



Constraints on the $\eta\eta'$ decay rate of a scalar glueball from gauge/gravity duality

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Predictions of glueball decay rates in the holographic Witten-Sakai-Sugimoto model for low-energy QCD can be uniquely extended to include finite quark masses up to an as-yet-undetermined parameter in the coupling of glueballs to the nonanomalous part of the pseudoscalar mass terms. The assumption of a universal coupling of glueballs to mass terms of the full nonet of pseudoscalar mesons leads to flavor asymmetries in the decay rates of scalar glueballs that agree well with experimental data for the glueball candidate $f_0(1710)$ and implies a vanishing decay rate into $\eta\eta'$ pairs, for which only upper bounds for the $f_0(1710)$ meson are known at present from experiment. Relaxing this assumption, the holographic model gives a tight correlation between the decay rates into pairs of pseudo-Goldstone bosons of the same type and $\eta\eta'$ pairs. If $\Gamma(G \rightarrow KK)/\Gamma(G \rightarrow \pi\pi)$ is kept within the range reported currently by the Particle Data Group for the $f_0(1710)$ meson, the rate $\Gamma(G \rightarrow \eta\eta')/\Gamma(G \rightarrow \pi\pi)$ is predicted to be $\lesssim 0.04$. The corresponding situation for $f_0(1500)$ is also discussed; however, this is found to be much less compatible with the interpretation of a largely unmixed glueball.

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I. INTRODUCTION

While quantum chromodynamics (QCD) has been established beyond any reasonable doubt as the fundamental theory of hadrons, one of its most conspicuous predictions, i.e., the existence of bound states of gluons called glueballs or gluonia [1–3], as well as their nature, has still not been settled by experiment [4–7]. Lattice QCD [8,9] predicts the lightest glueball to be a scalar with mass in the range 1.5–1.8 GeV, with only small effects from unquenching [10], but predictions from first principles on the potential mixing of glueballs with scalar quark-antiquark states and the decay pattern of glueballs are hard to come by.

The expectation from QCD in the limit of large number of colors (N_c) is that glueballs should be comparatively narrow states, and also that mixing should be suppressed [11]. If this holds true for $N_c = 3$ QCD, holographic gauge/gravity duality might be useful to shed light on the nature of glueballs. Here a particularly attractive model is due to Sakai and Sugimoto [12,13] who have extended the Witten model [14] for nonsupersymmetric and nonconformal low-energy QCD based on D4 branes in type-IIA supergravity by $N_f \ll N_c$ D8 and anti-D8 branes, which introduce chiral quarks and allow for a purely geometric realization of chiral symmetry breaking $U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V$.

As reviewed in [15], with almost no free parameters the (massless) Witten-Sakai-Sugimoto model reproduces many features of low-energy QCD and turns out to work remarkably well even on a quantitative level, although the model has a Kaluza-Klein mass scale M_{KK} at the order of glueball masses beyond which the dual theory is actually

a five-dimensional super-Yang-Mills theory (whose extra degrees of freedom are discarded by consistent truncation). Fixing M_{KK} through the experimental value of the ρ meson mass and varying the 't Hooft coupling $\lambda = 16.63\dots 12.55$ such that either the pion decay constant (as originally done in [12,13]) or the string tension in large- N_c lattice simulations [16] is matched, the decay rate of the ρ and the ω meson into pions is obtained as [17]

$$\Gamma(\rho \rightarrow 2\pi)/m_\rho = 0.1535\dots 0.2034, \quad (1)$$

$$\Gamma(\omega \rightarrow 3\pi)/m_\omega = 0.0033\dots 0.0102, \quad (2)$$

which matches well with the respective experimental values, 0.191(1) and 0.0097(1) [18]. The Witten-Sakai-Sugimoto model might therefore be capable of giving useful information on the decay patterns of glueballs, in particular if they exist without being mixed too strongly with $q\bar{q}$ states.

A large number of phenomenological studies assume the lowest scalar glueball to be responsible for the super-numerary state among the three isoscalar mesons $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$, for which the quark model provides just $u\bar{u} + d\bar{d}$ and $s\bar{s}$, but there is no agreement whether the glueball is to be found predominantly in $f_0(1500)$ or $f_0(1710)$ [19–31]. Most of the earlier analysis described $f_0(1500)$ as having a large glueball component with sizable mixing, while later studies (though not all of them) appear to prefer the interpretation of $f_0(1710)$ as a glueball with rather small mixing.

Experimentally, the decay pattern of the isoscalar $f_0(1710)$ is still not known as precisely as that of $f_0(1500)$. However, it is well known that $f_0(1710)$ decays preferentially into kaons and η mesons and much less frequently into pions, while naively one would expect flavor blindness in glueball decays. The flavor asymmetry in the decay of $f_0(1710)$ has been attributed to the mechanism of “chiral suppression” [32–34] according to which (part of the) decay amplitudes of a scalar glueball should be proportional to quark masses. However, in view of chiral symmetry breaking and the resulting large constituent quark masses this argument seems to be questionable [31].

In extension of our previous work on glueball decay in the Witten-Sakai-Sugimoto model [17], we have recently shown [35] that an effectively equivalent mechanism we called nonchiral enhancement could explain these flavor asymmetries. The assumption of a universal coupling of glueballs to anomalous and nonanomalous mass terms of the full nonet of pseudoscalar mesons leads to a remarkably good agreement with experimental data for the glueball candidate $f_0(1710)$ while implying a vanishing decay rate into $\eta\eta'$ pairs, for which only upper bounds for the $f_0(1710)$ meson are at present available from experiment.¹

In this paper we relax this assumption and study the predictions of the Witten-Sakai-Sugimoto model with finite quark masses when the as-yet-undetermined parameter in the glueball coupling to the nonanomalous mass terms is kept free. This will in particular lead to constraints on the $\eta\eta'$ decay rate of $f_0(1710)$ when interpreted as a nearly unmixed glueball state.

We shall also confirm our conclusion in [17] that the glueball candidate $f_0(1500)$ is disfavored by the holographic model, although the mass of the lowest predominantly dilatonic mode that we identify with the lightest scalar glueball of QCD² is 1487 MeV and thus very close to $f_0(1500)$. However, the resulting decay pattern [17] does not fit well to that of the $f_0(1500)$ meson when interpreted as a pure glueball. The decay into two pions is underestimated by a factor of 2, and the rate into four pions, which is the dominant decay mode of $f_0(1500)$, is too small by almost an order of magnitude.

¹An enhancement of the glueball decay rate into pseudoscalars according to the mass of the latter (instead of a complete suppression in the chiral limit) was also found in the model of Ref. [36], albeit just large enough to compensate approximately for kinematic suppression.

²In the first attempt to calculate the decay rate of the lightest glueball in [37] the lightest mode of the spectrum of glueballs obtained originally in [38,39] was identified with the lightest glueball, although when M_{KK} is fixed by the experimental value of the ρ meson mass it comes out at 855 MeV and thus much too light compared to lattice results. In [17] we have argued that the lowest-lying mode, which involves an “exotic” [38] polarization along the compactification direction, should be discarded and replaced by the next-highest, predominantly dilatonic mode.

This leaves the $f_0(1710)$ as a candidate for a nearly unmixed glueball. After all, the mass of $f_0(1710)$ is only 16% heavier, and we cannot expect the holographic model to be more accurate than some 10%–30%. Indeed, the decay rate of $f_0(1710)$ into two pions seems to be of roughly the right magnitude, while the significantly higher rates into kaons and eta mesons may be attributed to the nonchiral enhancement effect found in [35] under the assumption of a universal coupling of glueballs to pseudoscalar mass terms.

Before studying the effect of finite quark masses on glueball decay in the Witten-Sakai-Sugimoto model in maximal generality, we discuss the mass term arising from the $U(1)_A$ anomaly and its interplay with finite quark masses.

II. WITTEN-VENEZIANO MASS AND PSEUDOSCALAR MIXING

In the Witten-Sakai-Sugimoto model, the $U(1)_A$ flavor symmetry is broken by anomaly contributions of order $1/N_c$, which give rise to a Witten-Veneziano [40,41] mass term for the singlet η_0 pseudoscalar that is local with respect to the effective 3 + 1-dimensional boundary theory,

$$\mathcal{L}_m^{\text{WV}} = -\frac{1}{2} m_0^2 \eta_0^2(x), \quad (3)$$

where η_0 is obtained by an integration over the bulk coordinate z ,

$$\eta_0(x) = \frac{f_\pi}{\sqrt{2N_f}} \text{Tr} \int dz A_z(z, x). \quad (4)$$

The Witten-Veneziano mechanism relates m_0^2 to the topological susceptibility. It has been calculated by Sakai and Sugimoto in their model (following earlier work by [42,43]) with the result [12]

$$m_0^2 = \frac{N_f}{27\pi^2 N_c} \lambda^2 M_{\text{KK}}^2. \quad (5)$$

For $N_f = N_c = 3$, and the choice of parameters considered by us in [17,35], namely $M_{\text{KK}} = 949$ MeV, and λ varied from 12.55 to 16.63, one finds $m_0 = 730$ –967 MeV.

The massless Witten-Sakai-Sugimoto model can in principle be deformed to include mass terms for the entire pseudoscalar nonet by either world-sheet instantons [44,45] or non-normalizable modes of bifundamental fields corresponding to open-string tachyons [46–49]. These scenarios have only been demonstrated on a qualitative level, but they all agree in the form of those mass terms,

$$\mathcal{L}_m^{\text{M}} \propto \int d^4x \text{Tr}(\mathcal{M}U(x) + \text{H.c.}), \quad (6)$$

$$U(x) = \text{P exp} i \int_{-\infty}^{\infty} dz A_z(z, x) = e^{i\Pi^a \lambda^a / f_\pi}, \quad (7)$$

with tunable $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$, so that Gell-Mann-Oakes-Renner relations are realized.

For simplicity, and because we are not going to include electromagnetic interactions, we shall keep isospin symmetry and set $m_u = m_d = \hat{m}$. The mass terms resulting from \mathcal{M} then read

$$\begin{aligned} \mathcal{L}_m^{\mathcal{M}} = & -\frac{1}{2}m_\pi^2\pi_0^2 - m_\pi^2\pi_+\pi_- \\ & - m_K^2(K_0\bar{K}_0 + K_+K_-) \\ & -\frac{1}{2}m_1^2\eta_0^2 - \frac{1}{2}m_8^2\eta_8^2 - \frac{2\sqrt{2}}{3}(m_K^2 - m_\pi^2)\eta_0\eta_8 \end{aligned} \quad (8)$$

with

$$m_\pi^2 = 2\hat{m}\mu, \quad m_K^2 = (\hat{m} + m_s)\mu, \quad (9)$$

$$m_1^2 = \frac{2}{3}m_K^2 + \frac{1}{3}m_\pi^2, \quad m_8^2 = \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2, \quad (10)$$

and μ being the overall scale in (6).

With the addition of mass terms of the form (6), η_0 and η_8 are no longer mass eigenstates. Defining the mass eigenstates in terms of the pseudoscalar mixing angle θ_P [18],

$$\begin{aligned} \eta &= \eta_8 \cos \theta_P - \eta_0 \sin \theta_P \\ \eta' &= \eta_8 \sin \theta_P + \eta_0 \cos \theta_P, \end{aligned} \quad (11)$$

one obtains

$$\begin{aligned} m_{\eta,\eta'}^2 &= \frac{1}{2}m_0^2 + m_K^2 \\ &\mp \sqrt{\frac{m_0^4}{4} - \frac{1}{3}m_0^2(m_K^2 - m_\pi^2) + (m_K^2 - m_\pi^2)^2}, \end{aligned} \quad (12)$$

$$\theta_P = \frac{1}{2} \arctan \frac{2\sqrt{2}}{1 - \frac{3}{2}m_0^2/(m_K^2 - m_\pi^2)}. \quad (13)$$

In the absence of the Witten-Veneziano mass term (3), one would find $m_\eta^2 = m_\pi^2$ and $m_{\eta'}^2 = 2m_K^2 - m_\pi^2$, whereas $m_0 \rightarrow \infty$ with $\theta_P \rightarrow 0$ corresponds to a decoupling of η' and $m_\eta^2 \rightarrow m_8^2$.

In the Witten-Sakai-Sugimoto model, m_0^2 is given by (5), and our inclusion of quark masses through $\hat{m}\mu$ and $m_s\mu$ does not add any free parameters, as those can be fixed by the experimental values of m_π and m_K . In order to obtain an optimal match of \hat{m} , the average of m_u and m_d , without isospin breaking mass contributions from the electromagnetic interactions, we shall use $m_\pi^2 = m_{\pi_0}^2 \approx (135 \text{ MeV})^2$ in (9), while m_s will be fixed by

$$m_K^2 = \frac{1}{2}(m_{K_\pm}^2 + m_{K_0}^2) - \frac{1}{2}(m_{\pi_\pm}^2 - m_{\pi_0}^2) \approx (495 \text{ MeV})^2, \quad (14)$$

corresponding to a ratio $m_s/\hat{m} \approx 25.9$ slightly below the current-quark mass ratio 27.5 ± 1.0 of Ref. [18].

This setup for the masses in fact never reaches the experimental mass ratio $m_\eta/m_{\eta'} \approx 0.572$ for any values of m_u, m_d, m_s, m_0 [50,51]. With our isospin symmetric choice of quark masses a maximum value of $m_\eta/m_{\eta'} \approx 0.535$ is attained at $m_0 \approx 660.6 \text{ MeV}$ and $\theta_P \approx -28^\circ$, which is, however, outside the range considered above for the Witten-Sakai-Sugimoto model. But at this optimal point with respect to the ratio of the masses, both m_η and $m_{\eta'}$ are about 100 MeV below their physical values.

The range of values for $m_\eta, m_{\eta'}$, and θ_P obtained for $m_0 \in (730, 967) \text{ MeV}$ are shown in Fig. 1. It turns out that just around the center value of this range one can achieve various other possible optimizations of m_η and $m_{\eta'}$: The sum $m_\eta + m_{\eta'}$ matches the experimental value at $m_0 \approx 876 \text{ MeV}$ and $\theta_P \approx -17.4^\circ$; for the sum of squared masses this is the case at $m_0 \approx 852 \text{ MeV}$ and $\theta_P \approx -18.3^\circ$, and a least-square value of errors for the masses is obtained at $m_0 \approx 835 \text{ MeV}$ and $\theta_P \approx -19^\circ$.

The total range of θ_P in Fig. 1 coincides with most of the range considered in the phenomenological literature: Values of θ_P around -14° appear to be favored by the analysis of light meson decays [52,53], while radiative charmonium decay instead points to $\theta_P \approx -21^\circ$ [51,54]. Also the ratio $\Gamma(\eta' \rightarrow 2\gamma)/\Gamma(\eta \rightarrow 2\gamma)$ leads to larger values [18], $\theta_P = (-18 \pm 2)^\circ$, which happens to include the various optimization points of η and η' masses listed above.

In the following we shall use $m_0 = 850 \text{ MeV}$ as a central value, and $m_0 \in (730, 967) \text{ MeV}$ as a range for estimating a theoretical error bar for our semiquantitative study of the Witten-Sakai-Sugimoto model. It should of course be kept in mind that we have no control over subleading order

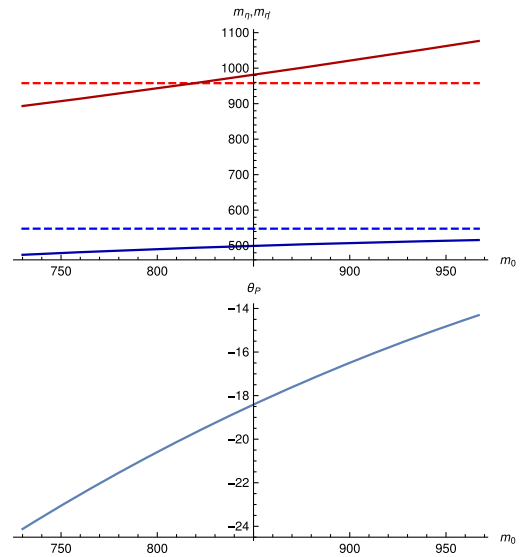


FIG. 1 (color online). Masses of the η and η' meson (blue and red lines in upper plot) and pseudoscalar mixing angle θ_P (lower plot) as a function of the Witten-Veneziano mass m_0 in the range obtained for the latter in the Witten-Sakai-Sugimoto model.

corrections in this model, but our hope is that errors continue to remain in the 10%–30% range seen in various previous applications [15]. In fact, if one considers the values of m_η and $m_{\eta'}$ as another prediction of the Witten-Sakai-Sugimoto model, the errors are, encouragingly, in the $\lesssim 10\%$ range only.

III. GLUEBALL COUPLINGS AND DECAY RATES IN THE WITTEN-SAKAI-SUGIMOTO MODEL WITH QUARK MASSES

For the chiral Witten-Sakai-Sugimoto model the vertices of glueball fields with pseudoscalar and vector mesons have been worked out in detail in [17]. The vertex for an on-shell scalar glueball corresponding to the lowest predominantly dilatonic mode G_D with mass M and two pseudoscalar mesons reads

$$\mathcal{L}_{G_D\pi\pi}^{\text{chiral}} = \frac{1}{2} d_1 \text{Tr}(\partial_\mu \boldsymbol{\pi} \partial_\nu \boldsymbol{\pi}) \left(\eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{M^2} \right) G_D, \quad (15)$$

where $d_1 \approx 17.226 \lambda^{-1/2} N_c^{-1} M_{\text{KK}}^{-1}$ and $\boldsymbol{\pi} = \Pi^a \lambda^a / \sqrt{2}$ [with the λ^a including the U(1) generator through $a = 0, \dots, 8$].

An additional contribution to the vertex of the U(1) pseudoscalar with G_D arises from the Witten-Veneziano mass term which has been calculated in [35] as

$$\mathcal{L}_{G_D\eta_0\eta_0}^{\text{chiral}} = \frac{3}{2} d_0 m_0^2 \eta_0^2 G_D \quad (16)$$

with $d_0 \approx 17.915 \lambda^{-1/2} N_c^{-1} M_{\text{KK}}^{-1}$. This coupling follows unambiguously from the original Witten-Sakai-Sugimoto model with massless quarks.

The coupling of G_D to the mass terms of the pseudo-Goldstone bosons induced by quark masses through tachyon condensation or world-sheet instantons is, however, not known. Given the similarity of the holographic expressions for the anomalous and the explicit mass terms, it is, however, clear that the latter will also have a coupling to the glueball modes which are concentrated around the minimal value of the radial bulk coordinate. In [35] the simplest possibility was explored, namely that of a universal coupling of the glueball to both kinds of meson mass terms, for which plausibility arguments were given. This most symmetric scenario implies that the mass terms of all pseudo-Goldstone bosons can be diagonalized simultaneously with their glueball couplings so that no $G_D \eta \eta'$ vertex arises. In this paper we shall study the consequences of having an undetermined prefactor d_m in the glueball couplings to the pseudoscalar mass terms arising from nonzero quark masses, i.e.

$$\mathcal{L}_{G_D\pi\pi}^{\text{massive}} = \frac{3}{2} d_m G_D \mathcal{L}_m^M \quad (17)$$

with L_m^M given by (8). The deviation of d_m from d_0 will be denoted by

$$d_m \equiv x d_0$$

with a free parameter x .

The decay rate of a glueball into two pseudoscalar mesons then has a mass dependence according to

$$\begin{aligned} \Gamma(G_D \rightarrow PP)^{\text{massive}} &= \frac{n_P d_1^2 M^3}{512\pi} \left(1 - 4 \frac{m_P^2}{M^2} \right)^{1/2} \left(1 + \alpha_P \frac{m_P^2}{M^2} \right)^2 \end{aligned} \quad (18)$$

with

$$\alpha_P = 4 \left(3 \frac{d_0}{d_1} x - 1 \right) \approx 4(3.120x - 1) \quad (19)$$

when P refers to pions ($n_P = 3$) or kaons ($n_P = 4$). Note that in the (leading-order) result (19) the dependence on λ and M_{KK} has dropped out; it only depends on the $O(1)$ parameter x , which parametrizes the undetermined ratio d_m/d_0 .

For η mesons ($n_P = 1$) the modification factor α depends on the $\eta - \eta'$ mixing (and thus through m_0 on λ and M_{KK}). It reads

$$\alpha_\eta = 4 \left(3 \frac{d_0}{d_1} \left[x + \sin^2 \theta_P \frac{m_0^2}{m_\eta^2} (1 - x) \right] - 1 \right). \quad (20)$$

(For glueballs heavy enough to be able to decay into two η' mesons the corresponding quantity would involve $\cos^2 \theta_P$ in place of $\sin^2 \theta_P$.)

With $d_m \neq d_0$, a scalar glueball can also decay into an $\eta - \eta'$ pair through the vertex

$$\mathcal{L}_{G_D\eta\eta'}^{\text{massive}} = -\frac{3}{2} (d_0 - d_m) \sin(2\theta_P) m_0^2 G_D \eta \eta' \quad (21)$$

with rate

$$\begin{aligned} \Gamma(G_D \rightarrow \eta\eta') &= \frac{|\mathbf{p}|}{8\pi M^2} \left(\frac{3}{2} (d_0 - d_m) \sin(2\theta_P) m_0^2 \right)^2, \\ |\mathbf{p}| &= \frac{\sqrt{[M^2 - (m_\eta + m_{\eta'})^2][M^2 - (m_\eta - m_{\eta'})^2]}}{2M}. \end{aligned} \quad (22)$$

Figure 2 displays the holographic results for the ratios $(4/3) \times \Gamma(G_D \rightarrow \pi\pi) / \Gamma(G_D \rightarrow KK)$ (green curve) and $4 \times \Gamma(G_D \rightarrow \eta\eta') / \Gamma(G_D \rightarrow KK)$ (blue curve), which describe the deviation from flavor symmetry in the decay into two pseudoscalars, and the ratio $\Gamma(G_D \rightarrow \eta\eta') / \Gamma(G_D \rightarrow \pi\pi)$ when the mass of the glueball is set to that of the $f_0(1710)$ meson. The rates involving the η and η' mesons depend on m_0 , which is varied over the range 730–967 MeV corresponding to the range of the 't Hooft coupling considered by us. These results are compared with the corresponding experimental data and error bars reported by the Particle Data Group [18] for $f_0(1710)$ (blue and green lines within light-blue and light-green bands) and the

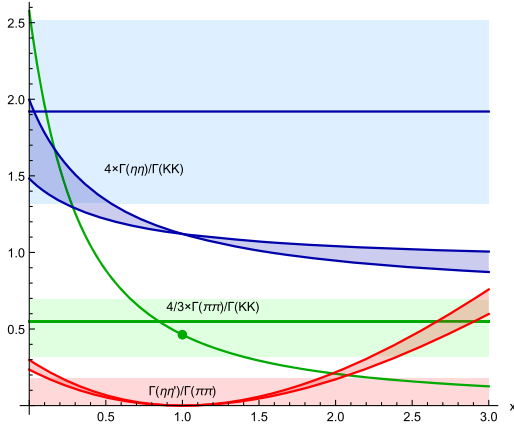


FIG. 2 (color online). Flavor asymmetries in the decay rates of the glueball G_D of the Witten-Sakai-Sugimoto model with mass set to that of $f_0(1710)$ as a function of $x = d_m/d_0$ and with m_0 varied over the range implied by $M_{KK} = 949$ MeV and $\lambda = 12.55 \dots 16.63$. The case $x = 1$ studied in [35] is marked by a dot. The light-green and light-blue bands around the straight horizontal lines give the current experimental results for $f_0(1710)$ reported in [18], and the light-red band corresponds to the upper limit on the $\eta\eta'$ decay rate of WA102 [55].

upper limit on the $\eta\eta'$ decay rate reported by the WA102 experiment [55].

The value $x \approx 0.32$ corresponds to the situation where kaon and pion decay rates have no mass dependence other than through phase space factors; $x > 0.32$ means non-chiral enhancement, $x < 0.32$ the opposite. The most symmetric choice $x = 1$ considered by us in [35] is highlighted by the green dot in Fig. 2. It is found to be within experimental error of the current experimental result [18]. If one varies x such the result for $\Gamma(G_D \rightarrow \pi\pi)/\Gamma(G_D \rightarrow KK)$ does not leave this error band, one finds that the Witten-Sakai-Sugimoto model with quark masses predicts $\Gamma(G_D \rightarrow \eta\eta')/\Gamma(G_D \rightarrow \pi\pi) \lesssim 0.04$, which is well below the upper limit of 0.18 from WA102. [This can be contrasted by the recent phenomenological study in [31] which predicts $\eta\eta'$ decay rates for $f_0(1710)$ that are several times higher than the upper limit reported by WA102.]

In Fig. 3 the analogous comparison is made for a glueball with mass set to that of the $f_0(1500)$ meson. Curiously enough, for $x \approx 0$, which corresponds in fact to a significant nonchiral suppression instead of enhancement, the experimental results for the various ratios of decay into pairs of pseudoscalars are approximately reproduced.³ However, as mentioned in the Introduction, we have found this glueball candidate disfavored by the absolute value of $\Gamma(G_D \rightarrow \pi\pi)$ (which depends only weakly on x) and even more so by

³In the case of $\Gamma(G_D \rightarrow \eta\eta')/\Gamma(G_D \rightarrow \pi\pi)$ a Breit-Wigner distribution for the glueball mass was used in order to produce a nonzero result, as the nominal mass of $f_0(1500)$ is below the $\eta\eta'$ threshold. [In the case of the $f_0(1710)$ this does not make much difference.]

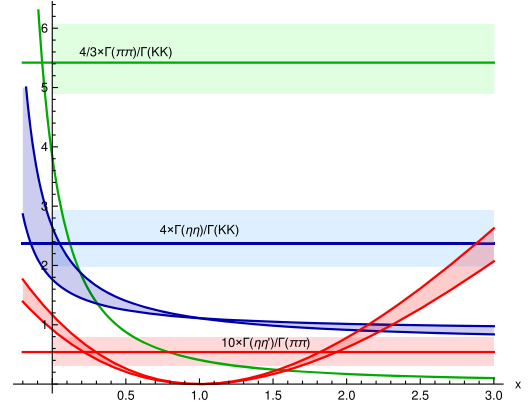


FIG. 3 (color online). Same as Fig. 2, but with glueball mass set to the mass of the $f_0(1500)$ meson. For the latter, a nonzero decay rate into $\eta\eta'$ is reported by the Particle Data Group (red horizontal line and light-red band) [18], here multiplied by a factor of 10 for better visibility.

$\Gamma(G_D \rightarrow 4\pi)$, both underestimating the experimental data significantly.

IV. CONCLUSION

Based on the available experimental data on the decay rates for the glueball candidate $f_0(1710)$, we conclude that the results of the Witten-Sakai-Sugimoto with finite quark masses are compatible with a nearly pure glueball interpretation of $f_0(1710)$, while disfavoring $f_0(1500)$.⁴ The observed flavor asymmetries in the decay rates to pairs of pseudoscalars are remarkably well reproduced for a range of the undetermined parameter $x = d_m/d_0$ over which the decay rate into $\eta\eta'$ remains well below the current upper limit reported by WA102 [55]. If the current experimental error [18] on $\Gamma(G_D \rightarrow \pi\pi)/\Gamma(G_D \rightarrow KK)$ holds up, our holographic model predicts $\Gamma(G_D \rightarrow \eta\eta')/\Gamma(G_D \rightarrow \pi\pi) \lesssim 0.04$. Moreover, a ratio $\Gamma(G_D \rightarrow \eta\eta')/\Gamma(G_D \rightarrow KK)$ somewhat below the current experimental value would be favored.

Additional predictions of the Witten-Sakai-Sugimoto model are that a pure glueball with a mass around 1.7 GeV and thus above the 2ρ threshold should have a substantial branching ratio into four pions [17], $\Gamma(G \rightarrow 4\pi)/\Gamma(G \rightarrow 2\pi) \approx 2.5$ (previous studies have usually assumed this to be negligibly small [30]), and also into two ω mesons, $\Gamma(G \rightarrow 2\omega)/\Gamma(G \rightarrow 2\pi) \approx 1.1$. [Presently the 2ω decay mode has the status of “seen” only, but can be found to appear in radiative J/ψ decays [18] at the ratio of roughly 0.8(3) compared to the two pion rate.]

It would clearly be very interesting to see whether these predictions of the Witten-Sakai-Sugimoto model for the decay pattern of $f_0(1710)$ (under the assumption of its

⁴In fact, whenever the $f_0(1500)$ is considered as the preferred glueball candidate, it is usually with significant mixing with $q\bar{q}$ states.

nearly pure glueball nature⁵) hold up against future experimental evidence.

⁵Even if mixing with $q\bar{q}$ states is indeed small, it may have non-negligible effects on the final decay pattern. Absent a correspondingly improved holographic QCD model, one could consider a more phenomenological approach such as extended linear sigma models [28] and use the holographic results for the glueball-meson interactions as input instead of fixing those through fits to experimental data.

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