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Erratum: Radiative improvement of the lattice nonrelativistic QCD action using the background field method with applications to quarkonium spectroscopy [Phys. Rev. D 88, 014505 (2013)]

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Our paper contained several typographical errors and one mathematical error. The following corrections should be made: Equation (68) should read

$$Z_{\mathrm{sym}-\sigma}^{\mathrm{QCD},(1)} = -\frac{1}{4\pi} + \frac{1}{4\pi}\log(\mu/M) + \frac{M}{6\mu},$$
$$Z_{\mathrm{asym}-\sigma}^{\mathrm{QCD},(1)} = -\frac{1}{4\pi}\log(\mu/M). \tag{68}$$

Equations (69)–(72) contain several typographical errors and should read

$$I_{\mathrm{sym}}^{\mathrm{sub}}(\mu) = -\int \frac{d^4k}{(2\pi)^4}\frac{4\pi M^2(1+\mathbf{k}^2/4M^2)}{(k^2+\mu^2)^2(k_0^2+\mathbf{k}^4/4M^2)}$$
$$= \tilde{Z}_{\mathrm{sym}}^{\mathrm{sub}}(\mu) - \frac{1}{\pi}\log(\mu a) - \frac{5M}{16\mu} + \frac{M^2}{2\pi\mu^2} - \frac{M^3}{2\mu^3} \tag{69}$$

$$I_{\mathrm{asym}}^{\mathrm{sub}}(\mu) = -\int \frac{d^4k}{(2\pi)^4}\frac{4\pi M^2}{(k^2+\mu^2)^2(ik_0+\mathbf{k}^2/2M)^2}$$
$$= \tilde{Z}_{\mathrm{asym}}^{\mathrm{sub}}(\mu) + \frac{15}{8\pi}\log(\mu a) + \frac{3M}{8\mu} - \frac{M^2}{2\pi\mu^2} \tag{70}$$

$$I_{\mathrm{sym}-\sigma}^{\mathrm{sub}}(\mu) = -\int \frac{d^4k}{(2\pi)^4}\frac{4\pi}{(k^2+\mu^2)^2}\left(\frac{\mathbf{k}^2/3}{k_0^2+\mathbf{k}^4/4M^2}+\frac{1}{2}\right)$$
$$= \tilde{Z}_{\mathrm{sym}-\sigma}^{\mathrm{sub}}(\mu) - \frac{1}{4\pi}\log(\mu a) - \frac{M}{6\mu} \tag{71}$$

$$I_{\mathrm{asym}-\sigma}^{\mathrm{sub}}(\mu) = -\int \frac{d^4k}{(2\pi)^4}\frac{4\pi}{(k^2+\mu^2)^2}\left(\frac{\mathbf{k}^2/3}{(ik_0+\mathbf{k}^2/2M)^2}-\frac{1}{2}\right)$$
$$= \tilde{Z}_{\mathrm{asym}-\sigma}^{\mathrm{sub}}(\mu) + \frac{1}{4\pi}\log(\mu a). \tag{72}$$

In Figs. 7–8, the ordinate axis should be labeled $(-\tilde{Z}_{\mathrm{sym}-\sigma}^{\mathrm{sub}})$ and $(-\tilde{Z}_{\mathrm{asym}-\sigma}^{\mathrm{sub}})$, respectively, and in the corresponding captions the signs of $\tilde{Z}_{\mathrm{sym}-\sigma}^{\mathrm{sub}}(\mu)$ and $\tilde{Z}_{\mathrm{asym}-\sigma}^{\mathrm{sub}}(\mu)$ should be flipped accordingly. (Note that this error is purely presentational.)

As a consequence of the change in Eq. (68), the un-numbered equation at the beginning of Sec. VI.C should read

$$Z_{\mathrm{sym}-\sigma}^{(1)} = -\tilde{Z}_{\mathrm{sym}-\sigma}^{\mathrm{NR},(1)} - \frac{1}{4\pi} - \frac{1}{4\pi}\log Ma,$$
$$Z_{\mathrm{asym}-\sigma}^{(1)} = -\tilde{Z}_{\mathrm{asym}-\sigma}^{\mathrm{NR},(1)} + \frac{1}{4\pi}\log Ma.$$

Tables VI–VIII for the four-fermion coefficients b_1 and b_8 thus change as detailed below. This leads to a corresponding change in the one-loop corrected hyperfine splitting, and Tables IX–XI should be amended accordingly as detailed below.

In the conclusions, the sentence beginning "The resulting estimate for the hyperfine splitting" should therefore be changed to read "The resulting estimate for the hyperfine splitting of 68(3)(5)(3) MeV is then in good agreement with the experimental value of 69.3(2.8) MeV." The general conclusions of our paper remain unaffected.

We are grateful to the authors of [3] for drawing our attention to the error in Eq. (68).

TABLE VI. Renormalization parameters for spin-dependent four-fermion operators, $n = 2$ full v^4 nonrelativistic QCD (NRQCD).

Ma	1.95	2.8	4.0
b_1	0.0037(3)	−0.0201(4)	−0.0490(5)
b_8	0.0893(8)	0.0183(15)	−0.0832(26)

TABLE VII. Renormalization parameters for spin-dependent four-fermion operators, $n = 4$ full v^4 NRQCD.

Ma	1.9	2.65	3.4
b_1	0.0075(1)	−0.0148(6)	−0.0341(2)
b_8	0.0997(5)	0.0353(1)	−0.0290(8)

TABLE VIII. Renormalization parameters for spin-dependent four-fermion operators, $n = 4$ full-spin v^6 NRQCD.

Ma	1.9	2.65	3.4
b_1	−0.0223(2)	−0.03599(2)	−0.0504(2)
b_8	−0.0280(9)	−0.0624(7)	−0.1071(9)

TABLE IX. Estimates of the corrections to the bottomonium hyperfine splitting results of [1] arising from the radiative improvement of the $n = 2$ full v^4 NRQCD action. The errors given in the last column are statistical, $O(\alpha^2)$, and relativistic corrections, in that order.

Ma	β	α_V	c_4 correction	Box correction	Total	Old hfs	New hfs
1.95	7.09	0.216	+31.4(3)%	−10.4(1)%	+21.0(3)%	56(2)	68(3)(5)(6)
2.8	6.76	0.249	+39.8(3)%	+1.3(2)%	+41.1(4)%	50(2)	71(3)(6)(5)
4.0	6.458	0.293	+49.3(3)%	+23.2(3)%	+72.5(4)%	41(2)	71(3)(7)(4)

TABLE X. The corrections to the bottomonium hyperfine splitting arising from the radiative improvement of the $n = 4$ full v^4 NRQCD action, as found in [2]. Only the four-fermion contributions are *post hoc* estimates. The errors given in the last column are statistical, $O(\alpha^2)$, and relativistic corrections, in that order.

		Hfs (MeV)		Correction	Hfs (MeV)
Ma	α_V	$c_4 = 1$	Improved c_4	four-fermion	corrected
1.9	0.225	56.1(1)	72.1(1)	−12.6(1)%	65.0(1)(2.8)(5.6)
2.65	0.253	50.5(1)	69.8(1)	−1.8(1)%	68.9(1)(3.2)(5.0)
3.4	0.275	45.6(1)	65.6(1)	+11.0(1)%	70.6(1)(3.4)(4.6)

TABLE XI. Corrections to the bottomonium hyperfine splitting arising from the radiative improvement of the $n = 4$ full NRQCD action including spin-dependent terms at order v^6.

Ma	α_V	c_4 correction	Box correction
1.9	0.225	+37(4)%	+7.7(1)%
2.65	0.253	+40(4)%	+15.4(1)%
3.4	0.275	+26(4)%	+25.9(1)%

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