Spontaneous-scalarization-induced dark matter and variation of the gravitational constant

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We propose a new scalar-tensor model which induces significant deviation from general relativity inside dense objects like neutron stars, while passing the Solar System and terrestrial experiments, extending a model proposed by Damour and Esposito-Farese. Unlike their model, we employ a massive scalar field, dubbed the "asymmetron," that not only realizes proper cosmic evolution but can also account for the cold dark matter. In our model, the asymmetron undergoes spontaneous scalarization inside dense objects, which results in the reduction of the gravitational constant by a factor of order unity. This suggests that observational tests of the constancy of the gravitational constant in the high-density phase are effective ways to study the asymmetron model.

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I. INTRODUCTION

General relativity(GR), which describes gravity in terms of a massless spin 2 field, has been tested and passed all the precision experimental tests such as the Solar System and the terrestrial experiments [1]. No observations which clearly contradict with predictions of GR have been found. This does not guarantee that GR remains valid under extreme physical conditions beyond the present experimental limits. Indeed, it is known that GR cannot tell what happens at the center of black holes and at the very beginning of the Universe if energy conditions are to be satisfied [2]. In addition to this, the explanation of the accelerating expansion of the late time universe may require the modification of GR on very large scales. Motivated by these considerations, GR may be viewed as an effective theory which is valid only in some domain of space spanned by physical parameters such as length, energy, and density, although the boundary of such a domain is not yet well defined. Many possibilities have been proposed in the literature in various contexts (see references in [3–6]).

In the near future, direct detection of gravitational waves will become possible by using the laser interferometers such as the advanced laser interferometric gravitational wave observatory (aLIGO) [7], advanced Virgo (aVirgo) [8], and KAGRA [9]. Target gravitational waves originate from the vicinity of the compact objects, such as neutron stars, where the matter density is much larger than any other place in the Universe. Observation and analysis of such gravitational waves should enable us to probe the laws of gravity in such previously unexplored domains.

Scalar-tensor (ST) theories are well studied and natural alternatives to GR [10–13]. Observations of gravitational waves enable us to probe ST theories in the high-density and strong-gravity regime. Interesting targets relevant to gravitational wave observations are a class of ST theories which mimic GR in the low-density (or weak-gravity) regime but significantly deviate from GR in the high-density (or strong-gravity) regime [14,15]. One natural way to construct such a model is to introduce interaction between the standard model particles and the scalar field by the conformal factor so that the effective potential for the scalar field depends on the matter density. If the system is static, the scalar field takes a value that minimizes the energy of the system. This expectation value depends on the matter density and controls the interaction strength between the standard model particles [16]. Then it is possible that the expectation value vanishes if matter density is low, and the spontaneous scalarization occurs if matter density exceeds a critical value. In such a case, modification of GR occurs only in the high-density region exceeding the critical density. This mechanism is completely opposite to the symmetron model proposed in [17] in which scalarization occurs only when the matter density becomes smaller than the critical density. In that model, matter density inside the Solar System is supposed to be larger than the critical density, and GR is recovered, but deviation from GR appears in the cosmological environment due to the low background density. For this reason, we call the scalar field that acquires a nonvanishing expectation value only in high-density environments the "asymmetron."

In this context, there is an interesting scalar-tensor theory proposed by Damour and Esposito-Farese (DEF) [18,19] in which significant deviation from GR occurs only in the vicinity and the inside of neutron stars and safely passes the Solar System experiments. In the DEF model, the scalar field in the high matter density region becomes tachyonic due to a particular form of the conformal coupling with the standard model particles (see left figure of Fig. 1). As a result, the scalar field takes a large nonvanishing value inside the neutron star and approaches a nonvanishing but much smaller value at a distance far away from the star. The value at infinity is fixed to match the cosmological value just as in the case of the Fierz-Jordan-Brans-Dicke theory [20–23], and this value must be small enough to satisfy the Solar System and terrestrial observational constraints. Since the magnitude of the scalar field controls the amount of deviation from GR, significant deviation from GR occurs only in the inside or vicinity of the neutron stars. Because of this, the structure of the neutron stars differs from that under GR, and this suggests that studying neutron stars and deriving observational consequences is the most effective way to test the DEF model, as has been pursued in the literature [24–33].

However, it is known that the DEF model faces the difficulty of embedding it in the cosmic history [28,34,35]. During inflationary and matter-dominated epochs, the coupling between the scalar field and the matter field forces the scalar field to take a nonvanishing value, and the law of gravity in the present Universe deviates from GR to an extent incompatible with the existing constraints. Our main motivation in this paper is to extend the DEF model to incorporate it in the cosmological context. We achieve this by dropping the two restrictions imposed in the DEF model. The first is the mass of the scalar field, and the second is the energy scale appearing in the conformal factor. In the DEF model, the scalar field is assumed to be exactly massless, and the energy scale in the conformal factor is taken to be around the Planck mass. We do not

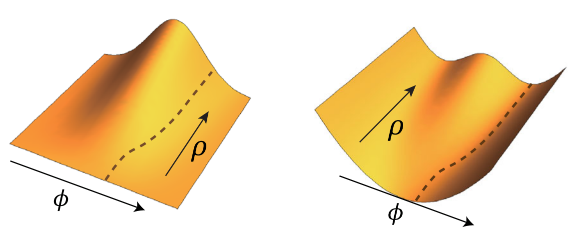

FIG. 1 (color online). Density-dependent effective potential of the scalar field for the Damour-Esposito-Farese model (left figure) and the asymmetron model we consider in this paper (right figure). Dotted curve in each figure represents the expectation value of ϕ inside the compact object with density ρ.

impose these conditions and assume that the scalar field is massive and the energy scale in the conformal factor differs from the Planck scale. Because of these assumptions, the effective potential of the asymmetron has a global minimum for any value of the matter density ρ, whereas the effective potential of the original DEF model does not have such a property (see Fig. 1).

Let us first briefly explain how the extended DEF model can be consistently embedded in the cosmology before describing the quantitative analysis in the subsequent sections. As is the case with the original DEF model, in the extended DEF model, the scalar field at the origin in the presence of matter becomes unstable and should, in principle, be pushed away from the origin. Thanks to the mass term, there exists a global minimum of the effective potential, which helps the asymmetron to settle down at this point. Assuming the universal conformal coupling, the scalarization should happen during inflation. Due to the nonvanishing value of the asymmetron, the gravitational constant would be different from the one we measure in the laboratory, and in this sense the law of gravity would be different from GR, as we know. After inflation, the Universe is reheated and dominated by radiation. Since the trace of the radiation energy-momentum tensor is zero, the asymmetron decouples from the matter and the global minimum shifts back to the origin of the effective potential. As the Universe further expands, the Hubble parameter gradually decreases and eventually becomes smaller than the mass of the asymmetron. By then, the asymmetron undergoes damped oscillation, and the Universe gradually approaches GR. That is, GR is a cosmological attractor in this model. As a result, GR is recovered to a good approximation in the present Universe. We will further show that the oscillating component of the asymmetron, which interacts only gravitationally with standard model particles, is a good candidate for cold dark matter. Therefore, not only is our extended DEF model cosmologically viable, but it also provides a dynamical mechanism for dark matter generation via asymmetron production during inflation.

Of course, cosmology is not the only arena where the extended DEF model becomes relevant to observations. When the matter density inside a compact astrophysical object exceeds the critical density, the asymmetron would undergo spontaneous scalarization and the laws of gravity might deviate from GR considerably. This phenomenon itself is similar to the original DEF model, but our asymmetron model provides additional new features as follows. Due to the mass term, the asymmetron outside the compact object where no matter exists diminishes exponentially on the length scale of the inverse of the mass. This is in clear contrast to the original DEF model where the asymptotic value is arbitrary and fixed by the boundary condition. Furthermore, the critical density beyond which the spontaneous scalarization occurs is not necessarily around the matter density of the neutron star (it could be either higher or lower), whereas in the original DEF model the spontaneous scalarization occurs inside the neutron star where the gravitational energy becomes comparable to its rest mass energy. This opens up a new possibility that not only neutron stars but also less compact astrophysical objects are the best targets to search for the deviation from GR.

In this paper, we analyze in detail the spontaneous scalarization in the asymmetron model and how the gravitational field changes outside the compact star before and after the spontaneous scalarization. We also show that inflation, assuming the universal conformal coupling to all the matter fields, induces the spontaneous scalarization and the asymmetron undergoes coherent oscillations in the later time Universe. As mentioned above, such an oscillating field can be a candidate for cold dark matter. We show that there is a parameter space where the production of the asymmetron can saturate the dark matter content. In the last section, we further comment on the possibility of the asymmetron as dark energy.

II. SPONTANEOUS SCALARIZATION IN THE HIGH-DENSITY REGION

A. Model

We introduce a real massive scalar field ϕ which is universally coupled to all the matter fields including the standard model particles through the metric $\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}$ (Thus, $\tilde{g}_{\mu\nu}$ is the Jordan metric). This ensures that the weak equivalence principle is satisfied. We assume that $g_{\mu\nu}$ satisfies the Einstein equations. Therefore, the basic action is given by

$$\begin{aligned} S &= S_g[g_{\mu\nu}] + S_\phi[g_{\mu\nu},\phi] + S_m[\tilde{g}_{\mu\nu},\phi] \\ &= \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\mu^2}{2}\phi^2\right) \\ &\quad + \int d^4x\sqrt{-\tilde{g}}\mathcal{L}_m(\tilde{g},\psi_m), \end{aligned} \tag{1}$$

where G_N is the Newton's constant and $\mathcal{L}_m$ is the matter Lagrangian of all the matter fields including the standard model fields. The corresponding equations of motion are given by

$$\Box\phi - \mu^2\phi + A^3(\phi)A_{,\phi}\tilde{T} = 0, \tag{2}$$

$$\begin{aligned} G_{\mu\nu} = 8\pi G_N\bigg[&-\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \frac{\mu^2}{2}\phi^2\right)g_{\mu\nu} \\ &+ \partial_\mu\phi\partial_\nu\phi + A^2(\phi)\tilde{T}_{\mu\nu}\bigg], \end{aligned} \tag{3}$$

where

$$\tilde{T}_{\mu\nu} \equiv -\frac{2}{\sqrt{-\tilde{g}}}\frac{\delta S_m}{\delta\tilde{g}^{\mu\nu}} \tag{4}$$

is the energy-momentum tensor with respect to $\tilde{g}_{\mu\nu}$ and $\tilde{T} \equiv \tilde{g}^{\mu\nu}\tilde{T}_{\mu\nu}$. Since all experiments are done with respect to $\tilde{g}_{\mu\nu}$, $\tilde{T}_{\mu\nu}$ is the normal energy-momentum tensor we use in the standard general relativity. For this reason, we call $\tilde{T}_{\mu\nu}$ the physical energy-momentum tensor.[1] Since $\tilde{T}$ is independent of ϕ, the first EOM states that the effective potential for ϕ is

$$V_{\rm eff}(\phi) = \frac{\mu^2}{2}\phi^2 - \frac{1}{4}A^4(\phi)\tilde{T}, \tag{5}$$

for which we have $\Box\phi - V_{{\rm eff},\phi} = 0$.

Now, let us choose the function $A^2(\phi)$ such that it is an even function and it monotonically decreases for $\phi > 0$ and asymptotically approaches a constant value. One simple form that satisfies all these conditions is given by

$$A^2(\phi) = 1 - \varepsilon + \varepsilon e^{-\frac{\phi^2}{2M^2}}, \tag{6}$$

with $0 < \varepsilon < 1$. In order to derive the quantitative results, we consider this form of $A(\phi)$ throughout this paper under the assumptions that $\varepsilon = \mathcal{O}(1)$ and is not very close to 0 nor 1. However, our conclusions are also qualitatively valid for other forms of $A^2(\phi)$ as long as it satisfies the required properties mentioned above.

With this choice, the effective potential for ϕ in the presence of non-relativistic matter, for which $\tilde{T} = -\tilde{\rho}$, becomes

$$V_{\rm eff}(\phi) = \frac{\mu^2}{2}\phi^2 + \frac{1}{4}\left(1 - \varepsilon + \varepsilon e^{-\frac{\phi^2}{2M^2}}\right)^2\tilde{\rho}. \tag{7}$$

The shape of $V_{\rm eff}$ is shown in the right panel of Fig. 1. When $\tilde{\rho}$ is uniform and the system is static, ϕ would take a constant value $\bar{\phi}$ which minimizes the effective potential. Taylor-expanding $V_{\rm eff}(\phi)$ around $\phi = 0$, we have

$$V_{\rm eff}(\phi) = \frac{1}{4}\tilde{\rho} + \frac{1}{2}\left(\mu^2 - \frac{\varepsilon\tilde{\rho}}{2M^2}\right)\phi^2 + \mathcal{O}(\phi^4). \tag{8}$$

We find $\phi = 0$ is stable for $\tilde{\rho} < \rho_{\rm PT} \equiv 2\mu^2M^2/\varepsilon$, but becomes unstable when $\tilde{\rho}$ exceeds $\rho_{\rm PT}$. When $\tilde{\rho} > \rho_{\rm PT}$, $\bar{\phi}$ is given by

[1] As expected, it can be verified by explicit computation that the conservation law $\tilde{\nabla}_\mu\tilde{T}^\mu{}_\nu = 0$ is an automatic consequence of the combination of Eqs. (2) and (3).

$$\frac{\bar{\phi}^2}{2M^2} = \ln f(\varepsilon, \rho_{\rm PT}/\tilde{\rho}),$$
$$f(\varepsilon,\eta) \equiv \frac{2\varepsilon}{1-\varepsilon}\left(\sqrt{1+\frac{4\varepsilon\eta}{(1-\varepsilon)^2}}-1\right)^{-1}. \quad (9)$$

We see that $\bar{\phi}$ depends only logarithmically on $\tilde{\rho}$. Thus, unless $\tilde{\rho}$ takes extremely huge values, $\bar{\phi}$ is $\mathcal{O}(M)$. To conclude, the scalar field undergoes spontaneous scalarization when $\tilde{\rho} > \rho_{\rm PT}$ is realized.

Let us next consider how the gravity behaves in the symmetric phase where $\bar{\phi} = 0$ is satisfied. In the symmetric phase, the interactions between ϕ and the other matter fields are, in the leading order, written as $\sim\phi^2 T_{\mu\nu}/M^2$. As we will see later, for observationally interesting cases, M is typically much larger than TeV scale, *i.e.*, far beyond the energy scale accessible by any terrestrial experiments. In this sense, ϕ completely decouples from the other matter fields and behaves as a free massive scalar field. Since $A^2(\bar{\phi}) = 1$, assuming there is no excitation of ϕ field, Eqs. (3) reduce to the Einstein equations. If ϕ is excited around $\phi = 0$, excitation will be observed as dark component interacting only gravitationally with ordinary matter. It is then natural to suppose that such excitation constitutes (a part of) dark matter. More detailed analysis of this possibility including its production mechanism will be discussed later. Therefore, at low density region $\tilde{\rho} < \rho_{\rm PT}$, GR is recovered.

B. Gravity in spontaneous scalarization phase

Contrary to the symmetric phase, deviation from GR occurs in the scalarization phase, which we will investigate in the following. In the scalarization phase, due to a nonvanishing $\bar{\phi}$, matter fields interact with ϕ with interaction strength proportional to $\bar{\phi}$. This acts as a fifth force between matter fields. Since ϕ is massive, the interaction range of the fifth force is limited to $\sim 1/\mu$. In addition to the emergence of the fifth force, field equations for gravity are also modified. Assuming no excitation of the ϕ field around $\bar{\phi}$, Eqs. (3), rewritten in terms of the Jordan-frame metric $\tilde{g}_{\mu\nu}$, become

$$\tilde{G}_{\mu\nu} + \Lambda_{\rm eff}\tilde{g}_{\mu\nu} = 8\pi G_{\rm eff}\tilde{T}_{\mu\nu}, \quad (10)$$

where

$$\Lambda_{\rm eff} = 4\pi G_N \mu^2 \bar{\phi}^2 A^{-2}(\bar{\phi})$$
$$= 4\pi G_N \varepsilon\rho_{\rm PT} \ln f(\varepsilon,\rho_{\rm PT}/\tilde{\rho})\left(1-\varepsilon+\frac{\varepsilon}{f(\varepsilon,\rho_{\rm PT}/\tilde{\rho})}\right)^{-1}, \quad (11)$$

$$G_{\rm eff} = A^2(\bar{\phi})G_N = \left(1-\varepsilon+\frac{\varepsilon}{f(\varepsilon,\rho_{\rm PT}/\tilde{\rho})}\right)G_N. \quad (12)$$

We find that $\tilde{g}_{\mu\nu}$ satisfies the Einstein equations with the gravitational constant replaced by $G_{\rm eff}$ and with the effective cosmological constant $\Lambda_{\rm eff}$. Contrary to the case of the standard Higgs mechanism, for which a smaller cosmological constant is realized in the symmetry-breaking phase compared to that in the symmetric phase, the opposite phenomenon happens in the current model. Namely, if there is no (or very tiny) cosmological constant in the symmetric phase, then a positive vacuum energy of $\mathcal{O}(\rho_{\rm PT})$ emerges in the spontaneous scalarization phase. Therefore, if the matter density is larger than $\rho_{\rm PT}$ but is still the same order of magnitude as $\rho_{\rm PT}$, the effective cosmological constant will also play a non-negligible role in gravitational physics. In the very high-density region in which $\tilde{\rho} \gg \rho_{\rm PT}$, we have

$$\Lambda_{\rm eff} \approx 4\pi G_N \frac{\varepsilon}{1-\varepsilon}\rho_{\rm PT} \ln\left((1-\varepsilon)\frac{\tilde{\rho}}{\rho_{\rm PT}}\right),$$
$$G_{\rm eff} \approx (1-\varepsilon)G_N. \quad (13)$$

We find that $\Lambda_{\rm eff}$ is enhanced only logarithmically from $\rho_{\rm PT}$. Thus, in the very high-density region, the effect of the effective cosmological constant is much smaller than the right-hand side of (10) and does not significantly affect the dynamics. The effective gravitational constant is reduced by $(1-\varepsilon)$. Thus, gravity is weakened by this amount.

In the above argument, we have ignored the contribution of the scalar force and focused only on the change in the pure gravity sector. In order to evaluate the scalar force, let us consider a test point source of its physical mass M_S immersed in the static and uniform matter distribution in which spontaneous scalarization occurs. The presence of the point source distorts the scalar field from $\bar{\phi}$ by the amount $\delta\phi$ as well as the Einstein-frame metric from $\eta_{\mu\nu}$ by the amount $h_{\mu\nu}$.[2] We assume M_S is so small that both $\delta\phi$ and $h_{\mu\nu}$ can be obtained by linear perturbation analysis. Then the equation for $\delta\phi$ is obtained by linearizing Eq. (2) on the background $\phi = \bar{\phi}$ given by Eq. (9). On this background, we have

$$A^3 A_{,\phi}|_{\phi=\bar{\phi}} = -\frac{\bar{A}^2\xi}{M}, \quad (14)$$

where $\bar{A} \equiv A(\bar{\phi})$, and we have introduced a dimensionless parameter ξ defined by

$$\xi \equiv \frac{\varepsilon}{\sqrt{2}}\frac{\sqrt{\ln f(\varepsilon,\rho_{\rm PT}/\tilde{\rho})}}{f(\varepsilon,\rho_{\rm PT}/\tilde{\rho})}. \quad (15)$$

Notice that in the deep scalarization phase for which $\tilde{\rho} \gg \rho_{\rm PT}$, this parameter is suppressed by a small factor

[2] For simplicity, we do not take into account the cosmological constant term given by Eq. (11) which exists in the background. Inclusion of it is straightforward.

$\rho_{\rm PT}/\tilde{\rho}$. Since ξ controls the coupling between the asymmetron and matter fields, the coupling is weak in the deep scalarization phase.

Using this quantity, the equation for $\delta\phi$ becomes

$$(\Delta - \mu^2)\delta\phi = -\frac{\bar{A}^2\xi}{M}\tilde{\rho}_S, \qquad \tilde{\rho}_S = \frac{M_S}{\bar{A}^3}\delta(\vec{x}). \quad (16)$$

The solution of this equation is given by

$$\delta\phi(r) = \frac{\xi}{4\pi}\frac{M_S}{M}\frac{e^{-\mu r}}{\bar{A}r}. \quad (17)$$

The metric perturbation $h_{\mu\nu}$ can be obtained in the standard manner. Noting that the gravitational constant is $\bar{A}^2 G_N$ in the scalarization phase and $\bar{A}r$ is the physical distance, we have

$$h_{00} = 2U, \qquad h_{ij} = 2U\delta_{ij}, \qquad U \equiv \frac{\bar{A}G_N M_S}{r} \quad (18)$$

in the isotropic coordinates (or the PPN coordinates).[3]

The Jordan-frame metric with first-order deviation from the background is given by

$$\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu} = \bar{A}^2(\eta_{\mu\nu} + h_{\mu\nu} + (\ln A^2)_{,\phi}|_{\phi=\bar{\phi}}\delta\phi\eta_{\mu\nu}). \quad (19)$$

Since the constant overall factor $\bar{A}^2$ is irrelevant to the following discussion, we will omit it. Substituting the above results in $\tilde{g}_{\mu\nu}$, we find

$$\tilde{g}_{00} = -1 + 2U + \frac{\bar{A}\xi^2 M_S}{2\pi M^2}\frac{e^{-\mu r}}{r}, \quad (20)$$

$$\tilde{g}_{ij} = \left(1 + 2U - \frac{\bar{A}\xi^2 M_S}{2\pi M^2}\frac{e^{-\mu r}}{r}\right)\delta_{ij}. \quad (21)$$

We find that the scalar force described by the Yukawa potential contributes to the metric perturbation in the Jordan frame which does not match the form predicted by the pure GR. We can translate this contribution to the PPN parameter γ (see, for instance, [1]). This parameter is defined by $\tilde{g}_{ij} = (1 + 2\gamma\tilde{U})\delta_{ij}$ where $\tilde{U}$ is the metric perturbation of the 00 component, $\tilde{g}_{00} = -1 + 2\tilde{U}$ (in GR, $\gamma = 1$). In the present case, γ becomes

$$\gamma = 1 - \frac{2\lambda}{1+\lambda}, \qquad \lambda \equiv \frac{\xi^2 e^{-\mu r}}{4\pi M^2 G_N}. \quad (22)$$

Since M appears in the denominator of $A_{,\phi}$ in Eq. (2), naively one would expect that if M is comparable or smaller than the Planck scale $\sim G_N^{-1/2}$, then the scalar force would become stronger than the gravitational force within the range $\sim\mu^{-1}$. The above result shows that this naive expectation is not correct since it is $\xi^2/(M^2 G_N)$ that determines the magnitude of the deviation from GR. As we mentioned earlier, ξ becomes small in the deep scalarization phase, and the system can become close to GR ($|\gamma - 1| \ll 1$) even when $M \lesssim G_N^{-1/2}$.

[3] If we are living in the scalarization phase, we have to replace $\bar{A}^2 G_N$ by G_N to satisfy the local gravity experiments. See the last paragraph of the last section for relevant discussion.

C. Spontaneous scalarization only inside a compact object

Having explained the basic picture of the spontaneous scalarization, it is intriguing to analyze a situation where a dense object, inside which spontaneous scalarization occurs, resides in a vacuum. To capture the essence of the phenomena, we make the following simplification that the object is static, uniform, and spherically symmetric and is made of nonrelativistic matter and its size is much larger than the Schwarzschild radius so that the metric in the Einstein frame can be taken to be the Minkowski one, but density is much larger than $\rho_{\rm PT}$. These assumptions will be inappropriate in a quantitative sense for dealing with realistic astrophysical objects such as normal stars, white dwarfs, neutron stars, and so on, but we believe that the following result remains qualitatively correct.

With the above assumptions, the equation for ϕ becomes

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{dV_{\rm eff}}{d\phi} = 0. \quad (23)$$

As is done in [36], let us perform the change of variables as

$$r \to \tau, \qquad \phi \to x, \qquad V_{\rm eff} \to -U. \quad (24)$$

Then, the above equations become

$$\frac{d^2x}{d\tau^2} + \frac{2}{\tau}\frac{dx}{d\tau} = -\frac{dU}{dx}, \quad (25)$$

which represents the motion of a point mass under the potential U associated with time-dependent friction. Denoting R by the radius of the object, U changes its shape at $\tau = R$ as shown in Fig. 2. What we want is a solution $x(\tau)$ with a boundary condition,

$$x(0) = x_c, \qquad \dot{x}(0) = 0, \qquad x(\tau \to \infty) = 0. \quad (26)$$

We follow [36] to construct the approximate analytic solution for this kind of problem.

When R is large enough, x stays near x_c for a long time. This means that x_c is very close to $\bar{\phi}$ at which $U' = 0$. At $\tau = R$, the friction had become negligible and the kinetic energy of x is just enough to be compensated for by the difference of potential energy between $x = x(R)$ and $x = 0$

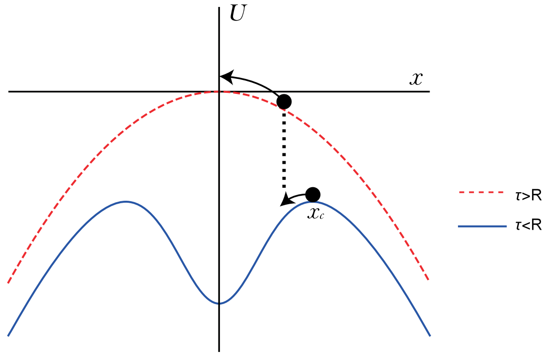

FIG. 2 (color online). Potential U for $\tau < R$ and $\tau > R$. Initially, x is at x_c and asymptotically approaches zero for $\tau \to \infty$.

so that x asymptotically approaches zero. Since the transition at $\tau = R$ happens near x_c, let us replace $U(x)$ before $\tau = R$ by the quadratic form around $\bar{\phi}$,

$$U(x) \approx -\frac{1}{2} m^2 (x - \bar{\phi})^2, \tag{27}$$

where m^2 is the second derivative of $V_{\rm eff}$ evaluated at $\bar{\phi}$,

$$\begin{aligned} m^2 &\equiv V_{{\rm eff},\phi\phi}(\bar{\phi}) \\ &= \frac{2\mu^2}{\rho_{\rm PT}} \left(2\rho_{\rm PT} - (1-\varepsilon) \frac{\tilde{\rho}}{f(\varepsilon, \rho_{\rm PT}/\tilde{\rho})} \right) \ln f(\varepsilon, \rho_{\rm PT}/\tilde{\rho}). \end{aligned} \tag{28}$$

Then Eq. (25) becomes

$$\frac{d^2 x}{d\tau^2} + \frac{2}{\tau} \frac{dx}{d\tau} = m^2 (x - \bar{\phi}), \tag{29}$$

whose solution with the initial condition $x(0) = x_c$, $\dot{x}(0) = 0$ is given by

$$x(\tau) = \bar{\phi} + (x_c - \bar{\phi}) \frac{\sinh(m\tau)}{m\tau}. \tag{30}$$

On the other hand, $x(\tau)$ for $\tau > R$ with the boundary condition $x \to 0$ for $\tau \to \infty$ is given by

$$x(\tau) = C \frac{e^{-\mu(\tau - R)}}{\tau}, \tag{31}$$

where C is the integration constant. Requiring that x and $\dot{x}$ are continuous at $\tau = R$ determines x_c and C as

$$x_c = \frac{-m(1 + R\mu) + m\cosh(mR) + \mu\sinh(mR)}{m\cosh(mR) + \mu\sinh(mR)} \bar{\phi}, \tag{32}$$

$$C = \frac{mR\cosh(mR) - \sinh(mR)}{m\cosh(mR) + \mu\sinh(mR)} \bar{\phi}. \tag{33}$$

In the high-density limit $\tilde{\rho} \gg \rho_{\rm PT}$, m^2 becomes

$$m^2 = 2\mu^2 \ln\left(\frac{(1-\varepsilon)\tilde{\rho}}{\rho_{\rm PT}} \right), \tag{34}$$

which is enhanced by the log factor compared to μ^2. Then, neglecting μ terms in x_c yields

$$x_c \approx \frac{-(1 + R\mu) + \cosh(mR)}{\cosh(mR)} \bar{\phi}. \tag{35}$$

Thus, if $R \gg m^{-1} (\gg \mu^{-1})$ is satisfied, then ϕ stays very close to $\bar{\phi}$ until the surface of the object and then decays exponentially over the length scale μ^{-1} outside the object. In other words, we can say that spontaneous scalarization occurs inside the object when the size of the object is much greater than the Compton wavelength of ϕ in the symmetric phase (in addition to the trivial condition that density is greater than $\rho_{\rm PT}$).

D. Gravity outside the scalarized compact object

Let us consider the metric perturbation outside a compact object inside which spontaneous scalarization occurs. As in the previous subsection, we assume that the compact object is made of nonrelativistic matter. We assume that the matter density is high enough so that spontaneous scalarization occurs inside the object but not compact enough so that gravity is weak everywhere. From Eqs. (3), we see that this amounts to performing a perturbative expansion of the metric in the Einstein frame around the Minkowski metric in terms of a dimensionless quantity given by (Schwarzshild radius)/(distance).[4] In this subsection, we consider only first-order corrections and treat the linearized Einstein equations.

We decompose the metric in the Einstein frame as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1, \tag{36}$$

[4]One may wonder why we do not consider linear perturbation in the Jordan frame. In order to see why this is not feasible, let us express the field equations (3) in terms of the Jordan-frame metric. They are given by

$$\begin{aligned} &\tilde{G}_{\mu\nu} + \tilde{g}_{\mu\nu}(\tilde{\nabla}^\alpha \ln A \tilde{\nabla}_\alpha \ln A - 2\tilde{\nabla}^\alpha \tilde{\nabla}_\alpha \ln A) + 2\tilde{\nabla}_\mu \tilde{\nabla}_\nu \ln A \\ &\quad = 8\pi G_N \left[-\left(\frac{1}{2} \tilde{\nabla}^\alpha \phi \tilde{\nabla}_\alpha \phi + \frac{\mu^2}{2} \phi^2 \right) \tilde{g}_{\mu\nu} + \tilde{\nabla}_\mu \phi \tilde{\nabla}_\nu \phi + A^2 \tilde{T}_{\mu\nu} \right]. \end{aligned}$$

We find that terms containing $\ln A$ on the left-hand side of the above equation are not associated with G_N. This makes sense since they come from $G_{\mu\nu}$ for the Einstein frame. It is now clear that the Jordan-frame metric cannot be expanded in terms of (Schwarzshild radius)/(distance). Indeed, since A changes by $\mathcal{O}(\varepsilon)$ from inside to outside of the compact star, a large variation of the Jordan-frame metric [exceeding $\mathcal{O}(G_N)$] is induced near the surface of the compact object.

where $h_{\mu\nu}$ is proportional to G_N. As usual, we introduce $\bar{h}_{\mu\nu}$ by $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$, and impose a gauge condition $\partial^\mu \bar{h}_{\mu\nu} = 0$. Then the linearized field equations become

$$\Box \bar{h}_{\mu\nu} = -16\pi G_N A^2(\phi)\tilde{T}_{\mu\nu}. \tag{37}$$

For the nonrelativistic matter, we have

$$\tilde{T}_{\mu\nu} \approx \mathrm{diag}(A^2\tilde{\rho}, 0, 0, 0). \tag{38}$$

Thus, only the $t-t$ component becomes nontrivial,

$$\Delta \bar{h}_{00} = -16\pi G_N A^4 \tilde{\rho}. \tag{39}$$

Then, the gravitational potential Φ defined by $\Phi = -\frac{1}{2}h_{00}$ becomes

$$\Phi(\vec{x}) = -G_N \int d^3x' \frac{A^4\tilde{\rho}(\vec{x}')}{|\vec{x}-\vec{x}'|}. \tag{40}$$

In particular, when $\vec{x}$ is very far from the object, this becomes

$$\Phi(\vec{x}) \approx -\frac{G_N}{r}\int d^3x' A^4\tilde{\rho}(\vec{x}'). \tag{41}$$

Since symmetry is restored ($\phi = 0$) outside the star, the Einstein frame is equivalent to the Jordan frame in such a region. Thus, the physical gravitational potential $\tilde{\Phi}$ is also given by Eq. (41). The *distance* r approaches the physical distance when r is much larger than the size of the star. Noting that $A^3 d^3x$ is the physical volume element, the physical mass M_S is given by

$$M_S = \int d^3x A^3 \tilde{\rho}(\vec{x}). \tag{42}$$

If the size of the star is much bigger than μ^{-1}, spontaneous scalarization occurs inside the star and ϕ takes the uniform value $\bar{\phi}$ given by Eq. (9) everywhere inside the star except for the thin shell region near the surface. Thus, it is reasonable to approximate A to be uniform inside the star ($A = A(\bar{\phi}) = A_{\rm in}$) and to have a step-function-like transition at the surface of the star and to become unity outside the star. With this simplification, we have

$$\Phi(\vec{x}) \approx -\frac{G_N A_{\rm in} M_S}{r}. \tag{43}$$

For $\tilde{\rho} \gg \rho_{\rm PT}$, we have $A_{\rm in} \approx \sqrt{1-\varepsilon}$. Therefore, from the observer outside the star, M_S appears to be decreased by $A_{\rm in}$ or, equivalently, G_N appears to be decreased by $A_{\rm in}$.

Taking a component parallel to $\tilde{u}^\mu$ of the conservation law $\tilde{\nabla}_\mu \tilde{T}^\mu{}_\nu = 0$ for the nonrelativistic matter and for the metric $\tilde{g}_{\mu\nu} = A^2\eta_{\mu\nu}$, we have

$$\frac{\partial}{\partial t}(\tilde{\rho}A^3) + \frac{\partial}{\partial x^i}(\tilde{\rho}A^3 v^i) = 0, \tag{44}$$

where v^i is defined by $\tilde{u}^i = v^i/A$. Thus, the mass M_S defined by Eq. (42), which is the sum of mass of each particle that constitutes the star, is conserved unless no matter escapes or enters the star. This implies that the gravitational potential far from the star changes by $A_{\rm in}$ after the star undergoes the spontaneous scalarization. At first glance, it appears that this conclusion is inconsistent with the Birkhoff's theorem. In order to understand this in more detail, let us consider a spherically symmetric star whose density is initially smaller than $\rho_{\rm PT}$. Let us assume that, at some time for some reason, a reduction of the radiation pressure occurs due to depletion of fuel to produce thermal energy, and the star starts to shrink and the density eventually exceeds $\rho_{\rm PT}$ before the star settles down to a new stable configuration. By the time the star becomes static again, the spontaneous scalarization is realized inside the star. This final state is already described in previous subsections. Let us write the Einstein-frame metric describing the transition as

$$ds^2 = -(1+2\Phi(t,r))dt^2 + (1+2\Lambda(t,r))dr^2 + r^2 d\Omega, \tag{45}$$

where both Φ and Λ are treated as linear perturbations just as in the previous subsection. The scalar field also respects the spherical symmetry and, hence, $\phi = \phi(t,r)$. Outside the star, the $t-r$ component of the Einstein equations (3) becomes

$$\dot{\Lambda} = 4\pi G_N r \dot{\phi}\phi'. \tag{46}$$

By integrating this equation along time with fixed r, we have

$$r[\Lambda(t\to\infty, r) - \Lambda(t\to -\infty, r)] = 4\pi G_N r^2 \int_{-\infty}^{\infty} dt \dot{\phi}\phi'. \tag{47}$$

From the argument of the previous subsection, the left-hand side of the above equation is equal to $(A_{\rm in}-1)G_N M_S$ when r is much bigger than the radius of the star. Thus, we have

$$(A_{\rm in}-1)M_S = \int_{-\infty}^{\infty} dt S_r \dot{\phi}\phi', \tag{48}$$

where $S_r \equiv 4\pi r^2$ is the surface area of the sphere of radius r. This result shows that the change of the gravitational potential before and after the spontaneous scalarization is compensated by the emission of the scalar wave whose flux is given by $\dot{\phi}\phi'$. Neglecting the metric perturbation, the equation of motion for ϕ is given by

$$-\ddot{\phi} + \phi'' + \frac{2}{r}\phi' + V_{\mathrm{eff},\phi} = 0. \tag{49}$$

Before the star starts to shrink, since the density of the star is less than ρ_{PT}, $\phi = 0$ everywhere. After the star starts to shrink and when the density exceeds ρ_{PT}, $V_{\mathrm{eff},\phi}$ at $\phi = 0$ becomes unstable inside the star and this acts as a force to push ϕ into the stable point. In this way, ϕ inside the star changes its value.[5] This change also excites the change of ϕ outside the star, and it propagates as a wave which decays as $\sim 1/r$. The contribution of the scalar wave to the Jordan-frame metric far from the star is given by

$$\tilde{h}_{\mu\nu} \supset -\varepsilon \frac{\phi^2}{2M^2}\eta_{\mu\nu} \propto r^{-2}. \tag{50}$$

Thus, this contribution is more suppressed for large r compared to the gravitational potential and gravitational wave, both of which decay as $\sim 1/r$, although the latter is absent in the present case from the beginning due to the simplified assumption that the system is spherically symmetric. For a distant observer, the dominant deviation from GR caused by the spontaneous scalarization is the change of the gravitational constant.

III. ASYMMETRON AS DARK MATTER

Having introduced a new scalar field ϕ which interacts with standard matter only gravitationally in the symmetric phase, it is natural to identify it with dark matter. As we will show, the spontaneous scalarization also provides a natural mechanism for fixing the abundance of dark matter within the framework of primordial inflation.

Let us consider the effective potential during inflation. Making the phenomenological approximation that inflation is caused by the fluid with its equation of state $\tilde{P}_{\mathrm{inf}} = -\tilde{\rho}_{\mathrm{inf}}$, we have

$$V_{\mathrm{eff}}(\phi) = \frac{\mu^2}{2}\phi^2 + \left(1 - \varepsilon + \varepsilon e^{-\frac{\phi^2}{2M^2}}\right)^2 \tilde{\rho}_{\mathrm{inf}}. \tag{51}$$

The true effective potential differs from this potential by the amount of slow-roll parameters multiplied to the second term, which is small enough for our present purpose and we ignore it. Due to the contribution of the pressure, the coefficient of the second term on the right-hand side is enhanced by a factor of 4 compared to the case of the nonrelativistic matter. As a result, $\bar{\phi}$ when the spontaneous scalarization occurs is given by

$$\frac{\bar{\phi}^2}{2M^2} = \ln f(\varepsilon, \rho_{\mathrm{PT}}/(4\tilde{\rho}_{\mathrm{inf}})). \tag{52}$$

From this, we find that spontaneous scalarization occurs for $\tilde{\rho}_{\mathrm{inf}} > \rho_{\mathrm{PT}}/4$, which we assume to be satisfied. The critical density is not equal to ρ_{PT} because ρ_{PT} is defined as the critical density for the case of the nonrelativistic matter [see below Eq. (8)].

During inflation, at the classical level, the ϕ field eventually settles down to the value given by Eq. (52) even if it deviated from that value initially. Roughly speaking, $\bar{\phi}$ divides a domain of ϕ into two regions according to the different time scale for approaching $\bar{\phi}$ from a given ϕ. For $\phi \ll \bar{\phi}$, we find from Eq. (51) $V_{\mathrm{eff}}'' \approx -\frac{2\varepsilon\tilde{\rho}_{\mathrm{inf}}}{M^2} + \mathcal{O}(\phi^2)$. Then we have $|V_{\mathrm{eff}}''|/H_{\mathrm{inf}}^2 \simeq 1/(M^2 G_N)$, where H_{inf} is the Hubble parameter during inflation in the Einstein frame. For our case, where $M \lesssim G_N^{-1/2}$, this ratio is larger than unity. This shows that $\phi = 0$ is unstable and rolls down to $\bar{\phi}$ within the Hubble time. On the other hand, for $\phi \gg \bar{\phi}$, the second term in Eq. (51) becomes exponentially suppressed up to the irrelevant constant part, and we have $V''_{\mathrm{eff}} \approx \mu^2 \ll H_{\mathrm{inf}}^2$. Solving the equation of motion for ϕ under the slow-roll approximation yields $\phi \propto e^{-\mu^2 t/(3H_{\mathrm{inf}})}$. Thus, the time scale for ϕ to approach $\bar{\phi}$ is given by H_{inf}/μ^2, and this is much larger than the Hubble time since the range of μ we are interested in is far below H_{inf} (see Fig. 3). This implies that $\bar{\phi}$ and a value larger than $\bar{\phi}$ are equally likely during inflation (especially for the last sixty e-folds relevant to the observable Universe). For the moment, we consider the case where ϕ stays near $\bar{\phi}$ during inflation. As we will explicitly demonstrate, extension to the case where ϕ is larger than $\bar{\phi}$ can be performed in a straightforward manner.

After inflation, the Universe is reheated and dominated by radiation. When this happens, $\tilde{T}$ vanishes and the effective potential reduces to the bare potential.[6] As the Universe expands, the Hubble parameter gradually decreases and at some point becomes equal to μ. Before this time, ϕ keeps its initial value fixed during inflation. After this time, ϕ oscillates around the origin like

[5]Since scalarization with a positive $\bar{\phi}$ and a negative one are equally allowed, scalarization occurs randomly on a distance over the correlation length. As a result, the compact star just after the spontaneous scalarization may be a mixture of positive and negative $\bar{\phi}$, and the two regions are separated by a domain wall. Though this may lead to interesting phenomena, the process of spontaneous scalarization with this effect being taken into account is complicated and we do not consider it in this paper.

[6]Strictly speaking, this is not correct since there exists a nonrelativistic baryon component even in the radiation-dominated era after the QCD phase transition which occurs around temperature $T_{\mathrm{QCD}} \approx 200$ MeV. The baryon density at this temperature is estimated as $\rho_b(T_{\mathrm{QCD}}) \approx 6 \times 10^{-12}\ \mathrm{GeV}^4$ for $\Omega_b = 0.04$, $g_{s*} = 20$, $T_{\mathrm{QCD}} = 200$ MeV. If ρ_{PT} is smaller than $\rho_b(T_{\mathrm{QCD}})$, which is the case for $\mu < 10^{-11}$ eV (see Fig. 3) when we require the asymmetron to be dark matter, the baryon forces the asymmetron to undergo the spontaneous scalarization at the time of the QCD phase transition. As a result, the result (56) cannot be applied straightforwardly and we need to modify it in an appropriate way. Since $\rho_b(T_{\mathrm{QCD}})$ is much smaller than the nuclear density which is an interesting target for ρ_{PT}, we do not consider this case in this paper and set $\mu > 10^{-11}$ eV.

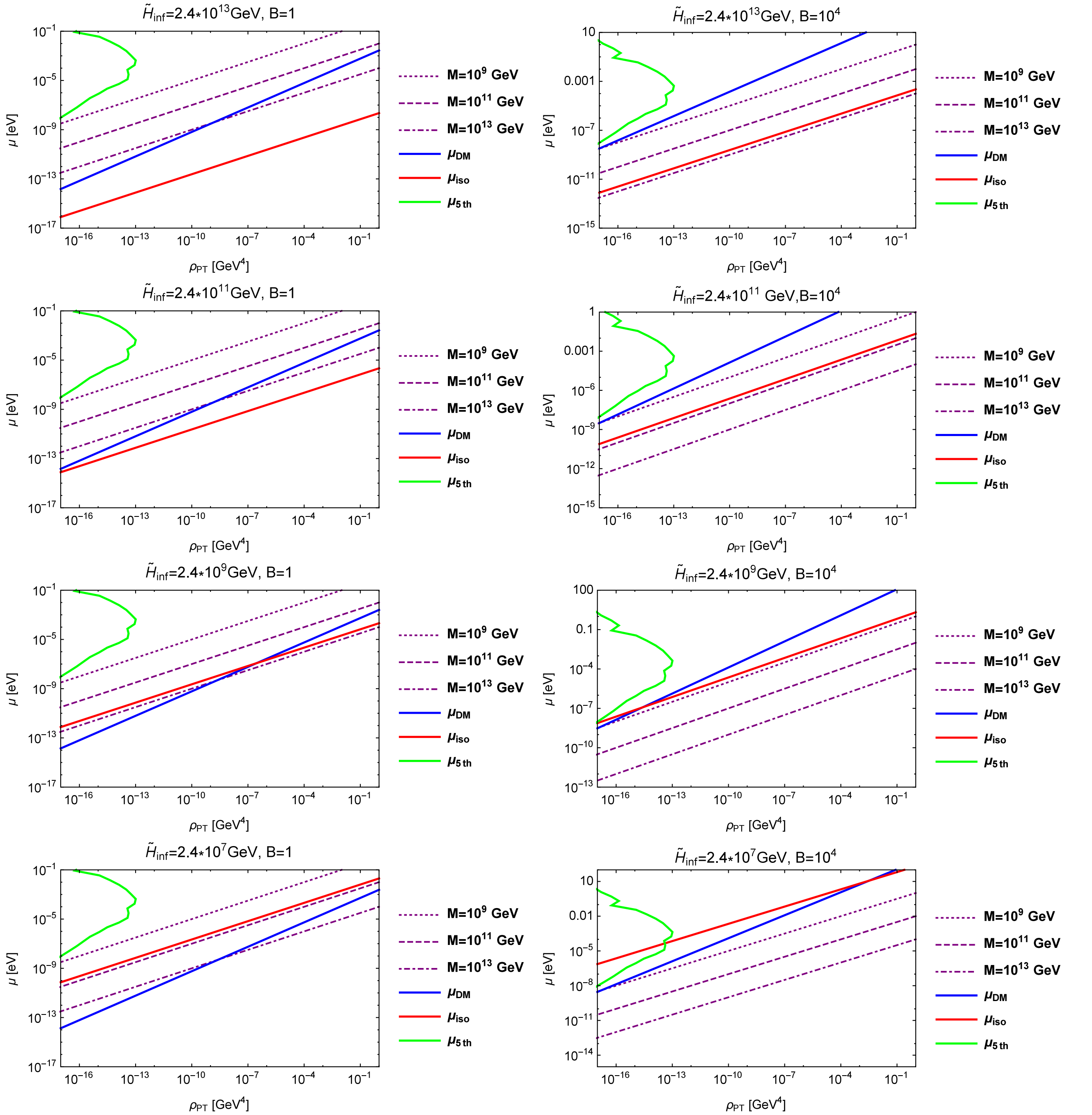

FIG. 3 (color online). Each panel shows three curves $\mu_{\rm DM}$, $\mu_{\rm iso}$ and $\mu_{\rm 5th}$ as well as contours of constant M for $B = 1$ (left side) and $B = 10^4$ (right side). Right to each curve of $\mu_{\rm iso}$ and $\mu_{\rm 5th}$ is the allowed region satisfying the observational constraints.

$\phi(t) \sim \sin(\mu t)/t$ and ρ_ϕ behaves as nonrelativistic matter. Thus, ρ_ϕ decreases as $1/a^3$ in the Einstein frame (a is the scale factor in the Einstein frame). Then the energy density of ϕ at present time is given by

$$\rho_{\phi,0} = \frac{1}{(1+z_{\rm eq})^3}\frac{a_{\rm osc}^3}{a_{\rm eq}^3}\rho_{\phi,{\rm osc}} = \frac{1}{(1+z_{\rm eq})^3}\frac{a_{\rm osc}^3}{a_{\rm eq}^3}\frac{\mu^2}{2}\bar{\phi}^2, \quad (53)$$

where the subscript "osc" in any quantity means that it is evaluated when ϕ starts oscillations; i.e., $\mu = H_{\rm osc}$ and $a_{\rm eq} = 1/(1+z_{\rm eq})$ is the scale factor at the time of matter radiation equality. We assume that there is no additional entropy production after inflation. Therefore, the entropy density of radiation decays as $1/\tilde{a}^3 = A^3 a^3$ ($\tilde{a}$ is the scale factor in the Jordan frame). With this assumption, we have

$$\rho_{\phi,0} = \frac{1}{(1+z_{\rm eq})^3}\frac{g_{*s,{\rm eq}}}{g_{*s,{\rm osc}}}\left(\frac{g_{*,{\rm osc}}}{g_{*,{\rm eq}}}\right)^{3/4}\left(\frac{\rho_{r,{\rm eq}}}{\tilde{\rho}_{r,{\rm osc}}}\right)^{3/4}\frac{1}{A_{\rm inf}^3}\frac{\mu^2}{2}\bar{\phi}^2, \quad (54)$$

where g_{*s} and g_* represent the effective degrees of freedom entering in the entropy density and energy density of radiation, respectively. By the time of matter-radiation equality, the amplitude of ϕ has decreased enough so that there is little difference between the Einstein frame and the Jordan frame, i.e., $A_{\rm eq} \approx 1$ to a very good approximation. Now, by using the Friedmann equation in the Einstein frame,

$$\mu^2 = H_{\rm osc}^2 = \frac{8\pi G_N}{3} A_{\rm inf}^4 \tilde{\rho}_{r,\rm osc}, \tag{55}$$

to eliminate $\tilde{\rho}_{r,\rm osc}$ and Eq. (52) to eliminate $\bar{\phi}$, we end up with

$$\Omega_{\phi,0} = \frac{\rho_{\phi,0}}{\rho_{c,0}} = \frac{g_{*s,\rm eq}}{g_{*s,\rm osc}} \left(\frac{g_{*,\rm osc}}{g_{*,\rm eq}}\right)^{3/4} \frac{\varepsilon \rho_{\rm PT}}{2\rho_{c,0}} \times \ln f(\varepsilon, \rho_{\rm PT}/(4\tilde{\rho}_{\rm inf})) \frac{1}{A_{\rm inf}^2} \left(\frac{H_0}{\mu}\right)^{3/2} \Omega_{r,0}^{3/4}. \tag{56}$$

If this quantity is equal to the observed $\Omega_{m,0}$, then ϕ constitutes the whole dark matter. This requirement yields a relation between μ and $\rho_{\rm PT}$, which is shown as $\mu_{\rm DM} = \mu_{\rm DM}(\rho_{\rm PT})$ for four different values of $\tilde{H}_{\rm inf}$ in Fig. 3 [and for two different values of B. For the definition of B, see discussion below Eq. (64)]. In this figure, the parameters are fixed as $g_{*,\rm eq} = g_{*s,\rm eq} = 100$, and $\varepsilon = 1/2$.

Since there is a strong upper limit on the deviation from general relativity by the Solar System experiments as well as the terrestrial ones, we require that the spontaneous scalarization occurs at a density larger than the Earth density, i.e., $\rho_{\rm PT} \gg \rho_{\rm Earth} \approx 5 \times 10^{-17}$ GeV^4.[7] Combining this with footnote 6, our primary interest for $\rho_{\rm PT}$ is $\rho_{\rm PT} \gtrsim 10^{-11}$ GeV^4. Then, from Fig. 3, we find that the corresponding restriction on μ is given by $\mu \gtrsim 10^{-11}$ eV which we regard as the possible minimum value of our interest.

A. Isocurvature constraint

We saw in the previous subsection that spontaneous scalarization occurs during the primordial inflation and this provides a mechanism for preparing the nonzero value of the asymmetron to realize its coherent oscillations, which behave as nonrelativistic matter interacting only gravitationally with other matter fields. There is indeed a parameter range of μ and M where the energy density of the asymmetron is equal to that of dark matter. But before the asymmetron can be considered as a candidate of dark matter, it must satisfy other observational constraints. There are two nontrivial observational constraints, which we will consider below.

The first constraint is the nondetection of the dark matter isocurvature perturbation. Since the ϕ field is almost massless during inflation, this field acquires almost scale-invariant classical fluctuations during inflation when each wavelength mode crosses the Hubble horizon. In addition to this, the standard adiabatic perturbations are also generated from classical fluctuations of either the inflaton or other light fields, which are equally shared by all the existing particle species such as photons, baryons, and dark matter. On top of this, dark matter has its own fluctuations coming from the fluctuations of the ϕ field itself, and these fluctuations act as isocurvature perturbations having no correlation with the adiabatic ones. Since there is a strong upper limit on the amplitude of the isocurvature perturbations imposed by CMB observations, this limit can be converted to the constraint on the domain of the $(\mu, \rho_{\rm PT})$ plane. To see this in a more quantitative manner, let us first introduce the dark matter isocurvature perturbation $\mathcal{S}_{\rm DM}$ by [37]

$$\mathcal{S}_{\rm DM} = \frac{\delta\rho_{\rm DM}}{\rho_{\rm DM}} - \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma}, \tag{57}$$

where $\delta\rho_\gamma$ is the density perturbation of photons. This quantity is conserved as long as the scale considered is the super-Hubble scale. In the present case, $\rho_{\rm DM} = \rho_\phi$.

As is done in [38] (but for a different model), we adopt the approximation of the sudden transition that ϕ is completely frozen before $H = \mu$ (hence, ρ_ϕ is constant in time) and starts to oscillate exactly when $H = \mu$ and behaves as nonrelativistic matter $\rho_\phi \propto a^{-3}$ [39]. Then, the hypersurface on which ϕ starts to oscillate coincides with the one with constant total energy density. Since $\mathcal{S}_{\rm DM}$ is independent of the choice of time slicing, we can compute δ_r and δ_ϕ in any time slicing, and we take the $H = m$ hypersurface for this purpose. On this hypersurface, we have

$$\rho_r(\vec{x}) + \rho_\phi(\vec{x}) = \rho_{\rm tot} = \frac{3\mu^2}{8\pi G_N}. \tag{58}$$

Decomposing this relation into the background part and perturbation part and extracting the perturbation part, we have

$$\delta_r(\vec{x}) = -\frac{\Omega_\phi}{1-\Omega_\phi}\delta_\phi(\vec{x}), \tag{59}$$

where $\Omega_\phi = \rho_\phi/\rho_{\rm tot}$, evaluated at the time when $H = \mu$. Plugging this relation into the definition of $\mathcal{S}_{\rm DM}$, we have

[7] It is possible that $\rho_{\rm PT} < \rho_{\rm Earth}$ and we are living in the spontaneous scalarization phase. One possibility is that ε is very tiny $\varepsilon \ll 1$. Since a large deviation from GR never happens in any situation for such a case, we do not consider this possibility in this paper. The second possibility is that $\rho_{\rm PT}$ is the order of the critical density of the Universe. In this case, the asymmetron behaves not as dark matter but as dark energy. We will briefly discuss this scenario in the last section.

$$\mathcal{S}_{\rm DM}(\vec{x}) = \left(1 + \frac{3}{4}\frac{\Omega_\phi}{1-\Omega_\phi}\right)\delta_\phi(\vec{x}) \approx \delta_\phi(\vec{x}), \tag{60}$$

where we have used $\Omega_\phi \ll 1$ since the time when the asymmetron starts to oscillate for the range of μ of our interest is much earlier than the time of matter-radiation equality. Since $\rho_\phi = \mu^2\phi^2/2$ in the radiation-dominated era, we finally have

$$\mathcal{S}_{\rm DM}(\vec{x}) = \frac{2\delta\phi}{\bar{\phi}}. \tag{61}$$

Here, $\delta\phi$ is the perturbation quantum mechanically generated during inflation. This is uncorrelated with the (adiabatic) curvature perturbation which is sourced by other fields. The corresponding power spectrum of $\mathcal{S}_{\rm DM}$ is given by

$$\mathcal{P}_{\rm CDM} = \frac{4}{\bar{\phi}^2}\left(\frac{H_{\rm inf}}{2\pi}\right)^2 = \frac{8G_N\mu^2}{3\pi}A^4_{\rm inf}\frac{\tilde{\rho}_{\rm inf}}{\varepsilon\rho_{\rm PT}}\frac{1}{\ln f(\varepsilon, \rho_{\rm PT}/(4\tilde{\rho}_{\rm inf}))}, \tag{62}$$

where the modified Friedmann equation,

$$3H^2_{\rm inf} = 8\pi G_N A^4(\bar{\phi})\rho_{\rm inf}, \tag{63}$$

is used to obtain the final expression. The upper limit on the uncorrelated dark matter isocurvature perturbation by WMAP 9 yr is given by [37]

$$\frac{\mathcal{P}_{\rm CDM}}{\mathcal{P}_{\mathcal{R}}} < \frac{\alpha}{1-\alpha}, \qquad \alpha < 0.047 \quad (95\%\ \text{C.L.}), \tag{64}$$

where $\mathcal{P}_{\mathcal{R}}$ is the power spectrum of the adiabatic perturbation. For fixed $\rho_{\rm PT}$ and $\tilde{\rho}_{\rm inf}$, this bound can be converted into the upper bound on μ which is shown as a line of $\mu = \mu_{\rm iso}$ in Fig. 3. The region in each panel above $\mu = \mu_{\rm iso}$ is excluded.

Before moving to the explanation of Fig. 3, let us comment on the case where the value of ϕ during inflation is greater than $\bar{\phi}$ as mentioned in the paragraph between Eqs. (52) and (53). Writing the background value of ϕ during inflation as $B\bar{\phi}(B \geq 1)$, we find that $\mu_{\rm DM}$ as a solution of Eq. (56) and $\mu_{\rm iso}$ as a solution of Eq. (62) combined with the upper limit of the WMAP constraint (64) scale as $\mu_{\rm DM} \propto B^{4/3}$ and $\mu_{\rm iso} \propto B$, respectively. Given these scaling properties under $\phi \to B\phi$, we see that the isocurvature constraint becomes more stringent for larger B, which is indeed observed in Fig. 3.

Let us first consider the panels on the left side, for which $B = 1$. We find that $\mu = \mu_{\rm DM}$ lies above $\mu = \mu_{\rm iso}$ in the relevant range of $\tilde{\rho}_{\rm PT}$ for $\tilde{H}_{\rm inf} \gtrsim 2.4 \times 10^{11}$ GeV. Therefore, inflation models with $\tilde{H}_{\rm inf}$ higher than this value are definitely not consistent with the asymmetron model being responsible for the total dark matter. If $\tilde{H}_{\rm inf}$ decreases to 2.4×10^9 GeV, there appears a range of $\rho_{\rm PT}$ for which the isocurvature constraint is safely satisfied. Indeed, we can understand that $\mu = \mu_{\rm DM}$ comes below the isocurvature constraint line if the inflation energy scale is lowered sufficiently from the expression of $\mathcal{P}_{\rm DM}$ given by Eq. (62). The equation shows that $\mathcal{P}_{\rm DM}$ is basically proportional to $\tilde{\rho}_{\rm inf}$ (the denominator depends only logarithmically on $\tilde{\rho}_{\rm inf}$). We see if $\tilde{H}_{\rm inf}$ is as low as 10^7 GeV, the isocurvature constraint is evaded for all the range of $\rho_{\rm PT}$ in which we are interested (i.e., nuclear energy density). The qualitative feature remains the same for the panels on the right side, for which $B = 10^4$. The only difference from the case of the left panels is that, as mentioned above, the value of $\tilde{H}_{\rm inf}$, below which the isocurvature constraint is satisfied, is lowered compared to the case of $B = 1$.

B. Constraint from the fifth force experiments

As is already mentioned, we are interested in the case where $\rho_{\rm PT}$ is between the stellar density and the nuclear density realized in the neutron stars so that spontaneous scalarization takes place in compact astrophysical objects and the $\mathcal{O}(\varepsilon)$ deviation from general relativity occurs only in such regions. In the asymmetron model, the Solar System is in the symmetric phase ($\bar{\phi} = 0$). As we saw in the previous section, GR is exactly recovered in this phase, and this model passes the Solar System and terrestrial experiments that have placed a very strong limit on deviation from GR. However, this conclusion must be reconsidered more carefully if we take the scenario of the asymmetron being dark matter. In this case, ϕ is coherently oscillating in time with the angular frequency μ, which describes the cold dark matter. The value of ϕ averaged over a time longer than the oscillation period is zero, but the value at each time is different from zero. Therefore, the assumption of no excitation of ϕ in the symmetric phase is violated if we require ϕ to be dark matter.

When ϕ is oscillating in the symmetric phase, the interaction between matter fields and the ϕ field given by $A^3A_\phi\tilde{T}$ [see Eq. (2)] also oscillates as $-\varepsilon a\sin(\mu t)/M^2$ ($A \approx 1$ is assumed), where a is the amplitude of the oscillations of ϕ and is given by $a^2 = 2\langle\phi^2\rangle$ ($\langle\cdots\rangle$ represents the time average over the oscillation period). The amplitude a is determined from the requirement that $\rho_\phi = \mu^2\langle\phi^2\rangle$ coincides with the dark matter density. This condition yields

$$a^2 = \frac{2\rho_{\rm DM}}{\mu^2}. \tag{65}$$

The effect of the oscillating ϕ can be observed as the periodically time-varying gravitational force (period is π/μ) acting on two massive bodies with a maximum given by $\sim\varepsilon a/M^2$ on top of the standard gravitational force. In order to see this, let us determine the gravitational potential in the Jordan frame. To simplify the analysis, based on the fact that the time scale of interest (e.g., time duration of experiments) is much larger than the oscillation period $2\pi/\mu$,

$$\frac{2\pi}{\mu} \approx 4\times 10^{-4} s \left(\frac{\mu}{10^{-11}\ \mathrm{eV}}\right)^{-1}, \tag{66}$$

for a range of interest $\mu \gtrsim 10^{-11}$ eV, we make the approximation that only the averaged value of ϕ enters the measurable gravitational potential and the asymmetron, both of which are simultaneously generated by the source object such as the Earth. Denoting $\delta\phi$ by the small deviation from the background ϕ caused by the presence of the point mass with its mass M_s ($\tilde{\rho} = M_s \delta(\vec{x})$), the equation for $\delta\phi$ is obtained by linearizing Eq. (2):

$$\Delta\delta\phi - \mu^2 \delta\phi + \frac{\varepsilon\sqrt{\langle\phi^2\rangle}}{2M^2}\tilde{\rho} = 0. \tag{67}$$

The solution of this equation is given by

$$\delta\phi(r) = \frac{\varepsilon\sqrt{\langle\phi^2\rangle}}{8\pi M^2}\frac{M_s}{r} e^{-\mu r}. \tag{68}$$

As a result, the time-time component of the Jordan-frame metric is given by

$$\tilde{g}_{00} = -1 + \frac{2G_N M_s}{r}F(r), \qquad F(r) \equiv 1 + \frac{\varepsilon^2 \langle\phi^2\rangle}{16\pi M^4 G_N} e^{-\mu r}. \tag{69}$$

The function $F(r)$ represents the modification from the standard gravitational potential.[8] Eliminating $\langle\phi^2\rangle$ by Eq. (65), we have

$$F(r) = 1 + \frac{\mu^2}{4\pi G_N \varepsilon}\frac{\rho_{\mathrm{DM}}}{\rho_{\mathrm{PT}}^2} e^{-\mu r}. \tag{70}$$

As expected, the ϕ field contributes to the Yukawa-type force between two bodies. Various experiments have been performed to test the inverse square law of gravity. One of the typical modifications of the inverse square law which is actively tested by experiments is exactly the form given by $F(r)$. In [40], deviation from the inverse square law of a form $F(r) = 1 + \alpha e^{-r/\lambda}$ is assumed and the upper limit on α is summarized for a wide range of λ from 10^{-9} m to 10^{15} m. We converted this constraint in the $\lambda - \alpha$ plane to the constraint in the $\rho_{\mathrm{PT}} - \mu$ plane. The result is shown as a green (dotted) curve in Fig. 3. The region satisfying the fifth force experiments is right to the green (dotted) curve. We find that the constraint from the fifth force experiment is much weaker than the isocurvature one and is always satisfied for any interesting range of ρ_{PT}.

[8] In addition to the modification by $F(r)$, change of the Newton's constant caused by the change of A ($\langle A^2\rangle \neq 1$) also modifies the gravitational. However, this correction is negligibly small, and we do not take this effect into account in Eq. (69).

C. Nonminimal coupling of the asymmetron to the Ricci scalar

So far, we have assumed that the asymmetron ϕ is minimally coupled to gravity. It is possible to add the nonminimal coupling term $\xi R\phi^2$ to the action (1), namely,

$$S \to S + \int d^4x\sqrt{-g}\left(-\frac{1}{2}\xi R\phi^2\right), \tag{71}$$

where ξ is a dimensionless parameter that we assume to be positive and $\mathcal{O}(1)$. In the presence of this term, field equations (2) and (3) are modified as

$$\Box\phi - \xi R\phi - \mu^2\phi + A^3(\phi)A_{,\phi}\tilde{T} = 0, \tag{72}$$

$$\begin{aligned}(1 - 16\pi G_N \xi\phi^2)G_{\mu\nu} &= 8\pi G_N\Big[-\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \frac{\mu^2}{2}\phi^2\right)g_{\mu\nu} \\ &\quad + \partial_\mu\phi\partial_\nu\phi + A^2(\phi)\tilde{T}_{\mu\nu} \\ &\quad + 2\xi(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \phi\Box\phi)g_{\mu\nu} \\ &\quad - 2\xi(\partial_\mu\phi\partial_\nu\phi + \phi\nabla_\mu\nabla_\nu\phi)\Big]. \end{aligned} \tag{73}$$

The first equation shows that the effective potential for the asymmetron now takes the form of

$$V_{\mathrm{eff}}(\phi) = \frac{\xi}{2}R\phi^2 + \frac{\mu^2}{2}\phi^2 - \frac{1}{4}A^4(\phi)\tilde{T}. \tag{74}$$

Having derived the basic equations, let us evaluate how the cosmological evolution of the asymmetron we have derived changes due to the nonminimally coupled term. During inflation, approximating the Hubble parameter to be constant, we have $R \approx 12H_{\mathrm{inf}}^2$. Since $H_{\mathrm{inf}} \gg \mu$ for the case of our interest, the Ricci scalar part becomes the dominant component of the mass term. Expanding V_{eff} up to second order in ϕ around $\phi = 0$ yields

$$V_{\mathrm{eff}}(\phi) = \tilde{\rho}_{\mathrm{inf}} + 6\xi H_{\mathrm{inf}}^2\left(1 - \frac{\varepsilon\tilde{\rho}_{\mathrm{inf}}}{6\xi M^2 H_{\mathrm{inf}}^2}\right)\phi^2 + \mathcal{O}(\phi^4). \tag{75}$$

The coefficient of the term quadratic in ϕ is negative for $M \ll G_N^{-1/2}$. Thus, the spontaneous scalarization also occurs during inflation in the presence of the nonminimally coupled term (for $\xi > 0$). The value of ϕ that minimizes the effective potential in the present case is obtained just by replacing μ with $\xi^{1/2}R^{1/2}$ in Eq. (52):

$$\frac{\bar{\phi}^2}{2M^2} = \ln f\left(\varepsilon, \frac{6\xi M^2 H_{\rm inf}^2}{\varepsilon \tilde{\rho}_{\rm inf}}\right). \tag{76}$$

Contrary to Eq. (52), this equation still contains $\bar{\phi}$ on the right-hand side through $H_{\rm inf}$ and obtaining the precise value of $\bar{\phi}$ requires numerical computation. Nevertheless, due to the logarithmic dependence of the right-hand side on $\bar{\phi}$, we see that $\bar{\phi}$ is $\mathcal{O}(M)$ for any physically reasonable case. Significant difference from the case of the minimal coupling arises for ϕ greater than $\bar{\phi}$. In the case of the minimal coupling, the effective potential is dominated by the bare mass term $\frac{\mu^2}{2}\phi^2$, and this term is not large enough to drive ϕ toward $\bar{\phi}$ within Hubble time. This is the reason why, in the previous subsections, we have also considered the possibility that the value of ϕ during inflation is larger than $\bar{\phi}$ and isocurvature perturbation from the fluctuation of the ϕ field is generated. On the other hand, in the present case, the nonminimally coupled term gives the effective mass to the asymmetron which is the order of $H_{\rm inf}$. As a result, even if the value of ϕ is greater than $\bar{\phi}$ initially, ϕ rolls down to $\bar{\phi}$ within Hubble time. Thus, it is natural to assume that ϕ stays $\bar{\phi}$ throughout inflation. In addition to this, fluctuation of the asymmetron is strongly suppressed on super-Hubble scales due to the heaviness of the field. This means there is little amount of isocurvature perturbation left on scales relevant to CMB observations and the isocurvature constraint used in the previous subsection becomes ineffective. In particular, inflation models with a relatively high-energy scale, which are not allowed in the minimally coupled case because of the isocurvature constraint, now become compatible with the asymmetron model as dark matter.

Having clarified the dynamics of the asymmetron during inflation, let us next evaluate the dynamics in the post-inflationary epochs. To simplify the situation, we assume that the Universe is instantly reheated and becomes radiation dominated soon after inflation. Using $\tilde{T} = 0$ for radiation, we find that the Ricci scalar is given by

$$R \approx 8\pi G_N[-(1-6\xi)\dot{\phi}^2 + 2\mu^2\phi^2 - 6\xi\phi\Box\phi]. \tag{77}$$

This R contributes to the equation of motion for ϕ. Although the resultant equation is a nonlinear differential equation for ϕ, the nonlinear contribution is suppressed compared to the bare mass term. Thus, the nonminimally coupled term does not play an important role in the postinflationary epochs and evolution of the asymmetron is in practice the same as that in the minimally coupled case which we have already discussed in detail. This shows that the effect of the nonminimally coupled term on the present energy density of the asymmetron is only to replace $\bar{\phi}$ in

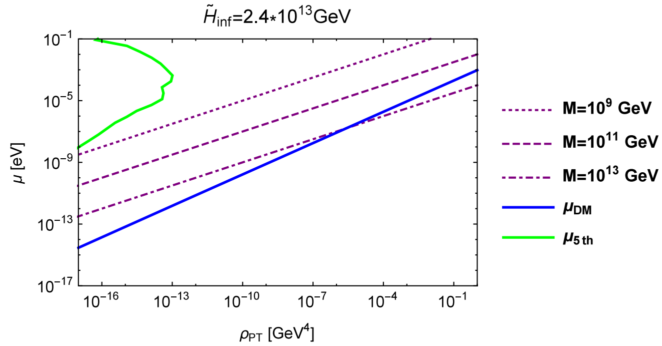

FIG. 4 (color online). Two curves $\mu_{\rm DM}$ and $\mu_{\rm 5th}$, as well as contours of the constant M, in the presence of the nonminimally coupled term ($\xi = 1$). As is explained in the main text, the isocurvature constraint becomes ineffective in the present case.

Eq. (53) with the solution of Eq. (76). Since $\bar{\phi}$ as the solution of Eq. (76) differs only by a factor of the order of unity from the one in the minimally coupled case, we expect that the curve $\mu_{\rm DM}$ in each panel on the left side in Fig. 3 remains qualitatively the same even in the nonminimally coupled case. This is indeed explicitly confirmed from Fig. 4 in which $\mu = \mu_{\rm DM}$ is plotted for $\tilde{H}_{\rm inf} = 2.4 \times 10^{13}$ GeV with $\xi = 1$ (and similar results for other values of $\tilde{H}_{\rm inf}$).

IV. DISCUSSION AND CONCLUSION

We have proposed the asymmetron model, a class of scalar-tensor theories, in which the significant deviation from GR occurs only in the high matter density region. This is an extended version of the Damour-Esposito model proposed in [18], adding a mass term and allowing the energy scale appearing in the conformal factor to differ from the Planck scale. We have shown that the asymmetron model can be consistently embedded in the cosmological framework. In particular, spontaneous scalarization caused by the inflaton in a dynamical way provides the initial condition for the subsequent coherent oscillations of the asymmetron. The damped oscillation has nice properties in that it not only makes the asymmetron behave as cold dark matter but also makes GR a cosmological attractor. The oscillating asymmetron yields a periodically varying fifth force, but its magnitude is far below the current experimental sensitivities and the model we studied in this paper is, in practice, indistinguishable from GR in the present Universe except inside dense compact objects and easily passes the Solar System and terrestrial experiments. There is a range of parameter space where the asymmetron can saturate the whole dark matter component and, at the same time, significant deviation from GR in the present Universe occurs only inside dense compact objects such as neutron stars.

In the spontaneous scalarization phase, the gravitational constant becomes smaller than that in the symmetric phase, namely, the value determined in laboratories. Thus, the gravity is weakened only inside dense compact objects, which is a dominant modification from GR. The scalar force also operates among matter with strength given by $\sim\xi^2/(M^2 G_N)$ (see Sec. II B) compared with the gravitational force. In the deep scalarization phase where $\xi \ll 1$, the scalar force can become tiny even when M is smaller than the Planck mass $G_N^{-1/2}$. Furthermore, the interaction range is limited by $\sim\mu^{-1}$, and for the range of our interest this scale is rather short. As a result, the weakening of gravity is the dominant feature representing deviation from GR when the density of the compact object is much bigger than the critical density above which spontaneous scalarization occurs. This suggests that the size of the compact star in the asymmetron model becomes larger than that in GR. Since the Chandrasekhar mass is proportional to $G_N^{-3/2}$, we expect that the Chandrasekhar mass in our model should be larger than that in GR for compact stars undergoing spontaneous scalarization.

There are many issues that we did not consider in this paper and that deserve further investigation. In this paper, we mainly focused on the mechanism of spontaneous scalarization in the asymmetron model, its basic properties, and embedding it in the cosmological framework. Obviously, the next thing to do is to investigate how to test this model in astrophysics, particularly in connection to gravitational wave observations. In this context, it is first interesting to clarify how the stellar structure (such as the mass-radius relation and the Chandrasekhar mass) in the asymmetron model is modified from GR. Gravitational waves from compact binaries are the main target of the laser interferometers. The dynamics of binaries, waveforms of the gravitational waves, and their detectability using the interferometers for the original DEF model have been studied [24–33]. Performing a similar analysis for the asymmetron model will help to elucidate which observation is the best probe for exploring the asymmetron model.

Another intriguing thought is the possibility of the asymmetron being responsible for dark energy. In this paper, we have focused on the case where $\rho_{\rm PT} \gg \rho_{\rm Earth}$ and the spontaneous scalarization occurs only in the extremely high density region. On the other hand, if $\rho_{\rm PT}$ is the order of the current critical density of the Universe, we expect that the scalarization persists until the present epoch, and the mass term $\frac{1}{2}\mu^2\phi^2$ approximately plays the role of the cosmological constant. Taking M to be the Planck mass, this is achieved if μ is chosen to be around the Hubble constant H_0. This means that locally, such as in the Solar System, the asymmetron mediates a long-range force in addition to the gravitational force. However, since the Solar System is in the deep scalarization phase, the scalar force is suppressed by the factor ξ given by Eq. (15). Indeed, if we take $\rho_{\rm PT}$ to be the present critical density of the Universe and $\tilde{\rho}$ the density of solar wind (we assume one proton per cubic centimeter) and $\varepsilon = 1/2$, we have $\xi \approx 5\times10^{-5}$ and $1-\gamma \approx 2\times10^{-10}$. This value is much below the current constraint $|\gamma - 1| \lesssim 10^{-5}$ obtained from the time delay measurement [1]. Thus, the asymmetron as dark energy can safely pass the Solar System constraints. The most characteristic feature would be time dependence of the gravitational constant [see Eq. (12)] through the time dependence of the matter density $\tilde{\rho}$ due to the cosmic expansion. On cosmological scales, the gravitational constant gradually increases as the Universe expands, and it is interesting to investigate how the large-scale structure is affected by such a time-varying gravitational constant.

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