Annihilation-type rare radiative $B_{(s)} \to V\gamma$ decays

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We obtain predictions for a number of radiative decays $B_{(s)} \to V\gamma$, V the vector meson, which proceed through the weak-annihilation mechanism. Within the factorization approximation, we take into account the photon emission from the B-meson loop and from the vector-meson loop; the latter subprocesses were not considered in the previous analyses but are found to have a sizeable impact on the $B_{(s)} \to V\gamma$ decay rate. The highest branching ratios for the weak-annihilation reactions reported here are $\mathcal{B}(\bar{B}_s^0 \to J/\psi\gamma) = 1.5 \times 10^{-7}$ and $\mathcal{B}(B^- \to \bar{D}_s^{*-}\gamma) = 1.7 \times 10^{-7}$, the estimated accuracy of these predictions being at the level of 20%.

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I. INTRODUCTION

The investigation of rare B decays forbidden at the tree level in the Standard Model provides the possibility to probe the electroweak sector at large mass scales. Interesting information about the structure of the theory is contained in the Wilson coefficients entering the effective Hamiltonian which take different values in different theories with testable consequences in rare B decays.

There is an interesting class of rare radiative B decays which proceed merely through the weak-annihilation mechanism. These processes have very small probabilities and have not been observed. So far, only upper limits on the branching ratios of these decays have been obtained; in 2004, the *BABAR* Collaboration provided the upper limit $\mathcal{B}(B^0 \to J/\psi\gamma) < 1.6 \times 10^{-6}$ [1]. Very recently, the LHCb Collaboration reached the same sensitivity to the B^0 decay and set the limit on the B_s^0 decay: $\mathcal{B}(B^0 \to J/\psi\gamma) < 1.7 \times 10^{-6}$ and $\mathcal{B}(B_s^0 \to J/\psi\gamma) < 7.4 \times 10^{-6}$ at 90% C.L. [2]. Obviously, with the increasing statistics, the prospects to improve the limits on the branching ratios by 1 order of magnitude or eventually to observe these decays in the near future seem very favorable.

The annihilation-type B decays are promising from the perspective of obtaining theoretical predictions since the QCD dynamics of these decays is relatively simple [3,4]. These decays have been addressed in the literature, but—in spite of their relative simplicity—the available theoretical predictions turned out to be rather uncertain; for instance, the predictions for $\mathcal{B}(B_s^0 \to J/\psi\gamma)$ decay vary from 5.7×10^{-8} [5] to 5×10^{-6} [6]. The situation is clearly unsatisfactory and requires clarification. We did not find any of these results convincing and present in this paper a more detailed analysis of the $B \to V\gamma$ decays.

The annihilation-type $B \to V\gamma$ decays proceed through the four-quark operators of the effective weak Hamiltonian. In the factorization approximation, the amplitude can be represented as the product of meson leptonic decay constants and matrix elements of the weak current between the meson and photon; the latter contain the meson-photon transition form factors. The photon can be emitted from the loop containing the B-meson [Fig. 1(a)], and this contribution is described by the $B\gamma$ transition form factors. The photon can be also emitted from the vector-meson V-loop [Fig 1(b)]; this contribution is described by the $V\gamma$ transition form factors. The latter were erroneously believed to give a small contribution to the amplitude and have not been considered in the previous analyses.

The main new ingredient of this paper is the analysis of the photon emission from the V-loop. First, we show that this contribution has no parametric suppression compared to the photon emission from the B-loop. Then, we calculate the $B\gamma$ and $V\gamma$ form factors within the relativistic dispersion approach based on the constituent quark picture [7]. As shown in Ref. [8], the form factors from this approach satisfy all rigorous constraints which emerge in QCD in the limit of heavy-to-heavy and heavy-to-light transitions; as demonstrated in Refs. [9–11], the numerical results for the weak transition form factors from this approach exhibit an excellent agreement with the results from lattice QCD and QCD sum rules.

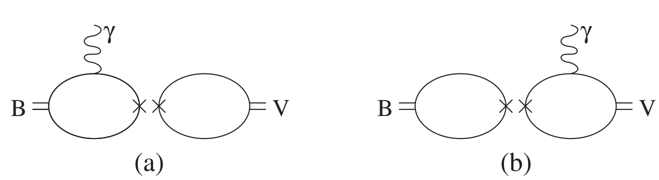

FIG. 1. Diagrams describing the weak-annihilation process for $B \to V\gamma$ in the factorization approximation: (a) The photon is emitted from the B-loop, and (b) the photon is emitted from the vector-meson V-loop.

Numerically, we report here that the V-loop contribution to the amplitude turns out to be comparable with the B-loop contribution and has a sizeable impact on the probability of the weak-annihilation $B \to V\gamma$ decay.

The paper is organized as follows. In Sec. II, the effective weak Hamiltonian and the structure of the amplitude are recalled. We discuss the general structure of the $B \to V\gamma$ amplitude and work out the constraints coming from gauge invariance. In Sec. III, we consider the photon emission from the B-loop and present the $B\gamma$ transition form factors within the relativistic dispersion approach based on the constituent quark picture. Section IV contains the analysis of the $V\gamma$ transition form factors. Finally, in Sec. V, the numerical estimates are given. The concluding Sec. VI summarizes our results and presents a critical discussion of other results existing in the literature.

II. EFFECTIVE HAMILTONIAN, THE AMPLITUDE, AND THE DECAY RATE

We consider the weak-annihilation radiative $B \to V\gamma$ transition, where V is the vector meson containing at least one charm quark, i.e., having the quark content $\bar{q}c$ $(q = u, d, s, c)$. The corresponding amplitude is given by the matrix element of the effective Hamiltonian [12]

$$A(B \to V\gamma) = \langle \gamma(q_1)V(q_2)|H_{\rm eff}|B(p)\rangle, \qquad (2.1)$$

where p is the B momentum, q_2 is the vector-meson momentum, and q_1 is the photon momentum, $p = q_1 + q_2$, $q_1^2 = 0$, $q_2^2 = M_V^2$, $p^2 = M_B^2$. The effective weak Hamiltonian relevant for the transition of interest has the form (we provide in this section formulas for the effective Hamiltonian with the flavor structure $\bar{d}c\bar{u}b$, but all other decays of interest may be easily described by an obvious replacement of the quark flavors and the corresponding Cabibbo-Kobayashi-Maskawa (CKM) factors $\xi_{\rm CKM}$)

$$H_{\rm eff} = -\frac{G_F}{\sqrt{2}}\xi_{\rm CKM}(C_1(\mu)\mathcal{O}_1 + C_2(\mu)\mathcal{O}_2), \qquad (2.2)$$

G_F is the Fermi constant, $\xi_{\rm CKM} = V_{cd}^* V_{ub}$, $C_{1,2}(\mu)$ are the scale-dependent Wilson coefficients [12], and we only show the relevant four-quark operators

$$\mathcal{O}_1 = \bar{d}_\alpha\gamma_\nu(1-\gamma_5)c_\alpha\bar{u}_\beta\gamma_\nu(1-\gamma_5)b_\beta,$$
$$\mathcal{O}_2 = \bar{d}_\alpha\gamma_\nu(1-\gamma_5)c_\beta\bar{u}_\beta\gamma_\nu(1-\gamma_5)b_\alpha. \qquad (2.3)$$

We use notations $e = \sqrt{4\pi\alpha_{\rm em}}$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$, $\epsilon^{0123} = -1$, and ${\rm Sp}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta) = 4i\epsilon^{\mu\nu\alpha\beta}$.

The amplitude can be written as

$$A(B \to V\gamma) = -\frac{G_F}{\sqrt{2}}\xi_{\rm CKM}a_{\rm eff}(\mu)\langle V(q_2)\gamma(q_1)|\bar{d}\gamma_\nu \times (1-\gamma_5)c\cdot\bar{u}\gamma_\nu(1-\gamma_5)b|B(p)\rangle, \qquad (2.4)$$

where $a_{\rm eff}(\mu)$ is an effective scale-dependent Wilson coefficient appropriate for the decay under consideration.

It is convenient to isolate the parity-conserving contribution which emerges from the product of the two equal-parity currents and the parity-violating contribution which emerges from the product of the two opposite-parity currents. The amplitude may then be parametrized as follows,

$$A(B \to V\gamma) = \frac{eG_F}{\sqrt{2}}[\epsilon_{q_1\epsilon_1^* q_2\epsilon_2^*}F_{\rm PC} + i\epsilon_2^{*\nu}\epsilon_1^{*\mu}(g_{\nu\mu}pq_1 - p_\mu q_{1\nu})F_{\rm PV}], \qquad (2.5)$$

where $F_{\rm PC}$ and $F_{\rm PV}$ are the parity-conserving and parity-violating invariant amplitudes, respectively. Hereafter, ϵ_2 (ϵ_1) is the vector-meson (photon) polarization vector. We use the shorthand notation $\epsilon_{abcd} = \epsilon_{\alpha\beta\mu\nu}a^\alpha b^\beta c^\mu d^\nu$ for any 4-vectors a, b, c, d.

For the decay rate, one finds

$$\Gamma(B \to V\gamma) = \frac{G_F^2\alpha_{em}}{16}M_B^3(1 - M_V^2/M_B^2)^3(|F_{\rm PC}|^2 + |F_{\rm PV}|^2). \qquad (2.6)$$

Neglecting the nonfactorizable soft-gluon exchanges, i.e., assuming vacuum saturation, the complicated matrix element in Eq. (2.4) is reduced to simpler quantities—the meson-photon matrix elements of the bilinear quark currents and the meson decay constants. The latter are defined as usual:

$$\langle V(q_2)|\bar{d}\gamma_\nu c|0\rangle = \epsilon_{2\nu}^* M_V f_V, \qquad f_V > 0,$$
$$\langle 0|\bar{u}\gamma_\nu\gamma_5 b|B(p)\rangle = ip_\nu f_B, \qquad f_B > 0. \qquad (2.7)$$

A. Parity-violating amplitude

The parity-violating contribution to the weak-annihilation amplitude has the form

$$A_{\rm PV}(B \to V\gamma) = \frac{G_F}{\sqrt{2}}\xi_{\rm CKM}a_{\rm eff}(\mu)\{\langle V\gamma|\bar{d}\gamma_\nu c|0\rangle\langle 0|\bar{u}\gamma_\nu\gamma_5 b|B\rangle + \langle V|\bar{d}\gamma_\nu c|0\rangle\langle\gamma|\bar{u}\gamma_\nu\gamma_5 b|B\rangle\}. \qquad (2.8)$$

It is convenient to denote

$$A_{\rm PV}^{(1)} = \langle V(q_2)|\bar{d}\gamma_\nu c|0\rangle\langle\gamma(q_1)|\bar{u}\gamma_\nu\gamma_5 b|B(p)\rangle \qquad (2.9)$$

and

$$A_{\rm PV}^{(2)} = \langle V(q_2)\gamma(q_1)|\bar{d}\gamma_\nu c|0\rangle\langle 0|\bar{u}\gamma_\nu\gamma_5 b|B(p)\rangle: \qquad (2.10)$$

(1) Let us start with $A^{(1)}_{\rm PV}$. One can write

$$\langle\gamma(q_1)|\bar u\gamma_\nu\gamma_5 b|B(p)\rangle = e\epsilon_1^{*\mu}T^B_{\mu\nu}, \quad (2.11)$$

where

$$T^B_{\mu\nu}(p,q_1)=i\int dx e^{iq_1x}\langle 0|T(J^{\rm e.m.}_\mu(x),\bar u\gamma_\nu\gamma_5 b)|B(p)\rangle, \quad (2.12)$$

and

$$J^{\rm e.m.}_\mu(x)=\frac{2}{3}(\bar u\gamma_\mu u+\bar c\gamma_\mu c+\bar t\gamma_\mu t) - \frac{1}{3}(\bar d\gamma_\mu d+\bar s\gamma_\mu s+\bar b\gamma_\mu b) \quad (2.13)$$

is the electromagnetic quark current.

The amplitude $T^B_{\mu\nu}$ in general contains five independent Lorentz structures and can be parametrized in various ways [4,11,13]. There is, however, the unique parametrization of the amplitude, which provides a distinct separation of the amplitude: form factors in the gauge-invariant transverse part of the amplitude and contact terms in its longitudinal part [7]:

$$T^B_{\mu\nu}=T^\perp_{\mu\nu}+\frac{iq_{1\mu}p_\nu}{q_1^2}R_1+\frac{iq_{1\mu}q_{1\nu}}{q_1^2}R_2, \quad (2.14)$$

with

$$T^\perp_{\mu\nu}=i\left(g_{\mu\nu}-\frac{q_{1\mu}q_{1\nu}}{q_1^2}\right)pq_1F_{A1}(q_1^2) + i\left(p_\mu-\frac{pq_1}{q_1^2}q_{1\mu}\right)q_{1\nu}F_{A2}(q_1^2) + i\left(p_\mu-\frac{pq_1}{q_1^2}q_{1\mu}\right)p_\nu F_{A3}(q_1^2). \quad (2.15)$$

The invariant amplitudes R_1 and R_2 in the longitudinal structure can be determined using the conservation of the electromagnetic current $\partial_\mu J^{\rm e.m.}_\mu=0$ [10], which leads to

$$q_{1\mu}T^B_{\mu\nu}(p,q_1)=-\langle 0|[\hat Q,\bar u\gamma_\nu\gamma_5 b]|B(p)\rangle = iQ_Bf_Bp_\nu \quad (2.16)$$

and thus to

$$R_1=Q_Bf_B,\qquad R_2=0. \quad (2.17)$$

The parametrization (2.14) of the amplitude is prompted by the structure of the Feynman diagram; let us rewrite the usual electromagnetic coupling of the quark as follows ($q_1=k-k'$):

$$(m+\hat k')\gamma_\mu(m+\hat k)=(m+\hat k')\left\{\gamma_\mu-\hat q_1\frac{q_{1\mu}}{q_1^2}\right\}(m+\hat k) + \frac{q_{1\mu}}{q_1^2}[(k^2-m^2)(m+\hat k')-(k'^2-m^2)(m+\hat k)]. \quad (2.18)$$

The first term is explicitly transverse with respect to $q_{1\mu}$ and leads to $T^\perp_{\mu\nu}$. The second term, containing the factors (k^2-m^2) and (k'^2-m^2), leads to the contact term $ip_\nu\frac{q_{1\mu}}{q_1^2}f_B$. The Lorentz structures in (2.15) have singularities at $q_1^2=0$, but the full amplitude $T^B_{\mu\nu}$ should be regular at $q_1^2=0$. So the singularities must cancel each other, yielding the constraints on the form factors at $q_1^2=0$:

$$F_{A1}(0)=-F_{A2}(0),\qquad F_{A3}(0)=\frac{f_BQ_B}{pq_1}. \quad (2.19)$$

Hereafter, when evaluating the invariant amplitudes at $q_1^2=0$, one should make use of relation $pq_1=\frac{1}{2}(M_B^2-M_V^2)$. By virtue of (2.19), for the amplitude $A_1^{\rm PV}$ at $q_1^2=0$, we find

$$A^{(1)}_{\rm PV}=ief_VM_V\epsilon_1^{*\mu}\epsilon_2^{*\nu}\{g_{\mu\nu}pq_1F_{A1}(0)+p_\mu q_{1\nu}F_{A2}(0) + p_\mu p_\nu F_{A3}(0)\} = ief_VM_V\epsilon_1^{*\mu}\epsilon_2^{*\nu} \times\left\{(g_{\mu\nu}pq_1-p_\mu q_{1\nu})\frac{F_A}{M_B}+p_\mu q_{1\nu}\frac{f_BQ_B}{pq_1}\right\}, \quad (2.20)$$

with $F_A=M_BF_{A1}(0)$. Notice that the contact term does not contribute to the amplitude directly but nevertheless determines the value of the form factor $F_{3A}(0)$.

(2) Let us now turn to $A^{(2)}_{\rm PV}$. Using the equation of motion for the quark fields

$$i\gamma_\nu\partial^\nu q(x)=mq(x)-Q_qA_\nu\gamma^\nu q(x), \quad i\partial^\nu\bar q(x)\gamma_\nu=-m\bar q(x)+Q_qA_\nu\bar q(x)\gamma^\nu, \quad (2.21)$$

one obtains

$$i\partial_\nu(\bar d\gamma_\nu c)=j+(Q_d-Q_c)\bar d\gamma_\nu cA_\nu, \quad (2.22)$$

where

$$j(x)=(m_c-m_d)\bar d(x)c(x) \quad (2.23)$$

is the scale-independent scalar current. Then, for the amplitude $A_{\rm PV}^{(2)}$, we find

$$\begin{aligned} A_{\rm PV}^{(2)} &= i p_\nu f_B \langle V\gamma|\bar{d}\gamma_\nu c|0\rangle \\ &= -i f_B \langle V\gamma|j|0\rangle - i f_B (Q_d - Q_c)\epsilon_1^{*\nu}\langle V|\bar{d}\gamma_\nu c)|0\rangle \\ &= -i f_B \langle V\gamma|j|0\rangle - i f_B \epsilon_1^{*\mu}\epsilon_2^{*\nu} g_{\mu\nu} Q_V f_V M_V. \end{aligned} \tag{2.24}$$

We have taken into account here the charge-conservation relation

$$Q_b - Q_u = Q_B = Q_V = Q_d - Q_c. \tag{2.25}$$

The amplitude $\langle V\gamma|j|0\rangle$ may be written as

$$\begin{aligned} \langle V(q_2)\gamma(q_1)|j|0\rangle &= ie\epsilon_1^{*\mu}(q_1)\langle V(q_2)| \\ &\quad\times \int dx e^{iq_1 x} T(J_\mu^{\rm e.m.}(x) j(0))|0\rangle \\ &\equiv ie\epsilon_1^{*\mu}(q_1) T_\mu^V, \end{aligned} \tag{2.26}$$

and for T_μ^V, one can write the decomposition

$$\begin{aligned} T_\mu^V = i\epsilon_2^{*\nu}(q_2)\Bigg\{ &\left(g_{\mu\nu} - \frac{q_{1\mu}q_{1\nu}}{q_1^2}\right) pq_1 H_{S1}(q_1^2) \\ &+ \left(p_\mu - \frac{pq_1}{q_1^2} q_{1\mu}\right) q_{1\nu} H_{S2}(q_1^2)\Bigg\}. \end{aligned} \tag{2.27}$$

Making use of the electromagnetic current conservation, one finds that the contact terms in T_μ^V are absent due to the relation $\langle V|j|0\rangle = 0$. Again, the singularities at $q_1^2 = 0$ of the transverse Lorentz projectors should cancel in the amplitude which is free from the singularity at $q_1^2 = 0$, leading to

$$H_{S1}(0) = -H_{S2}(0). \tag{2.28}$$

Then, for the radiative decay $q_1^2 = 0$, one obtains

$$\langle V(q_2)\gamma(q_1)|j|0\rangle = ief_B \epsilon_1^{*\mu}\epsilon_2^{*\mu}(g_{\mu\nu} pq_1 - p_\mu q_{1\nu}) H_S, \tag{2.29}$$

with $H_S = H_{S1}(0)$. Finally, using charge conservation $Q_V = Q_B$, we arrive at

$$\begin{aligned} A_{\rm PV}^{(2)} &= ief_B \epsilon_1^{*\mu}\epsilon_2^{*\nu}(g_{\mu\nu} pq_1 - p_\mu q_{1\nu}) H_S \\ &\quad - if_B \epsilon_1^{*\mu}\epsilon_2^{*\mu} g_{\mu\nu} Q_V f_V M_V. \end{aligned} \tag{2.30}$$

(3) We are ready now to obtain the parity-violating contribution to the amplitude in the factorization approximation. First, let us mention that making use of the charge-conservation $Q_B = Q_V$, the sum of the separately gauge-noninvariant terms in (2.20) and (2.30) yields a gauge-invariant combination

$$-ie\epsilon_1^{*\mu}\epsilon_2^{*\mu}\left(g_{\mu\nu} - \frac{p_\mu q_{1\nu}}{pq_1}\right) f_B Q_B f_V M_V. \tag{2.31}$$

For the sum $A_{\rm PV}^{(1)} + A_{\rm PV}^{(2)}$, we then find an explicitly gauge-invariant expression

$$\begin{aligned} A_{\rm PV}^{(1)} + A_{\rm PV}^{(2)} &= ie\epsilon_1^{*\mu}\epsilon_2^{*\nu}(g_{\mu\nu} pq_1 - p_\mu q_{1\nu}) \\ &\quad\times \left[\frac{F_A}{M_B} f_V M_V + f_B H_S - \frac{Q_B f_B f_V M_V}{pq_1}\right], \end{aligned} \tag{2.32}$$

such that the parity-violating amplitude of (2.5) is

$$\begin{aligned} F_{\rm PV} &= \xi_{\rm CKM} a_{\rm eff}(\mu) \\ &\quad\times \left[\frac{F_A}{M_B} f_V M_V + f_B H_S - \frac{2Q_B f_B f_V M_V}{M_B^2 - M_V^2}\right]. \end{aligned} \tag{2.33}$$

B. Parity-conserving amplitude

This amplitude reads

$$\begin{aligned} A_{\rm PC}(B\to V\gamma) = -\frac{G_F}{\sqrt{2}}\xi_{\rm CKM} a_{\rm eff}(\mu)\{&\langle V|\bar{d}\gamma_\nu c|0\rangle\langle\gamma|\bar{u}\gamma_\nu b|B\rangle \\ &+ \langle\gamma V|\bar{d}\gamma_\nu\gamma_5 c|0\rangle\langle 0|\bar{u}\gamma_\nu\gamma_5 b|B\rangle\}: \end{aligned} \tag{2.34}$$

(1) The first contribution to the amplitude, corresponding to the photon emission from the B-meson loop, reads

$$\begin{aligned} A_{\rm PC}^{(1)} &= \langle V|\bar{d}\gamma_\nu c|0\rangle\langle\gamma|\bar{u}\gamma_\nu b|B\rangle \\ &= -eM_V f_V \epsilon_{q_1\epsilon_1^* q_2\epsilon_2^*}\frac{F_V}{M_B}, \end{aligned} \tag{2.35}$$

where F_V is the form factor describing the $B\to\gamma$ transition induced by the vector weak current

$$\langle\gamma(q_1)|\bar{u}\gamma_\nu b|B(p)\rangle = -e\epsilon_{q_1\epsilon_1^* q_2\nu}\frac{F_V}{M_B}. \tag{2.36}$$

(2) The second term in (2.34), describing the photon emission from the vector-meson loop, may be reduced to the divergence of the axial-vector current. Making use of the equations of motion (2.21), one finds

$$i\partial_\nu(\bar{d}\gamma_\nu\gamma_5 c) = -j^5 + (Q_d - Q_c)\bar{d}\gamma_\nu\gamma_5 c A^\nu \quad (2.37)$$

with the scale-independent pseudoscalar current

$$j^5 = (m_d + m_c)\bar{d}\gamma_5 c. \quad (2.38)$$

Taking into account that $\langle V|\bar{d}\gamma_\nu\gamma_5 c|0\rangle = 0$, we find

$$\begin{aligned} A^{(2)}_{\rm PC} &= \langle 0|\bar{u}\gamma_\nu\gamma_5 b|B\rangle\langle\gamma V|\bar{d}\gamma_\nu\gamma_5 c|0\rangle \\ &= f_B\langle\gamma V|\partial_\nu(\bar{d}\gamma_\nu\gamma_5 c)|0\rangle \\ &= -ef_B\epsilon_{q_1\epsilon_1^* q_2\epsilon_2^*}H_P, \end{aligned} \quad (2.39)$$

where the form factor H_P is defined as

$$\langle\gamma(q_1)V(q_2)|j_5|0\rangle = ie\epsilon_{q_1\epsilon_1^* q_2\epsilon_2^*}H_P. \quad (2.40)$$

(3) Finally, the parity-conserving invariant amplitude of (2.5) takes the form

$$F_{\rm PC} = \xi_{\rm CKM}a_{\rm eff}(\mu)\left[\frac{F_V}{M_B}f_V M_V + f_B H_P\right]. \quad (2.41)$$

Summing up this section, within the factorization approximation, the weak-annihilation amplitude may be expressed in terms of four form factors: F_A, F_V, H_P, and H_S. It should be emphasized that each of the form factors F_A, F_V, H_P, and H_S actually depends on two variables: The B-meson transition form factors F_A, F_V depend on q_1^2 and q_2^2, and $F_{A,V}(q_1^2, q_2^2)$ should be evaluated at $q_1^2 = 0$ and $q_2^2 = M_V^2$. The vector-meson transition form factors H_P and H_S depend on q_1^2 and p^2, and $H_{S,P}(q_1^2, p^2)$ should be evaluated at $q_1^2 = 0$ and $p^2 = M_B^2$.

III. PHOTON EMISSION FROM THE B-MESON LOOP AND THE FORM FACTORS F_A AND F_V

In this section, we calculate the form factors $F_{A,V}$ within the relativistic dispersion approach to the transition form factors based on the constituent quark picture. This approach has been formulated in detail in Ref. [8] and applied to the weak decays of heavy mesons in Ref. [9].

The pseudoscalar meson in the initial state is described in the dispersion approach by the following vertex [7]: $\bar{q}_1(k_1)i\gamma_5 q(-k_2)G(s)/\sqrt{N_c}$, with $G(s) = \phi_P(s)(s - M_P^2)$, $s = (k_1 + k_2)^2$, $k_1^2 = m_1^2$ and $k_2^2 = m_2^2$. The pseudoscalar-meson wave function ϕ_P is normalized according to the relation [7]

$$\frac{1}{8\pi^2}\int_{(m_1+m_2)^2}^\infty ds\phi_P^2(s)(s-(m_1-m_2)^2)\frac{\lambda^{1/2}(s,m_1^2,m_2^2)}{s} = 1. \quad (3.1)$$

The decay constant is represented through $\phi_P(s)$ by the spectral integral

$$f_P = \sqrt{N_c}\int_{(m_1+m_2)^2}^\infty ds\phi_P(s)(m_1+m_2) \times\frac{\lambda^{1/2}(s,m_1^2,m_2^2)}{8\pi^2 s}\frac{s-(m_1-m_2)^2}{s}. \quad (3.2)$$

Here, $\lambda(a,b,c) = (a-b-c)^2 - 4bc$ is the triangle function.

Recall that the form factors $F_{A,V}$ describe the transition of the B-meson to the photon with the momentum q_1, $q_1^2 = 0$, induced by the axial-vector (vector) current with the momentum q_2, $q_2^2 = M_V^2$. We derive the double spectral representations for the form factor in p^2 and q_2^2; this allows us to avoid the appearance of the unphysical polynomial terms in the amplitudes which otherwise should be killed by appropriate subtractions.

A. Form factor F_A

The form factor F_A is given by the diagrams of Fig. 2. Figure 2(a) shows $F_A^{(b)}$, the contribution to the form factor of the process when the b-quark interacts with the photon; Fig. 2(b) describes the contribution of the process when the quark u interacts while b remains a spectator.

It is convenient to change the direction of the quark line in the loop diagram of Fig. 2(b). This is done by performing the charge conjugation of the matrix element and leads to a sign change for the $\gamma_\nu\gamma_5$ vertex. Now both diagrams in Figs. 2(a) and 2(b) are reduced to the diagram of Fig. 3 which defines the form factor $F_A^{(1)}(m_1, m_2)$; setting

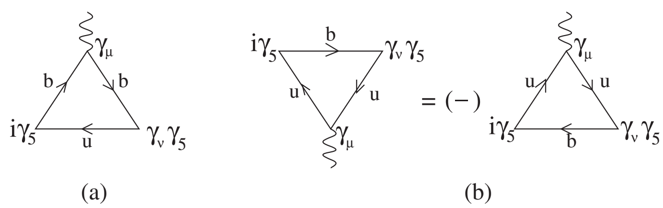

FIG. 2. Diagrams for the form factor F_A: a) $F_A^{(b)}$ and b) $F_A^{(u)}$.

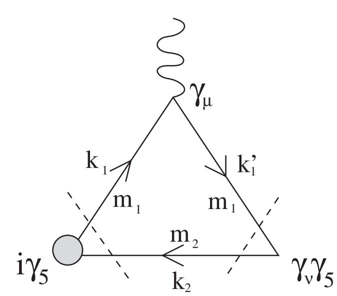

FIG. 3. The triangle diagram for $F_A^{(1)}(m_1, m_2)$. The cuts correspond to calculating the double spectral density in p^2 and q_2^2.

$m_1 = m_b$, $m_2 = m_u$ gives $F_A^{(b)}$, while setting $m_1 = m_u$, $m_2 = m_b$ gives $F_A^{(u)}$ such that

$$F_A = Q_b F_A^{(b)} - Q_u F_A^{(u)}. \tag{3.3}$$

For the diagram of Fig. 3 (quark 1 emits the photon, quark 2 is the spectator, and all quark lines are on their mass shell), the trace reads

$$\begin{aligned} &-\mathrm{Sp}(i\gamma_5(m_2 - \hat{k}_2)\gamma_\nu\gamma_5(m_1 + \hat{k}'_1)\gamma_\mu(m_1 + \hat{k}_1)) \\ &\quad = 4i(k_1 + k'_1)_\mu(m_1 k_2 + m_2 k_1)_\nu \\ &\qquad + 4i(g_{\mu\nu}q_{1\alpha} - g_{\mu\alpha}q_{1\nu})(m_1 k_2 + m_2 k_1)_\alpha. \end{aligned} \tag{3.4}$$

The double spectral density of the form factor $F_A^{(1)}(m_1, m_2)$ in the variables p^2, $p = k_1 + k_2$, and q_2^2, $q_2 = k'_1 + k_2$ is obtained as the coefficient of the structure $g_{\mu\nu}$ after the integration of the trace over the quark phase space. At $q_1^2 = 0$, the double spectral representation for the elastic form factor is reduced to a single spectral representation, which is given below.

The easiest was to derive this spectral representation is to use the light-cone variables [14]. Performing the necessary calculations, we arrive at the following representation:

$$\begin{aligned} \frac{1}{M_B}F_A^{(1)}(m_1,m_2) &= \frac{\sqrt{N_c}}{4\pi^2}\int\frac{dx_1dx_2dk_\perp^2}{x_1^2x_2}\delta(1-x_1-x_2)\frac{\phi_B(s)}{s-M_V^2} \\ &\quad\times\left(m_1x_2+m_2x_1+(m_1-m_2)\frac{2k_\perp^2}{M_B^2-M_V^2}\right). \end{aligned} \tag{3.5}$$

Here, x_i is the fraction of the B-meson light-cone momentum carried by the quark i, and

$$s = \frac{m_1^2}{x_1} + \frac{m_2^2}{x_2} + \frac{k_\perp^2}{x_1 x_2}. \tag{3.6}$$

This expression may be cast in the form of a single dispersion integral

$$\begin{aligned} \frac{1}{M_B}F_A^{(1)}(m_1, m_2) &= \frac{\sqrt{N_c}}{4\pi^2}\int_{(m_1+m_2)^2}^{\infty}\frac{ds\phi_B(s)}{(s - M_V^2)} \\ &\times\left(\rho_+(s, m_1, m_2) + 2\frac{m_1 - m_2}{M_B^2 - M_V^2}\rho_{k_\perp^2}(s, m_1, m_2)\right), \end{aligned} \tag{3.7}$$

where

$$\begin{aligned} \rho_+(s, m_1, m_2) &= (m_2 - m_1)\frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{s} \\ &\quad + m_1\log\left(\frac{s + m_1^2 - m_2^2 + \lambda^{1/2}(s, m_1^2, m_2^2)}{s + m_1^2 - m_2^2 - \lambda^{1/2}(s, m_1^2, m_2^2)}\right), \end{aligned} \tag{3.8}$$

$$\begin{aligned} \rho_{k_\perp^2}(s, m_1, m_2) &= \frac{s + m_1^2 - m_2^2}{2s}\lambda^{1/2}(s, m_1^2, m_2^2) \\ &\quad - m_1^2\log\left(\frac{s + m_1^2 - m_2^2 + \lambda^{1/2}(s, m_1^2, m_2^2)}{s + m_1^2 - m_2^2 - \lambda^{1/2}(s, m_1^2, m_2^2)}\right). \end{aligned} \tag{3.9}$$

Making use of the light-cone representation (3.5) and the light-cone representation of the pseudoscalar-meson decay constant (3.2)

$$\begin{aligned} f_P &= \frac{\sqrt{N_c}}{4\pi^2}\int\frac{dx_1dx_2dk_\perp^2}{x_1x_2}\delta(1-x_1-x_2)\delta \\ &\quad\times\left(s-\frac{m_1^2}{x_1}-\frac{m_2^2}{x_2}-\frac{k_\perp^2}{x_1x_2}\right)\phi_P(s)(m_1x_2+m_2x_1) \end{aligned} \tag{3.10}$$

and employing the fact that the wave function $\phi_P(s)$ is localized near the threshold in the region $\sqrt{s} - m_b - m_u \leq \bar{\Lambda}$, it is easy to show that in the limit $m_b \to \infty$ the photon emission from the light quark dominates over the emission from the heavy quark [15],

$$\frac{1}{M_B}F_A^{(u)} = \frac{f_B}{\bar{\Lambda}m_b} + \cdots, \qquad \frac{1}{M_B}F_A^{(b)} = \frac{f_B}{m_b^2} + \cdots \tag{3.11}$$

B. Form factor F_V

The consideration of the form factor F_V is very similar to the form factor F_A. F_V is determined by the two diagrams shown in Fig. 4. Figure 4(a) gives $F_V^{(b)}$, the contribution of the process when the b-quark interacts with the photon; Fig. 4(b) describes the contribution of the process when the quark u interacts.

It is again convenient to change the direction of the quark line in the loop diagram of Fig. 4(b) by performing the charge conjugation of the matrix element. For the vector current γ_ν in the vertex, the sign does not change (in contrast to the $\gamma_\nu\gamma_5$ case considered above). Then, both diagrams in Figs. 4(a) and 4(b) are reduced to the diagram of Fig. 5 which gives the form factor $F_V^{(1)}(m_1, m_2)$; setting $m_1 = m_b$, $m_2 = m_u$ gives $F_V^{(b)}$, while setting $m_1 = m_u$, $m_2 = m_b$ gives $F_V^{(u)}$ such that

$$F_V = Q_b F_V^{(b)} + Q_u F_V^{(u)}. \tag{3.12}$$

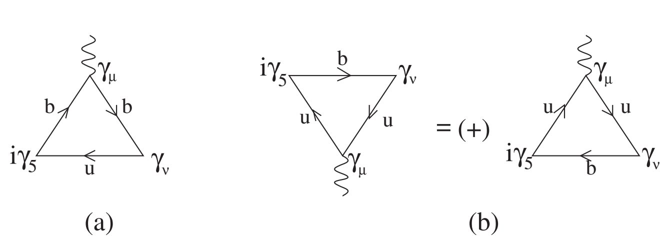

FIG. 4. Diagrams for the form factor F_V: a) $F_V^{(b)}$ and b) $F_V^{(u)}$.

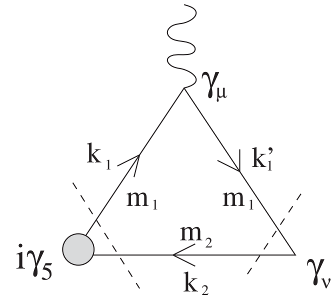

FIG. 5. The triangle diagram for $F_V^{(1)}(m_1, m_2)$. The cuts correspond to calculating the double spectral density in p^2 and q_2^2.

The trace corresponding to the diagram of Fig. 4 (1—active quark, 2—spectator) reads

$$-\mathrm{Sp}(i\gamma_5(m_2-\hat{k}_2)\gamma_\nu(m_1+\hat{k}_1')\gamma_\mu(m_1+\hat{k}_1)) = 4\epsilon_{\mu q_1 \nu\alpha}(m_1 k_2 + m_2 k_1)_\alpha.$$

The light-cone representation of the form factor corresponding to Fig. 5 takes the form

$$\frac{1}{M_B}F_V^{(1)}(m_1,m_2) = -\frac{\sqrt{N_c}}{4\pi^2}\int\frac{dx_1dx_2dk_\perp^2}{x_1^2x_2}\delta(1-x_1-x_2) \times\frac{\phi_B(s)}{s-M_V^2}(m_1x_2+m_2x_1), \quad (3.13)$$

which may be written as a single spectral integral,

$$\frac{1}{M_B}F_V^{(1)}(m_1,m_2) = -\frac{\sqrt{N_c}}{4\pi^2}\int_{(m_1+m_2)^2}^\infty\frac{ds\phi_B(s)}{(s-M_V^2)}\rho_+(s,m_1,m_2). \quad (3.14)$$

The function $\rho_+(s,m_1,m_2)$ is given in (3.8). In the heavy-quark limit $m_b\to\infty$, one finds

$$\frac{1}{M_B}F_V^{(u)} = -\frac{f_B}{\bar{\Lambda}m_b}+\cdots, \qquad \frac{1}{M_B}F_V^{(b)} = -\frac{f_B}{m_b^2}+\cdots \quad (3.15)$$

The dominant contribution in the heavy-quark limit again comes from the process when the light quark emits the photon. As seen from Eqs. (3.11) and (3.15), one finds $F_A = F_V$ in the heavy-quark limit, in agreement with the large-energy effective theory [16].

IV. PHOTON EMISSION FROM THE VECTOR-MESON LOOP. THE FORM FACTORS H_S AND H_P

We now calculate the form factors $H_{P,S}$ using the relativistic dispersion approach. The vector meson in the final state is described in this approach by the vertex $\bar{q}_2(-k_2)\Gamma_\beta q_1(k_1')$, $\quad\Gamma_\beta = (-\gamma_\beta + \frac{(k_1'-k_2)_\beta}{\sqrt{s}+m_1+m_2})G(s)/\sqrt{N_c}$, with $G(s)=\phi_V(s)(s-M_V^2)$, $s=(k_1'+k_2)^2$, $k_1'^2=m_1^2$, and $k_2^2=m_2^2$. The vector-meson wave function ϕ_V is normalized according to [8]

$$\frac{1}{8\pi^2}\int_{(m_1+m_2)^2}^\infty ds\phi_V^2(s)(s-(m_1-m_2)^2)\frac{\lambda^{1/2}(s,m_1^2,m_2^2)}{s} = 1. \quad (4.1)$$

Its decay constant is represented through $\phi_V(s)$ by the spectral integral

$$f_V = \sqrt{N_c}\int_{(m_1+m_2)^2}^\infty ds\phi_V(s) \times\frac{2\sqrt{s}+m_1+m_2}{3}\frac{\lambda^{1/2}(s,m_1^2,m_2^2)}{8\pi^2 s}\frac{s-(m_1-m_2)^2}{s}. \quad (4.2)$$

Now, the form factors $H_{S,P}$ describe the transition of the current with momentum p, $p^2=M_B^2$, to the photon with momentum q_1, $q_1^2=0$, and the vector meson with the momentum q_2, $q_2^2=M_V^2$. Similar to the previous section, we derive the double spectral representations for the form factor in p^2 and q_2^2.

A. Form factor H_S

The form factor H_S is given by the diagrams of Fig. 6. Figure 6(a) shows $H_S^{(d)}$, the contribution to the form factor of the process when the d-quark interacts with the photon; Fig. 6(b) describes the contribution of the process when the quark u interacts while d remains spectator.

Changing the direction of the quark line in the loop diagram of Fig. 6(b) leads to a sign change for the scalar current $j=(m_c-m_d)\bar{d}c$ in the vertex, such that both diagrams in Figs. 6(a) and 6(b) are reduced to the diagram of Fig. 7 which defines the form factor $H_S^{(1)}(m_1,m_2)$; setting $m_1=m_d$, $m_2=m_c$ gives $H_S^{(d)}$, while setting $m_1=m_c$, $m_2=m_d$ gives $H_S^{(c)}$ such that

$$H_S = Q_d H_S^{(d)} - Q_c H_S^{(c)}. \quad (4.3)$$

For the diagram of Fig. 7 (quark 1 emits the photon, quark 2 is the spectator, and all quark lines are on their mass shell), the trace for $q_1^2=0$ reads

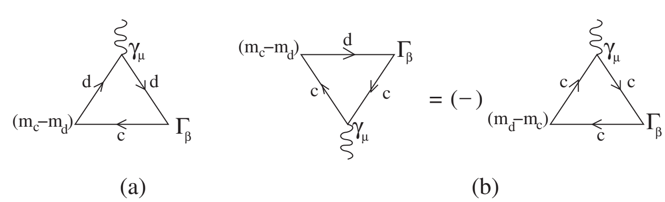

FIG. 6. Diagrams for the form factor H_S: a) $H_S^{(d)}$ and b) $H_S^{(c)}$.

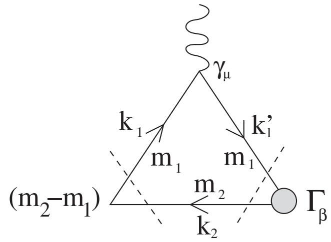

FIG. 7. The triangle diagram for $H_S^{(1)}(m_1, m_2)$.

$$-\mathrm{Sp}((m_1+\hat{k}_1')\gamma_\mu(m_1+\hat{k}_1)(m_2-\hat{k}_2)\Gamma_\beta) = -4(k_1+k_1')_\mu \times (m_1k_2-m_2k_1)_\beta - 4(g_{\mu\beta}q_\alpha - g_{\mu\alpha}q_\beta)(m_1k_2-m_2k_1)_\alpha + 2\frac{(k_1'-k_2)_\beta}{\sqrt{s}+m_1+m_2}(k_1+k_1')_\mu(s-(m_1+m_2)^2). \quad (4.4)$$

The double spectral density of $H_S^{(1)}(m_1, m_2)$ in p^2, $p = k_1 + k_2$, and q_2^2, $q_2 = k_1' + k_2$, is obtained as the coefficient of the structure $g_{\mu\nu}$ after the integration of the trace over the quark phase space. The light-cone representation for the form factor reads

$$H_S^{(1)}(m_1, m_2) = \frac{\sqrt{N_c}}{4\pi^2}\int \frac{dx_1 dx_2 dk_\perp^2}{x_1^2 x_2}\delta(1-x_1-x_2) \times \frac{\phi_V(s)}{s-p^2-i0}(m_2-m_1)\left(m_1x_2 - m_2x_1 + \frac{2k_\perp^2\sqrt{s}}{p^2-M_V^2}\right), \quad (4.5)$$

with s given in terms of $x_{1,2}$ and $k_\perp^2$ by (3.6). The corresponding single dispersion integral has the form

$$H_S^{(1)}(m_1, m_2) = \frac{\sqrt{N_c}}{4\pi^2}\int_{(m_1+m_2)^2}^{\infty} \frac{ds\phi_V(s)}{(s-p^2-i0)}(m_2-m_1) \times \left(\rho_+(s, m_1, -m_2) + \frac{2\sqrt{s}}{p^2-M_V^2}\rho_{k_\perp^2}(s, m_1, m_2)\right), \quad (4.6)$$

where $\rho_+(s, m_1, m_2)$ and $\rho_{k_\perp^2}(s, m_1, m_2)$ were determined earlier in (3.8) and (3.9).

The light-cone representation (4.5) allows us to obtain the behavior of the form factor in the limit $m_Q \to \infty$ for the heavy-light vector meson $\bar{Q}q$ (and assuming $p^2 \sim m_Q^2$),

$$H_S^{(q)} \propto f_V/\bar{\Lambda}, \qquad H_S^{(Q)} \propto f_V/m_Q, \quad (4.7)$$

but one expects a strong numerical suppression because of the partial cancellation of the leading-order contributions.

B. Form factor H_P

The form factor H_P is determined by the two diagrams shown in Fig. 8: Fig. 8(a) gives $H_P^{(d)}$, the contribution of the process when the d-quark interacts with the photon;

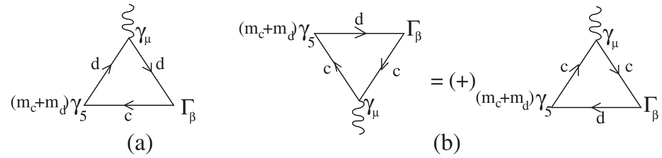

FIG. 8. Diagrams for the form factor H_P: a) $H_P^{(d)}$ and b) $H_P^{(u)}$.

Fig. 8(b) describes the contribution of the process when the c-quark interacts.

We again change the direction of the quark line in the loop diagram of Fig. 8(b) by performing the charge conjugation of the matrix element. For the pseudoscalar current $(m_c + m_d)\bar{d}\gamma_5 c$ in the vertex, the sign does not change, and both diagrams in Figs. 8(a) and 8(b) are reduced to the diagram of Fig. 9 which gives the form factor $H_P^{(1)}(m_1, m_2)$; setting $m_1 = m_d$, $m_2 = m_c$ gives $H_P^{(d)}$, while setting $m_1 = m_c$, $m_2 = m_d$ gives $H_P^{(c)}$ such that

$$H_P = Q_d H_P^{(d)} + Q_c H_P^{(c)}. \quad (4.8)$$

The trace corresponding to the diagram of Fig. 9 (1—active quark, 2—spectator) reads

$$-\mathrm{Sp}((m_1+\hat{k}_1')\gamma_\mu(m_1+\hat{k}_1)\gamma_5(m_2-\hat{k}_2)\Gamma_\beta) = 4i\epsilon_{\mu q_1\beta q_2}(m_1k_2+m_2k_1)_\alpha + 4i\epsilon_{\mu q_1\alpha q_2}\frac{k_{2\alpha}(k_1'-k_2)_\beta}{\sqrt{s}+m_1+m_2}.$$

The light-cone representation of the form factor corresponding to the diagram of Fig. 5 takes the form

$$H_P^{(1)}(m_1, m_2) = \frac{\sqrt{N_c}}{4\pi^2}\int \frac{dx_1dx_2dk_\perp^2}{x_1^2x_2}\delta(1-x_1-x_2) \frac{\phi_V(s)}{s-p^2-i0}(m_1+m_2)\left(m_1x_2+m_2x_1+\frac{k_\perp^2}{\sqrt{s}+m_1+m_2}\right), \quad (4.9)$$

which may be written as a single spectral integral,

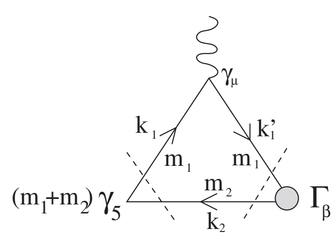

FIG. 9. The triangle diagram for $H_P^{(1)}(m_1, m_2)$. The cuts correspond to calculating the double spectral density in p^2 and q_2^2.

$$H_P^{(1)}(m_1,m_2)=\frac{\sqrt{N_c}}{4\pi^2}\int_{(m_1+m_2)^2}^{\infty}\frac{ds\phi_V(s)}{(s-p^2-i0)}(m_1+m_2)\times\left(\rho_+(s,m_1,m_2)+\frac{\rho_{k_\perp^2}}{\sqrt{s}+m_1+m_2}\right),\tag{4.10}$$

with $\rho_+(s,m_1,m_2)$ and $\rho_{k_\perp^2}$ given in (3.8) and (3.9).

The light-cone representation (4.9) leads to the following large-m_Q behavior of $H_P^{(Q,q)}$ for the heavy-light vector meson $\bar{Q}q$:

$$H_P^{(q)}\to\frac{f_V}{\bar{\Lambda}}\frac{m_Q^2}{m_Q^2-p^2},\qquad H_P^{(Q)}\to\frac{f_V m_Q}{m_Q^2-p^2}.\tag{4.11}$$

For the B decays of interest, we need the value of the form factors $H_{P,S}(p^2,q_1^2=0)$ at $p^2=M_B^2$, which lies above the threshold $(m_c+m_q)^2$. The spectral representations for $H_{P,S}(p^2=M_B^2)$ develop the imaginary parts which occur due to the quark-antiquark intermediate states in the p^2-channel. It should be emphasized that no anomalous cuts emerge in the double spectral representation at $q_1^2\le 0$ [17]. In all cases considered in this paper, the value of $p^2=M_B^2$ lies far above the region of resonances which occur in the quark-antiquark channel. Far above the resonance region, local quark-hadron duality works well, and the calculation of the imaginary part based on the quark diagrams is trustable. The imaginary part turns out to be orders of magnitude smaller than the real part of the form factor and for the practical purpose of the decay-rate calculation may be safely neglected.

V. NUMERICAL RESULTS

The derived spectral representations for the form factors allow one to obtain numerical predictions for the form factors of interest as soon as the parameters of the model—the meson wave functions and the quark masses—are fixed.

A. Parameters of the model

The wave function $\phi_i(s)$, $i=P,V$ can be written as

$$\phi_i(s)=\frac{\pi}{\sqrt{2}}\frac{\sqrt{s^2-(m_1^2-m^2)^2}}{\sqrt{s-(m_1-m)^2}}\frac{w_i(k^2)}{s^{3/4}},\qquad k^2=\lambda(s,m_1^2,m^2)/4s,\tag{5.1}$$

with $w_i(k^2)$ normalized as follows:

$$\int w_i^2(k^2)k^2dk=1.\tag{5.2}$$

The meson weak transition form factors from the dispersion approach reproduce correctly the structure of the heavy-quark expansion in QCD for heavy-to-heavy and heavy-to-light meson transitions, as well as for the meson-photon transitions, if the radial wave functions $w(k^2)$ are localized in a region of the order of the confinement scale, $k^2\le\Lambda^2$ [8].

Following Ref. [9], we make use of a simple Gaussian parametrization of the radial wave function

$$w_i(k^2)\propto\exp(-k^2/2\beta_i^2),\tag{5.3}$$

which satisfies the localization requirement for $\beta\simeq\Lambda_{\mathrm{QCD}}$ and proves to provide a reliable picture of a large family of the transition form factors [9].

In Ref. [9], we fixed the parameters of the quark model—constituent quark masses and the wave-function parameters β_i of the Gaussian wave functions—by requiring that the dispersion approach reproduces (i) meson decay constants and (ii) some of the well-measured lattice QCD results for the form factors at large q^2. The analysis of Ref. [9] demonstrated that a simple Gaussian ansatz for the radial wave functions allows one to reach this goal (to great extent due to the fact that the dispersion representations satisfy rigorous constrains from nonperturbative QCD in the heavy-quark limit). With these few model parameters, Ref. [9] gave predictions for a great number of weak-transition form factors in the full kinematical q^2-region of weak decays; these results were shown to agree with the available results from lattice QCD and QCD sum rules within 10% accuracy in the full q^2-region. We therefore assign a 10% uncertainty to our form-factor estimates in this work.

We use here the same values of the constituent quark masses as obtained in Ref. [9]:

$$m_d=m_u=0.23\ \mathrm{GeV},\qquad m_s=0.35\ \mathrm{GeV},\qquad m_c=1.45\ \mathrm{GeV},\qquad m_b=4.85\ \mathrm{GeV}.\tag{5.4}$$

With the quark masses (5.4) and the meson wave-function parameters β quoted in Table I, the decay constants from our dispersion approach reproduce the best-known decay constants of pseudoscalar and vector mesons also summarized in Table I.

B. $B\to\gamma$ and $\gamma\to V$ form factors

Before turning to the numerical estimates, let us emphasize that, as obvious from (3.11),(3.15), (4.11), and (4.7), the photon emission from the V-loop and from the B-loop have the same scaling behavior in the heavy-quark limit. Therefore, *a priori*, there is no valid reason to neglect the V-loop contributions. We shall see that indeed the photon emission from the vector-meson loop gives the contribution of a similar size as the photon emission from the B-meson loop. Our numerical estimates for the necessary form factors are summarized in Tables II and III.

TABLE I. Meson masses from Ref. [18], leptonic decay constants, and the corresponding wave-function parameters β [19].

	B	B_s	D^*	D_s^*	J/ψ
M (GeV)	5.279	5.370	2.010	2.11	3.097
f (MeV)	192 ± 8 [20]	226 ± 15 [20]	248 ± 2.5 [21]	311 ± 9 [21]	405 ± 7 [18,22]
β (GeV)	0.565	0.62	0.48	0.54	0.68

TABLE II. The form factors $F_A(M_V^2)$ and $F_V(M_V^2)$ describing the $B \to \gamma$ and $B_s \to \gamma$ transition for $V = J/\psi, D_s^*, D^*$.

$B_s \to \gamma$	$M_V^2 = M_\psi^2$	$M_V^2 = M_{D_s^*}^2$	$M_V^2 = M_{D^*}^2$	$B \to \gamma$	$M_V^2 = M_\psi^2$	$M_V^2 = M_{D_s^*}^2$	$M_V^2 = M_{D^*}^2$
$F_V^{(b)}(M_V^2)$	−0.060	−0.048	−0.046	$F_V^{(b)}(M_V^2)$	−0.054	−0.044	−0.043
$F_V^{(s)}(M_V^2)$	−0.410	−0.328	−0.322	$F_V^{(u,d)}(M_V^2)$	−0.388	−0.316	−0.310
$F_A^{(b)}(M_V^2)$	0.074	0.059	0.058	$F_A^{(b)}(M_V^2)$	0.066	0.052	0.050
$F_A^{(s)}(M_V^2)$	0.324	0.279	0.276	$F_A^{(u,d)}(M_V^2)$	0.304	0.268	0.264

TABLE III. The form factors $H_P(p^2)$ and $H_S(p^2)$, describing the $\gamma \to V$ transition ($V = J/\psi, D_s^*, D^*$) for $p^2 = M_B^2$. The difference between the form factors at $p^2 = M_B^2$ and $p^2 = M_{B_s}^2$ is negligible and may be safely ignored. One finds $\mathrm{Im}H_{P,V}(M_B^2) \ll \mathrm{Re}H_{P,V}(M_B^2)$, and thus $\mathrm{Im}H_{P,V}(M_B^2)$ may be safely neglected for the decay-rate calculations.

	$\gamma \to J/\psi$		$\gamma \to D_s^*$ ($q = s$)		$\gamma \to D^*$ ($q = u, d$)	
	$p^2 = M_B^2$	$p^2 = M_{B_s}^2$	$p^2 = M_B^2$	$p^2 = M_{B_s}^2$	$p^2 = M_B^2$	$p^2 = M_{B_s}^2$
$H_P^{(c)}(M_B^2)$	−0.196	−0.183	−0.044	−0.042	−0.032	−0.030
$H_P^{(q)}(M_B^2)$	…	…	−0.096	−0.092	−0.081	−0.078
$H_S^{(c)}(M_B^2)$	0	0	0.016	0.015	0.014	0.013
$H_S^{(q)}(M_B^2)$	…	…	−0.007	−0.006	−0.002	−0.001

C. Decay rates

We have now everything for the calculation of the amplitudes and the decay rates. We consider several annihilation-type B decays which have the highest probabilities; the weak-annihilation quark diagrams which induce these decays are shown in Fig. 10.

The corresponding F_{PC} and F_{PV} and the decay rates are summarized in Table IV. To highlight the contribution to the amplitudes coming from the photon emission from the V-meson loop, we multiply it by a coefficient r which is set to unity in the decay-rate calculations. Obviously, for some modes, the photon emission from the vector-meson loop is comparable or even exceeds the photon emission from the B-meson loop and thus should be taken into account.

For the scale-dependent Wilson coefficients $C_i(\mu)$ and $a_{1,2}(\mu)$ at the renormalization scale $\mu \simeq 5$ GeV, we use the following values [12]: $C_1 = 1.1$, $C_2 = -0.241$, $a_1 = C_1 + C_2/N_c = 1.02$, and $a_2 = C_2 + C_1/N_c = 0.15$. Similar values are used for numerical estimates in Ref. [5]: e.g., for $B_{(s)} \to J/\psi\gamma$ decay, $a_2 = 0.15$ in our analysis corresponds to the effective Wilson coefficient $\bar{a}_q = 0.163$.

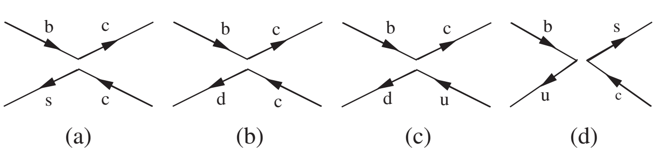

FIG. 10. Four-quark operators inducing the annihilation B decays listed in Table IV. (a) $\bar{B}_s^0 \to J/\psi\gamma$; (b) $\bar{B}_d^0 \to J/\psi\gamma$; (c) $\bar{B}_d^0 \to D^{*0}\gamma$; (d) $B^- \to D_s^{*-}\gamma$.

VI. DISCUSSION AND CONCLUSIONS

We have analyzed the annihilation-type radiative B decays in the naive factorization approximation, taking into account both the photon emission from the B-meson loop and the vector-meson loop (V-loop). The latter contribution was not taken into account in all previous analyses and is therefore the novel feature of this paper. We have shown that, in general, the photon emission from the V-loop is not suppressed compared to the photon emission from the B-loop and gives a comparable contribution. Our main results are as follows:

(i) We calculated the form factors F_A and F_V describing the photon emission from the B-loop and the form factors H_P and H_S describing the photon emission from the V-loop, making use of the relativistic dispersion approach based on the constituent quark picture. The form factors from this method satisfy all

TABLE IV. The amplitudes and the branching ratio for the annihilation-type decay of B and B_s.

Reaction	CKM factor	$F_{\rm PC}$ (GeV)	$F_{\rm PV}$ (GeV)	Br
$\bar{B}_s^0 \to J/\psi\gamma$	$a_2 V_{cb} V_{cs}^*$	$0.036 - 0.052r$	0.020	$1.43 \times 10^{-7} (\frac{a_2}{0.15})^2$
$\bar{B}_d^0 \to J/\psi\gamma$	$a_2 V_{cb} V_{cd}^*$	$0.035 - 0.050r$	0.021	$7.54 \times 10^{-9} (\frac{a_2}{0.15})^2$
$\bar{B}_d^0 \to D^{*0}\gamma$	$a_2 V_{cb} V_{ud}^*$	$0.012 - 0.014r$	$0.007 + 0.002r$	$4.33 \times 10^{-8} (\frac{a_2}{0.15})^2$
$B^- \to D_s^{*-}\gamma$	$a_1 V_{ub} V_{cs}^*$	$-0.025 + 0.001r$	$-0.014 + 0.002r$	$1.68 \times 10^{-7} (\frac{a_1}{1.02})^2$

rigorous constraints from QCD in the heavy-quark limit for heavy-to-heavy, heavy-to-light, and heavy-meson-photon transition form factors. The numerical parameters of the model such as the effective constituent quark masses and the nonperturbative meson wave functions have been fixed in Ref. [9] by fitting to well-known leptonic decay constants of heavy mesons and a few well-measured form factors from lattice QCD. The predictions from the dispersion approach to the transition form factors were then tested in many B and D decays and agree quite well with the available results from lattice QCD and QCD sum rules with the accuracy of a few percent [7]. So, we assign a 10% uncertainty in the decay rate related to the form-factor uncertainties.

We emphasize that the photon emission from the V-loop has no parametric suppression compared to the photon emission from the B-loop and therefore cannot be neglected. Moreover, the numerical impact of the photon emission from the V-loop is substantial; for instance, in the case of the $\bar{B}_s \to J/\psi\gamma$ decay, taking into account the photon emission from both s- and b-quarks in the B-loop and the photon emission from the V-loop leads to a strong 60% suppression of the decay rate compared to the result based on merely the photon emission by the light quark of the B-meson.

(ii) Making use of our results for the form factors and employing naive factorization for the complicated amplitudes of the four-quark operators, we obtain predictions for the annihilation-type decays with the largest branching fractions:

$$\mathcal{B}(\bar{B}_s^0 \to J/\psi\gamma) = 1.43 \times 10^{-7}\left(\frac{a_2}{0.15}\right)^2, \quad (6.1)$$

$$\mathcal{B}(\bar{B}_d^0 \to J/\psi\gamma) = 7.54 \times 10^{-9}\left(\frac{a_2}{0.15}\right)^2, \quad (6.2)$$

$$\mathcal{B}(\bar{B}_d^0 \to D^{0*}\gamma) = 4.33 \times 10^{-8}\left(\frac{a_2}{0.15}\right)^2, \quad (6.3)$$

$$\mathcal{B}(B^- \to \bar{D}_s^{*-}\gamma) = 1.68 \times 10^{-7}\left(\frac{a_1}{1.02}\right)^2. \quad (6.4)$$

We would like to emphasize a relatively large branching ratio of the $\bar{B}^- \to \bar{D}_s^{*-}\gamma$ decay which makes this mode a prospective candidate for the experimental studies in the near future.

(iii) Uncertainties in our predictions listed above come from the two sources: (a) as just mentioned above, an approximate model for the form factors which yields an error in the decay rate at the level of 10%–15% and (b) naive factorization of the complicated four-quark operators. The accuracy of the naive factorization for the decay rates may be probed to some extent by variations of the scale μ in the scale-dependent Wilson coefficients $C_i(\mu)$ (recall that the amplitudes $A_{\rm PV}$ and $A_{\rm PC}$ are scale independent). Another way to access the size of the nonfactorizable corrections was indicated in Ref. [3], where the nonfactorizable corrections in heavy-to-heavy radiative decays have been related to nonfactorizable corrections in the $B - \bar{B}$ oscillations. The latter have been found to be at the level of a few percent [23]. On the basis of these arguments, one does not expect corrections to factorization larger than 5%–10%. We therefore assign here a 10% uncertainty to the branching ratios related to nonfactorizable contributions.

In view of this argument, huge negative corrections to factorization in $B_s^0 \to J/\psi\gamma$ reported in Ref. [5], which lead to a suppression of the decay rate by almost a factor 30, seem unrealistic. The correction to naive factorization in Ref. [5] has been calculated within the formalism of Ref. [24]. Reference [5] reported a strong cancellation between the factorizable contribution and the radiative correction calculated using QCD factorization [24]. However, the authors of Ref. [5] have not taken into account several other contributions to the amplitude (e.g., the photon emission from the charm loop). Therefore, the huge reduction of the branching ratio reported in Ref. [5] does not seem to us trustable, and the analysis of nonfactorizable effects should be revised. Reference [5] reports also the branching ratios based on factorization approximation; however, the factorization results of Ref. [5] neglect several effects (photon emission from the heavy quark of the B-loop and photon emission from the V-loop) which lead to a visible suppression of the branching ratio. So we also do not confirm the factorization results of Ref. [5].

Reference [6] reported another estimate for $\bar{B}_s \to J/\psi\gamma$ based on naive factorization, neglecting the photon emission from the V-loop.[1] However, the huge form factors $F_{A,V}$ reported in Ref. [6] clearly contradict the results from the large-energy effective theory [13,15], and therefore the results of Ref. [6] cannot be trusted.

In conclusion, we believe that in comparison with the existing estimates [5,6] credit should be given to our results. First, we take into account those contributions which have been neglected in Refs. [5,6] but which are shown to give sizeable contributions to the amplitude. Second, our calculation of the form factors is based on a more detailed model for the B-meson structure than the models employed in Refs. [5,6]. Taking into account the uncertainties mentioned above (the scale in the Wilson coefficients, making use of the factorization approximation for the weak-annihilation amplitude, and uncertainties in the form factors), we estimate the accuracy of our theoretical predictions for the branching ratios (6.1)–(6.4) to be at the level of 20%.

[1]Reference [6] contains an erroneous statement that the photon emission from the V-loop vanishes for the equal quark masses in the vector meson. The cancellation between the photon emission from the quark and from the antiquark of the V-meson occurs for the form factor H_S, whereas for the form factor H_P, the amplitudes of these two subprocesses add to each other.

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