| = 1$ mesonlike excitations carrying strangeness $|S| = 1$ and $C = 1$ baryonlike excitations with $|S| = 1$, 2, i.e.,

$$
p^{C,S}(T, \mu_B, \mu_S, \mu_C)
$$

= $p_M^{C,S=1}(T) \cosh(\hat{\mu}_S + \hat{\mu}_C)$
+ $\sum_{j=1}^{2} p_B^{C,S=j}(T) \cosh(\mu_B - j\mu_S + \mu_C)$. (3)

Thus, the partial pressures of the strange-charm hadronlike excitations can be obtained as $p_M^{C,S=1} = \chi_{13}^{SC} - \chi_{112}^{BSC}$, $p_B^{C,S=1} =$ $\chi_{13}^{SC} - \chi_{22}^{SC} - 3\chi_{112}^{BSC}$, and $p_B^{C,S=2} = (2\chi_{112}^{BSC} + \chi_{22}^{SC} - \chi_{13}^{SC})/2$. In Fig. [2](#page-1-1) (bottom) we show the fractional contributions of these partial pressures towards the total charm partial pressure $p^{C}(T) = \chi_2^{C}$. Even in this subsector, contributions from the hadronlike excitations are significant for $T \le 200$ MeV. However, partial pressure for the $S = 2$ charm-baryon-like excitations is negligible.

Having shown that there can be significant contributions from charm-meson- and baryonlike excitations to the charm partial pressure in the QGP, it is important to ask whether the addition of only these charm degrees of freedom, besides the charm-quark-like excitations, is sufficient to describe all available lattice QCD results for upto-fourth-order charm susceptibilities. As discussed pre-viously in Ref. [\[12\],](#page-3-5) the constraints $\chi_4^C = \chi_2^C$, $\chi_{11}^{BC} = \chi_{13}^{BC}$, $\chi_{11}^{SC} = \chi_{13}^{SC}$ are due to negligible contributions from $|C| = 2$, 3 hadronlike states and they do not provide any independent constraint specific to our proposed model. The remaining four independent fourth-order generalized charm susceptibilities, χ_2^C , χ_{13}^{BC} , χ_{22}^{BC} , and χ_{31}^{BC} , allow us to define the three partial pressures, p_q^C , p_M^C , and p_B^C , and one constraint

FIG. 3. Lattice OCD results for four constraints (c_i) normalized by the total charm pressure (see text).

$$
c_1 \equiv \chi_{13}^{BC} - 4\chi_{22}^{BC} + 3\chi_{31}^{BC} = 0 \tag{4}
$$

that has to hold if the model is correct. If we consider the strange-charm subsector, we have six generalized susceptibilities, χ_{13}^{SC} , χ_{22}^{SC} , χ_{31}^{SC} , χ_{112}^{BSC} , χ_{121}^{BSC} , and χ_{211}^{BSC} . We can use three of these to estimate the partial pressures $p_M^{C,S=1}$, $p_B^{C,S=1}$, and $p_B^{C,S=2}$ defined above, while the remaining ones will provide three additional constraints that can be used to validate our proposed model. These constraints can be written as

$$
c_2 \equiv 2\chi_{121}^{BSC} + 4\chi_{112}^{BSC} + \chi_{22}^{SC} - 2\chi_{13}^{SC} + \chi_{31}^{SC} = 0, \quad (5a)
$$

$$
c_3 \equiv 3\chi_{112}^{BSC} + 3\chi_{121}^{BSC} - \chi_{13}^{SC} + \chi_{31}^{SC} = 0, \qquad (5b)
$$

$$
c_4 \equiv \chi_{211}^{BSC} - \chi_{112}^{BSC} = 0.
$$
 (5c)

Note that the above constraints hold trivially for a free charm-quark gas. It is assuring that our proposed model also smoothly connects to the HRG at T_c . In Fig. [3](#page-2-0) we show the lattice QCD data for c_i 's. Despite large errors on the presently available lattice data, all the c_i 's are, in fact, consistent with zero. Note that, since a possible strangecharm diquarklike excitation will carry $|C| = |S| = 1$ but $|B| = 2/3$, the QCD data being consistent with the constraint $c_4 = 0$ actually tells us that the thermodynamic contributions of possible diquarklike excitations are negligible in the deconfined phase of QCD.

One may speculate on the nature of these charm-hadronlike excitations and, in particular, why their partial pressures vanish gradually with increasing temperature. A likely explanation may be that with increasing temperature the spectral functions of these excitations gradually broaden. A detailed treatment of thermodynamics of quasiparticles with finite width was developed in Refs. [25–[27\].](#page-4-4) It was shown that broad asymmetric spectral functions lead to partial pressures that are considerably smaller than those obtained with zero-width quasiparticles of the same mass, and for sufficiently large width the partial pressures can be made arbitrarily small. Thus, the smallness of the partial pressure of charm-quark-like excitations for $T \sim T_c$ may imply that they have a large width for those temperatures, while the widths of the charm-hadron-like excitations increase with the temperatures; these excitations become very broad for $T \gtrsim 200$ MeV. Such a gradual melting picture is also consistent with the gradual changes of the screening correlators of open-charm-meson-like excitations with increasing temperature [\[24\]](#page-4-3).

Finally, one may wonder whether the rich structure of the up-to-fourth-order generalized charm susceptibilities can be described only in terms of the charm quasiquarks without invoking presence of any other type of charm degrees of freedom. In terms of charm quasiquarks alone, the charm partial pressure will be $p^{C}/T^4 = 6/$ $\pi^2 \hat{m}_c^2 K_2(\hat{m}_c) \cosh(\hat{\mu}_c + \hat{\mu}_B/3)$, where $\hat{m}_c = m_c/T$ with m_c being the mass of the charm quasiparticle. The lattice QCD results for the charm susceptibilities, for example, the nonvanishing values of χ_{mn}^{SC} , can only be described if the charm-quasiquark mass depends of the chemical potentials of all the quark flavors, i.e., $m_c \equiv m_c(T, \mu_B, \mu_S, \mu_C)$. For simplicity, one may imagine Taylor expanding m_c in terms of the chemical potentials and treating these coefficients as parameters for fitting all the lattice QCD results on the generalized charm susceptibilities. Obviously, such a quasiquark model will contain at least as many tunable parameters as the number of susceptibilities. Moreover, in order to satisfy various other constraints observed in the lattice QCD data, such as $\chi_4^C = \chi_2^C$, $\chi_{11}^{BC} = \chi_{13}^{BC}$, these parameters must also be very finely tuned. For example, in order to satisfy the constraint $c_4 = 0$, the coefficients of the $\mathcal{O}(\mu_B^2 \mu_S \mu_C)$ term of m_c must be equal to coefficient of the $\mathcal{O}(\mu_B \mu_S \mu_C^2)$ term. Even if one chooses to use such finely tuned parameters for the chemical potential dependence of the quasiquark mass, the charm partial pressure is not guaranteed to go smoothly over to the HRG values, as observed in the lattice data.

To conclude, using the lattice QCD results for up-tofourth-order generalized charm susceptibilities [\[12\]](#page-3-5) we have shown that the weakly coupled charm quasiquarks become the dominant charm degrees of freedom only above $T \gtrsim 200$ MeV. To investigate the nature of charm degrees of freedom in the intermediate temperature regime, $T_c \lesssim T \lesssim 200$ MeV, we postulated the presence of noninteracting charm-meson- and baryonlike excitations in the QGP, along with the charm-quark-like excitations. We have shown that such a picture is consistent with the presently available lattice QCD results. We have isolated the individual partial pressures of these excitations and found that just above T_c open-charm-meson- and baryonlike excitations provide the dominant contribution to the thermodynamics of charm sector. We also do not observe presence of diquarklike excitations in the s-c sector at these temperatures. Our study hints at possible resonant scattering of the heavy quarks in the medium until around $1.2T_c$, as first advocated in Ref. [\[28\].](#page-4-5) These findings may have important consequences for the heavy-quark phenomenology of heavy-ion collision experiments, especially in understanding the experimentally observed elliptic flow and nuclear modification factor of heavy flavors at small and moderate values of transverse momenta [5–[8,28,29\].](#page-3-2)

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