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Beyond six parameters: Extending ΛCDM

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Cosmological constraints are usually derived under the assumption of a six-parameter ΛCDM theoretical framework or simple one-parameter extensions. In this paper we present, for the first time, cosmological constraints in a significantly extended scenario, varying up to 12 cosmological parameters simultaneously, including the sum of neutrino masses, the neutrino effective number, the dark energy equation of state, the gravitational wave background and the running of the spectral index of primordial perturbations. Using the latest Planck 2015 data release (with polarization), we found no significant indication for extensions to the standard ΛCDM scenario, with the notable exception of the angular power spectrum lensing amplitude, $A_{\rm lens}$, which is larger than the expected value at more than 2 standard deviations, even when combining the Planck data with BAO and supernovae type Ia external data sets. In our extended cosmological framework, we find that a combined Planck + BAO analysis constrains the value of the rms density fluctuation parameter to $\sigma_8 = 0.781^{+0.065}_{-0.063}$ at 95 % C.L., helping to relieve the possible tensions with the CFHTlenS cosmic shear survey. We also find a lower value for the reionization optical depth $\tau = 0.058^{+0.040}_{-0.043}$ at 95 % C.L. with respect to the one derived under the assumption of ΛCDM. The scalar spectral index n_S is now compatible with a Harrison-Zeldovich spectrum to within 2.5 standard deviations. Combining the Planck data set with the Hubble Space Telescope prior on the Hubble constant provides a value for the equation of state $w < -1$ at more than 2 standard deviations, while the neutrino effective number is fully compatible with the expectations of the standard three neutrino framework.

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I. INTRODUCTION

In the past 20 years, measurements of the cosmic microwave background (CMB) anisotropy angular power spectrum have witnessed one of the most impressive technological advances in experimental physics. Following the first detection of CMB temperature anisotropies at large angular scales by the COBE satellite in 1992 [1], the angular power spectrum has been measured with increasing precision by balloon-borne experiments such as BOOMERanG [2] and MAXIMA [3] and by ground-based experiments as DASI [4], showing the unambiguous presence of a "first peak" and subsequent oscillations in the angular power spectrum at intermediate angular scales ($\theta \sim 0.2°$). The spectacular measurements obtained by the WMAP satellite mission [5] have not only confirmed the presence of these acoustic oscillations but also provided the first precise measurement of the cross temperature-polarization angular spectrum and the first constraints on the epoch of reionization. The very small-scale part of the angular temperature power spectrum, and especially the damping tail, has been accurately determined by such experiments as ACT [6] and SPT [7]. This impressive progress in the measurement of the CMB anisotropy temperature angular spectrum has culminated with the cosmic-variance limited measurements of the Planck experiment that has now also provided exquisite results on the polarization and cross temperature-polarization spectra.

Despite this impressive progress on the experimental side, it is interesting to note that the constraints on cosmological parameters are still presented (as in the latest Planck 2015 data release, [8]) under the assumption of a simple ΛCDM model, based on the variation of just six cosmological parameters. While this model still provides a good fit to the data, it is the same model used, for example, in the analysis of the BOOMERanG 1998 data (see [2]), i.e., more than 15 years ago. While this "minimal" approach is justified by the good fit to the data that the ΛCDM provides, we believe that it does not do adequate justice to the high quality of the most recent data sets. In light of the new precise data, some of the assumptions made in the six-parameter approach are indeed no longer fully justified. For example, fixing the total neutrino mass to zero or to some small value is completely arbitrary since we know that neutrinos must have masses and that current cosmological data sets are sensitive to variations in the absolute neutrino mass scale of order ~100 meV. At the same time, considering that a cosmological constant offers difficulties in any theoretical interpretation, it seems reasonable to incorporate in any analysis a possible dynamical dark energy component. This is certainly plausible (and even preferred if one wants to address the "why now?" problem) and, indeed, fixing the dark energy equation of state to -1 is not favored by any theoretical arguments. Most inflationary models predict a sizable contribution of gravitational waves. Given the progress made in the search for B-mode polarization, especially by the recent combined BICEP2 + Planck analysis [9], it is an opportune moment to allow any such contribution to be directly constrained by the data, without assuming a null contribution as in the six-parameter model. A similar argument can be made for the running of the scalar spectral index $dn_s/dlnk$. Moreover, the neutrino effective number, $N_{\rm eff}$, could easily be different than the standard expected value of 3.046. Even assuming the standard three neutrino framework, nonstandard decoupling, inflationary reheating, dark matter decay, and many other physical processes could alter its value. Finally, the Planck 2015 release still hints at an anomalous value for the lensing amplitude $A_{\rm lens}$ [10]. While this parameter is purely phenomenological, one should clearly consider it and check to see whether the cosmology obtained is consistent with other data sets. The goal of this paper is to constrain the cosmological parameters in this extended parameter space.

II. METHOD

As was discussed in the Introduction, besides the six parameters of the "standard" ΛCDM model—i.e., the Hubble constant H_0, the baryon $\Omega_b h^2$, and CDM energy densities $\Omega_c h^2$—the primordial amplitude and spectral index of the scalar perturbations A_s and n_s (at pivot scale $k_0 = 0.05h\ \mathrm{Mpc}^{-1}$), and the reionization optical depth τ, we also consider variations in six additional parameters: the total mass for the three standard neutrinos, Σm_ν, the dark energy equation of state w assumed to be constant with redshift, the tensor-scalar ratio of amplitude r at pivot scale $k_0 = 0.05h\ \mathrm{Mpc}^{-1}$, the running of the scalar spectral index $dn_s/dlnk$, at pivot scale $k_0 = 0.05h\ \mathrm{Mpc}^{-1}$, the amplitude of the lensing signal in the CMB angular spectra, $A_{\rm lens}$ as defined in [10], and the effective number of relativistic neutrinos, $N_{\rm eff}$. In what follows, we refer to this model as extended cold dark matter (*E*CDM).

We let all these parameters vary freely simultaneously in a range of external, conservative priors listed in Table I.

We produce constraints on these cosmological parameters by making use of several recent data sets. First, we use the full Planck 2015 release on temperature and polarization CMB angular power spectra. This data set includes the large angular scale temperature and polarization measured by the Planck LFI experiment and the small-scale temperature and polarization spectra measured by Planck HFI. We refer to this data set simply as Planck [11]. We also include information on CMB lensing from Planck trispectrum detection (see [12]). We refer to this data set as *lensing*. We add baryonic acoustic oscillation data from 6dFGS [13], SDSS-MGS [14], BOSSLOWZ [15], and CMASS-DR11 [15] surveys as in [8]. We refer to this data set as BAO. We impose a constraint on the Hubble constant from the Hubble Space Telescope [16] data set. Recently, this constraint has been criticized in [17] where a more conservative value was suggested, a choice adopted in the recent Planck analysis [8]. For reasons that will appear below, we choose to use the less conservative [16] determination. We refer to this data set as HST. We use luminosity distances of supernovae type IA from the "joint light-curve analysis" derived from the SNLS and SDSS catalogs [18]. We refer to this data set as JLA. We add weak lensing galaxy data from the CFHTlenS [19] survey with the priors and conservative cuts to the data described in [8]. We refer to this data set as WL. We consider redshift space distortions from [20] with the prescription given in [8]. We refer to this data set as RSD. Finally, we include upper limits on CMB polarization B modes as recently placed by a common analysis of Planck, BICEP2, and Keck Array data [9]. We refer to this data set as BKP.

TABLE I. External priors on the cosmological parameters assumed in this paper.

Parameter	Prior
$\Omega_{\rm b} h^2$	[0.005,0.1]
$\Omega_{\rm cdm} h^2$	[0.001,0.99]
$\Theta_{\rm s}$	[0.5,10]
τ	[0.01,0.8]
n_s	[0.8,1.1]
$\log[10^{10} A_s]$	[2,4]
$\sum m_\nu$ (eV)	[0,3]
w	[−3.5, 0.5]
$\frac{dn_s}{dlnk}$	[−0.5, 0.5]
r	[0,0.5]
$N_{\rm eff}$	[0.05,10]
$A_{\rm lens}$	[0,10]

We use the publicly available Monte Carlo Markov chain package COSMOMC [21] with a convergence diagnostic based on the Gelman and Rubin statistic. We use the July 2015 version which includes support for the Planck data release 2015 likelihood [11,22] and implements an efficient sampling using the fast/slow parameter decorrelations [23]. While in this paper we will focus our attention on cosmological parameters, we also vary foreground parameters using the same technique and parametrization described in [11] and [8].

III. RESULTS

The results of our analysis are reported in Table II, where we also include, for comparison, the constraints obtained

TABLE II. Constraints at 95 % C.L. on the cosmological parameters assuming the standard six-parameter ΛCDM model and the extended, 12-parameter EΛCDM model.

Model Data set	$\Omega_b h^2$	$\Omega_c h^2$	H_0	τ	n_s	σ_8	$\frac{dn_s}{d\ln k}$	r	w	Σm_ν[eV]	$N_{\rm eff}$	$A_{\rm lens}$
ΛCDM Planck	$0.02226^{+0.00031}_{-0.00029}$	$0.1198^{+0.0028}_{-0.0028}$	$67.3^{+1.3}_{-1.3}$	$0.079^{+0.034}_{-0.035}$	$0.9646^{+0.0092}_{-0.0092}$	$0.831^{+0.026}_{-0.026}$	…	…	…	…	…	…
ΛCDM Planck + BAO	$0.02229^{+0.00028}_{-0.00027}$	$0.1193^{+0.0021}_{-0.0020}$	$67.52^{+0.93}_{-0.93}$	$0.082^{+0.031}_{-0.032}$	$0.9662^{+0.0078}_{-0.0079}$	$0.832^{+0.025}_{-0.025}$	…	…	…	…	…	…
ECDM Planck	$0.02239^{+0.00060}_{-0.00056}$	$0.1186^{+0.0071}_{-0.0068}$	>51.2	$0.058^{+0.040}_{-0.043}$	$0.967^{+0.025}_{-0.025}$	$0.81^{+0.24}_{-0.26}$	$-0.003^{+0.020}_{-0.019}$	<0.183	$-1.32^{+0.98}_{-0.85}$	<0.959	$3.08^{+0.57}_{-0.51}$	$1.21^{+0.27}_{-0.24}$
ECDM Planck + BAO	$0.02251^{+0.00056}_{-0.00052}$	$0.1185^{+0.0069}_{-0.0069}$	$68.4^{+4.3}_{-4.1}$	$0.058^{+0.041}_{-0.043}$	$0.972^{+0.024}_{-0.024}$	$0.781^{+0.065}_{-0.063}$	$-0.004^{+0.018}_{-0.018}$	<0.187	$-1.04^{+0.20}_{-0.21}$	<0.534	$3.11^{+0.52}_{-0.48}$	$1.20^{+0.19}_{-0.19}$
ECDM Planck + lensing	$0.02214^{+0.00053}_{-0.00052}$	$0.1176^{+0.0069}_{-0.0066}$	>54.5	$0.058^{+0.040}_{-0.043}$	$0.959^{+0.024}_{-0.024}$	$0.85^{+0.21}_{-0.24}$	$-0.005^{+0.018}_{-0.018}$	<0.178	$-1.45^{+0.96}_{-0.83}$	<0.661	$2.93^{+0.51}_{-0.48}$	$1.04^{+0.16}_{-0.15}$
ECDM Planck + HST	$0.02239^{+0.00059}_{-0.00057}$	$0.1187^{+0.0072}_{-0.0070}$	$74.4^{+5.1}_{-5.1}$	$0.057^{+0.040}_{-0.045}$	$0.966^{+0.025}_{-0.025}$	$0.81^{+0.10}_{-0.11}$	$-0.003^{+0.020}_{-0.019}$	<0.186	$-1.32^{+0.29}_{-0.31}$	<0.957	$3.09^{+0.58}_{-0.55}$	$1.18^{+0.19}_{-0.18}$
ECDM Planck + JLA	$0.02242^{+0.00058}_{-0.00056}$	$0.1188^{+0.0071}_{-0.0067}$	$67.4^{+4.4}_{-4.2}$	$0.058^{+0.040}_{-0.043}$	$0.968^{+0.025}_{-0.025}$	$0.759^{+0.088}_{-0.089}$	$-0.004^{+0.020}_{-0.019}$	<0.183	$-1.06^{+0.13}_{-0.14}$	<0.854	$3.10^{+0.57}_{-0.54}$	$1.20^{+0.19}_{-0.17}$
ECDM Planck + WL	$0.02251^{+0.00056}_{-0.00055}$	$0.1188^{+0.0073}_{-0.0069}$	>54.2	<0.0835	$0.972^{+0.024}_{-0.024}$	$0.82^{+0.22}_{-0.25}$	$0.000^{+0.020}_{-0.019}$	<0.197	$-1.41^{+0.98}_{-0.79}$	<0.974	$3.16^{+0.58}_{-0.56}$	$1.24^{+0.23}_{-0.22}$
ECDM Planck + BAO − RSD	$0.02253^{+0.00052}_{-0.00050}$	$0.1184^{+0.0069}_{-0.0067}$	$68.6^{+4.2}_{-3.9}$	$0.056^{+0.038}_{-0.042}$	$0.972^{+0.023}_{-0.023}$	$0.774^{+0.055}_{-0.058}$	$-0.004^{+0.018}_{-0.018}$	<0.188	$-1.05^{+0.17}_{-0.19}$	<0.626	$3.12^{+0.51}_{-0.48}$	$1.22^{+0.18}_{-0.17}$
ECDM Planck + BKP	$0.02237^{+0.00057}_{-0.00056}$	$0.1186^{+0.0072}_{-0.0069}$	>52.3	$0.058^{+0.039}_{-0.044}$	$0.966^{+0.026}_{-0.026}$	$0.81^{+0.23}_{-0.25}$	$-0.003^{+0.019}_{-0.018}$	<0.101	$-1.31^{+0.96}_{-0.89}$	<0.876	$3.07^{+0.57}_{-0.55}$	$1.20^{+0.24}_{-0.22}$

assuming the standard six parameters in ΛCDM. The significant increase in the number of parameters produces, as expected, a relaxation in the constraints on the six ΛCDM parameters. Considering the allowable volume of the six-dimensional ΛCDM parameter space to be proportional to the square root of the determinant of the 6×6 parameter covariance, we find that moving from ΛCDM to EΛCDM expands this volume by a factor of ~63000. The parameters that are mostly affected are the Hubble constant and the rms amplitude of density fluctuations that are now practically undetermined from Planck measurements alone and have significantly larger errors with respect to ΛCDM even when external data sets such as BAO are included. The main reason for this relaxation is the inclusion in the analysis of the dark energy equation of state w, which introduces a geometrical degeneracy with the matter density and the Hubble constant. Moreover, marginalizing over the lensing amplitude $A_{\rm lens}$ removes the lensing information in the CMB spectra that could potentially break this geometrical degeneracy. In this respect, it is interesting to note that a Planck + HST analysis provides a value for the equation of state w less than the cosmological constant value −1 at more than 95 % C.L. Constraints on the baryon and cold dark matter densities, the scalar spectral index n_S and the optical depth τ are also much weaker, mainly due to degeneracies between these parameters and $A_{\rm lens}$ and $N_{\rm eff}$. Apart from the increase in the errors, it is interesting that parameters such as σ_8 and the optical depth τ are shifted toward lower values with respect to ΛCDM. These shifts are clear in Figs. 1 and 2 where we plot the 68 % and 95 % C.L. contour plots in the σ_8 vs Ω_m and τ vs n_S planes, respectively. This is mainly due to the anomalous value of $A_{\rm lens}$ and persists when external data sets as BAO, JLA, WL, and RSD are included. Looking at

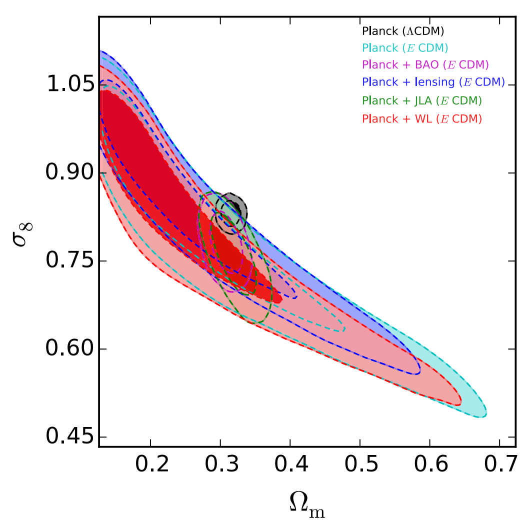

FIG. 1 (color online). Constraints at 68 % and 95 % confidence levels on the σ_8 vs Ω_m plane under the assumption of EΛCDM and different data sets. The black contours are the constraints under ΛCDM.

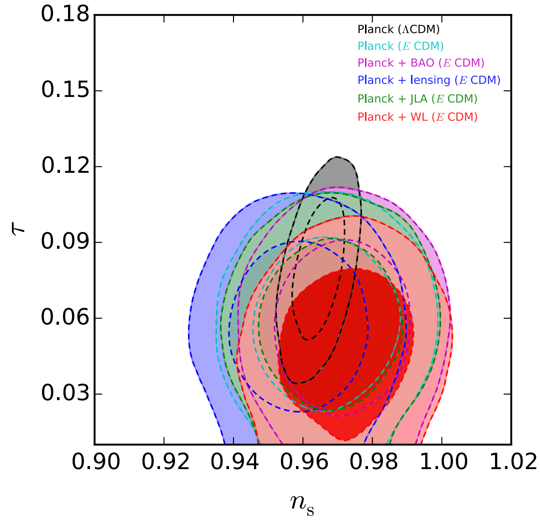

FIG. 2 (color online). Constraints at 68 % and 95 % confidence levels on the τ vs n_S plane under the assumption of $E\Lambda$CDM and different data sets. The black contours are the constraints under ΛCDM.

the results in Table II, the value of $A_{\rm lens}$ is always different from the standard value at more than 95 % C.L. when the Planck CMB data set is combined with external data sets, with the only notable exception being the lensing information from the Planck trispectrum that pushes the value of $A_{\rm lens}$ back into agreement with unity. The nature of the Planck $A_{\rm lens}$ anomaly could be different from the lensing determination but since it also persists in our extended $E\Lambda$CDM scenario, this clearly deserves further investigation. Moreover, $A_{\rm lens}$ is the only parameter that hints at a tension with standard ΛCDM. Again, by looking at Table II, apart from $A_{\rm lesn}$, we see no evidence of "new physics": we just have (weaker) upper limits on the neutrino mass, the running of the spectral index is compatible with zero, the dark energy equation of state is compatible with $w = -1$ (expected when we use the HST prior), and the neutrino effective number is remarkably close to the standard value $N_{\rm eff} = 3.046$. It is impressive that even in a 12-parameter space, the neutrino effective number is still constrained with exquisite precision. This is mainly due to the inclusion of the Planck HFI small angular scale polarization data in the analysis. Removing this data set but keeping the low angular scale LFI polarization, we get a much weaker constraint from Planck + BAO of $N_{\rm eff} = 4.35^{+1.8}_{-1.6}$ at 95 % C.L. The Planck + BAO constraint on a neutrino mass of $\Sigma m_{nu} < 0.534$ eV at 95 % C.L. is significantly weaker with respect to the constraint $\Sigma m_{nu} < 0.174$ eV at 95 % C.L. obtained with the same data set but assuming ΛCDM. The constraint on the tensor-scalar amplitude r is about a factor of 2 larger than in ΛCDM. However, when the BKP data set is included, the 95 % C.L. upper limit of $r < 0.108$ is recovered. This clearly shows how a measurement of primordial B modes is crucial to constraining the tensor amplitude in a model-independent way. The inclusion of the BKP data set affects only the constraint on the tensor amplitude and leaves the other constraints virtually unchanged.

IV. CONCLUSIONS

In this paper we have presented, for the first time, constraints on cosmological parameters in the framework of an "extended" cold dark matter model ($E\Lambda$CDM) that is based on 12 parameters instead of the usual six assumed in the ΛCDM model. In this extension some of the parameters usually well constrained under ΛCDM—such as the Hubble constant and the amplitude of matter density fluctuations σ_8—are now unconstrained by CMB observations. Combining the CMB data with several other data sets reveals no statistically significant evidence of any tension. More specifically, we have found no evidence for new physics beyond the standard ΛCDM model, i.e., there is no island of parameters in our extended theoretical framework that could be preferred to the standard ΛCDM territory. However $E\Lambda$CDM prefers a lower value of σ_8 relative to that obtained for six-parameter ΛCDM but still requires a slightly anomalous value of $A_{\rm lens} > 1$. The lower value of σ_8 in $E\Lambda$CDM brings the Planck data into more agreement with the results of the CFHTlenS survey [24]. This result motivates further studies that could explain the physical nature of the $A_{\rm lens} > 1$ anomaly.

The tension between the Planck and HST values of the Hubble parameter in the $E\Lambda$CDM scenario is solved by a value of the dark energy equation of state $w < -1$, while the neutrino effective number remains compatible with the standard value of 3.04.

Of course, the number of parameters can be further extended by considering, for example, nonzero curvature, isocurvature primordial perturbations, features in the primordial spectrum, a varying (with redshift) dark energy equation of state, nonstandard big bang nucleosynthesis and a change in the primordial helium abundance Y_p, and so on. Further extensions, however, may be premature because of degeneracies. For example, most effects of varying curvature are degenerate with a variation in w. CDM isocurvature modes have a spectrum similar to a gravitational wave background and the $A_{\rm lens}$ parameter could account for undetected features in the angular spectrum. Moreover, a change in $N_{\rm eff}$ could account for a change in Y_p.

We find it impressive that despite the increase in the number of parameters, some of the constraints on key parameters are relaxed but not significantly altered. The cold dark matter *ansatz* remains robust, the baryon density is compatible with BBN predictions, and the neutrino effective number is compatible with standard expectations. The excellent quality of the new data motivates our exploration beyond the overexploited territory of ΛCDM towards new and uncharted frontiers.

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[1] G. F. Smoot, C. L. Bennett, A. Kogut *et al.*, Structure in the COBE differential microwave radiometer first year maps, Astrophys. J. **396**, L1 (1992).

[2] P. de Bernardis *et al.* (Boomerang Collaboration), A flat universe from high resolution maps of the cosmic microwave background radiation, Nature (London) **404**, 955 (2000).

[3] S. Hanany *et al.*, MAXIMA-1: A measurement of the cosmic microwave background anisotropy on angular scales of 10 arcminutes to 5 degrees, Astrophys. J. **545**, L5 (2000).

[4] N. W. Halverson *et al.*, DASI first results: A measurement of the cosmic microwave background angular power spectrum, Astrophys. J. **568**, 38 (2002).

[5] D. N. Spergel *et al.* (WMAP Collaboration), First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Determination of cosmological parameters, Astrophys. J. Suppl. Ser. **148**, 175 (2003).

[6] J. Dunkley *et al.*, The Atacama Cosmology Telescope: Cosmological parameters from the 2008 power spectra, Astrophys. J. **739**, 52 (2011).

[7] R. Keisler *et al.*, A measurement of the damping tail of the cosmic microwave background power spectrum with the South Pole Telescope, Astrophys. J. **743**, 28 (2011).

[8] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2015 results. XIII. Cosmological parameters, arXiv:1502.01589.

[9] P. A. R. Ade *et al.* (BICEP2 and Planck Collaborations), Joint Analysis of BICEP2/Keck Array and Planck Data, Phys. Rev. Lett. **114**, 101301 (2015).

[10] E. Calabrese, A. Slosar, A. Melchiorri, G. F. Smoot, and O. Zahn, Cosmic microwave weak lensing data as a test for the dark universe, Phys. Rev. D **77**, 123531 (2008).

[11] N. Aghanim *et al.* (Planck Collaboration), Planck 2015 results. XI. CMB power spectra, likelihoods, and robustness of parameters, arXiv:1507.02704.

[12] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2015 results. XV. Gravitational lensing, arXiv:1502.01591.

[13] F. Beutler, C. Blake, M. Colless, D. H. Jones, L. Staveley-Smith, L. Campbell, Q. Parker, W. Saunders, and F. Watson, The 6dF Galaxy Survey: Baryon acoustic oscillations and the local Hubble constant, Mon. Not. R. Astron. Soc. **416**, 3017 (2011).

[14] A. J. Ross, L. Samushia, C. Howlett, W. J. Percival, A. Burden, and M. Manera, The clustering of the SDSS DR7 main Galaxy sample. I. A 4 per cent distance measure at $z = 0.15$, Mon. Not. R. Astron. Soc. **449**, 835 (2015).

[15] L. Anderson *et al.* (BOSS Collaboration), The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: Baryon acoustic oscillations in the Data Releases 10 and 11 Galaxy samples, Mon. Not. R. Astron. Soc. **441**, 24 (2014).

[16] A. G. Riess, L. Macri, S. Casertano, H. Lampeitl, H. C. Ferguson, A. V. Filippenko, S. W. Jha, W. Li, and R. Chornock, A 3% solution: Determination of the Hubble constant with the Hubble Space Telescope and Wide Field Camera 3, Astrophys. J. **730**, 119 (2011); **732**, 129(E) (2011).

[17] G. Efstathiou, H0 revisited, Mon. Not. R. Astron. Soc. **440**, 1138 (2014).

[18] M. Betoule *et al.* (SDSS Collaboration), Improved cosmological constraints from a joint analysis of the SDSS-II and SNLS supernova samples, Astron. Astrophys. **568**, A22 (2014).

[19] C. Heymans *et al.*, CFHTLenS: The Canada-France-Hawaii Telescope Lensing Survey, Mon. Not. R. Astron. Soc. **427**, 146 (2012).

[20] L. Samushia *et al.*, The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: Measuring growth rate and geometry with anisotropic clustering, Mon. Not. R. Astron. Soc. **439**, 3504 (2014).

[21] A. Lewis and S. Bridle, Cosmological parameters from CMB and other data: A Monte Carlo approach, Phys. Rev. D **66**, 103511 (2002).

[22] See http://cosmologist.info/cosmomc/.

[23] A. Lewis, Efficient sampling of fast and slow cosmological parameters, arXiv:1304.4473.

[24] N. MacCrann, J. Zuntz, S. Bridle, B. Jain, and M. R. Becker, Cosmic discordance: Are Planck CMB and CFHTLenS weak lensing measurements out of tune?, Mon. Not. R. Astron. Soc. **451**, 2877 (2015).