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Temporal instability enables neutrino flavor conversions deep inside supernovae

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We show that a self-interacting neutrino gas can spontaneously acquire a nonstationary pulsating component in its flavor content, with a frequency that can exactly cancel the "multiangle" refractive effects of dense matter. This can then enable homogeneous and inhomogeneous flavor conversion instabilities to exist even at large neutrino and matter densities, where the system would have been stable if the evolution were strictly stationary. Large flavor conversions, especially close to a supernova core, are possible via this novel mechanism. This may have important consequences for the explosion dynamics, nucleosynthesis, as well as for neutrino observations of supernovae.

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I. INTRODUCTION

Inside a supernova (SN), neutrino densities are so high that neutrino flavor oscillations are affected not only by ordinary matter, but also by the neutrinos themselves [1–4]. Neutrino-neutrino interactions lead to highly correlated collective flavor conversions and unexpected effects, which completely change the physics of supernova neutrinos [5–23]. See Refs. [24,25] for recent reviews. In this article, we show that time-dependent fluctuations lead to a novel effect that can enable flavor conversion deeper in the SN than previously realized.

Above the neutrinosphere, in the absence of collisions, the dynamics of a dense neutrino gas is characterized in terms of "density matrices" in flavor space, $\varrho(t, \mathbf{x}, E, \mathbf{v})$ for neutrinos with energy E and velocity $\mathbf{v}$ at position $\mathbf{x}$ and time t. These obey the kinetic equations [21,26,27],

$$i(\partial_t + \mathbf{v}\cdot\nabla)\varrho = [\mathsf{H}, \varrho]. \qquad (1)$$

On the left-hand side (lhs), the first term accounts for explicit time dependence, while the second, proportional to the neutrino velocity $\mathbf{v}$, is the drift term due to neutrino free streaming. On the right-hand side (rhs), $\mathsf{H}(t, \mathbf{x}, E, \mathbf{v})$ is the Hamiltonian matrix in flavor space containing the neutrino mass-square matrix and potentials due to matter and neutrinos.

Flavor evolution of the dense neutrino gas, as governed by Eq. (1), has a highly complex structure. It depends on the four time and space coordinates, the four energy and velocity coordinates (with $|\mathbf{v}| = 1$, in our ultrarelativistic approximation), as well as the flavor states of all neutrinos. In order to reduce this complexity, symmetries in the neutrino flavor evolution have often been assumed. For neutrinos in a SN environment, all of the previous literature is based on the assumption that the evolution is stationary, i.e., there is no explicit time dependence, or only a slow/small time dependence that does not significantly affect the flavor evolution. Additionally, under the assumption of a spherically symmetric neutrino emission, the dynamics reduces to a one-dimensional evolution along the radial coordinate. This is the rationale behind the often-used "bulb model" [5,7].

These symmetry assumptions, viz., temporal stationarity and spatial homogeneity, have been recently criticized because self-interacting neutrinos can spontaneously break these space-time symmetries. Indeed, studies on simple toy models show that the translation symmetries in time [28,29] and space [30–34] are not stable. Even tiny inhomogeneities may lead to new flavor instabilities [30–32] that can develop also at large neutrino densities, as above the SN core, where oscillations are otherwise expected to be suppressed due to synchronization. However, large neutrino densities in a SN are typically accompanied by a large matter density [34], which produces "multiangle matter effects" [35] that suppress both homogeneous and inhomogeneous instabilities. The current understanding is then that neutrinos cannot change their flavor too close to the SN core.

Flavor conversions at small distances from the SN core would have major consequences for SN explosions, nucleosynthesis, as well as neutrino observations of nearby supernovae (SNe). If conversions are possible below the shock radius, neutrinos can provide a net positive energy to the shock and assist SN explosions [36–39]. Similarly, the neutron-to-proton ratio can be changed deeper inside a star, affecting the yield of heavier nuclei created through the r-process mechanism [40,41]. Also, in order to interpret any potential observation of neutrinos from SNe, current and proposed neutrino experiments depend crucially on understanding where and how the flavor-dependent neutrino fluxes have converted to each other [25,42–44]. Now that Gd-doping in Super-Kamiokande [45] has been approved

[46], the imminent observation of the diffuse background of SN neutrinos may raise this issue [47], even without a galactic SN.

In the following, using linear stability analysis we show the presence of an unstable pulsating mode that leads to flavor conversion at high neutrino and matter density. The key insight is that the frequency of pulsation can undo the phase dispersion due to a large matter density. As a result, flavor instabilities, which would have grown only if matter effects were small, can now develop at large neutrino and matter densities. Then, to demonstrate that this linear instability survives in the nonlinear regime, we numerically calculate the flavor evolution in a simplified model and show that flavor conversions indeed occur at large neutrino and matter densities when there are space and time-dependent fluctuations. Finally, we discuss the implications for SN neutrinos and conclude.

II. LINEAR ANALYSIS FOR A GENERAL SCENARIO

Assuming that the neutrinos are initially in flavor eigenstates, their density matrices $\varrho(t,\mathbf{x},E,\mathbf{v})$ can be written in a two-flavor framework as

$$\varrho = \frac{\mathrm{Tr}(\varrho)}{2} + \frac{n_\nu}{2} g \begin{pmatrix} 1 & S \\ S^* & -1 \end{pmatrix}, \tag{2}$$

to linear order in $S(t,\mathbf{x},E,\mathbf{v})$ [48]. The quantity $g(t,\mathbf{x},E,\mathbf{v})$ is the energy and angular distribution of neutrinos from the source and n_ν is an arbitrary normalization constant, with dimensions of number density, for making S dimensionless. A nonzero off-diagonal element S represents flavor conversions. For antineutrinos, $\bar{\varrho}(E) \equiv -\varrho(-E)$, extending the physical range of E from $-\infty$ to $+\infty$. The Hamiltonian for the flavor evolution is

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_F \mathsf{N}_l + \sqrt{2}G_F \int d\Gamma' (1 - \mathbf{v}\cdot\mathbf{v}')\varrho', \tag{3}$$

where sans-serif quantities are 2×2 matrices in flavor space. Namely, M^2 is the neutrino mass-squared matrix, while $\sqrt{2}G_F\mathsf{N}_l$ and the last term on the rhs appear due to forward scattering on matter [49] and neutrinos [1,2], respectively. The integral is over all neutrino energies and velocities, i.e., $\int d\Gamma' = \int_{-\infty}^{+\infty} dE' E'^2 \int d\mathbf{v}'/(2\pi)^3$.

In a nonisotropic neutrino gas, as in the case of neutrinos streaming off a SN core, there is a net neutrino current so that neutrinos moving in different directions, i.e., with different $\mathbf{v}$, acquire different phases via the velocity-dependent terms, i.e., $(1-\mathbf{v}\cdot\mathbf{v}')$ in H and $\mathbf{v}\cdot\nabla$ from the drift term in Eq. (1). These are multiangle effects that arise due to the current-current nature of the low-energy weak interactions and the source geometry. Typically, they inhibit the collective behavior of the flavor evolution, but can also lead to flavor decoherence [50–52].

Using Eqs. (2)–(3) in Eq. (1), and taking a vanishing mixing angle, we find the equation for flavor evolution,

$$i(\partial_t + \mathbf{v}\cdot\nabla)S = [-\omega + \lambda + \mu \int d\Gamma'(1-\mathbf{v}\cdot\mathbf{v}')g']S - \mu \int d\Gamma'(1-\mathbf{v}\cdot\mathbf{v}')g'S', \tag{4}$$

where the relevant energy scales are the neutrino oscillation frequency in vacuum $\omega = \Delta m^2/(2E)$, the matter potential $\lambda = \sqrt{2}G_F n_e$, and the neutrino potential $\mu = \sqrt{2}G_F n_\nu$. Note that μ always appears in product with S, making the precise choice of n_ν immaterial.

Let us consider the evolution of S along the radial distance r, while Fourier decomposing it in t and the spatial coordinates transverse to $\hat{r}$, viz., $\mathbf{r}_T$. We take the spectrum $g'(t,\mathbf{x},E,\mathbf{v})$ to be independent of time and space, i.e., $g'(t,\mathbf{x},E,\mathbf{v}) \equiv g'(E,\mathbf{v})$, so that it does not get Fourier transformed. Explicitly,

$$S = \int_{-\infty}^{+\infty} dp d\mathbf{k} e^{-i(pt+\mathbf{k}\cdot\mathbf{r}_T)} Q_{p,\mathbf{k}} e^{-i\Omega_{p,\mathbf{k}} r}, \tag{5}$$

where $Q_{p,\mathbf{k}} e^{-i\Omega_{p,\mathbf{k}} r}$ is the Fourier coefficient of a flavor evolution mode with temporal pulsation p and inhomogeneity wave vector $\mathbf{k}$. Inserting this ansatz into Eq. (4), using $\mathbf{v}\cdot\nabla = v_r\partial_r + \mathbf{v}_T\cdot\nabla_T$, and dividing by the radial velocity v_r, we find an eigenvalue equation for $Q_{p,\mathbf{k}}(E,\mathbf{v})$,

$$\left[\frac{-\omega + \bar{\lambda} - p - \mathbf{v}_T\cdot\mathbf{k}}{v_r} - \Omega_{p,\mathbf{k}}\right] Q_{p,\mathbf{k}} = \frac{\mu}{v_r}\int d\Gamma'(1-\mathbf{v}\cdot\mathbf{v}')g'Q'_{p,\mathbf{k}}, \tag{6}$$

where $\bar{\lambda} = \lambda + \mu\int d\Gamma'(1-\mathbf{v}\cdot\mathbf{v}')g'$ encodes "matter" effects from both matter and neutrinos. Note that in the linear regime, different Fourier modes are not coupled. A growing solution to this equation, with $\mathrm{Im}(\Omega) > 0$, signals that there is an instability.

For a stationary system one finds growing solutions even at a large neutrino density, if inhomogeneities are present, i.e., $\mathbf{k} \neq 0$, as long as $\bar{\lambda} \ll \mu$ [30,31,34]. In a SN however, $\bar{\lambda}$ is also large when μ is large and these instabilities are typically not realized [34]. The reason for this is clear: One cannot simultaneously obtain growing solutions for all velocities, because the inhomogeneous term, the matter term, and the μ-dependent neutrino-neutrino interaction term on the rhs are all large, i.e., $\mathbf{v}_T\cdot\mathbf{k}, \bar{\lambda}, \mu \gg \omega$, but have different velocity dependences which cannot completely cancel against each other.

The nonstationary system has an innocuous-looking but important difference with respect to the stationary system. nonstationarity lowers $\bar{\lambda}$ by p, i.e., $\bar{\lambda} \to \bar{\lambda} - p$, as also seen in Ref. [29]. More importantly, a fact not realized so far is that this $p \neq 0$ term has the same multiangle dependence as

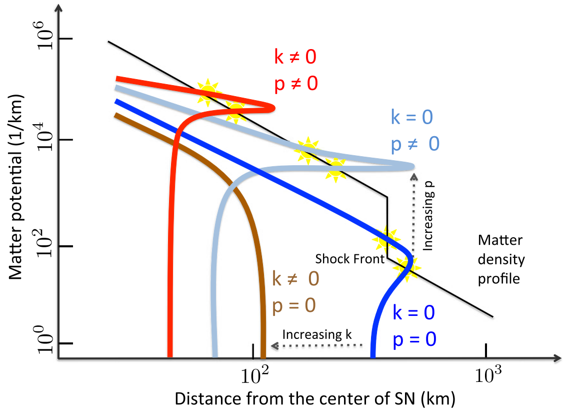

FIG. 1 (color online). Schematic of the SN matter potential (thin line) and densities where neutrino flavor composition is unstable to small oscillations with spatial inhomogeneity wave number k and frequency of time dependence p (thick lines) [53]. Time-dependent fluctuations can be unstable at higher matter density and lead to neutrino flavor conversions deeper in a SN.

$\bar{\lambda}$. Therefore, if one allows for a nonstationary solution, the neutrino system with a pulsation $p \simeq \bar{\lambda}$ can undo the phase dispersion due to a large matter term for all velocities. Thus, one can find growing solutions, with $\mathrm{Im}(\Omega) > 0$, to the eigenvalue equation [Eq. (6)], as previously for the $\bar{\lambda} \ll \mu$ scenario. The eigenvalues are identical to those in Sec. IV B 4 of Ref. [34] with the shift $\bar{\lambda} \to \bar{\lambda} - p$. These solutions are highly oscillatory in space ($\mathbf{k} \neq 0$) and time ($p \neq 0$), and would lead to flavor averaging. *This is our main result.*

In Fig. 1, we illustrate this general idea in the context of SN neutrinos. The thin black line shows a SN density profile, while the thick colored lines schematically show where flavor instabilities with certain pulsation p and wave number $k = |\mathbf{k}|$ can grow, i.e., have $\mathrm{Im}(\Omega) > 0$. The inhomogeneous instability, i.e., with $k \neq 0$, is always *below* the homogeneous $k = 0$ instability [34], and never occurs for the physically available matter density. However, $p \simeq \bar{\lambda}$ can *raise* these $k = 0$ and $k \neq 0$ instabilities, making them unstable at low radii. They can now develop at large μ and λ, i.e., at a small radius, with a temporal oscillation of frequency $\simeq p$. This can happen for both normal and inverted neutrino mass ordering.

How does this linear instability evolve when linear theory is no longer appropriate? What is its impact? To answer these questions, we must numerically solve the equations of motion (EoMs), Eq. (1), in the fully nonlinear regime. This is what we do next.

III. NONLINEAR ANALYSIS FOR THE TWO-BEAMS MODEL

The new effect, relevant to SN neutrinos, is that a stationary system, which is stable to both homogeneous and inhomogeneous perturbations at large μ and λ, becomes unstable therein when nonstationarity is allowed. This requires simulating a system with temporal nonstationarity, spatial inhomogeneity, and multiangle matter effects, which is extremely challenging and has not been attempted so far. Here, we present the first simulation with these three features.

The simplest model that can accommodate the required features is the neutrino "line model" [30,32,34]. In this model, one considers monochromatic neutrinos emitted in two directions, "L" and "R," from an infinite plane at $z = 0$. Assuming translational invariance along the y-direction, the flavor evolution along $z > 0$ can be characterized on the two-dimensional plane spanned by the x and z coordinates. The neutrino emission modes L and R are labeled in terms of their velocities, i.e., $\mathbf{v}_L = (v_{x,L}, 0, v_{z,L}) = (\cos\vartheta_L, 0, \sin\vartheta_L)$, where $\vartheta_L \in [0, \pi]$ is the emission angle, and similarly for $\mathbf{v}_R$. Thus, for the L mode, the differential operator on the lhs in Eq. (1) takes the form

$$\partial_t + \mathbf{v}_L \cdot \nabla = \partial_t + v_{x,L}\partial_x + v_{z,L}\partial_z, \tag{7}$$

while the Hamiltonian in Eq. (3) becomes

$$\mathsf{H}_L = \frac{-\omega + \lambda}{2}\sigma_3 + \mu(1 - \mathbf{v}_L \cdot \mathbf{v}_R)[(1+\epsilon)\varrho_R - \bar{\varrho}_R], \tag{8}$$

where σ_3 is the diagonal Pauli matrix, and ϵ is the neutrino-antineutrino asymmetry, i.e., $1+\epsilon = (n_{\nu_e} - n_{\nu_x})/(n_{\bar{\nu}_e} - n_{\bar{\nu}_x})$, with ν_x being a nonelectron flavor. The normalization $n_\nu = n_{\bar{\nu}_e} - n_{\bar{\nu}_x}$ is used to define $\mu = \sqrt{2}G_F(n_{\bar{\nu}_e} - n_{\bar{\nu}_x})$.

The differential operator in Eq. (7) shows that the flavor evolution is determined by a partial differential equation in one temporal and two spatial dimensions. By Fourier transforming the EoMs in t and x, as in Eq. (5), one obtains a tower of ordinary differential equations in the z coordinate for the different Fourier modes $\varrho_{p,k}$ with temporal pulsation p and spatial wave number k. In the nonlinear regime, the EoMs for the different Fourier modes have a convolution term due to interactions between the different modes [32].

If $v_{z,L} = v_{z,R}$, as assumed in Refs. [30,32], even a very large matter term λ can be rotated away from the EoMs by studying the flavor evolution in a suitable corotating frame [6]. Conversely, if $v_{z,L} \neq v_{z,R}$ the matter term leads to frequencies $\lambda/v_{z,L}$ for the L mode and $\lambda/v_{z,R}$ for the R mode. Their difference cannot be removed and gives multiangle matter effects [35]. If $\lambda \gg \mu$, this phase difference between L and R modes is so large that it suppresses the self-induced flavor conversions from both $k = 0$ and $k \neq 0$ instabilities [34,35]. Conversely, if one allows for a nonstationary solution, the neutrino system selects the pulsations $p \simeq \bar{\lambda}$ that compensate the phase dispersion due to a large matter term, and generate growing instabilities, in particular, at small scales associated with spatial inhomogeneities.

To quantitatively illustrate this claim, we take the source at $z = 0$ to emit only ν_e and $\bar{\nu}_e$, with a factor of two excess of ν_e over $\bar{\nu}_e$, i.e., $\epsilon = 1$. We choose $\theta = 10^{-3}$ and a normal mass ordering, i.e., $\omega > 0$, but the result would be similar for the inverted ordering, i.e., $\omega < 0$. The overall frequency scale is set by $\omega = 1$. A large $\mu = 40$ is chosen, so that oscillations are suppressed in the homogeneous case, as in a SN. We take the L and R modes to have two different angles $\vartheta_R = 5\pi/18$ and $\vartheta_L = 7\pi/9$, so that a large matter potential, $\lambda = 4 \times 10^4$, suppresses the inhomogeneous modes, mimicking the similar effect in a SN.

In order to make k and p dimensionless, k is expressed in multiples of the vacuum oscillation frequency, i.e., $k = n_k\omega$, while p is expressed in multiples of the matter potential $\bar{\lambda}$, i.e., $p = n_p\bar{\lambda}$. For simplicity, we limit ourselves to the $n_p = 1$ mode. In this way, in the nonlinear regime we only have to consider the convolution among the different Fourier modes associated with spatial inhomogeneities, $\sim\sum_{j_k}[((1+\epsilon)\varrho_{R,n_k-j_k} - \bar{\varrho}_{R,n_k-j_k}), \varrho_{L,j_k}]$, and analogously for the R mode [32]. We include modes up to $n_k = 600$ but ensure that $n_k > 400$ remain empty. This trick avoids "spectral blocking" that leads to a spurious rise of the Fourier coefficients at large n_k due to truncation of the tower of equations [54]. To seed the spatial inhomogeneity, we use numerical noise of $\mathcal{O}(10^{-8})$ for all modes.

Figure 2 shows the flavor dynamics of this two-beams model. Amplitudes of flavor conversion, $\log_{10}|\varrho^{e\mu}_{n_k}(z)|$, are shown at distance z for various Fourier modes n_k of inhomogeneity along x, in the presence of time-dependent fluctuations with frequency $p = \bar{\lambda}$. As linear theory predicts [$\mathrm{Im}(\Omega)$ vs n_k shown in the inset], modes around $n_k \simeq 100$ are the most unstable and grow first with the predicted rate $\mathrm{Im}(\Omega)/\omega \simeq 3$. However, while only the modes $n_k \simeq \mathcal{O}(10^2)$ are unstable according to linear analysis, a cascade in Fourier space develops in both smaller and larger n_k due to the convolution enforced by the neutrino-neutrino interaction. Modes with $n_k < 10^2$ also grow fast when nonlinearity sets in. The flavor composition begins to oscillate with many frequencies and appears to be "averaged out."

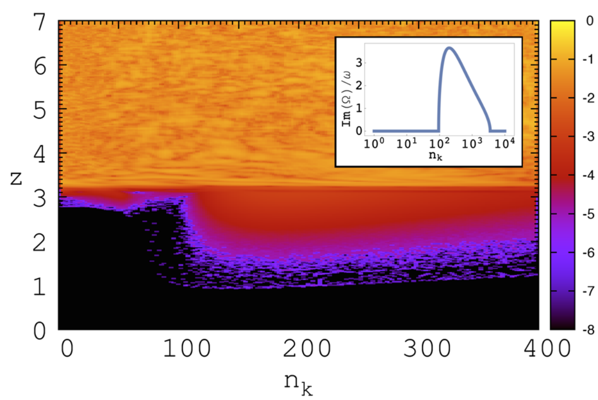

FIG. 2 (color online). Amplitudes of flavor conversion at distance z in our two-beams model, $\log_{10}|\varrho^{e\mu}_{n_k}(z)|$, for the n_k Fourier modes of inhomogeneity along the transverse direction. Time-dependent pulsations of frequency $p = \bar{\lambda}$ and large neutrino and matter densities are included. Inset: Linear growth rates $\mathrm{Im}(\Omega)/\omega$ for a larger range of n_k (note the log scale). Modes $n_k \simeq 10^2$ grow first and quickly excite all other modes, leading to large flavor conversion.

Thus, we find that neutrinos can change flavor at large λ and μ, if nonstationary solutions with frequency $p \simeq \lambda$ are allowed. We expect that the cascade in Fourier space leads to "flavor decoherence," or approximate equilibration between all flavors [55]. In a SN, this may have important consequences, which we discuss below.

IV. DISCUSSION AND CONCLUSIONS

An important question is whether this effect can be important in a SN. Here, we provide a back-of-the-envelope estimate. To aid shock revival, flavor instability has to occur below the shock front at $r \simeq 200$ km and above the gain radius at $r \simeq 100$ km (see, e.g., Fig. 4 in Ref. [38]). One thus needs pulsations of high frequency $p \simeq \lambda \sim 10^{3-6}\ \mathrm{km}^{-1} \sim 3 \times 10^{8-11}$ Hz. Between $r \simeq 100$ km and 150 km, a typical instability with growth rate $\mathrm{Im}(\Omega)/\omega \simeq 3$ then grows by $\sim$60 e-foldings, i.e., a factor of $\simeq 10^{26}$, for 15 MeV neutrinos with $\omega \sim 0.4\ \mathrm{km}^{-1}$, assuming constant growth. Can such high-frequency fluctuations occur in a SN with even tiny amplitudes? In this context, we find it intriguing that pair correlations of the neutrino field, which are many-body corrections to the single-particle density matrices, lead to relative number fluctuations of a size $\kappa^2 \sim (\lambda\beta/E)^2 \sim 10^{-22}$, where $\beta \simeq 10^{-2}c$ is the typical speed of ordinary matter in SN, which oscillates with a frequency $\sim 2E \sim 10^{22}$ Hz for 15 MeV neutrinos, as shown in Ref. [23]. However, in a realistic SN the density behind the shock, though often much flatter than that shown in Fig. 1, is not *exactly* constant and the growth rates decrease when $p \not\simeq \bar{\lambda}$. On the other hand, nearby p modes are then excited and nonlinear coupling of modes makes all instabilities grow. More detailed studies are needed to study these effects [56], the impact on nucleosynthesis, and overall flavor conversion.

Potentially, the consequences of our finding augur another paradigm shift in the understanding of self-induced conversions and on their impact on the SN dynamics. The possibility of low-radii conversions behind the stalled shock wave during the accretion phase, suppressed by the large matter term in the stationary and homogeneous case [57–61], implies that the flavor dynamics may need to be taken into account in the revitalization of the shock wave [36,38]. Also, the impact on nucleosynthesis in a SN would be important [40,41]. With flavor equilibration, the interpretation of observed SN fluxes may also become simpler [55].

However, the possibility of flavor conversions close to the neutrinosphere in a SN also questions the assumption that flavor conversions safely occur outside it. This assumption allowed one to replace the full Boltzmann equations, containing both oscillations and scatterings, with the flavor oscillation equations for free-streaming neutrinos. In a nonstationary situation, this assumption may no longer be guaranteed and may imply the necessity to simultaneously perform the neutrino transport and flavor evolution [21]. This is a formidable problem that would require new computational techniques. Furthermore, it is possible that these instabilities (inhomogeneity and nonstationarity) appear in a regime where the coarse-grained description adopted using density matrices [2] is insufficient. Although we have used this standard description here, as a first step, this is a more fundamental aspect that needs further study.

In conclusion, we have presented the first study of nonlinear effects of nonstationarity in a dense neutrino gas. We have pointed out novel temporal instabilities that can dramatically affect flavor evolution, and raise the possibility of self-induced flavor conversions deep in a SN. The discovery of the role of symmetry breaking in the flavor evolution of SN neutrinos is opening completely new directions of investigation. We foresee that many surprises are still in store.

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