Strong $D_s^* D_s \eta^{(\prime)}$ and $B_s^* B_s \eta^{(\prime)}$ vertices from QCD light-cone sum rules

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The strong $D_s^* D_s \eta^{(\prime)}$ and $B_s^* B_s \eta^{(\prime)}$ vertices are studied and the relevant couplings are calculated in the context of the light-cone QCD sum rule method with twist-4 accuracy by including the next-to-leading-order corrections. In the analysis, both the quark and gluon components of the η and η' mesons and the axial-anomaly-corrected higher-twist distributions are included.

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I. INTRODUCTION

During the last several years, the investigation of the electromagnetic, weak, and strong decay channels of heavy mesons, with the computation of their numerous transition form factors and their strong couplings with different hadrons, has become a rapidly growing branch of hadronic physics. Progress in understanding the nature of such mesons, including bottom-strange (and charm-strange) ones, has been achieved from both the experimental and theoretical sides.

Thus, experimental measurements of hadronic processes and the extraction of the parameters of bottom/charm-strange mesons have been performed by different collaborations [1–6]. Theoretical calculations of parameters related to these mesons were fulfilled by applying various nonperturbative approaches and schemes, such as the lattice QCD calculations [7], the QCD and three-point sum rule methods (for instance, see Refs. [8–15]), and different quark models [16,17]. In this way, the masses, strong couplings, and form factors of some bottom/charm-strange mesons were obtained.

Studies of the vertices consisting of interacting bottom/charm-strange and light mesons have also attracted considerable interest. In fact, the strong couplings determined by the vertices $D_s^* D_s \eta^{(\prime)}$ and $B_s^* B_s \eta^{(\prime)}$ have been recently calculated in Ref. [12], where the three-point sum rule approach is used. The present work is devoted to the analysis of these vertices, but within the context of the QCD light-cone sum rule (LCSR) method [18]. The latter provides more elaborate theoretical tools to perform detailed analysis of the aforementioned problems. Indeed, the light-cone sum rule method invokes such quantities of the eta mesons as their distribution amplitudes (DAs) of different twists and partonic contents. This allows one to take into account the quark-gluon structure of particles in a more clear form than other approaches.

It should be noted that the $\eta - \eta'$ system of light pseudoscalar mesons accumulate important properties of the particle phenomenology, like the mixing of the $SU(3)$ flavor group singlet η_1 and octet η_8 states to form the physical mesons, the problem of axial $U(1)$ anomaly, and its impact on the relevant distribution amplitudes of the eta mesons. To this list of features, one should add also the complicated quark-gluon structure of the η and η' mesons and subtleties in the treatment of their gluon components that contribute to exclusive processes, the vertices under consideration being sample ones, at the next-to-leading order (NLO) of the perturbative QCD. These features of the $\eta - \eta'$ system, as well as new experimental data, have triggered numerous theoretical works devoted to the analysis of the mesons' mixing problems and computations of various exclusive processes to extract some constraints on the parameters of their distribution amplitudes, including the two-gluon ones [19–31]. The aim of this work is to study the bottom/charm-strange meson strong couplings and consider the vertices $D_s^* D_s \eta^{(\prime)}$ and $B_s^* B_s \eta^{(\prime)}$ by including in the analysis a gluon component of the η and η' mesons. The computation of a gluonic contribution to such strong couplings is a new issue that is considered in the present study.

This paper is structured in the following manner: In Sec. II, we present rather comprehensive information on the quark-gluon structure of the η and η' mesons and details of their leading and higher-twist distribution amplitudes. Existing singlet-octet and quark-flavor mixing schemes of the $\eta - \eta'$ system are briefly outlined, along with their advantages and drawbacks. In Sec. III, the light-cone sum rules for the strong couplings are derived. Here, the mesons' leading and higher-twist DAs up to twist 4 are utilized. In this section, we calculate the NLO corrections to the leading twist term, and also include in the light-cone sum rules contributions appearing due to the gluon component of the eta mesons. In Sec. IV, we perform numerical computations to find the values of the corresponding strong couplings. In this section we also make our brief conclusions. In the Appendix, the QCD two-point sum rule expressions to determine some of the parameters in higher-twist DAs of the $\eta - \eta'$ system are collected.

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II. MIXING SCHEMES AND DISTRIBUTION AMPLITUDES OF η, η' MESONS

Computation of the strong couplings $D_s^*D_s\eta^{(\prime)}$ and $B_s^*B_s\eta^{(\prime)}$, and relevant matrix elements within the framework of the QCD LCSR method, requires knowledge of the η and η' mesons' distribution amplitudes. In this work we use the mixing scheme for the eta mesons' DAs elaborated in Ref. [31] and relevant expressions presented there by adding the necessary formulas for the three-particle twist-3 DAs $\Phi_{3M}^{(s)}(\alpha)$.

Below we concentrate mainly on the s-quark distributions, because only s valence quarks from the heavy $D_s^{(*)}$ and $B_s^{(*)}$ mesons contribute to quark-antiquark and quark-gluon-antiquark DAs of the eta mesons. Nevertheless, when necessary, we provide some information also on q components of the corresponding DAs.

Hence, we define two-particle DAs for the s-quark flavor as

$$\langle M(q)|\bar{s}(x)\gamma_\mu\gamma_5 s(0)|0\rangle = -iq_\mu F_M^{(s)}\int_0^1 du e^{iqxu}\phi_M^{(s)}(u,\mu), \tag{1}$$

where $M(q)$ is the $\eta(q)$ or $\eta'(q)$ meson state. In this expression $\phi_M^{(s)}(u)$ is the leading twist, i.e. the twist-2 DA of the $M(q)$ meson. For brevity, in the matrix element, the gauge link is not shown explicitly. The normalization is chosen such that

$$\int_0^1 du\phi_M^{(s)}(u,\mu) = 1. \tag{2}$$

Similar distribution amplitudes can be defined for $q = u, d$ quarks as well, with evident replacement $s \to q$ in Eqs. (1) and (2). Then, assuming exact isospin symmetry and denoting $m_q = (m_u + m_d)/2$, we can determine the couplings $F_M^{(u)} = F_M^{(d)}$, $F_M^{(s)}$ as the matrix elements

$$\langle 0|J_{\mu 5}^{(i)}|M(q)\rangle = if_M^{(i)}q_\mu, \qquad i = q, s \tag{3}$$

of flavor-diagonal axial vector currents $J_{\mu 5}^{i}$:

$$J_{\mu 5}^{(q)} = \frac{1}{\sqrt{2}}[\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d], \qquad J_{\mu 5}^{(s)} = \bar{s}\gamma_\mu\gamma_5 s. \tag{4}$$

The couplings $F_M^{(u)}$, $F_M^{(d)}$, and $F_M^{(s)}$ are connected with $f_M^{(i)}$ couplings by means of the following simple expressions:

$$F_M^{(u)} = F_M^{(d)} = \frac{f_M^{(q)}}{\sqrt{2}}, \qquad F_M^{(s)} = f_M^{(s)}.$$

This definition of the distributions corresponds to the quark-flavor (QF) basis introduced to describe mixing in the $\eta - \eta'$ system. In the QF basis, mixing of the q and s states forms the physical η and η' mesons. Alternatively, one can determine DAs of the eta mesons starting from the singlet-octet (SO) basis of the $SU(3)$ flavor group. To this end, one introduces the $SU(3)$ flavor singlet $J_{\mu 5}^{(1)}$ and octet $J_{\mu 5}^{(8)}$ currents

$$J_{\mu 5}^{(1)} = \frac{1}{\sqrt{3}}[\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s],$$
$$J_{\mu 5}^{(8)} = \frac{1}{\sqrt{6}}[\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s] \tag{5}$$

and defines the corresponding matrix elements as

$$\langle 0|J_{\mu 5}^{(i)}|M(q)\rangle = if_M^{(i)}q_\mu, \qquad i = 1, 8. \tag{6}$$

The eta mesons' quark-flavor and singlet-octet combinations of the distributions are connected with each other as

$$\begin{pmatrix} f_M^{(8)}\phi_M^{(8)}(u,\mu) \\ f_M^{(1)}\phi_M^{(1)}(u,\mu) \end{pmatrix} = U(\varphi_0)\begin{pmatrix} f_M^{(q)}\phi_M^{(q)}(u,\mu) \\ f_M^{(s)}\phi_M^{(s)}(u,\mu) \end{pmatrix}. \tag{7}$$

Here

$$U(\varphi_0) = \begin{pmatrix} \cos\varphi_0 & -\sin\varphi_0 \\ \sin\varphi_0 & \cos\varphi_0 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix}, \tag{8}$$

with $\varphi_0 = \arctan(\sqrt{2})$.

In the singlet-octet basis, the scale dependence of the DAs is considerably simpler than in the QF approach. In fact, SO couplings and DAs do not mix with each other via renormalization. Moreover, the octet coupling $f_M^{(8)}$ is scale independent, whereas the singlet coupling $f_M^{(1)}$ evolves due to the $U(1)$ anomaly [32]:

$$f_M^{(1)}(\mu) = f_M^{(1)}(\mu_0)\left\{1 + \frac{2n_f}{\pi\beta_0}[\alpha_s(\mu) - \alpha_s(\mu_0)]\right\}, \tag{9}$$

where n_f is the number of light quark flavors.

This basis is also preferable for solution of the evolution equations. Thus, the quark-antiquark DAs in the singlet-octet basis can be expanded in terms of Gegenbauer polynomials $C_n^{3/2}(2u-1)$ that are eigenfunctions of the one-loop flavor-nonsinglet evolution equation:

$$\phi_M^{(1,8)}(u,\mu) = 6u\bar{u}\left[1 + \sum_{n=2,4,\ldots} a_{n,M}^{(1,8)}(\mu)C_n^{3/2}(2u-1)\right]. \tag{10}$$

The sum in Eq. (10) runs over polynomials of even dimension $n = 2, 4, \ldots$, implying that the quark-antiquark DAs are symmetric functions under the interchange of the quark momenta

$$\phi_M^{(1,8)}(u,\mu) = \phi_M^{(1,8)}(\bar{u},\mu). \tag{11}$$

Another twist-2 DA of the $\eta - \eta'$ system is connected with its two-gluon component. This distribution can be defined as a nonlocal matrix element

$$\langle M(p)|G_{\mu\nu}(x)\tilde{G}^{\mu\nu}(0)|0\rangle = \frac{C_F}{2\sqrt{3}} f_M^{(1)} (qx)^2 \int_0^1 du e^{iqxu} \phi_M^{(g)}(u,\mu), \tag{12}$$

where $G_{\mu\nu} = G_{\mu\nu}^a \lambda^a/2$ with $\mathrm{tr}[\lambda^a \lambda^b] = 2\delta^{ab}$. The dual gluon field strength tensor is defined as $\tilde{G}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}$, and $C_F = 4/3$.

The gluon DA is antisymmetric,

$$\phi_M^{(g)}(u,\mu) = -\phi_M^{(g)}(\bar{u},\mu), \tag{13}$$

and can be expanded in a series of Gegenbauer polynomials $C_{n-1}^{5/2}(2u-1)$ of odd dimension:

$$\phi_M^{(g)}(u,\mu) = 30u^2\bar{u}^2 \sum_{n=2,4,\ldots} a_{n,M}^{(g)}(\mu) C_{n-1}^{5/2}(2u-1). \tag{14}$$

It should be emphasized that the octet components of the eta mesons' DAs are renormalized multiplicatively to the leading order and mix with the gluon components only at the next-to-leading order, whereas the singlet components mix with gluon ones already in the LO (see Appendix B in Ref. [31] for details). The values of the parameters $a_{n,M}^{(1,8,g)}$ at a certain scale μ_0 determine all nonperturbative information on the DAs.

In the exact $SU(3)$ flavor symmetry limit, $\eta = \eta_8$, and η' is a flavor singlet, $\eta' = \eta_1$. In this limit $f_\eta^{(q)} = f_\pi$, with $f_\pi = 131$ MeV being equal to the pion decay constant. However, it is known empirically that the $SU(3)$-breaking corrections are large and, as a result, the relation of physical η, η' mesons to the basic octet and singlet states becomes complicated and involves two different mixing angles; see, e.g., a discussion in Ref. [19].

To avoid these problems and reduce a number of free parameters necessary to treat the $\eta - \eta'$ system, a new mixing scheme (FKS) was proposed [19]. It uses the QF basis and is founded on the observation that vector mesons ω and ϕ are to a very good approximation pure $\bar{u}u + \bar{d}d$ and $\bar{s}s$ states, and the same is true also for tensor mesons. The smallness of mixing corresponds to the OZI rule that is phenomenologically very successful. Therefore, if the axial $U(1)$ anomaly is the only effect that makes the situation in the pseudoscalar channel different, it is natural to suggest that the physical states are related to the flavor ones by an orthogonal transformation

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = U(\varphi) \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}, \quad U(\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}. \tag{15}$$

The assumption on the state mixing implies that the same mixing pattern applies to the decay constants and to the wave functions as well. In other words,

$$\begin{pmatrix} f_\eta^{(q)} & f_\eta^{(s)} \\ f_{\eta'}^{(q)} & f_{\eta'}^{(s)} \end{pmatrix} = U(\varphi) \begin{pmatrix} f_q & 0 \\ 0 & f_s \end{pmatrix}, \tag{16}$$

and

$$\begin{pmatrix} f_\eta^{(q)}\phi_\eta^{(q)} & f_\eta^{(s)}\phi_\eta^{(s)} \\ f_{\eta'}^{(q)}\phi_{\eta'}^{(q)} & f_{\eta'}^{(s)}\phi_{\eta'}^{(s)} \end{pmatrix} = U(\varphi) \begin{pmatrix} f_q\phi_q & 0 \\ 0 & f_s\phi_s \end{pmatrix} \tag{17}$$

are held with the same mixing angle φ.

This conjecture allows one to reduce four DAs of physical states η, η' to the two DAs, $\phi_q(u,\mu)$ and $\phi_s(u,\mu)$, of the flavor states:

$$\phi_\eta^{(q)}(u) = \phi_{\eta'}^{(q)}(u) = \phi_q(u),$$
$$\phi_\eta^{(s)}(u) = \phi_{\eta'}^{(s)}(u) = \phi_s(u). \tag{18}$$

The singlet and octet DAs is this scheme are given by

$$\begin{pmatrix} f_\eta^{(8)}\phi_\eta^{(8)} & f_\eta^{(1)}\phi_\eta^{(1)} \\ f_{\eta'}^{(8)}\phi_{\eta'}^{(8)} & f_{\eta'}^{(1)}\phi_{\eta'}^{(1)} \end{pmatrix} = U(\varphi) \begin{pmatrix} f_q\phi_q & 0 \\ 0 & f_s\phi_s \end{pmatrix} U^T(\varphi_0), \tag{19}$$

and the same relation is valid for the couplings $f_M^{(i)}$ and the couplings multiplied by the parameters $f_M^{(i)} a_{n,M}^{(i)}$. The couplings f_q and f_s, as well as mixing angle φ in the quark-flavor scheme have been determined in Ref. [19] from the fit to the experimental data

$$f_q = (1.07 \pm 0.02) f_\pi,$$
$$f_s = (1.34 \pm 0.06) f_\pi,$$
$$\varphi = 39.3^\circ \pm 1.0^\circ. \tag{20}$$

It is worth noting that the flavor-singlet and flavor-octet couplings have different scale dependence, and Eq. (19) cannot hold at all scales. It is natural to assume that the scheme refers to a low renormalization scale $\mu_0 \sim 1$ GeV and the DAs at higher scales are obtained by the QCD evolution.

Then, for the gluon DA, we assume that

$$\langle\eta_q|G_{\mu\nu}(x)\tilde{G}^{\mu\nu}(0)|0\rangle = \langle\eta_s|G_{\mu\nu}(x)\tilde{G}^{\mu\nu}(0)|0\rangle$$

and as a result get

$$\phi_\eta^{(g)}(u) = \phi_{\eta'}^{(g)}(u). \tag{21}$$

We define two-particle twist-3 DAs for the strange quarks in the following way:

$$2m_s\langle M(q)|\bar{s}(x)i\gamma_5 s(0)|0\rangle = \int_0^1 du e^{iqxu}\phi_{3M}^{(s)p}(u) \tag{22}$$

and

$$2m_s\langle M(q)|\bar{s}(x)\sigma_{\mu\nu}\gamma_5 s(0)|0\rangle = \frac{i}{6}(q_\mu x_\nu - q_\nu x_\mu)\int_0^1 du e^{iqxu}\phi_{3M}^{(s)\sigma}(u), \tag{23}$$

with the normalization

$$\int_0^1 du\phi_{3M}^{(s)p}(u) = \int_0^1 du\phi_{3M}^{(s)\sigma}(u) = h_M^{(s)}. \tag{24}$$

Here [22,31]

$$h_M^{(s)} = m_M^2 f_M^{(s)} - A_M,$$
$$A_M = \langle 0|\frac{\alpha_s}{4\pi}G_{\mu\nu}^a\tilde{G}^{a,\mu\nu}|M(p)\rangle, \tag{25}$$

which follows from the anomaly relation

$$\partial^\mu J_{\mu5}^{(s)} = 2m_s\bar{s}i\gamma_5 s + \frac{\alpha_s}{4\pi}G_{\mu\nu}^a\tilde{G}^{a,\mu\nu}.$$

Twist-3 DAs for the light $q = u$ or d quark can be defined by similar expressions with substitutions $s \to q$; e.g. $H_M^{(q)} = m_M^2 F_M^{(q)} - A_M$, where

$$H_M^{(u)} = H_M^{(d)} = \frac{h_M^{(q)}}{\sqrt{2}}. \tag{26}$$

Writing the normalization of the twist-3 DAs in this form [see Eqs. (22)–(25)], we follow Refs. [22,25,31]. Note that this definition formally remains correct in the chiral $m_s \to 0$ limit. As mentioned above, in this case η and η' are purely flavor octet and flavor singlet, respectively, so that η becomes massless and η' remains massive due to the axial anomaly [33,34]. Equation (25) is then satisfied trivially for η, because all three terms vanish, and for η' the cancellation of the two terms on the rhs implies the well-known relation for the η' mass in terms of the anomaly matrix element. The ratio h_s/m_s (and the similar ratios for light quarks) remains finite, so the contribution of twist-3 DAs to correlation functions remains finite in the case in which they enter the coefficients without a quark mass factor. For further discussion and examples, we refer to Ref. [25].

We assume that at low scales, the FKS mixing scheme is valid for all quantities and distributions, and introduce two new parameters h_q and h_s [22]:

$$\begin{pmatrix} h_\eta^{(q)}, h_\eta^{(s)} \\ h_{\eta'}^{(q)}, h_{\eta'}^{(s)} \end{pmatrix} = U(\varphi)\begin{pmatrix} h_q, 0 \\ 0, h_s \end{pmatrix} \tag{27}$$

with numerical values (in GeV^3)

$$h_q = 0.0016 \pm 0.004, \qquad h_s = 0.087 \pm 0.006. \tag{28}$$

Within the FKS scheme, we can rewrite four DAs $\phi_{3M}^{(q,s)p}$ in terms of two functions $\phi_{3s}^p(u)$ and $\phi_{3q}^p(u)$. The same argumentation is valid for the distribution amplitudes $\phi_{3M}^{(q,s)\sigma}$, as well. Let us note that for calculation of the strong couplings of interest we need only the s components of the DAs. Therefore, we get

$$\phi_{3\eta'}^{(s)p}(u) = \phi_{3s}^p(u)\cos\varphi, \qquad \phi_{3\eta}^{(s)p}(u) = -\phi_{3s}^p(u)\sin\varphi,$$
$$\phi_{3\eta'}^{(s)\sigma}(u) = \phi_{3s}^\sigma(u)\cos\varphi, \qquad \phi_{3\eta}^{(s)\sigma}(u) = -\phi_{3s}^\sigma(u)\sin\varphi, \tag{29}$$

where

$$\phi_{3s}^p(u) = h_s + 60m_s f_{3s}C_2^{1/2}(2u-1),$$
$$\phi_{3s}^\sigma(u) = 6\bar{u}u[h_s + 10m_s f_{3s}C_2^{3/2}(2u-1)]. \tag{30}$$

The coupling f_{3s} is defined as

$$\langle 0|\bar{s}\sigma_{n\xi}\gamma_5 gG^{n\xi}s|\eta_s(p)\rangle = 2i(pz)^2 f_{3s},$$

and we assume that

$$f_{3\eta'}^{(s)} = f_{3s}\cos\varphi, \qquad f_{3\eta}^{(s)} = -f_{3s}\sin\varphi. \tag{31}$$

For the coupling f_{3s}, as an estimate, we adopt a value of the similar parameter obtained for the pion. The latter at the scale $\mu_0 = 1$ GeV is equal to

$$f_{3s}(\mu_0) \simeq f_{3\pi}(\mu_0) = (0.0045 \pm 0.0015)\ \mathrm{GeV}^2.$$

The scale dependence of $f_{3s}(\mu)$ is determined by the formula

$$f_{3s}(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{55/9\beta_0} f_{3s}(\mu_0). \tag{32}$$

Here some comments are in order. Let us explain our choice of the parameters in the higher-twist DAs. First of all, there is not any information on flavor-singlet contributions to these parameters. Moreover, computation of these parameters using the QCD sum rule method by taking into account only quark contents of η and η' mesons leads to numerical values that are very close to parameters of the pion DAs. In fact, calculations of the parameters f_{3s} and $\delta_M^{2(s)}$ presented in the Appendix illustrate the correctness of this choice. Therefore, in what follows we will use parameters from the pion DAs, keeping in mind that the approximation accepted here does not encompass the flavor-singlet effects.

The eta mesons' three-particle twist-3 DAs are defined in accordance with Ref. [35]:

$$\begin{aligned}&\langle M(q)|\bar{s}(x) g G_{\mu\nu}(vx)\sigma_{\alpha\beta}\gamma_5 s(0)|0\rangle\\ &\quad = i f_{3M}^{(s)}[q_\alpha(q_\mu g_{\nu\beta} - q_\nu g_{\mu\beta}) - (\alpha\leftrightarrow\beta)]\\ &\quad\times\int D\underline{\alpha} e^{iqx(\alpha_1+v\alpha_3)}\Phi_{3M}^{(s)}(\alpha),\end{aligned} \tag{33}$$

where

$$\int \mathcal{D}\underline{\alpha} = \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right).$$

The expansion of the function $\Phi_{3M}^{(s)}(\alpha)$ in the conformal spin leads to the known expression

$$\Phi_{3M}^{(s)}(\alpha) = 360\alpha_1\alpha_2\alpha_3^2\left[1+\frac{1}{7}\omega_{3s}(7\alpha_3-3)\right], \tag{34}$$

with

$$\omega_{3s}(\mu_0) \simeq \omega_{3\pi}(\mu_0) = (-1.5\pm 0.7)\ \mathrm{GeV}^2 \tag{35}$$

and

$$(f_{3s}\omega_{3s})(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{104/9\beta_0}(f_{3s}\omega_{3s})(\mu_0).$$

Finally, we will need the DAs of twist 4, which are rather numerous. First of all, there are 4 two-particle twist-4 distribution amplitudes of the $\eta-\eta'$ system stemming from the matrix element

$$\begin{aligned}&\langle M(q)|\bar{s}(x)\gamma_\mu\gamma_5 s(0)|0\rangle\\ &\quad = -iq_\mu F_M^{(s)}\int_0^1 du e^{iqxu}\left[\phi_M^{(s)}(u)+\frac{x^2}{16}\phi_{4M}^{(s)}(u)\right]\\ &\quad - i\frac{x_\mu}{qx}F_M^{(s)}\int_0^1 du e^{iqxu}\psi_{4M}^{(s)}(u).\end{aligned} \tag{36}$$

Other three-particle twist-4 distributions are given by the expressions

$$\begin{aligned}&\langle M(q)|\bar{s}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)s(0)|0\rangle\\ &\quad = F_M^{(s)}\frac{q_\mu}{qx}(q_\alpha x_\beta - q_\beta x_\alpha)\int\mathcal{D}\underline{\alpha} e^{iqx(\alpha_1+v\alpha_3)}\Phi_{4M}^{(s)}(\alpha)\\ &\quad + F_M^{(s)}\left[q_\beta\left(g_{\alpha\mu}-\frac{x_\alpha q_\mu}{qx}\right)-q_\alpha\left(g_{\beta\mu}-\frac{x_\beta q_\mu}{qx}\right)\right]\\ &\quad\times\int\mathcal{D}\underline{\alpha} e^{iqx(\alpha_1+v\alpha_3)}\Psi_{4M}^{(s)}(\alpha)\end{aligned} \tag{37}$$

and

$$\begin{aligned}&\langle M(q)|\bar{s}(x)\gamma_\mu\gamma_5 g_s \tilde{G}_{\alpha\beta}(vx)s(0)|0\rangle\\ &\quad = F_M^{(s)}\frac{q_\mu}{qx}(q_\alpha x_\beta - q_\beta x_\alpha)\int\mathcal{D}\underline{\alpha} e^{iqx(\alpha_1+v\alpha_3)}\tilde{\Phi}_{4M}^{(s)}(\alpha)\\ &\quad + F_M^{(s)}\left[q_\beta\left(g_{\alpha\mu}-\frac{x_\alpha q_\mu}{qx}\right)-q_\alpha\left(g_{\beta\mu}-\frac{x_\beta q_\mu}{qx}\right)\right]\\ &\quad\times\int\mathcal{D}\underline{\alpha} e^{iqx(\alpha_1+v\alpha_3)}\tilde{\Psi}_{4M}^{(s)}(\alpha).\end{aligned} \tag{38}$$

The distribution amplitudes $\Phi_{4M}^{(s)}$ and $\Psi_{4M}^{(s)}$ can be expanded in orthogonal polynomials that correspond to contributions of increasing spin in the conformal expansion. Taking into account contributions of the lowest and the next-to-lowest spin, one finds [31,35–37]

$$\begin{aligned}\Phi_{4M}^{(s)}(\alpha) &= 120\alpha_1\alpha_2\alpha_3[\phi_{1,M}^{(s)}(\alpha_1-\alpha_2)],\\ \tilde{\Phi}_{4M}^{(s)}(\alpha) &= 120\alpha_1\alpha_2\alpha_3[\tilde{\phi}_{0,M}^{(s)}+\tilde{\phi}_{2,M}^{(s)}(3\alpha_3-1)],\\ \tilde{\Psi}_{4M}^{(s)}(\alpha) &= -30\alpha_3^2\Big\{\psi_{0,M}^{(s)}(1-\alpha_3)\\ &\quad + \psi_{1,M}^{(s)}[\alpha_3(1-\alpha_3)-6\alpha_1\alpha_2]\\ &\quad + \psi_{2,M}^{(s)}\left[\alpha_3(1-\alpha_3)-\frac{3}{2}(\alpha_1^2+\alpha_2^2)\right]\Big\},\\ \Psi_{4M}^{(s)}(\alpha) &= -30\alpha_3^2(\alpha_1-\alpha_2)\Big\{\psi_{0,M}^{(s)}+\psi_{1,M}^{(s)}\alpha_3\\ &\quad + \frac{1}{2}\psi_{2,M}^{(s)}(5\alpha_3-3)\Big\}.\end{aligned} \tag{39}$$

The coefficients $\phi_{kM}^{(s)}, \psi_{kM}^{(s)}$ are related by QCD equations of motion (EOMs) [31]. From these EOMs one obtains

$$\tilde{\phi}_{0M}^{(s)} = \psi_{0M}^{(s)} = -\frac{1}{3}\delta_M^{2(s)} \tag{40}$$

and

$$\begin{aligned}
\tilde{\phi}_{2M}^{(s)} &= \frac{21}{8}\delta_M^{2(s)}\omega_{4M}^{(s)},\\
\phi_{1M}^{(s)} &= \frac{21}{8}\left[\delta_M^{2(s)}\omega_{4M}^{(s)} + \frac{2}{45}m_M^2\left(1-\frac{18}{7}a_{2M}^{(s)}\right)\right],\\
\psi_{1M}^{(s)} &= \frac{7}{4}\left[\delta_M^{2(s)}\omega_{4M}^{(s)} + \frac{1}{45}m_M^2\left(1-\frac{18}{7}a_{2M}^{(s)}\right) + 4m_s\frac{f_{3M}^{(s)}}{f_M^{(s)}}\right],\\
\psi_{2M}^{(s)} &= \frac{7}{4}\left[2\delta_M^{2(s)}\omega_{4M}^{(s)} - \frac{1}{45}m_M^2\left(1-\frac{18}{7}a_{2M}^{(s)}\right) - 4m_s\frac{f_{3M}^{(s)}}{f_M^{(s)}}\right].
\end{aligned} \tag{41}$$

Here the parameter $\delta_M^{2(s)}$ is defined as

$$\langle 0|\bar{s}\gamma^\rho ig\tilde{G}_{\rho\mu}s|M(p)\rangle = p_\mu f_M^{(s)}\delta_M^{2(s)}.$$

Its value at μ_0 is chosen to be equal to

$$\delta_M^{2(s)}(\mu_0) \simeq \delta_\pi^2(\mu_0) = (0.18 \pm 0.06)\ \mathrm{GeV}^2, \tag{42}$$

and its evolution is given by the formula

$$\delta_M^{2(s)}(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{10/\beta_0}\delta_M^{2(s)}(\mu_0).$$

We set the parameter $\omega_{4M}^{(s)}(\mu_0)$ equal to $\omega_{4\pi}(\mu_0)$:

$$\omega_{4M}^{(s)}(\mu_0) \simeq \omega_{4\pi}(\mu_0) = (0.2 \pm 0.1)\ \mathrm{GeV}^2, \tag{43}$$

with

$$(\delta_M^{2(s)}\omega_{4M}^{(s)})(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{32/9\beta_0}(\delta_M^{2(s)}\omega_{4M}^{(s)})(\mu_0).$$

The DAs $\phi_{4M}^{(s)}(u)$ and $\psi_{4M}^{(s)}(u)$ can be calculated in terms of the three-particle DAs of twist 4 and the DAs of lower twist. As a result, one obtains the expressions for the two-particle DAs $\psi_{4M}^{(s)}(u)$ and $\psi_{4M}^{(s)}(u)$ that can be separated in "genuine" twist-4 contributions and meson mass corrections as

$$\psi_{4M}^{(s)}(u) = \psi_{4M}^{(s)\mathrm{twist}}(u) + m_M^2\psi_{4M}^{(s)\mathrm{mass}}(u), \tag{44}$$

with

$$\begin{aligned}
\psi_{4M}^{(s)\mathrm{twist}}(u) &= \frac{20}{3}\delta_M^{2(s)}C_2^{1/2}(2u-1) + 30m_s\frac{f_{3M}^{(s)}}{f_M^{(s)}}\\
&\quad\times\left(\frac{1}{2} - 10u\bar{u} + 35u^2\bar{u}^2\right),\\
\psi_{4M}^{(s)\mathrm{mass}}(u) &= \frac{17}{12} - 19u\bar{u} + \frac{105}{2}u^2\bar{u}^2\\
&\quad + a_{2,M}^{(s)}\left(\frac{3}{2} - 54u\bar{u} + 225u^2\bar{u}^2\right),
\end{aligned} \tag{45}$$

and similarly

$$\phi_{4M}^{(s)}(u) = \phi_{4M}^{(s)\mathrm{twist}}(u) + m_M^2\phi_{4M}^{(s)\mathrm{mass}}(u), \tag{46}$$

where

$$\begin{aligned}
\phi_{4M}^{(s)\mathrm{twist}}(u) &= \frac{200}{3}\delta_M^{2(s)}u^2\bar{u}^2 + 21\delta_M^{2(s)}\omega_{4M}^{(s)}\{u\bar{u}(2+13u\bar{u})\\
&\quad + 2[u^3(10-15u+6u^2)\ln u + (u\leftrightarrow\bar{u})]\}\\
&\quad + 20m_s\frac{f_{3M}^{(s)}}{f_M^{(s)}}u\bar{u}[12 - 63u\bar{u} + 14u^2\bar{u}^2],\\
\phi_{4M}^{(s)\mathrm{mass}}(u) &= u\bar{u}\left[\frac{88}{15} + \frac{39}{5}u\bar{u} + 14u^2\bar{u}^2\right]\\
&\quad - a_{2,M}^{(s)}u\bar{u}\left[\frac{24}{5} - \frac{54}{5}u\bar{u} + 180u^2\bar{u}^2\right]\\
&\quad + \left(\frac{28}{15} - \frac{24}{5}a_{2,M}^{(s)}\right)[u^3(10-15u+6u^2)\ln u\\
&\quad + (u\leftrightarrow\bar{u})].
\end{aligned} \tag{47}$$

These expressions complete the list of the distribution amplitudes that are necessary for analyzing the strong vertices $D_s^* D_s \eta^{(\prime)}$ and $B_s^* B_s \eta^{(\prime)}$ with twist-4 accuracy.

It is worth noting that we have chosen parameters of the higher-twist DAs in order to obey the pattern of the state mixing accepted for the $\eta - \eta'$ system. In fact, it is not difficult to see that the relations in Eq. (17) are true for the DAs $\phi_{3M}^{(3)p}(u)$, $\phi_{3M}^{(3)\sigma}(u)$, and $f_{3M}^{(s)}\Phi_{3M}^{(s)}(\alpha)$ as well. This formula is fulfilled approximately for twist-4 DAs $F_M^{(s)}\phi_{4M}^{(s)}(u)$ and $F_M^{(s)}\psi_{4M}^{(s)}(u)$. The main sources of deviation from Eq. (17) are terms $\sim m_M^2$ in twist-4 DAs that, nevertheless, numerically have rather small effects on final results.

III. THE LCSR FOR STRONG COUPLINGS

In the context of the QCD sum rules on the light cone, heavy-heavy-light-meson strong couplings were analyzed already in Refs. [38–40], where the vertices $D^* D\pi$, $B^* B\pi$, as well as vertices with ρ mesons were considered. In the present work, we calculate within the QCD LCSR method the strong couplings that correspond to the vertices

$D_s^* D_s \eta^{(\prime)}$ and $B_s^* B_s \eta^{(\prime)}$. Below, we concentrate on the couplings $g_{B_s^* B_s M}$; results for $g_{D_s^* D_s M}$ can be easily obtained from relevant expressions by replacements $b \to c$, $B_s^0 \to D_s^-$, and $B_s^{0*} \to D_s^{*-}$.

A. Leading-order results

In the calculation of the leading-order contribution to the LCSR, we use technical tools and methods elaborated in the original paper [38]. We start from the correlation function

$$F_\mu(p,q) = i \int d^4x e^{ipx} \langle M(q)|\mathcal{T}\{\bar{s}(x)\gamma_\mu b(x), \times \bar{b}(0) i\gamma_5 s(0)\}|0\rangle. \tag{48}$$

It is well known that this correlator can be calculated in both hadronic and quark-gluon degrees of freedom. Within the QCD LCSR method obtained in this way, expressions should be matched in order to find the couplings $g_{B_s^* B_s \eta}$ and $g_{B_s^* B_s \eta'}$ and extract numerical estimates for them. In terms of hadronic quantities, the aforementioned correlation functions are given by the expression

$$F_\mu^h(p,q) = \frac{g_{B_s^* B_s M} m_{B_s}^2 m_{B_s^*} f_{B_s} f_{B_s^*}}{m_b (p^2 - m_{B_s^*}^2)[(p+q)^2 - m_{B_s}^2]} \times \left[q_\mu + \frac{1}{2}\left(1 - \frac{m_{B_s}^2 + m_M^2}{m_{B_s^*}^2}\right) p_\mu\right],$$

where we have defined the couplings $g_{B_s^* B_s M}$ and decay constants f_{B_s}, $f_{B_s^*}$ by means of the following matrix elements:

$$\langle B_s^{*0}(p) M(q)|B_s^0(p+q)\rangle = -g_{B_s^* B_s M} q_\mu \epsilon^\mu,$$
$$\langle B_s|\bar{b} i\gamma_5 s|0\rangle = \frac{m_{B_s}^2 f_{B_s}}{m_b},$$
$$\langle 0|\bar{s}\gamma_\mu b|B_s^*\rangle = m_{B_s^*} f_{B_s^*} \epsilon_\mu. \tag{49}$$

The correlation function depends on the invariants p^2, $(p+q)^2$ and can be written as a sum of invariant amplitudes

$$F_\mu(p,q) = F(p^2,(p+q)^2) q_\mu + \tilde{F}(p^2,(p+q)^2) p_\mu.$$

For our purposes, it is enough to consider the function $F(p^2,(p+q)^2)$.

Computation of the amplitude $F(p^2,(p+q)^2)$ in terms of the hadronic quantities leads to an expression that contains the contribution of the ground state and the contribution of the higher resonances and continuum states with relevant quantum numbers in the form of a double dispersion integral:

$$F^h(p^2,(p+q)^2) = \frac{g_{B_s^* B_s M} m_{B_s}^2 m_{B_s^*} f_{B_s} f_{B_s^*}}{m_b (p^2 - m_{B_s^*}^2)[(p+q)^2 - m_{B_s}^2]} + \int \frac{ds_1 ds_2 \rho^h(s_1,s_2)}{(s_1 - p^2)[s_2 - (p+q)^2]} + \cdots. \tag{50}$$

Here the dots stand for single dispersion integrals that, in general, should be included to make the expression finite.

Considering p^2 and $(p+q)^2$ as independent variables and applying the Borel transformation, we find

$$\mathcal{B}_{M_1^2}\mathcal{B}_{M_2^2} F^h(p^2,(p+q)^2) = \frac{g_{B_s^* B_s M} m_{B_s}^2 m_{B_s^*} f_{B_s} f_{B_s^*}}{m_b} e^{-\frac{m_{B_s^*}^2}{M_1^2} - \frac{m_{B_s^*}^2}{M_2^2}} + \int ds_1 ds_2 e^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} \rho^h(s_1,s_2). \tag{51}$$

In order to obtain the sum rules expression for the strong couplings, the double Borel transformation should be applied to the same invariant amplitude, but now calculated using the quark-gluon degrees of freedom. To this end, one needs to employ the general expression for the correlation function Eq. (48) and compute it by substituting the light-cone expansion for the b-quark propagator

$$\langle 0|\mathcal{T}\{b(x)\bar{b}(0)\}|0\rangle = \int \frac{d^4k}{(2\pi)^4 i} e^{-ikx} \frac{\not{k} + m_b}{m_b^2 - k^2} - i g_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \times \int_0^1 dv \left[\frac{1}{2}\frac{\not{k} + m_b}{(m_b^2 - k^2)^2} G^{\mu\nu}(vx)\sigma_{\mu\nu} + \frac{\not{k} + m_b}{m_b^2 - k^2} v x_\mu G^{\mu\nu}(vx)\gamma_\nu\right] \tag{52}$$

and expressing remaining nonlocal matrix elements in terms of distribution amplitudes of the eta mesons. The diagrams corresponding to the free b-quark propagator, and to the one-gluon field components in the expansion Eq. (52), are depicted in Figs. 1(a) and 1(b), respectively.

Technical details of similar calculations can be found in Ref. [38]. Therefore, we do not concentrate here on these procedures and provide below only final results. Thus, for the contribution arising from Fig. 1(a) we find

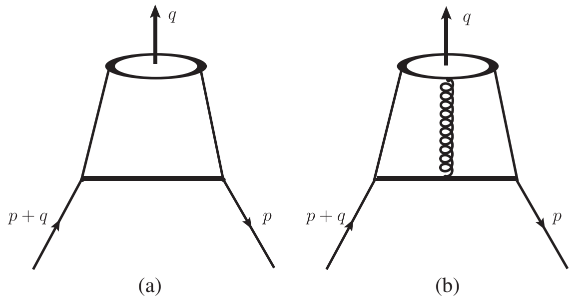

FIG. 1. Leading-order diagrams contributing to the correlation function. Thick lines correspond to a heavy quark. Diagram (a) describes quark-antiquark contributions of various twists to the correlator, whereas (b) shows the contribution coming from three-particle components of the meson distribution amplitude.

$$
\begin{aligned}
&F^{(a)}(p^2,(p+q)^2)\\
&=\int_0^1\frac{du}{\Delta(p,q,u)}\left\{m_bF_M^{(s)}\left[\phi_M^{(s)}(u)-\frac{m_M^2u\bar{u}}{\Delta(p,q,u)}\phi_M^{(s)}(u)\right.\right.\\
&\quad\left.+\frac{1}{\Delta(p,q,u)}\left(2uG_{4M}^{(s)}(u)-\frac{m_b^2\phi_{4M}^{(s)}(u)}{2\Delta(p,q,u)}\right)\right]\\
&\quad\left.+\frac{\phi_{3M}^{(s)p}(u)}{2m_s}u+\frac{\phi_{3M}^{(s)\sigma}(u)}{6m_s}+\frac{\phi_{3M}^{(s)\sigma}(u)}{12m_s}\frac{m_b^2+p^2}{\Delta(p,q,u)}\right\}.
\end{aligned}
\tag{53}
$$

In this expression we have introduced the shorthand notation for the denominator of the free b-quark propagator [see the first term in Eq. (52)],

$$
\Delta(p,q,u)=m_b^2-(1-u)p^2-u(p+q)^2,
$$

and also defined the new function $G_{4M}^{(s)}(u)$,

$$
G_{4M}^{(s)}(u)=-\int_0^u\psi_{4M}^{(s)}(v)dv.
$$

The meson mass correction $\sim m_M^2$ in Eq. (53) comes from the expansion of the leading-order twist-2 term.

Computations with one-gluon field components in the b-quark propagator lead to the following result:

$$
\begin{aligned}
&F^{(b)}(p^2,(p+q)^2)\\
&\quad=\int_0^1dv\int\mathcal{D}\alpha\left\{\frac{4f_{3M}^{(s)}\Phi_{3M}^{(s)}(\alpha)vpq}{[m_b^2-(p+q(\alpha_1+v\alpha_3))^2]^2}\right.\\
&\qquad\left.+F_M^{(s)}m_b\frac{2\Psi_{4M}^{(s)}(\alpha)-\Phi_{4M}^{(s)}(\alpha)+2\tilde{\Psi}_{4M}^{(s)}(\alpha)-\tilde{\Phi}_{4M}^{(s)}(\alpha)}{[m_b^2-(p+q(\alpha_1+v\alpha_3))^2]^2}\right\}.
\end{aligned}
\tag{54}
$$

Now, having applied the formula for the double Borel transformation

$$
\begin{aligned}
&\mathcal{B}_{M_1^2}\mathcal{B}_{M_2^2}\frac{(l-1)!}{[m_b^2-(1-u)p^2-u(p+q)^2]^l}\\
&\quad=(M^2)^{2-l}e^{-m_b^2/M^2}\delta(u-u_0),
\end{aligned}
$$

with

$$
u_0=\frac{M_1^2}{M_1^2+M_2^2},\qquad M^2=\frac{M_1^2M_2^2}{M_1^2+M_2^2},
$$

it is not difficult to find a desired expression for the Borel transformation of the invariant amplitude in terms of the quark-gluon degrees of freedom.

By this manner we obtain

$$
\begin{aligned}
&\mathcal{B}_{M_1^2}\mathcal{B}_{M_2^2}F^{\rm QCD}(p^2,(p+q)^2)\\
&=e^{-m_b^2/M^2}\times M^2\left\{m_bF_M^{(s)}\phi_M^{(s)}(u_0)\left(1-\frac{m_M^2u_0\bar{u}_0}{M^2}\right)\right.\\
&\quad+\frac{\phi_{3M}^{(s)p}(u_0)}{2m_s}u_0+\frac{\phi_{3M}^{(s)\sigma}(u_0)}{6m_s}+\frac{1}{12m_s}u_0\frac{d\phi_{3M}^{(s)\sigma}(u_0)}{du}\\
&\quad+\frac{m_b^2\phi_{3M}^{(s)\sigma}(u_0)}{6m_sM^2}+\frac{2F_M^{(s)}m_b}{M^2}u_0G_4(u_0)-\frac{F_M^{(s)}m_b^3}{4M^4}\phi_{4M}^{(s)}(u_0)\\
&\quad\left.+2f_{3M}^{(s)}I_M^{3(s)}(u_0)+F_M^{(s)}m_b\frac{I_M^{4(s)}(u_0)}{M^2}\right\}.
\end{aligned}
\tag{55}
$$

In Eq. (55) the new functions

$$
\begin{aligned}
I_M^{3(s)}(u_0)=&\int_0^{u_0}d\alpha_1\left[\frac{\Phi_{3M}^{(s)}(\alpha_1,1-u_0,u_0-\alpha_1)}{u_0-\alpha_1}\right.\\
&\left.-\int_{u_0-\alpha_1}^{1-\alpha_1}d\alpha_3\frac{\Phi_{3M}^{(s)}(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)}{\alpha_3^2}\right]
\end{aligned}
\tag{56}
$$

and

$$
\begin{aligned}
I_M^{4(s)}(u_0)=&\int_0^{u_0}d\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{d\alpha_3}{\alpha_3}[2\Psi_{4M}^{(s)}(\alpha)-\Phi_{4M}^{(s)}(\alpha)\\
&+2\tilde{\Psi}_{4M}^{(s)}(\alpha)-\tilde{\Phi}_{4M}^{(s)}(\alpha)]
\end{aligned}
\tag{57}
$$

are introduced.

Equation (55) is the required Borel transformed expression for the function $F^{\rm QCD}(p^2,(p+q)^2)$ given in the quark-gluon degrees of freedom. In order to derive the light-cone sum rule formulas for the couplings $g_{B_s^*B_s\eta}$ and $g_{B_s^*B_s\eta'}$, one should equate the Borel transformations of $F^{\rm h}(p^2,(p+q)^2)$ as in Eq. (51) and $F^{\rm QCD}(p^2,(p+q)^2)$ as written down in Eq. (55). Then the only unknown term is a contribution of higher resonances and continuum states represented in Eq. (51) as the integral with double spectral density $\rho^{\rm h}(s_1,s_2)$. To solve this problem, in accordance with the main idea of the sum rule methods, we suggest that above some threshold in the (s_1,s_2) plane, the double spectral density $\rho^h(s_1,s_2)$ can be replaced by

$\rho^{\mathrm{QCD}}(s_1, s_2)$. Then the continuum subtraction can be performed in accordance with the procedure developed in Refs. [18,38,41]. It is based on the observation that double spectral density in the leading contributions, i.e. in those proportional to the positive powers of the Borel parameter M^2, is concentrated (or can be expanded) near the diagonal $s_1 = s_2$. In this case, for the continuum subtraction, the simple expressions can be derived, which are not sensitive to the shape of the duality region [18,38,41]. The general formula in the case $M_1^2 = M_2^2 = 2M^2$ and $u_0 = 1/2$ reads

$$M^{2n} e^{-\frac{m_b^2}{M^2}} \to \frac{1}{\Gamma(n)} \int_{m_b^2}^{s_0} ds e^{-\frac{s}{M^2}} (s - m_b^2)^{n-1}, \qquad n \geq 1. \tag{58}$$

For terms $\sim M^2$, it leads to the simple prescription

$$M^2 e^{-m_b^2/M^2} \to M^2 (e^{-m_b^2/M^2} - e^{-s_0/M^2}) \tag{59}$$

adopted in our work, as well.

For the higher-twist terms, which are proportional to the zeroth or to the negative powers of M^2, on the one hand, continuum subtraction is not expected to have a large effect, and, on the other hand, it is not known how to perform it in a theoretically clean way. The difficulty here is that the quark-hadron duality is not expected to work pointwise in the two-dimensional plane (s_1, s_2), but, at best, after integration over the line $s_1 + s_2 = \text{const.}$ (see, for example, Refs. [42,43]). For this reason a naive subtraction using the "square" duality region $s_1 < s_0, s_2 < s_0$ does not have a strong theoretical basis. The spectral densities corresponding to the higher-twist terms under consideration are not concentrated near the diagonal $s_1 = s_2$; as a result, the required continuum subtractions take rather complicated forms. Because the higher-twist spectral densities decrease with s_1 and s_2 quickly enough and the impact of the subtracted terms on the final result is not significant, in a standard technique for the LCSRs of this type, one does not perform continuum subtractions in these terms at all [38]. Here we follow these procedures and subtract the continuum contributions only in the terms $\sim M^2$.

The masses of the B_s and B_s^* mesons are numerically close to each other, hence in our calculations we can safely set $M_1^2 = M_2^2$ and $u_0 = 1/2$. Then, it is not difficult to write down the following sum rule:

$$\begin{aligned} f_{B_s} f_{B_s^*} g_{B_s^* B_s M} = {} & \frac{m_b}{m_{B_s}^2 m_{B_s^*}} e^{\frac{m_{B_s}^2 + m_{B_s^*}^2}{2M^2}} \Bigg\{ M^2 (e^{-\frac{m_b^2}{M^2}} - e^{-\frac{s_0}{M^2}}) \Bigg[m_b F_M^{(s)} \phi_M^{(s)}(u_0) + \frac{\phi_{3M}^{(s)p}(u_0)}{2m_s} u_0 + \frac{\phi_{3M}^{(s)\sigma}(u_0)}{6m_s} \\ & + \frac{1}{12m_s} u_0 \frac{d\phi_{3M}^{(s)\sigma}(u_0)}{du} + 2 f_{3M}^{(s)} I_M^{3(s)}(u_0) \Bigg] + e^{-\frac{m_b^2}{M^2}} \Bigg[F_M^{(s)} m_b \Big(-m_M^2 u_0 \bar{u}_0 \phi_M^{(s)}(u_0) + 2u_0 G_{4M}^{(s)}(u_0) \\ & + I_M^{4(s)}(u_0) - \frac{m_b^2}{4M^2} \phi_{4M}^{(s)}(u_0) \Big) + \frac{m_b^2}{6m_s} \phi_{3M}^{(s)\sigma}(u_0) \Bigg] \Bigg\}_{u_0 = 1/2}. \end{aligned} \tag{60}$$

This result differs from the corresponding expression of Ref. [38] due to new definitions of the DAs and the additional mass term in the sum rule expression.

For self-consistent treatment of Eq. (60), one needs expressions for f_{B_s} and $f_{B_s^*}$ with NLO accuracy. A recent calculation of the heavy-light mesons' decay constants, performed in the context of QCD sum rules method by taking into account $O(\alpha_s^2)$ terms in the perturbative part and $O(\alpha_s)$ corrections to the quark-condensate contribution, can be found in Ref. [44]. For further details and explicit expressions, we refer to this work (see also Ref. [45]).

B. NLO corrections: Gluonic contributions to the strong couplings

The QCD LCSRs for the strong couplings [Eq. (60)] have been derived at the leading order of the perturbative QCD with twist-4 accuracy. In order to improve our results and make more precise theoretical predictions for the strong couplings, we need to find NLO perturbative corrections at least to the leading twist term, and in this way include in our analysis also the gluon component of the eta mesons. The NLO correction to the leading twist term and the relevant double spectral density for the strong vertices $B^* B \pi$ and $D^* D \pi$ were found in Ref. [40]. In this work, the authors demonstrated that, to this end, it is sufficient to utilize the NLO correction to the transition form factor $B \to \pi$ calculated in Ref. [46], and from the corresponding expression deduced the double spectral density for the coupling $g_{B^* B \pi}$. Because the pion is a pseudoscalar particle and has only a quark component, after some corrections that depend on the definitions of DAs and decay constants, results of this work can be used to find NLO corrections to the leading twist term in the LCSRs for strong couplings arising from the quark component of the η and η' mesons. Therefore, we borrow the corresponding expression for the NLO correction from Ref. [40], and for the asymptotic DAs $\phi_{\eta(\eta')}^{(s)}(u)$ we get

$$Q^{(s)}_{\eta(\eta')}(M^2, s_0^{B_s}) = \frac{\alpha_s C_F}{4\pi} \frac{F^{(s)}_{\eta(\eta')} m_b}{\sqrt{2}} \times \int_{2m_b^2}^{2s_0^{B_s}} f\left(\frac{s}{m_b^2} - 2\right) e^{-s/2M^2} ds, \quad (61)$$

where

$$f(x) = \frac{\pi^2}{4} + 3\ln\left(\frac{x}{2}\right)\ln\left(1+\frac{x}{2}\right) - \frac{3(3x^3+22x^2+40x+24)}{3(2+x)^3}\ln\left(\frac{x}{2}\right) + 6\mathrm{Li}_2\left(-\frac{x}{2}\right) - 3\mathrm{Li}_2(-x) - 3\mathrm{Li}_2(-1-x) - 3\ln(1+x)\ln(2+x) - \frac{3(3x^2+20x+20)}{4(2+x)^3} \quad (62)$$

$$+\frac{6x(1+x)\ln(1+x)}{(2+x)^2}. \quad (63)$$

In order to find the gluonic contributions to the LCSRs, one has to compute the quark box diagrams shown in Fig. 2. For the transitions $B \to \eta^{(\prime)}$ they were calculated in Ref. [25] (see also Ref. [47]). We adapt to our problem the relevant expressions obtained in Ref. [25] and use them in our calculations.

To derive the double spectral density, we start from the expression

$$F^{(g)}(p^2, (p+q)^2) = \frac{\alpha_s C_F}{4\pi} f_M^{(1)} m_b \int_{m_b^2}^{\infty} \frac{d\alpha g(\alpha, p^2)}{\alpha - (p+q)^2}, \quad (64)$$

where

$$g(\alpha, p^2) = \frac{25}{6\sqrt{3}} a^{(g)}_{2,M} \left\{ \frac{m_b^2 - \alpha}{(\alpha - p^2)^5} [59m_b^6 + 21p^6 - 63p^4\alpha - 19p^2\alpha^2 + 2\alpha^3 + m_b^2\alpha(164p^2 + 13\alpha) - m_b^4(82p^2 + 95\alpha)] + 6\frac{(m_b^2 - p^2)(\alpha - m_b^2)}{(\alpha - p^2)^5} \times [5m_b^4 + p^4 + 3p^2\alpha + \alpha^2 - 5m_b^2(p^2+\alpha)] \times \left[2\ln\frac{\alpha - m_b^2}{m_b^2} - \ln\frac{\mu^2}{m_b^2}\right] \right\}. \quad (65)$$

We employ a method described in detailed form in Ref. [43]. In other words, we first perform the double Borel transformations

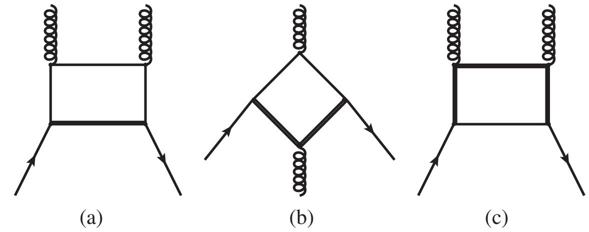

FIG. 2. Quark box diagrams that determine the gluonic contribution. Thick lines correspond to a heavy quark.

$$B_{t_1}(p^2) B_{t_2}((p+q)^2) F^{(g)}(p^2, (p+q)^2) \equiv \hat{F}^{(g)}(t_1, t_2) = \frac{1}{t_1 t_2} \int ds_1 ds_2 \rho(s_1, s_2) e^{-s_1/t_1 - s_1/t_2},$$

and then apply the Borel transformations in $\tau_1 = 1/t_1$ and $\tau_2 = 1/t_2$ in order to extract $\rho(s_1, s_2)$

$$B_{1/s_1}(\tau_1) B_{1/s_2}(\tau_2) \frac{1}{\tau_1\tau_2} \hat{F}^{(g)}(1/\tau_1, 1/\tau_2) = s_1 s_2 \rho(s_1, s_2).$$

Having subtracted the contribution of the resonances and continuum states, we get the gluonic correction as the double dispersion integral:

$$F_M(p^2, (p+q)^2) = \frac{\alpha_s C_F}{4\pi} f_M^{(1)} m_b \int_{m_b^2}^{s_0^{B_s}} \int_{m_b^2}^{s_0^{B_s}} \frac{ds_1 ds_2 \rho(s_1, s_2)}{(s_1 - p^2)(s_2 - (p+q)^2)}, \quad (66)$$

where

$$\rho(s_1, s_2) = \frac{25}{6\sqrt{3}} a^{(g)}_{2,M} [\rho_1(s_1, s_2) + 6\rho_2(s_1, s_2)].$$

Here

$$\rho_1(s_1, s_2) = 21\Delta^{(1)}(s_1 - s_2) - \frac{82}{6}\Delta^{(3)}(s_1 - s_2) - \frac{59}{24}\Delta^{(4)}(s_1 - s_2), \quad (67)$$

and

$$\rho_2(s_1, s_2) = L(s_1, \mu)\left[\Delta^{(2)}(s_1 - s_2) + \frac{1}{3}\Delta^{(3)}(s_1 - s_2) + \frac{1}{24}\Delta^{(4)}(s_1 - s_2)\right]. \quad (68)$$

In Eqs. (67) and (68),

$$\Delta^{(n)}(s_1 - s_2) = (s_1 - m_b)^n \delta^{(n)}(s_1 - s_2),$$
$$L(s,\mu) = 2\ln\frac{s - m_b^2}{m_b^2} - \ln\frac{\mu^2}{m_b^2}, \tag{69}$$

with $\delta^{(n)}(s_1 - s_2)$ being defined as

$$\delta^{(n)}(s_1 - s_2) = \frac{\partial^n}{\partial s_1^n}\delta(s_1 - s_2).$$

The Borel transformations in the variables p^2 and $(p+q)^2$ of the integral in Eq. (66) give us the desired gluonic contribution to the sum rules:

$$\mathcal{B}_{M_1^2}\mathcal{B}_{M_2^2}F_M((p+q)^2, p^2) = \frac{\alpha_s C_F}{4\pi} f_M^{(1)} m_b \int_{m_b^2}^{s_0^{B_s}} ds_1 \int_{m_b^2}^{s_0^{B_s}} ds_2 \rho(s_1, s_2) e^{-s_1/M_1^2} e^{-s_2/M_2^2}. \tag{70}$$

In the case $M_1^2 = M_2^2 = 2M^2$, by applying methods from Appendix B of Ref. [38], we calculate the integrals in Eq. (70),

$$\int_{m_b^2}^{s_0^{B_s}} ds_1 \int_{m_b^2}^{s_0^{B_s}} ds_2 \Delta^{(k)}(s_1 - s_2) e^{-(s_1+s_2)/2M^2} = \frac{(-1)^k}{2^{k+1}} \int_{2m_b^2}^{2s_0^{B_s}} ds e^{-s/2M^2} \left(\frac{d}{dv}\right)^k \left(v - \frac{m_b^2}{s}\right)^k_{v=1/2} \tag{71}$$

and

$$\int_{m_b^2}^{s_0^{B_s}} ds_1 \int_{m_b^2}^{s_0^{B_s}} ds_2 \ln(s_1 - m_b^2) \Delta^{(k)}(s_1 - s_2) \times e^{-(s_1+s_2)/2M^2} = \frac{(-1)^k}{2^{k+1}} \int_{2m_b^2}^{2s_0^{B_s}} ds e^{-s/2M^2} \left(\frac{d}{dv}\right)^k \times \left[\left(v - \frac{m_b^2}{s}\right)^k \ln(sv - m_b^2)\right]_{v=1/2}. \tag{72}$$

The integrations over s can be performed explicitly, allowing us to find the gluonic contribution in a rather simple form:

$$\tilde{Q}_{\eta(\eta')}(M^2, s_0^{B_s}) = \frac{\alpha_s C_F}{4\pi} f_{\eta(\eta')}^{(1)} m_b [r_1(M^2, s_0^{B_s}) + r_2(M^2, s_0^{B_s})], \tag{73}$$

where

$$r_1(M^2, s_0^{B_s}) = M^2(e^{-m_b^2/M^2} - e^{-s_0/M^2})\left(-\frac{51}{32}\right) \tag{74}$$

and

$$r_2(M^2, s_0^{B_s}) = \frac{3}{16} M^2 e^{-m_b^2/M^2}\Bigg[22 + 20\psi(7) - 20\Gamma\left(0, \frac{s_0^{B_s} - m_b^2}{M^2}\right) + 20\ln\frac{2M^2}{m_b^2} - 10\ln\frac{\mu^2}{m_b^2}\Bigg] + \frac{3}{16} M^2 e^{-s_0^{B_s}/M^2}\Bigg[-27 - 20\ln\frac{2(s_0^{B_s} - m_b^2)}{m_b^2} + 10\ln\frac{\mu^2}{m_b^2}\Bigg]. \tag{75}$$

Here $\psi(z) = (d/dz)\ln\Gamma(z)$ and $\Gamma(a,z)$ are digamma and incomplete gamma functions, respectively.

Then, the NLO corrections to LCSRs arising from the quark and gluonic components of the eta mesons are given by the expression

$$\frac{m_b}{m_{B_s}^2 m_{B_s^*}} e^{\frac{m_{B_s}^2 + m_{B_s^*}^2}{2M^2}} (Q_{\eta(\eta')}^{(s)} + \tilde{Q}_{\eta(\eta')}), \tag{76}$$

which should be added to Eq. (60).

It is interesting to note that strong couplings given by Eqs. (60) and (76) may be presented in the form

$$g_{B_s^* B_s \eta} \simeq -\sin\varphi G_{B_s^* B_s \eta}^{(s)},$$
$$g_{B_s^* B_s \eta'} \simeq \cos\varphi G_{B_s^* B_s \eta'}^{(s)}. \tag{77}$$

In fact, excluding some terms, the couplings with the high accuracy follow the mixing pattern discussed above that can be demonstrated explicitly.

IV. NUMERICAL RESULTS AND CONCLUSIONS

The LCSR expressions for $g_{B_s^* B_s \eta}$ and $g_{B_s^* B_s \eta'}$ in Eqs. (60) and (76) contain numerous parameters that should be fixed in accordance with the usual procedures. But apart from that, in numerical calculations there is also a necessity to utilize equalities to connect η and η' mesons' DAs and decay constants obtained using different bases. Indeed, as we have emphasized above, in order to solve renormalization group equations, it is convenient to use the singlet-octet basis. This basis was used in Ref. [31] to describe the evolution of the flavor-octet and flavor-singlet DAs with NLO accuracy. One should note that the gluon DA in Eq. (12) is normalized in terms of the decay constant $f_M^{(1)}$. From another side, the QF basis is more suitable to analyze the $\eta - \eta'$ mixing phenomena and solve equations of motion, which determine parameters in twist-4 DAs. The values of the decay constants in Eq. (20) were deduced within the QF mixing scheme, as well. The general expression for such transformations can be found in Eq. (19). Here we provide the formula for eta mesons' decay constants in the SO basis:

$$\begin{pmatrix} f_\eta^{(8)} & f_\eta^{(1)} \\ f_{\eta'}^{(8)} & f_{\eta'}^{(1)} \end{pmatrix} = \begin{pmatrix} \cos\theta_8 & -\sin\theta_1 \\ \sin\theta_8 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} f_8 & 0 \\ 0 & f_1 \end{pmatrix},$$

with the numerical values of the parameters

$$f_1 = (1.17 \pm 0.03) f_\pi, \qquad f_8 = (1.26 \pm 0.04) f_\pi,$$
$$\theta_1 = -(9.2^\circ \pm 1.7^\circ), \qquad \theta_8 = -(21.2^\circ \pm 1.6^\circ).$$

The B_s and B_s^* mesons' decay constants and masses enter into Eqs. (60) and (76) as input parameters. Their values are collected below (in MeV):

$$m_\eta = 547.86 \pm 0.02, \qquad m_{\eta'} = 957.78 \pm 0.06,$$
$$m_{B_s} = 5366.77 \pm 0.4, \qquad m_{B_s^*} = 5415.4 \pm 1.5.$$

The decay constants f_{B_s} and $f_{B_s^*}$ were calculated from the two-point QCD sum rules in Ref. [45] (in MeV):

$$f_{B_s} = 231 \pm 16, \qquad f_{B_s^*} = 213 \pm 18. \tag{78}$$

We employ the masses of the quarks in the $\overline{\mathrm{MS}}$ scheme (in GeV):

$$m_b(m_b) = 4.18 \pm 0.03, \qquad m_c(m_c) = 1.275 \pm 0.025. \tag{79}$$

Their scale dependencies are taken into account in accordance with the renormalization group evolution

$$m_q(\mu) = m_q(\mu_0) \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{\gamma_q},$$

with $\gamma_b = 12/23$ and $\gamma_c = 12/25$. The strange quark mass is $m_s = 0.137$ GeV. The renormalization scale is set equal to

$$\mu_b = \sqrt{m_{B_s}^2 - m_b^2} \simeq 3.4 \text{ GeV}. \tag{80}$$

The parameters and quantities are evolved to this scale, employing the two-loop QCD running coupling $\alpha_s(\mu)$ with $\Lambda^{(4)} = 326$ MeV. The same QCD two-loop coupling is used throughout this work, for example, to compute NLO corrections. The evolution of the leading twist DAs is calculated with NLO accuracy by taking into account quark-gluon mixing [31]. Calculations require us to fix the threshold parameter s_0 and a region within which it may be varied. For s_0 we employ

$$s_0^{B_s} \equiv s_0^{B_s^*} \simeq 36 \pm 2.5 \text{ GeV}^2.$$

Additionally, the eta mesons' DAs contain the Gegenbauer moments $a_n^{(1,8)}(\mu_0)$ and $a_n^{(g)}(\mu_0)$. In Ref. [31] they were extracted from the analysis of the eta mesons' electromagnetic transition form factors. In the present work, for $a_n^{(1,8)}$ and $a_2^{(g)}$ we utilize values that are compatible with ones from this work and accept the following models for DAs:

$$\text{I. } a_2^{(1,8)} = a_4^{(1,8)} = 0.1, \qquad a_2^{(g)} = -0.2,$$
$$\text{II. } a_2^{(1,8)} = a_4^{(1,8)} = 0.2, \qquad a_2^{(g)} = -0.2,$$
$$\text{III. } a_2^{(1,8)} = 0.2, \qquad a_4^{(1,8)} = 0, \qquad a_2^{(g)} = -0.2. \tag{81}$$

Results of the computations of the "scaled" couplings $f_{B_s} f_{B_s^*} g_{B_s^* B_s \eta'}$ and $f_{B_s} f_{B_s^*} |g_{B_s^* B_s \eta}|$ are depicted in Fig. 3. Calculations have been carried out employing model I. From analysis, we find the range of values of the Borel parameter 8 GeV$^2 < M^2 <$ 12 GeV^2, where the effects of the higher resonances and continuum states is less than 30% of the leading-order twist-2 contribution, and terms $\sim M^{-2}$ form only $\sim$5% of the sum rule. Additionally, in this interval the dependence of the couplings on M^2 is stable, and one may expect that the sum rule gives reliable predictions.

The sum rules receive contributions from the different terms, as shown in Fig. 4. The main component is the leading-order twist-2 term: it forms approximately 60% of the strong couplings. The effect of the NLO quark correction is also essential: in the explored range of the Borel parameter, it equals $\simeq$12.5% of the coupling $f_{B_s} f_{B_s^*} g_{B_s^* B_s \eta'}$. The same estimation is valid for $f_{B_s} f_{B_s^*} g_{B_s^* B_s \eta}$, as well. The correction originating from the gluon content of the meson is very small. In fact, it is only $\simeq -0.5\%$ of $f_{B_s} f_{B_s^*} g_{B_s^* B_s \eta'}$.

The higher-twist terms play an essential role in forming the couplings. Indeed, $\sim$28% of their values within the considered range of M^2 are due to HT corrections. The

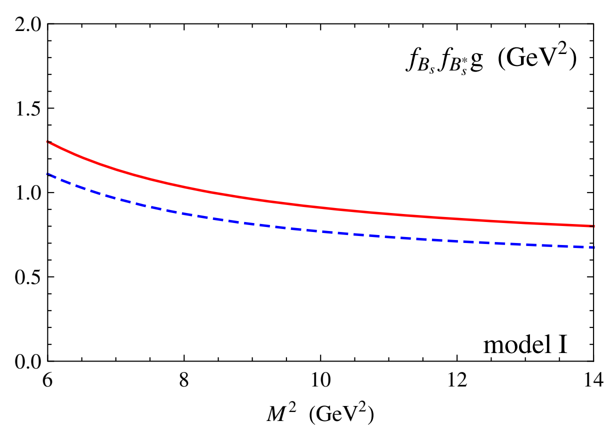

FIG. 3 (color online). The strong couplings as functions of the Borel parameter M^2. The solid (red) line describes $f_{B_s} f_{B_s^*} g_{B_s^* B_s \eta'}$, whereas the dashed (blue) curve corresponds to $f_{B_s} f_{B_s^*} |g_{B_s^* B_s \eta}|$. In computations the model I is used. The parameter $s_0^{B_s}$ is set equal to 36 GeV^2.

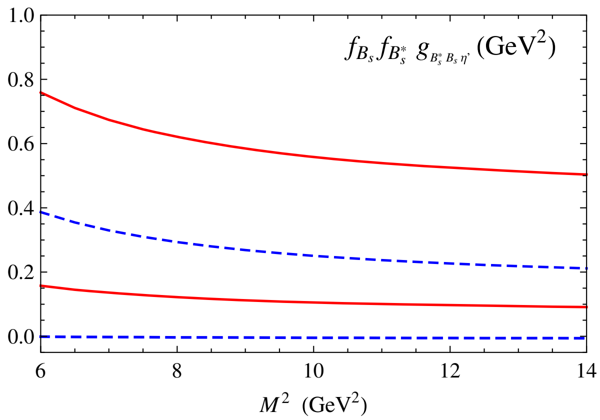

FIG. 4 (color online). Contributions to the coupling $f_{B_s} f_{B_s^*} g_{B_s^* B_s \eta'}$ originating from the leading, the higher-twist, and the NLO terms. The upper solid (red) line is the contribution of the LO twist-2 term, the upper dashed line (blue) shows the contribution of the higher-twist terms, the lower solid (red) curve is the NLO effect coming from the meson's quark component, and the lower dashed (blue) line is the gluonic contribution to the coupling. The parameters are the same as in Fig. 3.

main part of the HT corrections are determined by the two-particle twist-3 DAs $\phi_{3\eta'}^{(s)p}(u)$ and $\phi_{3\eta'}^{(s)\sigma}(u)$: they give ~33%, whereas the corrections of remaining HT terms are small, −5%.

The extracted couplings, in general, depend on the distribution amplitudes utilized in calculations. We have computed the couplings using the different model DAs and given the results in Fig. 5. Some of the DAs (models I and II) lead to almost identical predictions, such that corresponding lines become undistinguishable. Therefore, in Fig. 5 we show only the line corresponding to model I. At the same time, the results for couplings due to another pair of DAs (models I and III) differ from each other considerably.

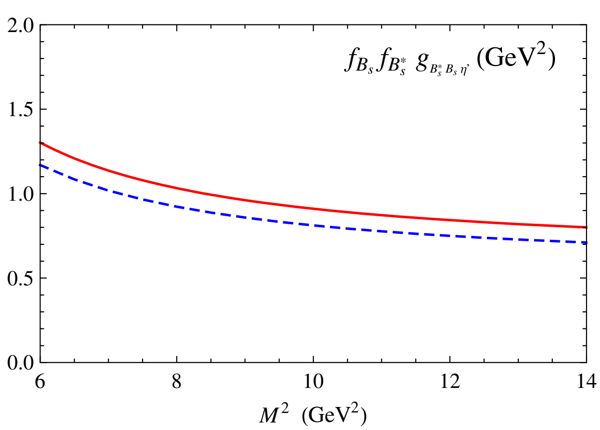

FIG. 5 (color online). The coupling $f_{B_s} f_{B_s^*} g_{B_s^* B_s \eta'}$ computed using the different model DAs. The solid (red) line shows model I, and the dashed (blue) line shows model III.

The predictions in the present work are made employing model I. By varying the parameters within the allowed ranges, we estimate the uncertainties of computations. The important sources of uncertainties are M^2 and $s_0^{B_s}$, as well as the decay constants f_{B_s} and $f_{B_s^*}$, calculated within the two-point QCD sum rules. Having changed M^2 and $s_0^{B_s}$ within 8 GeV^2 $< M^2 <$ 12 GeV^2, and 33.5 GeV^2 $< s_0^{B_s} <$ 38.5 GeV^2, respectively, and having taken into account uncertainties arising from the meson decay constants, we get

$$f_{B_s} f_{B_s^*} |g_{B_s^* B_s \eta}| = 0.837 \pm 0.08\ \mathrm{GeV}^2, \quad f_{B_s} f_{B_s^*} g_{B_s^* B_s \eta'} = 0.994 \pm 0.12\ \mathrm{GeV}^2. \tag{82}$$

Dividing the product of the couplings by the decay constants gives for the couplings the following predictions:

$$|g_{B_s^* B_s \eta}| = 17.08 \pm 1.63, \qquad g_{B_s B_s^* \eta'} = 20.2 \pm 2.44. \tag{83}$$

We proceed in our studies and extract the strong couplings $g_{D_s^* D_s \eta}$ and $g_{D_s^* D_s \eta'}$ (see Fig. 6). To this end, in all expressions we have to replace $b \to c$. The masses and decay constants in units of MeV are

$$m_{D_s} = 1969 \pm 1.4, \qquad m_{D_s^*} = 2112.1 \pm 0.4, \quad f_{D_s} = 240 \pm 10, \qquad f_{D_s^*} = 308 \pm 21. \tag{84}$$

All parameters should be adjusted to the new problem. This leads to the replacements

$$\mu_c = \sqrt{m_{D_s}^2 - m_c^2} \simeq 1.68\ \mathrm{GeV} \tag{85}$$

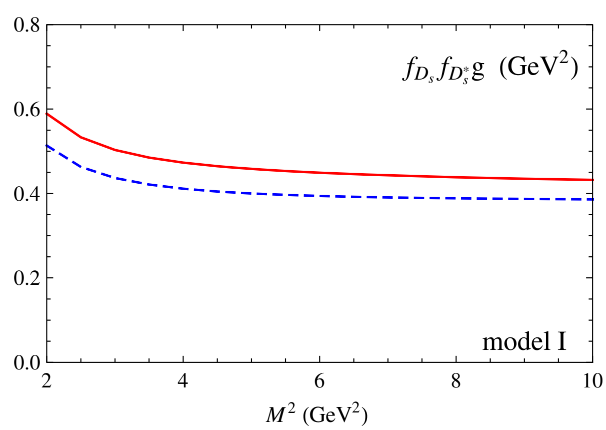

FIG. 6 (color online). The couplings as functions of the Borel parameter M^2. The solid (red) line corresponds to $f_{D_s} f_{D_s^*} g_{D_s^* D_s \eta'}$, and the dashed (blue) curve is the coupling $f_{D_s} f_{D_s^*} |g_{D_s^* D_s \eta}|$. In computations, the model I is used. The parameter $s_0^{D_s}$ is set equal to 7 GeV^2.

and $s_0^{D_s} = 7 \pm 1\ \mathrm{GeV}^2$. It has been found that the range of the Borel parameter $3\ \mathrm{GeV}^2 < M^2 < 5\ \mathrm{GeV}^2$ is suitable for evaluating the sum rules. From the relevant sum rules for the product of the decay constants and coupling we extract the following values:

$$f_{D_s} f_{D_s^*} |g_{D_s^* D_s \eta}| = 0.411 \pm 0.04\ \mathrm{GeV}^2, \\ f_{D_s} f_{D_s^*} g_{D_s^* D_s \eta'} = 0.473 \pm 0.042\ \mathrm{GeV}^2. \qquad (86)$$

Then, for the couplings we get

$$|g_{D_s^* D_s \eta}| = 4.51 \pm 0.44, \qquad g_{D_s^* D_s \eta'} = 5.19 \pm 0.46. \qquad (87)$$

Our results have been obtained within the quark-hadron duality ansatz of Ref. [38], where $g_{D^* D\pi}$ and $g_{B^* B\pi}$ were evaluated. But there is a discrepancy between the predictions for $g_{D^* D\pi}$ and the data of the CLEO Collaboration [48]. One of the main input parameters in these calculations is a value of the leading twist DA at $u_0 = 1/2$. In Ref. [38] it was chosen as $\phi_\pi(1/2) \simeq 1.2$, whereas recent analysis of the pion electromagnetic transition form factor performed in Refs. [49,50] predicts LT pion DAs enhanced at the middle point: these model DAs at $u_0 = 1/2$ are very close to the asymptotic DA with $\phi_{\mathrm{asy}}(1/2) = 1.5$. The usage of updated twist-3 DAs may also lead to sizeable corrections, because twist-3 terms contribute to $g_{D^* D\pi}$ at the level of 50%–60%, and are as important as the twist-2 term. All these questions necessitate new, updated investigation of the couplings $g_{D^* D\pi}$ and $g_{B^* B\pi}$ in the context of the LCSR method. The real accuracy of this method is not completely clear at present. On the one hand, it leads to results with 30%–50% deviation from experimental data as in the $g_{D^* D\pi}$ case; on the other hand, it gives rather precise predictions for radiative decays of mesons. Indeed, the LCSR prediction for $g_{D^* D\gamma}$ [51,52] correctly describes experimental data: the value of the quark condensate's magnetic susceptibility that enters into this sum rule as a nonperturbative parameter is known from both QCD sum rules and lattice computations [53], and the two agree with each other. As QCD lattice simulations of $g_{D^* D\pi}$ (see Ref. [54]) agree with the CLEO data, it will be instructive to compare our predictions for the strong couplings $g_{B_s^* B_s \eta^{(\prime)}}$ and $g_{D_s^* D_s \eta^{(\prime)}}$ with relevant lattice results, when they become available.

The couplings $g_{B_s^* B_s \eta^{(\prime)}}$ were calculated in Ref. [12] by applying the three-point sum rule method, as well. Differences in adopted definitions for the couplings, chosen structures, and explored kinematical regimes to extract their values make direct comparison of relevant findings rather problematic: we note only a sizeable numerical discrepancy between our predictions and the results of Ref. [12]. We emphasize also the advantage of the LCSR method compared to the three-point sum rule approach in calculations of the strong couplings and/or form factors. Indeed, in the three-point sum rules, the higher orders in the operator product expansion (OPE) are enhanced by powers of the heavy quark mass, and for sufficiently large masses the OPE breaks down. The LCSR method does not suffer from such problems: It is consistent with the heavy-quark limit and provides more elaborate tools for investigation than alternative approaches.

In the present work we have investigated the strong $D_s^* D_s \eta^{(\prime)}$ and $B_s^* B_s \eta^{(\prime)}$ vertices and calculated the relevant couplings using the method of QCD sum rules on the light cone. We have included in our analysis effects of the eta mesons' gluon components. The derived expressions have been explored, and numerical values of the strong couplings $g_{D_s^* D_s \eta^{(\prime)}}$ and $g_{B_s^* B_s \eta^{(\prime)}}$ have been evaluated. Studies have demonstrated that the direct contribution to the strong couplings arising from the two-gluon components of the η and η' is small. But owing to mixing, the gluon components affect the quark DAs, which cannot be ignored.

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APPENDIX

This appendix is devoted to the calculation of f_{3s} and $\delta_M^{2(s)}$, which enter as parameters into higher-twist DAs of the η and η' mesons. To this end, in the two-point sum rules written down below, we consider f_s and h_s, as well as the mixing angle φ, as input parameters; then only f_{3s} and $\delta_M^{2(s)}$ remain unknown.

f_{3s} and $\delta_M^{2(s)}$ can be defined in terms of matrix elements of some local operators. Indeed, the parameter f_{3s} can be defined through the matrix element of the following twist-3 operator:

$$\langle 0|\bar{s}\sigma_{z\nu}\gamma_5 g G_{z\nu} s|M(p)\rangle = 2if_{3M}^{(s)}(pz)^2.$$

In order to extract its value, we use the correlation function of nonlocal light-ray operators, which enter the definition of the three-particle distribution amplitude with the corresponding local operator. Such a so-called "nondiagonal" correlation function is given by the following expression [36]:

$$\Pi_{ND}^s = i\int d^4y e^{-ipy}\langle 0|\mathcal{T}\{[\bar{s}(z)\sigma_{\mu z}\gamma_5 g G_{\mu z}(vz)s(0)] \times [\bar{s}(y)\gamma_5 s(y)]\}|0\rangle \equiv (pz)^2\int D\underline{\alpha} e^{-ipz(\alpha_2+v\alpha_3)}\pi_{ND}^s(\underline{\alpha}). \quad (A1)$$

The sum rule for the coupling f_{3s} is derived by expanding the correlation function in powers of pz:

$$\Pi_{ND}^s = (pz)^4\{\Pi_{ND}^{(0)s} + i(pz)[\Pi_{ND}^{(1A)s} + (2v-1)\Pi_{ND}^{(1B)s}] + \cdots\}. \quad (A2)$$

The hadronic content of the function Π has been modeled employing "$\eta + \eta' +$ continuum" approximation. Then we get the following sum rule:

$$f_{3\eta}^{(s)}\frac{h_\eta^{(s)}}{m_s}e^{-\frac{m_\eta^2}{M^2}} + f_{3\eta'}^{(s)}\frac{h_{\eta'}^{(s)}}{m_s}e^{-\frac{m_{\eta'}^2}{M^2}} = \mathcal{B}_{M^2}[\Pi_{ND}^{(0)s}].$$

The left-hand side of this expression can be modified using information on mixing of the decay constants:

$$\frac{f_{3s}h_s}{m_s}(\sin^2\varphi e^{-\frac{m_\eta^2}{M^2}} + \cos^2\varphi e^{-\frac{m_{\eta'}^2}{M^2}}) = \mathcal{B}_{M^2}[\Pi_{ND}^{(0)s}]. \quad (A3)$$

Now, having applied the explicit expression for $\mathcal{B}_{M^2}[\Pi_{ND}^{(0)s}]$, we determine f_{3s} using the sum rule:

$$\frac{f_{3s}h_s}{m_s}(\sin^2\varphi e^{-\frac{m_\eta^2}{M^2}} + \cos^2\varphi e^{-\frac{m_{\eta'}^2}{M^2}}) = \frac{\alpha_s}{73\pi^3}\int_0^{s_0} dsse^{-\frac{s}{M^2}} + \frac{1}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle - \frac{4\alpha_s}{9\pi}m_s\langle\bar{s}s\rangle\left[\frac{19}{6} + \gamma_E - \ln\frac{M^2}{\mu^2} + \int_{s_0}^\infty\frac{ds}{s}e^{-\frac{s}{M^2}}\right] + \frac{80}{27}\frac{\alpha_s\pi}{M^2}\langle\bar{s}s\rangle^2 + \frac{1}{3M^2}m_s\langle\bar{s}\sigma gGs\rangle. \quad (A4)$$

Numerical calculations have been performed at the scale $\mu_0 = 1$ GeV. To evaluate a continuum contribution we set $s_0 = 1.5\ \text{GeV}^2$ and vary it within the limits $1.3\text{GeV}^2 < s_0 < 1.7\text{GeV}^2$ to estimate errors. The Borel parameter M^2 is changed in the interval $0.8\ \text{GeV}^2 < M^2 < 1.8\ \text{GeV}^2$. The parameters have been extracted at $M^2 = 1.3\ \text{GeV}^2$. For $f_{3s}(\mu_0)$ we have found

$$f_{3s} \simeq 0.0041\ \text{GeV}^2. \quad (A5)$$

The varying of s_0 in the allowed limits results in error ± 0.00005, which may be neglected.

We introduce the parameter $\delta_M^{2(s)}$ through the local matrix element

$$\langle 0|\bar{s}\gamma^\rho ig\tilde{G}_{\rho\mu}s|M(p)\rangle = p_\mu f_M^{(s)}\delta_M^{2(s)}, \quad (A6)$$

considering it as the universal one; i.e., we suggest that it does not depend on the particles η and η'. In the local matrix element, information on the mixing is contained in the decay constants $f_M^{(s)}$. Then we can write

$$f_s^2\delta_M^{4(s)}[\sin^2\varphi e^{-\frac{m_\eta^2}{M^2}} + \cos^2\varphi e^{-\frac{m_{\eta'}^2}{M^2}}] = \mathcal{B}_{M^2}[\Pi_0^{A(s)}],$$

where $\mathcal{B}_{M^2}[\Pi_0^{A(s)}]$ is given by the expression [36]

$$\mathcal{B}_{M^2}[\Pi_0^{A(s)}] = \frac{\alpha_s}{160\pi^3}\int_0^{s_0} dss^2 e^{-\frac{s}{M^2}} + \frac{1}{72}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \times \int_0^{s_0} dse^{-\frac{s}{M^2}} - \frac{\alpha_s}{9\pi}m_s\langle\bar{s}s\rangle\int_0^{s_0} dse^{-\frac{s}{M^2}} + \frac{8\pi\alpha_s}{9}\langle\bar{s}s\rangle^2 - \frac{13\alpha_s}{54\pi}m_s\langle\bar{s}\sigma gGs\rangle + \frac{59\pi\alpha_s}{81}\frac{m_0^2}{M^2}\langle\bar{s}s\rangle^2 + \frac{\pi}{9M^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle m_s\langle\bar{s}s\rangle - \frac{2\alpha_s}{\pi}m_s\langle\bar{s}\sigma gGs\rangle \times \left\{\gamma_E - \ln\frac{M^2}{\mu^2} + \int_{s_0}^\infty\frac{ds}{s}e^{-\frac{s}{M^2}}\right\}. \quad (A7)$$

Computations of $\delta_M^{2(s)}$ with the same input parameters as in the previous case lead to the following prediction:

$$\delta_M^{2(s)}(\mu_0) \simeq 0.1896 \pm 0.001\ \text{GeV}^2. \quad (A8)$$

As is seen, f_{3s} and $\delta_M^{2(s)}$ numerically are very close to the pion's parameters $f_{3\pi}$ and δ_π^2, respectively.

The values of the quark and quark-gluon condensates at μ_0 utilized in numerical calculations are listed below:

$$\langle\bar{q}q\rangle = (-0.24\pm 0.01)^3\text{GeV}^3, \quad \langle\bar{q}\sigma gGq\rangle = m_0^2\langle\bar{q}q\rangle,$$
$$m_0^2 = (0.8\pm 0.1)\text{GeV}^2, \quad \langle\bar{s}s\rangle = [1-(0.2\pm 0.2]\langle\bar{q}q\rangle,$$
$$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = (0.012\pm 0.006)\text{GeV}^4,$$
$$\langle\bar{s}\sigma gGs\rangle = [1-(0.2\pm 0.2)]\langle\bar{q}\sigma gGq\rangle. \quad (A9)$$

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