Gravitational waves from double hybrid inflation

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We present a two-stage hybrid inflationary scenario in nonminimal supergravity which can predict values of the tensor-to-scalar ratio of the order of a few $\times 10^{-2}$. For the parameters considered, the underlying supersymmetric particle physics model possesses two inflationary paths, the trivial and the semishifted one. The trivial path is stabilized by supergravity corrections and supports a first stage of inflation with a limited number of e-foldings. The tensor-to-scalar ratio can become appreciable while the value of the scalar spectral index remains acceptable as a result of the competition between the relatively mild supergravity corrections and the strong radiative corrections to the inflationary potential. The additional number of e-foldings required for solving the puzzles of hot big bang cosmology are generated by a second stage of inflation taking place along the semishifted path. This is possible only because the semishifted path is almost perpendicular to the trivial one, and thus not affected by the strong radiative corrections along the trivial path, and also because the supergravity effects remain mild. The requirement that the running of the scalar spectral index remain acceptable limits the possible values of the tensor-to-scalar ratio not to exceed about 5×10^{-2}. Our model predicts the formation of an unstable string-monopole network, which may lead to detectable gravity wave signatures in future space-based laser interferometer observations.

I. INTRODUCTION

Inflation (for a review, see e.g. Ref. [1]) is by now considered to be an integral part of standard cosmology thanks to a plethora of precise observations on the cosmic microwave background radiation (CMBR) and the large-scale structure in the Universe. Therefore, it is very important to construct realistic inflationary models based on particle theory and consistent with all the available cosmological and phenomenological requirements. Undoubtedly, hybrid inflation [2] is one of the most promising inflationary scenarios. It is [3,4] naturally realized in the context of supersymmetric (SUSY) grand unified theory (GUT) models based on gauge groups with rank greater than or equal to 5.

In standard SUSY hybrid inflation, however, the GUT gauge symmetry is spontaneously broken only at the end of inflation, and thus, if magnetic monopoles are predicted by this symmetry breaking, they are copiously produced [5], leading to a cosmological catastrophe. This disaster is avoided in the smooth [5] or shifted [6] variants of SUSY hybrid inflation, where the GUT gauge symmetry is broken already during inflation. These variants were based on nonrenormalizable superpotential terms. It was, though, subsequently shown that a new smooth [7] and a new

shifted [8] hybrid inflation scenario can be constructed with only renormalizable superpotential terms within an extended Pati-Salam (PS) SUSY GUT model, which was initially introduced [9] for solving a very different problem. Namely, the simplest SUSY PS model predicts (see Ref. [10]) exact Yukawa unification [11] and, if it is supplemented with universal boundary conditions, yields unacceptable b-quark mass values. This problem is solved in the extended model, where Yukawa unification is naturally and moderately violated.

After the first accurate measurement [12] of the scalar spectral index n_s, however, it has been realized that there is a tension between all these well-motivated and natural inflationary scenarios and the measured value of this index. Indeed, within the standard power-law cosmological model with cold dark matter and a cosmological constant, the data imply that n_s is clearly lower than unity—for the latest results on n_s see Ref. [13]. Inflationary scenarios, on the other hand, such as the ones mentioned above, within supergravity (SUGRA) with minimal Kähler potential, yield [14] n_s's which are very close to unity or even exceed it.

One idea [15] for reducing the predicted spectral index is based on the observation that n_s generally decreases with the number of e-foldings suffered by our present horizon scale during inflation. So, by reducing this number of e-foldings, we can achieve values of n_s compatible with the recent data without having to abandon the use of a minimal

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Kähler potential. The additional number of e-foldings required for solving the horizon and flatness problems of standard hot big bang cosmology can be provided by a subsequent second stage of inflation.

It is interesting to note that the extended SUSY PS model of Ref. [9], which can lead to new smooth [7] or new shifted [8] hybrid inflation, can also provide us with a double inflation scenario called standard-smooth hybrid inflation [16], which solves the above mentioned spectral index problem along the lines just discussed. The cosmological scales exit the horizon during the main stage of inflation, which is of the standard hybrid type and occurs as the system slowly rolls down a trivial classically flat direction on which the PS gauge group is unbroken. This direction is subsequently destabilized, giving its place to a classically nonflat valley of minima along which new smooth hybrid inflation takes place with the PS GUT gauge group being broken. Consequently, magnetic monopoles are produced only at the end of the first stage of inflation, but they are adequately diluted by the second stage, which also provides the extra e-foldings needed for solving the puzzles of hot big bang cosmology.

After the recent results of BICEP2 [17] on the B mode in the polarization of the CMBR at degree angular scales, it seems possible that the inflationary scenarios will have to face a new challenge. Namely, they should be able to accommodate appreciable values of the tensor-to-scalar ratio r, since a B mode of primordial origin could be due to the production of gravitational waves during inflation. We should, however, consider this possibility with reservation, since some serious criticism [18] to the original BICEP2 analysis has already appeared, claiming that the foreground from Galactic polarized dust emission has been underestimated. On the other hand, after the recently released Planck HFI 353 GHz dust polarization data [19], the first attempts to make a joint analysis of the Planck and BICEP2 data have been presented [20,21]. They showed that, although r is smaller than initially claimed, significant values of r—of order 0.01—cannot be excluded. The most recent joint analysis [21] yields an upper limit on r of about 0.12 at a 95% confidence level. Unfortunately, all the above mentioned variants of SUSY hybrid inflation predict negligible values of r. So, it is certainly worth investigating whether realistic SUSY hybrid inflation models accommodating appreciable values of r can be constructed.

In Ref. [22], a double inflation scenario has been proposed which is compatible with the BICEP2 data [17]. The first stage of inflation is of the SUSY hybrid type, while the second stage is left unspecified, which makes the scenario incomplete. The inflationary potential is supplemented with a mass-squared term for the inflaton attributed to SUGRA corrections and with a logarithmic term representing very strong radiative corrections due to the SUSY breaking during inflation. It is the competition between these two contributions which allows appreciable

values of r while n_s remains acceptable. The assumption, however, that an inflaton mass-squared term is the only relevant SUGRA correction during inflation in a scenario with Planck-scale inflaton field values seems totally unjustified. In addition, this paper follows the usual practice of only taking into account the radiative corrections in the derivatives of the inflationary potential when calculating the slow-roll parameters and neglecting them when calculating the potential itself. In the case of extremely strong radiative corrections, however, this may lead to erroneous results. Also, Ref. [23] attempts to accommodate the BICEP2 results in a double hybrid inflation model where the inflaton potential changes dynamically with the evolution of the inflaton fields. The particular implementation of this interesting idea, though, appears to have some problems of naturalness in the design of the superpotential. In addition, the treatment of SUGRA seems to be incomplete. Finally, Ref. [24] shows that, in SUSY hybrid inflation models, it is possible to obtain values of r close to 0.03 by employing an expansion of a nonminimal Kähler potential with appropriate coefficients. The validity of this approach may, however, be questionable, since the inflaton takes values close to the Planck scale.

In this paper, we will show that a reduced version of the extended SUSY PS model of Ref. [9] can yield a two-stage inflationary scenario which can predict values of r up to about 0.05. Larger values of the tensor-to-scalar ratio would lead to unacceptably large running of the scalar spectral index. In the range of the model parameters considered here, the model in the global SUSY limit possesses practically two classically flat directions, namely the trivial and the semishifted [25] one. After including SUGRA corrections, the trivial path, on which the full GUT gauge group is unbroken, is stabilized, and a first stage of inflation can occur as the system slowly rolls down this path. All the cosmological scales exit the horizon during this stage, and our present horizon undergoes a limited number of e-foldings. The obtained tensor-to-scalar ratio can be appreciable, while the scalar spectral index assumes acceptable values thanks to the competing effect of the sufficiently mild SUGRA corrections resulting from the construction of Ref. [26] and the strong radiative corrections to the inflationary potential.

Subsequently, a second inflationary stage occurs along the semishifted path, where $U(1)_{B-L}$ remains unbroken, and provides the additional number of e-foldings required for solving the standard problems of hot big bang cosmology. This is possible since, for our choice of parameters, the semishifted path is almost perpendicular to the trivial path, and thus is not affected by the strong radiative corrections on the trivial path. It is also important that the SUGRA corrections on the semishifted path are kept sufficiently mild again by the mechanism of Ref. [26].

We first present, in Sec. II, the salient features of the model in global SUSY. In Sec. III, we then calculate the

SUGRA and one-loop radiative corrections to the potential and discuss our double inflationary scenario. Finally, in Sec. IV, we summarize our conclusions. Throughout, we will use units where the reduced Planck mass $m_P = 2.4354 \times 10^{18}$ GeV is equal to unity.

II. THE MODEL IN GLOBAL SUSY

We consider a reduced version of the extended SUSY PS model of Ref. [9]. This version is based on the left-right-symmetric gauge group $G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, which is a subgroup of the PS group. The superfields of the model which are relevant for inflation are the following: a gauge singlet S, a pair of superfields Φ, $\bar{\Phi}$ belonging to the $(1,1,3)_0$ representation of G_{LR}, and a conjugate pair of Higgs superfields H and $\bar{H}$ belonging to the $(1,1,2)_1$ and $(1,1,2)_{-1}$ representations of G_{LR}, respectively. The field Φ acquires a vacuum expectation value (VEV) which breaks G_{LR} to $G_{SM} \times U(1)_{B-L}$, with G_{SM} being the standard model (SM) gauge group, while the VEVs of H and $\bar{H}$ cause the breaking of G_{LR} to G_{SM}. The full superfield content and superpotential, the global symmetries, and the charge assignments can be easily derived from the extended SUSY PS model of Ref. [9] by simply reducing its GUT gauge group to G_{LR}. The only global symmetry of the model which is relevant here is its $U(1)$ R symmetry under which S and $\bar{\Phi}$ have charge 1, with all the other superfields mentioned above being neutral.

The superpotential terms relevant for inflation are

$$W = \kappa S(M^2 - \Phi^2) - \gamma S H \bar{H} + m \Phi \bar{\Phi} - \lambda \bar{\Phi} H \bar{H}, \quad (1)$$

where M, m are superheavy masses and κ, γ, λ are dimensionless coupling constants. These parameters are normalized so that they correspond to the couplings between the SM singlet components of the superfields. The mass parameters M, m and any two of the three dimensionless parameters κ, γ, λ can always be made real and positive by appropriately redefining the phases of the superfields. The third dimensionless parameter, however, remains generally complex. For definiteness, we will choose this parameter to be real and positive too.

The F-term scalar potential obtained from the superpotential in Eq. (1) is given by

$$V_F^0 = |\kappa(M^2 - \Phi^2) - \gamma H \bar{H}|^2$$
$$+ |m\bar{\Phi} - 2\kappa S \Phi|^2 + |m\Phi - \lambda H \bar{H}|^2$$
$$+ |\gamma S + \lambda \bar{\Phi}|^2 (|H|^2 + |\bar{H}|^2), \quad (2)$$

where the complex scalar fields which belong to the SM singlet components of the superfields are denoted by the same symbol. From this potential and the vanishing of the D terms (which implies that $\bar{H}^* = e^{i\theta}H$), one finds [25] two distinct continua of SUSY vacua:

$$\Phi = \Phi_+, \bar{H}^* = H, |H| = \sqrt{\frac{m\Phi_+}{\lambda}}, S = \bar{\Phi} = 0, \quad (3)$$

$$\Phi = \Phi_-, \bar{H}^* = -H, |H| = \sqrt{\frac{-m\Phi_-}{\lambda}}, S = \bar{\Phi} = 0, \quad (4)$$

where

$$\Phi_\pm \equiv \pm M \left(\sqrt{1 + \left(\frac{\gamma m}{2\kappa\lambda M}\right)^2} \mp \frac{\gamma m}{2\kappa\lambda M} \right). \quad (5)$$

The potential in Eq. (2), generally, possesses [25] three flat directions. The first one is the usual trivial flat direction at

$$\Phi = \bar{\Phi} = H = \bar{H} = 0, \quad (6)$$

with

$$V_F^0 = V_{tr} \equiv \kappa^2 M^4. \quad (7)$$

On this direction, G_{LR} is unbroken. The second one, which appears at

$$\Phi = -\frac{\gamma m}{2\kappa\lambda}, \qquad \bar{\Phi} = -\frac{\gamma}{\lambda} S,$$
$$H\bar{H} = \frac{\kappa\gamma(M^2 - \Phi^2) + \lambda m\Phi}{\gamma^2 + \lambda^2},$$
$$V_F^0 = V_{nsh} \equiv \kappa^2 M^4 \left(\frac{\lambda^2}{\gamma^2 + \lambda^2}\right) \left(1 + \frac{\gamma^2 m^2}{4\kappa^2\lambda^2 M^2}\right)^2, \quad (8)$$

is the trajectory for the new shifted hybrid inflation [8]. On this direction, G_{LR} is broken to G_{SM}. The third flat direction, which exists only if $M^2 > m^2/2\kappa^2$, lies at

$$\Phi = \pm M\sqrt{1 - \frac{m^2}{2\kappa^2 M^2}}, \qquad \bar{\Phi} = \frac{2\kappa\Phi}{m} S, \qquad H = \bar{H} = 0. \quad (9)$$

It is the path along which semishifted hybrid inflation [25] takes place with

$$V_F^0 = V_{ssh} \equiv m^2 M^2 \left(1 - \frac{m^2}{4\kappa^2 M^2}\right). \quad (10)$$

Along this direction, G_{LR} is broken to $G_{SM} \times U(1)_{B-L}$.

We choose to consider the case where $M^2 > m^2/2\kappa^2$, and thus, the semishifted flat direction exists. One can show—see Ref. [25]—that, in this case, we always have $V_{ssh} < V_{nsh}$ and $V_{ssh} < V_{tr}$. Therefore, the semishifted flat direction, if it exists, always lies lower than both the trivial and the new shifted one. On the other hand, the new shifted flat

direction may lie either lower or higher than the trivial one depending on the values of the parameters. Here we will take $\kappa \sim 1$, $\gamma \ll \lambda \ll \kappa$, $m \ll M$, and $|S| < 1$. In this case, the new shifted flat direction practically coincides with the trivial one, and thus plays no independent role in our scheme.

III. THE DOUBLE INFLATIONARY SCENARIO

In this section, we will show that, after including SUGRA corrections, the trivial path becomes stable for large absolute values of the real canonically normalized inflaton. Thus, it can support a first stage of inflation during which the Universe undergoes a number of e-foldings which, although limited, is adequately large for all the cosmological scales to exit the horizon. Strong radiative corrections to the inflationary potential, which are controlled by the parameter κ, in conjunction with mild SURGA corrections then guarantee that an appreciable value of the tensor-to-scalar ratio can be achieved together with an acceptable value of the scalar spectral index. Our scenario can predict values of the tensor-to-scalar ratio only up to about 0.05, because larger values require unacceptably large running of the scalar spectral index.

A subsequent second stage of inflation along the semi-shifted path can provide the additional number of e-foldings required for solving the horizon and flatness problems of the standard hot big bang cosmology. This is possible since, for the parameters chosen, this direction is almost orthogonal to the trivial path, and thus, it is not affected by the strong radiative corrections present during the first stage of inflation. In this connection, it is also important that the SUGRA corrections on the semishifted path remain mild.

After the termination of the first stage of inflation, the system moves towards the semishifted path, and the group $SU(2)_R$ breaks spontaneously to a $U(1)$ subgroup, leading to the formation of magnetic monopoles. On the other hand, the spontaneous breaking of a linear combination of this $U(1)$ and $U(1)_{B-L}$, which takes place at the end of the second inflationary stage, leads to the production of open cosmic strings which connect these monopoles to anti-monopoles. Subsequently, the monopoles come into the postinflationary horizon and the whole system of strings and monopoles decays well before recombination without leaving any trace in the CMBR. The gravitational waves which are generated by the decaying strings may, though, be measurable in the future.

A. The first inflationary stage

We adopt here the following Kähler potential:

$$
\begin{aligned}
K = & -\ln\left(1 - |S|^2\right) - \ln\left(1 - |\bar{\Phi}|^2\right) + |\Phi|^2 + |H|^2 \\
& + |\bar{H}|^2 - 2\ln\left(-\ln|Z_1|^2\right) + |Z_2|^2
\end{aligned}
\tag{11}
$$

$(|S|, |\bar{\Phi}| < 1, 0 < |Z_1| < 1)$, where we include two extra G_{LR} singlet superfields Z_1 and Z_2, which do not enter the superpotential at all because they transform nontrivially under additional anomalous $U(1)$ gauge symmetries. The resulting F-term potential in SUGRA is given by

$$
V_F = \left[\sum_i |W_{X_i} + K_{X_i} W|^2 K_{X_i X_i^*}^{-1} - 3|W|^2 \right] e^K,
\tag{12}
$$

where a subscript X_i denotes derivation with respect to the field X_i, and the sum extends over all the seven fields S, $\bar{\Phi}$, Φ, H, $\bar{H}$, Z_1, Z_2. The values of Z_1 and Z_2 are assumed to be fixed [26] by anomalous D terms. Note that the superfields S, $\bar{\Phi}$, Z_1 possess Kähler potentials of the no-scale type, which for $Z_2 = 0$, in view of the relation

$$
|K_{Z_1}|^2 K_{Z_1 Z_1^*}^{-1} = 2,
\tag{13}
$$

guarantee the exact flatness of the potential along the trivial path [26] and its approximate flatness on the semishifted one—see below. These paths are, respectively, parametrized by the complex inflatons S and $\bar{\Phi}$ (approximately). However, as we shall see—cf. Ref. [26]—the relation

$$
|K_{Z_2}|^2 K_{Z_2 Z_2^*}^{-1} = |Z_2|^2 \equiv \beta
\tag{14}
$$

implies that these inflatons acquire squared masses proportional to β as soon as the value of Z_2 becomes nonzero.

Using the R and $U(1)_{B-L}$ symmetries of the model, we can rotate S and H on the real axis—cf. e.g. Ref. [25]. The fields Φ, $\bar{\Phi}$, $\bar{H}$ remain in general complex. However, for simplicity, we will also restrict them on the real axis. This is not expected to influence our results in any essential way, since these fields are anyway real in the vacuum and on all the flat directions of the model, given that the parameters of the model are chosen to be real—see Sec. II. Also, we can show that, everywhere on the trivial and the semishifted inflationary paths, the mass-squared matrices of the imaginary parts of the fields do not mix with the mass-squared matrices of their real parts and, during both inflations, have positive eigenvalues in the directions perpendicular to these paths. So there is no instability in the direction of the imaginary parts of the fields which are orthogonal to these inflationary trajectories. Moreover, as we can demonstrate, both the trivial and the semishifted inflationary paths are destabilized with the fields developing real values.

We define the canonically normalized real scalar fields σ, $\bar{\phi}$, ϕ, h, $\bar{h}$ corresponding to the Kähler potential in Eq. (11) as follows—cf. Ref. [26]:

$$
S = \tanh\frac{\sigma}{\sqrt{2}}, \qquad \bar{\Phi} = \tanh\frac{\bar{\phi}}{\sqrt{2}},
\tag{15}
$$

$$\Phi = \frac{\phi}{\sqrt{2}}, \qquad H = \frac{h}{\sqrt{2}}, \qquad \bar{H} = \frac{\bar{h}}{\sqrt{2}}. \qquad (16)$$

We can now evaluate the potential V_F in Eq. (12) with the overall factor $\exp\left[-2\ln\left(-\ln|Z_1|^2\right) + |Z_2|^2\right]$ absorbed into redefined parameters κ, γ, m, and λ and find

$$V_F = \left[A_1^2\cosh^2\frac{\bar{\phi}}{\sqrt{2}} - A_2^2\sinh^2\frac{\bar{\phi}}{\sqrt{2}} + \beta A_3^2 + A_4^2 + A_5^2 \right.$$
$$+ \frac{1}{2}(h^2 + \bar{h}^2)A_6^2 + \frac{1}{2}(\phi^2 + h^2 + \bar{h}^2)A_3^2$$
$$\left. + (\sqrt{2}\phi A_5 - 2h\bar{h}A_6)A_3\right]e^{\frac{1}{2}(\phi^2 + h^2 + \bar{h}^2)}. \qquad (17)$$

Here

$$A_1 = \kappa\left(M^2 - \frac{\phi^2}{2}\right) - \frac{\gamma}{2}h\bar{h}, \qquad (18)$$

$$A_2 = m\frac{\phi}{\sqrt{2}} - \frac{\lambda}{2}h\bar{h}, \qquad (19)$$

$$A_3 = A_1\sinh\frac{\sigma}{\sqrt{2}}\cosh\frac{\bar{\phi}}{\sqrt{2}} + A_2\cosh\frac{\sigma}{\sqrt{2}}\sinh\frac{\bar{\phi}}{\sqrt{2}}, \qquad (20)$$

$$A_4 = A_1\sinh\frac{\sigma}{\sqrt{2}}\sinh\frac{\bar{\phi}}{\sqrt{2}} + A_2\cosh\frac{\sigma}{\sqrt{2}}\cosh\frac{\bar{\phi}}{\sqrt{2}}, \qquad (21)$$

$$A_5 = m\cosh\frac{\sigma}{\sqrt{2}}\sinh\frac{\bar{\phi}}{\sqrt{2}} - \sqrt{2}\kappa\phi\sinh\frac{\sigma}{\sqrt{2}}\cosh\frac{\bar{\phi}}{\sqrt{2}}, \qquad (22)$$

and

$$A_6 = \gamma\sinh\frac{\sigma}{\sqrt{2}}\cosh\frac{\bar{\phi}}{\sqrt{2}} + \lambda\cosh\frac{\sigma}{\sqrt{2}}\sinh\frac{\bar{\phi}}{\sqrt{2}}. \qquad (23)$$

On the trivial trajectory where $\bar{\phi}$, ϕ, h, $\bar{h} = 0$, the F-term potential takes the form

$$V_F = \kappa^2 M^4\left[1 + \beta\sinh^2\frac{\sigma}{\sqrt{2}}\right]. \qquad (24)$$

The mass-squared eigenvalues in the directions perpendicular to this trajectory for $\sinh^2(\sigma/\sqrt{2}) \gg M^2/2$ can also be found from Eq. (17) to be

$$m_\phi^2 \simeq 4\kappa^2\sinh^2\frac{\sigma}{\sqrt{2}}, \qquad (25)$$

$$m_{\bar{\phi}}^2 \simeq \kappa^2 M^4\left(1 + (1+\beta)\sinh^2\frac{\sigma}{\sqrt{2}}\right), \qquad (26)$$

$$m_{\chi_1}^2 \simeq (\kappa M^2 - \gamma)\left[\kappa M^2 + ((1+\beta)\kappa M^2 - \gamma)\sinh^2\frac{\sigma}{\sqrt{2}}\right], \qquad (27)$$

and

$$m_{\chi_2}^2 \simeq (\kappa M^2 + \gamma)\left[\kappa M^2 + ((1+\beta)\kappa M^2 + \gamma)\sinh^2\frac{\sigma}{\sqrt{2}}\right], \qquad (28)$$

where $\chi_1 = (h + \bar{h})/\sqrt{2}$ and $\chi_2 = (h - \bar{h})/\sqrt{2}$. Thus, assuming that $\gamma < \kappa M^2$, we see that the trivial path, which is flat in the limit $\beta \to 0$, is stable for large absolute values of the inflaton σ.

Note that Eqs. (27) and (28) hold for any value of $\sinh^2(\sigma/\sqrt{2})$. On the contrary, one can show that as $\sinh^2(\sigma/\sqrt{2})$ decreases, the eigenvalues and eigenstates of the mass-squared matrix of the $\phi - \bar{\phi}$ system change. In particular, when $\sinh^2(\sigma/\sqrt{2}) \simeq M^2/2 + m^2/(2\kappa^2 M^2)$, the mass-squared matrix of the $\phi - \bar{\phi}$ system acquires a zero eigenvalue with $\bar{\phi}$ dominating the corresponding eigenstate. Subsequently, as $\sinh^2(\sigma/\sqrt{2})$ approaches the value $M^2/2$, the eigenvalues of the $\phi - \bar{\phi}$ mass-squared matrix become almost opposite to each other, with ϕ and $\bar{\phi}$ contributing almost equally to both the eigenstates. A further decrease of $\sinh^2(\sigma/\sqrt{2})$ leads to the domination of the unstable eigenstate by ϕ. Since the field ϕ is required to develop a nonzero VEV in order to cancel the false vacuum energy density $\kappa^2 M^4$ on the trivial trajectory—see Eq. (2) or Eqs. (17) and (18)—we will take as critical value σ_c of σ at which the trivial path is destabilized the one determined by the relation

$$\sinh^2\frac{\sigma_c}{\sqrt{2}} = \frac{M^2}{2}. \qquad (29)$$

To the F-term scalar potential V_F in Eq. (17) has to be added during the first stage of inflation (i.e. for $|\sigma| \geq |\sigma_c|$) the term

$$V_r^\phi = \kappa^2 M^4\left(\frac{N_\phi\kappa^2}{8\pi^2}\right)\ln\frac{2\tanh^2\frac{\sigma}{\sqrt{2}}}{M^2}, \qquad (30)$$

corresponding to the dominant one-loop radiative corrections to the inflationary potential due to the N_ϕ-dimensional supermultiplet Φ ($N_\phi = 3$). Notice that the renormalization scale in these radiative corrections is chosen such that V_r^ϕ vanishes at $|\sigma| = |\sigma_c|$ ($\tanh^2(\sigma_c/\sqrt{2}) \simeq \sinh^2(\sigma_c/\sqrt{2}) = M^2/2$). Setting

$$\delta_\phi = \frac{N_\phi\kappa^2}{2\pi^2}, \qquad (31)$$

we can rewrite the full inflationary potential and its derivatives (denoted by primes) with respect to the canonically normalized real inflaton field σ as follows:

$$\frac{V}{\kappa^2 M^4} = 1 + \beta \sinh^2 \frac{\sigma}{\sqrt{2}} + \frac{\delta_\phi}{4} \ln \frac{2\tanh^2 \frac{\sigma}{\sqrt{2}}}{M^2} \equiv C(\sigma), \quad (32)$$

$$\frac{V'}{\kappa^2 M^4} = \frac{1}{\sqrt{2}} \sinh(\sqrt{2}\sigma)\left(\beta + \frac{\delta_\phi}{\sinh^2(\sqrt{2}\sigma)}\right), \quad (33)$$

$$\frac{V''}{\kappa^2 M^4} = \cosh(\sqrt{2}\sigma)\left(\beta - \frac{\delta_\phi}{\sinh^2(\sqrt{2}\sigma)}\right), \quad (34)$$

and

$$\frac{V'''}{\kappa^2 M^4} = \sqrt{2}\sinh(\sqrt{2}\sigma)\left(\beta - \frac{\delta_\phi}{\sinh^2(\sqrt{2}\sigma)}\right)$$
$$+ \frac{2\sqrt{2}\delta_\phi}{\tanh^2(\sqrt{2}\sigma)\sinh(\sqrt{2}\sigma)}. \quad (35)$$

The usual slow-roll parameters for inflation are then written as

$$\epsilon = \frac{1}{2}\left(\frac{V'}{\kappa^2 M^4}\right)^2 \frac{1}{C^2(\sigma)}, \quad (36)$$

$$\eta = \left(\frac{V''}{\kappa^2 M^4}\right)\frac{1}{C(\sigma)}, \quad (37)$$

and

$$\xi = \left(\frac{V'}{\kappa^2 M^4}\right)\left(\frac{V'''}{\kappa^2 M^4}\right)\frac{1}{C^2(\sigma)} = 2|\tanh(\sqrt{2}\sigma)|\eta\sqrt{\epsilon}$$
$$+ \frac{4\delta_\phi\sqrt{\epsilon}}{C(\sigma)\tanh^2(\sqrt{2}\sigma)|\sinh(\sqrt{2}\sigma)|}. \quad (38)$$

Using these expressions, we can evaluate the scalar spectral index n_s, its running α_s, and the tensor-to-scalar ratio r from the formulas

$$n_s = 1 + 2\eta - 6\epsilon, \quad (39)$$

$$\alpha_s = 16\eta\epsilon - 24\epsilon^2 - 2\xi, \quad (40)$$

$$r = 16\epsilon. \quad (41)$$

Finally, the scalar potential on the trivial inflationary path can be written in terms of the scalar power spectrum amplitude A_s and r as follows:

$$V = \frac{3\pi^2}{2}A_s r. \quad (42)$$

As a numerical example, we take the value of the real inflaton field σ at the horizon exit of the pivot scale $k_* = 0.05$ Mpc^{-1} to be $\sigma_* = 1.45$. Also, we take $\kappa = 1.7$, $\beta = 0.022$, and the scalar power spectrum amplitude $A_s = 2.215 \times 10^{-9}$ at the same pivot scale [13]. With these input numbers, we then find $M = 3.493 \times 10^{-3}$, $C(\sigma_*) = 2.2941$, $\epsilon = 0.00188$, $\eta = -0.01389$, $n_s = 0.9609$, $\alpha_s = -0.01674$, and $r = 0.0301$.

As one can see, not only can our predictions be perfectly consistent with the latest data released by the Planck satellite experiment [13], but they can also accommodate large values of the tensor-to-scalar ratio r of the order of a few $\times 10^{-2}$. As is obvious from Eq. (41), such values of r require relatively large values of ϵ, which in turn reduce the scalar spectral index n_s in Eq. (39) below unity, but not quite adequately to make it compatible with the data. So we need an appreciable negative value of η, which requires that the parenthesis on the right-hand side of Eq. (34) be dominated by the second term. A similar parenthesis appears on the right-hand side of Eq. (33) too, but with the two terms added rather than subtracted. As it turns out, both these terms have to be appreciable, with the second one being larger, in order to be able to bring n_s near its best-fit value from the Planck data. This is certainly possible only for large values of the parameter κ controlling the radiative corrections on the trivial path. Note that the first stage of inflation ends before the system reaches the critical point in Eq. (29) by violating the slow-roll conditions, and the obtained number of e-foldings is limited due to the large values of ϵ involved and the fact that $\sigma_* \sim 1$.

B. The second inflationary stage

For the rest of the parameters of the model, we choose the values $m = 1.827 \times 10^{-5}$, $\lambda = 0.1$, and $\gamma = 10^{-6}$. We solve numerically the differential equations of the system with potential energy density given by the exact V_F in Eq. (17) supplemented with the relevant radiative corrections and the D terms involving the fields H, $\bar{H}$. The numerical investigation then reveals that there exist appropriate small initial absolute values of the scalar fields $\bar{\phi}$, ϕ, h, $\bar{h}$ for which, after the first stage of inflation and the elapse of a sufficient amount of cosmic time t for the energy density to approach $m^2 M^2$, we have $\phi^2 \simeq 2M^2$, $h, \bar{h} \simeq 0$, and the scalar fields σ and $\bar{\phi}$ take values such that $A_5 \simeq 0$ with $|\sigma| \ll 1$. So it is obvious that the system reaches the semishifted inflationary path in Eq. (9)—note that the second relation in this equation is equivalent to $A_5 = 0$. It is remarkable that $|\bar{\phi}|$, which at the end of the first inflationary stage is extremely small, manages to attain values of the order of a few $\times 10^{-1}$ at the onset of the second stage.

It is worth noticing, however, that the initial values of the fields which lead to a double inflation scenario, although quite frequent, do not seem to form well-defined connected regions. In other words, the solutions of the coupled system of differential equations exhibit a rather "chaotic" behavior in the sense that a slight change of the initial conditions may lead from a double to a single inflation scenario. Note that a similar situation is encountered [27] even in the simplest SUSY hybrid inflation scenario, where a slight change of initial conditions may ruin inflation, leading the system directly to the vacuum. Such a behavior, given the multidimensionality of the field space in our case, makes it very difficult to provide further details concerning the structure of the space of initial values leading to the desirable scenario.

For a negligible value of γ, $\phi^2 \simeq 2M^2$, $A_5 \simeq 0$, and $|\sigma| \ll 1$, we find that $A_1 \simeq 0$, $A_3 \simeq A_2 \sinh(\bar{\phi}/\sqrt{2})$, $A_4 \simeq A_2 \cosh(\bar{\phi}/\sqrt{2})$, $A_6 \simeq \lambda \sinh(\bar{\phi}/\sqrt{2})$, and the F-term scalar potential becomes

$$
V_F \simeq \left[A_2^2 + (\beta + M^2)A_2^2 \sinh^2 \frac{\bar{\phi}}{\sqrt{2}} \right.
$$
$$
+ \frac{1}{2}(h^2 + \bar{h}^2)(\lambda^2 + A_2^2)\sinh^2 \frac{\bar{\phi}}{\sqrt{2}}
$$
$$
\left. - 2h\bar{h}\lambda A_2 \sinh^2 \frac{\bar{\phi}}{\sqrt{2}} \right] e^{M^2 + \frac{1}{2}(h^2 + \bar{h}^2)} \quad (43)
$$

$$
\overset{h,\bar{h}\simeq 0}{\underset{M^2 \ll \beta}{\simeq}} m^2 M^2 \left[1 + \beta \sinh^2 \frac{\bar{\phi}}{\sqrt{2}} \right]. \quad (44)
$$

The expression in Eq. (44) gives approximately the F–term potential on the semishifted path. Notice the striking similarity of this expression to the one in Eq. (24) involving the same parameter β.

From $A_5 \simeq 0$ and the fact that $A_5 \propto m \tanh(\bar{\phi}/\sqrt{2}) - \sqrt{2}\kappa\phi \tanh(\sigma/\sqrt{2})$, it follows that the combination of S and $\bar{\Phi}$, which could remain large when the energy density approaches $m^2 M^2$ and plays the role of the complex inflaton in the second stage of inflation, is

$$
\frac{mS + 2\kappa\langle\Phi\rangle\bar{\Phi}}{\sqrt{m^2 + 4\kappa^2 M^2}} \simeq \bar{\Phi}, \quad (45)
$$

since the contribution of $\bar{\Phi}$ in this combination is about $2\kappa M/m \simeq 650$ times bigger than that of S.

From Eq. (43), we can construct the mass-squared matrix for the $h - \bar{h}$ system during the second stage of inflation. We find that the mass eigenstates are given by the combinations $\chi_1 = (h + \bar{h})/\sqrt{2}$ and $\chi_2 = (h - \bar{h})/\sqrt{2}$ with masses squared

$$
m_{\chi_1}^2 \simeq (\lambda - mM)\left[(\lambda - (1+\beta)mM)\sinh^2 \frac{\bar{\phi}}{\sqrt{2}} - mM \right], \quad (46)
$$

$$
m_{\chi_2}^2 \simeq (\lambda + mM)\left[(\lambda + (1+\beta)mM)\sinh^2 \frac{\bar{\phi}}{\sqrt{2}} + mM \right]. \quad (47)
$$

We see that χ_1 develops an instability which terminates the valley along which the second stage of inflation takes place with the critical value $\bar{\phi}_c$ of the real canonically normalized inflaton $\bar{\phi}$ being approximately determined from the relation

$$
\sinh^2 \frac{\bar{\phi}_c}{\sqrt{2}} = \frac{mM}{\lambda}. \quad (48)
$$

To the F-term scalar potential V_F during the second stage of inflation (i.e. for $|\bar{\phi}| \geq |\bar{\phi}_c|$ and $|\sigma| < |\sigma_c|$) has to be added the following term:

$$
V_r^h = m^2 M^2 \left(\frac{N_h \lambda^2}{16\pi^2} \right)
$$
$$
\times \ln \frac{(\tanh \frac{\sigma}{\sqrt{2}} + \sqrt{2}\kappa \frac{\langle\phi\rangle}{m} \tanh \frac{\bar{\phi}}{\sqrt{2}})^2}{(1 + 4\kappa^2 \frac{M^2}{m^2})(\frac{mM}{\lambda})}, \quad (49)
$$

which may be approximated as

$$
V_r^h \simeq m^2 M^2 \left(\frac{N_h \lambda^2}{16\pi^2} \right) \ln \frac{\lambda \tanh^2 \frac{\bar{\phi}}{\sqrt{2}}}{mM} \quad (50)
$$

and corresponds to the dominant one-loop radiative corrections due to the N_h-dimensional supermultiplets H, $\bar{H}$ ($N_h = 2$). Notice that the renormalization scale is chosen such that V_r^h vanishes at $|\bar{\phi}| = |\bar{\phi}_c|$ ($\tanh^2(\bar{\phi}_c/\sqrt{2}) \simeq \sinh^2(\bar{\phi}_c/\sqrt{2}) = mM/\lambda$).

The one-loop radiative corrections involving the Φ supermultiplet are neglected since they are relatively very small. This is because Φ couples to the combination which plays the role of the complex inflaton during the second stage of inflation only through S, and the contribution of S to this combination is severely suppressed. Indeed, the slope of the potential along the semishifted path generated by the radiative corrections involving the Φ supermultiplet is suppressed relative to the one involving the H, $\bar{H}$ supermultiplets by, approximately, a factor $(N_\phi/8N_h)(m/\lambda M)^2 \sim 5 \times 10^{-4}$. This is a very important property of our model resulting from the fact that, for the parameters chosen, the semishifted path is almost perpendicular to the trivial one. So the very strong radiative corrections on the trivial trajectory, which are controlled by the strong coupling constant κ and are needed, as we have seen, for accommodating appreciable values of r, do not

affect the second stage of inflation. This is very crucial, since otherwise the semishifted path would become too steep and there would be no way of generating the extra e-foldings required for solving the puzzles of hot big bang cosmology.

The number of e-foldings during the second stage of inflation between an initial value $\bar{\phi}_{\text{in}}$ and a final value $\bar{\phi}_{\text{f}}$ of the inflaton $\bar{\phi}$ is given, in the slow-roll approximation, by $N(\bar{\phi}_{\text{f}}) - N(\bar{\phi}_{\text{in}})$, where

$$N(\bar{\phi}) \simeq \frac{1}{2\beta\sqrt{1-(\delta_h/\beta)}} \ln \frac{\cosh(\sqrt{2}\bar{\phi}) + \sqrt{1-(\delta_h/\beta)}}{\cosh(\sqrt{2}\bar{\phi}) - \sqrt{1-(\delta_h/\beta)}},$$

$$(51)$$

with

$$\delta_h = \frac{N_h \lambda^2}{4\pi^2}. \tag{52}$$

The termination of slow-roll inflation is due to the radiative corrections in Eq. (50) and takes place at a value $\bar{\phi}_{\text{f}}$ ($|\bar{\phi}_{\text{f}}| \gg |\bar{\phi}_{\text{c}}|$) of $\bar{\phi}$ given by

$$\cosh(\sqrt{2}\bar{\phi}_{\text{f}}) \simeq \frac{\delta_h}{2} + \sqrt{1 + \frac{\delta_h^2}{4}}. \tag{53}$$

It turns out numerically that, with the chosen values of the parameters, the pivot scale $k_* = 0.05$ Mpc^{-1} suffers about 13 e-foldings during the first stage of inflation. As a consequence, approximately another 38–39 e-foldings (for reheat temperature $T_{\text{r}} = 10^9$ GeV) must be provided by the second inflationary stage, which requires a value of $|\bar{\phi}_{\text{in}}| \simeq 0.23$ at the onset of this stage. This requirement can indeed be fulfilled in our numerical example, as we have shown by extensive numerical studies. It is worth noticing that, due to the presence of mild but appreciable SUGRA corrections and not-too-weak radiative corrections, the second stage of inflation is able to generate a relatively limited number of e-foldings. Consequently, this number is not too sensitive to the value of $\bar{\phi}_{\text{in}}$.

In order to support the statements made above concerning the numerical part of our work, we depict, in Fig. 1, the evolution of the fields σ and $\bar{\phi}$ as functions of the number of e-foldings N starting from the point where the pivot scale k_* exits the horizon for a particular choice of initial conditions at this point. Namely, we start with $\sigma = 1.45$, $\bar{\phi} = 10^{-3}$, $\phi = 10^{-8}$, $h = 10^{-4}$, and $\bar{h} = 1.01 \times 10^{-4}$, corresponding to an almost D-flat direction. All the fields are given zero initial velocity except for σ, the velocity of which is taken to be -1.1074×10^{-6}. This is, indeed, the actual value of the velocity of σ on the trivial path, which was determined numerically.

We observe that σ assumes values above its critical value for about 13 e-foldings. Towards the end of the first
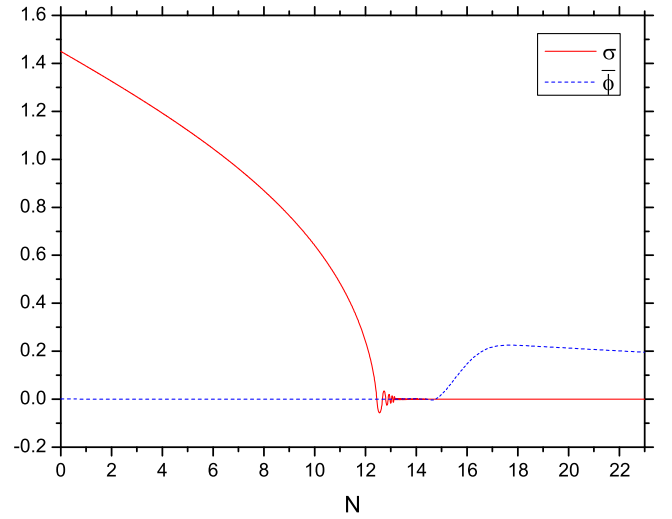

FIG. 1 (color online). The evolution of the fields σ and $\bar{\phi}$ for the case with $r = 0.0301$ versus the number N of e-foldings after the horizon exit of the pivot scale k_*, where we take $\sigma = 1.45$, $\bar{\phi} = 10^{-3}$, $\phi = 10^{-8}$, $h = 10^{-4}$, $\bar{h} = 1.01 \times 10^{-4}$, and $d\sigma/dt = -1.1074 \times 10^{-6}$.

inflationary stage, σ oscillates with appreciable amplitude around zero 4 times. We may consider that, during these oscillations, the first stage of inflation has not come to an end. When the amplitude of the oscillations falls below the critical value of σ, ϕ moves to its value on the semishifted path and $\bar{\phi}$ starts performing slow oscillations with variable amplitudes typically of order M. The size of $\bar{\phi}$ remains small for about 1.7 e-foldings before starting its growth and acquires its largest value $\simeq 0.225$ at $N \simeq 17.7$, when the second inflationary stage may be considered as having already started. For $N \gtrsim 20$, the evolution of $\bar{\phi}$ follows Eq. (51) closely.

If we allow for a stronger running of the scalar spectral index, we may obtain larger values of r. For example, taking $\sigma_* = 1.35$, $\kappa = 1.75$, and $\beta = 0.037$, we find $M = 3.891 \times 10^{-3}$, $C(\sigma_*) = 2.3479$, $\epsilon = 0.00314$, $\eta = -0.00844$, $n_{\text{s}} = 0.9643$, $\alpha_{\text{s}} = -0.03007$, and $r = 0.0502$. In addition, we choose $m = 3.891 \times 10^{-5}$, $\lambda = 0.1$, and $\gamma = 10^{-6}$. With these choices, the pivot scale suffers about 10 e-foldings during the first stage of inflation and, consequently, approximately another 41–42 e-foldings must be provided by the second stage. This implies that $|\bar{\phi}_{\text{in}}|$ lies in the range 0.38–0.40. We verified numerically that the fulfillment of this requirement is indeed feasible. In Fig. 2, we depict the evolution of the fields σ and $\bar{\phi}$ as functions of the number of e-foldings N, again starting from the point where the pivot scale k_* exits the horizon for a particular choice of initial conditions at this point.

Needless to say, by changing the values of the input parameters of the model, we can easily achieve successful solutions with smaller values of the tensor-to-scalar ratio,
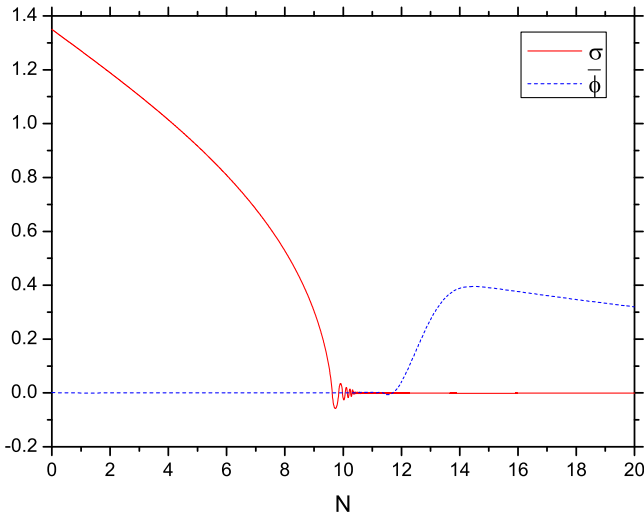
FIG. 2 (color online). The evolution of the fields σ and $\bar{\phi}$ for the case with $r = 0.0502$ versus the number N of e-foldings after the horizon exit of the pivot scale k_*, where we take $\sigma = 1.35$, $\bar{\phi} = 10^{-3}$, $\phi = 10^{-8}$, $h = 9 \times 10^{-4}$, $\bar{h} = 9.01 \times 10^{-4}$, and $d\sigma/dt = -1.8523 \times 10^{-6}$.

and there is no particular fine-tuning of the parameters required in our model.

C. The formation of monopoles and cosmic strings

After the termination of the first stage of inflation, the system moves towards the semishifted path and the group $SU(2)_{\rm R}$ breaks spontaneously to a $U(1)$ subgroup by the nonzero value which the field Φ develops. This leads to the formation of superheavy magnetic monopoles. We can obtain a rough order-of-magnitude estimate of the mean distance between the monopoles and the antimonopoles, which is adequate for our purposes here. We assume that, at production, this distance is $p(2\kappa M)^{-1}$ as determined by the relevant Higgs boson mass with $p \sim 1$ being a geometric factor. In the matter-dominated period between the two inflationary stages, this distance increases by a factor $\sim(\kappa^2 M^4/m^2 M^2)^{1/3}$, where $\kappa^2 M^4$ and $m^2 M^2$ are the classical potential energy densities on the trivial and the semishifted paths, respectively. The subsequent second inflationary stage stretches this distance by a factor $\exp N_2$, where N_2 is the number of e-foldings during this stage which is large but not huge—cf. Ref. [28]. In the period of damped oscillations of the inflaton field, the monopole-antimonopole distance increases by another factor $\sim(m^2 M^2/c(T_{\rm r})T_{\rm r}^4)^{1/3}$, where $T_{\rm r}$ is the reheat temperature, which we take to be 10^9 GeV, and $c(T) = \pi^2 g(T)/30$, with $g(T)$ being the effective number of massless degrees of freedom at cosmic temperature T. In the radiation-dominated period which follows reheating, the monopole-antimonopole distance is multiplied by another factor $\sim T_{\rm r}/T \sim (4c(T)/3)^{1/4}T_{\rm r}\sqrt{t}$, where t is the

cosmic time. So this distance, in the radiation-dominated period, becomes

$$\sim \left(\frac{4}{3}\right)^{\frac{1}{4}} c(T_{\rm r})^{-\frac{1}{3}} c(T)^{\frac{1}{4}} p(2\kappa M)^{-1} e^{N_2} \left(\frac{\kappa^2 M^4}{T_{\rm r}^4}\right)^{\frac{1}{3}} T_{\rm r} t^{\frac{1}{2}}. \quad (54)$$

Equating this distance with the postinflationary particle horizon $\sim 2t$, we find the time $t_{\rm H}$ (in $t_{\rm P} \equiv m_{\rm P}^{-1} \simeq 2.7027 \times 10^{-43}$ sec units) at which the monopoles enter this horizon:

$$t_{\rm H} \sim \frac{p^2}{8\sqrt{3}} c(T_{\rm r})^{-\frac{2}{3}} c(T_{\rm H})^{\frac{1}{2}} e^{2N_2} \left(\frac{M}{\kappa T_{\rm r}}\right)^{\frac{2}{3}}. \quad (55)$$

Here $T_{\rm H}$ is the cosmic temperature at cosmic time $t_{\rm H}$.

The formation of monopoles is not the whole story, though, since our scenario leads to the generation of cosmic strings too. Indeed, after the end of the second inflationary stage, the system settles in one of the two distinct continua of SUSY vacua in Eqs. (3) and (4) with $\Phi_\pm \simeq \pm M$ in our case, and a linear combination of the $U(1)_{B-L}$ gauge symmetry and the unbroken $U(1)$ subgroup of $SU(2)_{\rm R}$ breaks spontaneously, leading to the production of local cosmic strings. These strings, if they survived after recombination, could have a small contribution to the CMBR power spectrum parametrized [29] by the dimensionless string tension $G\mu_{\rm s}$, where G is Newton's gravitational constant and $\mu_{\rm s}$ is the string tension, i.e. the energy per unit length of the string. Applying to our case the results of Ref. [29], which considered local strings within the Abelian Higgs model in the Bogomol'nyi limit, we write, for the string tension,

$$\mu_{\rm s} = 4\pi |\langle H \rangle|^2, \quad (56)$$

where $\langle H \rangle$ is the VEV of H. Although the strings in our model are more complicated than in the Bogomol'nyi limit of the Abelian Higgs model, we think that the above estimate for the string tension is good enough for our purposes here.

In our case, the strings decay well before recombination, and thus do not affect the CMBR. The reason is that they are mostly open strings connecting monopoles to antimonopoles. This is easily understood if we realize that the breaking of $SU(2)_{\rm R} \times U(1)_{B-L}$ to $U(1)_Y$ by $\langle H \rangle$ and $\langle \bar{H} \rangle$ is similar to the breaking of the electroweak gauge group and thus cannot lead to any topologically stable monopoles or strings—see Ref. [30] for a more detailed argument. It can only lead to the existence of topologically unstable dumbbell configurations [31] consisting of an open string with a monopole and an antimonopole at the two ends.

The strings, at any given time after their formation, can be thought of as random walks with a step of the order of the particle horizon [32]—to describe the evolution of this string network, we will follow closely this reference. They typically connect monopoles to antimonopoles, but

unstable closed strings of limited size may also exist. As shown in Ref. [32], at all times before the entrance of the monopoles into the horizon, there is of the order of 1 string segment per horizon, and thus, the ratio of the energy density $\rho_s(t)$ of the string network to the total energy density $\rho_{tot}(t)$ of the Universe remains practically constant. At cosmic time t_H, we have approximately one monopole-antimonopole pair per horizon volume $\sim (4\pi/3)(2t_H)^3$ connected by an almost straight string segment of the size of the horizon and energy $\sim \mu_s 2t_H$. The energy density $\rho_s(t_H)$ of the strings at cosmic time t_H is then $\sim 3G\mu_s/2t_H^2$. After this time, more and more string segments enter the horizon, but the length of each segment remains constant. Consequently, the system of string segments behaves like pressureless matter, and thus $\rho_s(t) \sim 3G\mu_s/2(t_H t^3)^{1/2}$, which implies that the "relative string energy density"

$$\frac{\rho_s(t)}{\rho_\gamma(t)} \sim 2G\mu_s \left(\frac{t}{t_H}\right)^{\frac{1}{2}} \tag{57}$$

[$\rho_\gamma(t)$ is the photon energy density] increases with time—remember that we are in the radiation-dominated era of the Universe, and thus $\rho_{tot}(t) = \rho_\gamma(t) = c(T)T^4 = 3/4t^2$. The strings, finally, decay at cosmic time [33]

$$t_d \sim (\Gamma G\mu_s)^{-1}2t_H, \tag{58}$$

where $\Gamma \sim 50$, by emitting gravitational waves with energy density $\rho_{gw}(t_d)$ at production given by

$$\frac{\rho_{gw}(t_d)}{\rho_\gamma(t_d)} \sim 2\left(\frac{2}{\Gamma}\right)^{\frac{1}{2}}(G\mu_s)^{\frac{1}{2}}, \tag{59}$$

as one can infer from Eq. (57). Note that this formula also gives the maximal relative string energy density.

From Eq. (55), where we substitute the lowest value of the number of e-foldings N_2 and take $p = 2$, we find that, for the two numerical examples presented, $t_H \sim 4.76 \times 10^{-7}$ sec and 1.04×10^{-4} sec, respectively. In deriving these values, we take $g(T_r) = 228.75$, which corresponds to the spectrum of the minimal SUSY SM, and $g(T_H) = 40.75$ and 10.75 in our two numerical examples, respectively. The above values of $g(T_H)$ are consistent with the effective number of massless degrees of freedom at the cosmic temperatures T_H corresponding to the values of the cosmic time t_H obtained. We see that the strings enter the horizon well before the time of big bang nucleosynthesis. Their decay time is $t_d \sim 5.97 \times 10^{-2}$ sec and 5.49 sec in the two cases, as one can find from the corresponding dimensionless string tensions

$$G\mu_s = \frac{|\langle H \rangle|^2}{2} \simeq \frac{mM}{2\lambda} \simeq 3.19 \times 10^{-7} \quad \text{and} \quad 7.57 \times 10^{-7}. \tag{60}$$

This means that the cosmic strings decay around the time of nucleosynthesis, and thus, well before recombination, which takes place at a cosmic time $\sim 10^{13}$ sec. As a consequence, they do not affect the CMBR. Their maximal relative energy density in the Universe is $\sim 2.26 \times 10^{-4}$ and 3.48×10^{-4} for our two numerical examples. So the cosmic strings remain always subdominant. In particular, they do not disturb nucleosynthesis at all.

Had the strings been around until the present time, an upper bound would have to be imposed on the dimensionless string tension in order to keep their contribution to the CMBR power spectrum at an acceptable level. For the Abelian-Higgs field theory model, this bound is found to be [34]

$$G\mu_s \lesssim 3.2 \times 10^{-7}. \tag{61}$$

In our first numerical example, the dimensionless string tension $G\mu_s$ given in Eq. (60) almost saturates the upper bound in Eq. (61), but it violates the recent more stringent upper bound [35]

$$G\mu_s \lesssim 3.3 \times 10^{-8} \tag{62}$$

from pulsar timing arrays, which also holds for strings surviving until the present time. Our second numerical example violates both the bounds in Eqs. (61) and (62). Thus, both our examples are only possible because the strings decay sufficiently early.

The value of the ratio of the energy density of the gravitational waves produced by the strings to that of the photons at the present cosmic time t_0 is found from Eq. (59) to be—cf. Ref. [36]—

$$\frac{\rho_{gw}(t_0)}{\rho_\gamma(t_0)} \sim 2\left(\frac{2}{\Gamma}\right)^{\frac{1}{2}}(G\mu_s)^{\frac{1}{2}}\left(\frac{3.9}{10.75}\right)^{\frac{4}{3}}. \tag{63}$$

The present abundance of these gravitational waves is then given by

$$\Omega_{gw}h^2(t_0) \sim \left(\frac{\rho_{gw}(t_0)}{\rho_\gamma(t_0)}\right)\left(\frac{\rho_\gamma(t_0)}{\rho_c(t_0)}\right)h_0^2, \tag{64}$$

where $\rho_c(t_0)$ is the present critical energy density of the Universe and $h_0 \simeq 0.7$ the present value of the Hubble parameter in units of $\text{km sec}^{-1}\,\text{Mpc}^{-1}$. We find that, for our two numerical examples, $\Omega_{gw}h^2(t_0) \sim 2.18 \times 10^{-9}$ and 3.35×10^{-9}, respectively. The frequency $f(t_d)$ of these gravitational waves at production must be $\sim t_H^{-1}$, since the length of the decaying strings is $\sim 2t_H$—see Ref. [33]. The present value of this frequency is then

$$f(t_0) \sim t_H^{-1}\left(\frac{t_d}{t_{eq}}\right)^{\frac{1}{2}}\left(\frac{t_{eq}}{t_0}\right)^{\frac{2}{3}}, \tag{65}$$

where t_{eq} is the equidensity time at which matter starts dominating the Universe. For the two numerical examples, this frequency turns out to be $\sim 1.06 \times 10^{-4}$ Hz and 4.68×10^{-6} Hz, respectively. Note that the estimate of the frequencies and abundances of the gravity waves presented here cannot be made much more accurate, but it can certainly be considered good enough for our purposes.

We see that the predicted frequencies of the gravitational waves are too high to yield any restrictions from CMBR considerations [36]. They are also well above the range of frequencies probed by the pulsar timing array observations [37]. So, the most recent stringent bound [35] from these observations does not apply to our case. However, the frequency of the gravitational waves in our first numerical example lies marginally within the range to be probed by the future space-based laser interferometer gravitational-wave observatories such as the evolved laser interferometer space antenna/new gravitational-wave observatory (eLISA/NGO) [38], which is expected to be able to detect values of $\Omega_{gw}h^2(t_0)$ as low as 4×10^{-10}. Our overall conclusion is that the monopole-string system disappears without causing any trouble, but the gravitational waves that it generates may be probed by future space-based laser interferometer observations.

IV. CONCLUSIONS

In view of the recent results [20,21] indicating that appreciable values of the tensor-to-scalar ratio in the CMBR cannot be excluded, we addressed the question whether such values can be obtained in SUSY hybrid inflation models resulting from particle physics. To this end, we have considered a reduced version of the extended SUSY PS model of Ref. [9], which was initially constructed for solving the b-quark mass problem of the simplest SUSY PS model with universal boundary conditions. The reason for focusing on this model is that it is known to support successful versions of hybrid inflation like the standard-smooth one [16]. This scenario is compatible with all the recent data even with a minimal Kähler potential, but predicts negligible values of the tensor-to-scalar ratio.

In the context of this particular particle physics model, we demonstrated that a two-stage hybrid inflationary scenario which can predict values of the tensor-to-scalar ratio of the order of a few $\times 10^{-2}$ can be constructed. For the values of the parameters considered in this paper, the model in the global SUSY limit possesses practically two classically flat directions, the trivial and the semishifted [25] one. The SUGRA corrections to the potential stabilize the trivial flat direction so that it becomes able to support a first stage of inflation. All the cosmological scales exit the horizon during this inflationary stage, and our present horizon undergoes a limited number of e-foldings. The tensor-to-scalar ratio can acquire appreciable values as a result of sufficiently mild (i.e. noncatastrophic), but still appreciable, SUGRA corrections combined with strong radiative corrections to the inflationary potential, while the value of the scalar spectral index remains acceptable. We can obtain values of the tensor-to-scalar ratio up to about 0.05. Larger values would require unacceptably large running of the scalar spectral index.

The additional number of e-foldings required for solving the standard problems of hot big bang cosmology are generated by a second inflationary stage taking place along the semishifted path, where $U(1)_{B-L}$ is unbroken. This is possible since the semishifted direction, being almost orthogonal to the trivial path, is not affected by the strong radiative corrections on the trivial path, and also because the SUGRA corrections on the semishifted path remain mild.

At the end of the first inflationary stage, the group $SU(2)_R$ breaks spontaneously to a $U(1)$ subgroup, and thus magnetic monopoles are formed. The subsequent spontaneous breaking of a linear combination of this $U(1)$ and $U(1)_{B-L}$ at the end of the second inflationary stage leads to the production of cosmic strings connecting these monopoles to antimonopoles. At later times, the monopoles enter the horizon and the string-monopole system decays into gravity waves well before recombination without leaving any trace in the CMBR. The resulting gravity waves, however, may be measurable in the future.

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