R-parity conserving supersymmetric extension of the Zee model

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We extend the Zee model, where tiny neutrino masses are generated at the one-loop level, to a supersymmetric model with R-parity conservation. It is found that the neutrino mass matrix can be consistent with the neutrino oscillation data thanks to the nonholomorphic Yukawa interaction generated via one-loop diagrams of sleptons. We find a parameter set of the model, where in addition to the neutrino oscillation data, experimental constraints from the lepton flavor violating decays of charged leptons and current LHC data are also satisfied. In the parameter set, an additional CP-even neutral Higgs boson other than the standard-model-like one, a CP-odd neutral Higgs boson, and two charged scalar bosons are light enough to be produced at the LHC and future lepton colliders. If the lightest charged scalar bosons are mainly composed of the $\mathrm{SU}(2)_L$-singlet scalar boson in the model, they would decay into $e\nu$ and $\mu\nu$ with 50% of a branching ratio for each. In such a case, the relation among the masses of the charged scalar bosons and the CP-odd Higgs in the minimal supersymmetric standard model approximately holds with a radiative correction. Our model can be tested by measuring the specific decay patterns of charged scalar bosons and the discriminative mass spectrum of additional scalar bosons.

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I. INTRODUCTION

Neutrino oscillation data [1–9] have indicated the existence of tiny masses of neutrinos, which are absent in the standard model (SM) of particle physics. If the tiny neutrino masses are generated by a new physics at a very high energy scale (e.g., the seesaw mechanism [10]), such a new physics (heavy new particles) is not directly accessible by experiments. In contrast, in scenarios based on radiative generation of tiny neutrino masses [11–16], the smallness of neutrino masses is deduced by the quantum effect without introducing very heavy new particles. Therefore, such scenarios can consequently be tested by current and future collider experiments.

The model proposed by Zee [11] is the first model of radiative generation of neutrino masses, where an $\mathrm{SU}(2)_L$-singlet scalar field with a hypercharge $Y = 1$ and the second $\mathrm{SU}(2)_L$-doublet Higgs field with $Y = 1/2$ are introduced to construct the one-loop diagram for the neutrino mass. Studies on the phenomenology in the Zee model can be found in Refs. [17–25]. In the minimal version of the model (the so-called Zee-Wolfenstein model [17]), lepton flavor violating (LFV) Yukawa couplings with the second Higgs doublet are forbidden at the tree level. However, such a model has already been excluded by current neutrino oscillation data (see, e.g., Ref. [24]). In order to reproduce the neutrino data, lepton flavor violating interactions are necessary [22,24,25].

In the Zee model, there are several problems to be solved. Although the Zee model with LFV couplings is phenomenologically acceptable, they should be well controlled by some principle in order to suppress the dangerous flavor changing neutral current (FCNC) processes.[1,2] The model is also confronted by the quadratic divergence problem like the SMİn addition, there is no dark matter (DM) candidate in the Zee model. If we consider a supersymmetric (SUSY) extension of the Zee model, we may be able to solve these problems simultaneously. The quadratic divergence is automatically canceled by the loop contribution of SUSY partner particles. If the R-parity is conserved, the lightest SUSY particle becomes stable and it can be DM. Moreover, the LFV Yukawa couplings can naturally be induced.

The previous study of the SUSY extension of the Zee model is found in Ref. [20], where the R-parity violation is introduced to the minimal SUSY SM (MSSM). In this model, right-handed sleptons play the role of the charged singlet scalar in the Zee model. Since the sleptons carry lepton flavors in contrast with the singlet scalar in the Zee model, the flavor structure of the generated neutrino mass matrix becomes different from the one in the Zee-Wolfenstein model. SUSY models for the other scenarios of radiative neutrino masses can be found in Refs. [27–30].

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[1]It would be also required to introduce the third $\mathrm{SU}(2)_L$-doublet scalar field which is only for the quark Yukawa interactions without FCNC in the quark sector.

[2]An idea to control such FCNC well so that stringent constraints on $\mu \to \bar{e}ee$ and $\mu \to e\gamma$ are automatically satisfied is introducing the A_4 symmetry to the Zee model [26].

In this paper, we propose a SUSY extension of the Zee model with the R-parity conservation (the SUSY Zee model). The stability of the dark matter candidate is guaranteed. This is the simplest alternative example of the MSSM with the right-handed neutrino superfields (the SUSY seesaw model). In the SUSY Zee model, the MSSM is extended by introducing a pair of $SU(2)_L$-singlet superfields with hypercharge $Y = 1$ and -1, which also carry lepton numbers. The Higgs sector of the MSSM is the type-II two Higgs doublet model [31] at the tree level. The extra Higgs doublet can play the role of the second Higgs doublet of the Zee model. In SUSY models, nonholomorphic Yukawa interactions are generally induced by the one-loop effect of SUSY particles [32–37]. This mechanism may be utilized for generating the flavor violating interaction, which is required for the Zee model to satisfy the neutrino data. The structure of the LFV Yukawa matrix is determined by the flavor structure of the soft SUSY breaking slepton mass matrices. Such radiatively induced coupling constants are expected to be much smaller than the other Yukawa coupling constants. In this model, we study the neutrino mass matrix, and we find a benchmark point for model parameters, which satisfies the required structure of the neutrino mass matrix and the constraints from experimental searches for the LFV decays of charged leptons. On the benchmark point, phenomenological consequences of our model are discussed, and testability of our model at the LHC and future lepton colliders such as the International Linear Collider (ILC) is mentioned.

This paper is organized as follows. In Sec. II, our model is defined. Section III is devoted to showing how the neutrino masses are generated at the one-loop level. In the section, a benchmark set of model parameters which satisfy neutrino oscillation data are given. Phenomenology in the SUSY Zee model with the benchmark set is discussed in Sec. IV. Conclusions are given in Sec. V.

TABLE I. Superfields of the SUSY Zee model. The baryon number is zero for all particles in this table.

	Spin 0	Spin 1/2	$SU(2)_L$	$U(1)_Y$	Lepton #
$\hat{L}_\ell$	$\tilde{L}_\ell = \begin{pmatrix} \tilde{\nu}_{\ell L} \\ \tilde{\ell}_L \end{pmatrix}$	$L_\ell = \begin{pmatrix} \nu_{\ell L} \\ \ell_L \end{pmatrix}$	**2**	$-\frac{1}{2}$	1
$\hat{\ell}^c$	$\tilde{\ell}_R^*$	$(\ell_R)^c$	**1**	1	−1
$\hat{\Phi}_u$	$\Phi_u = \begin{pmatrix} \phi_u^+ \\ \phi_u^0 \end{pmatrix}$	$\tilde{\Phi}_u = \begin{pmatrix} \tilde{\phi}_u^+ \\ \tilde{\phi}_u^0 \end{pmatrix}$	**2**	$\frac{1}{2}$	0
$\hat{\Phi}_d$	$\Phi_d = \begin{pmatrix} \phi_d^0 \\ \phi_d^- \end{pmatrix}$	$\tilde{\Phi}_d = \begin{pmatrix} \tilde{\phi}_d^0 \\ \tilde{\phi}_d^- \end{pmatrix}$	**2**	$-\frac{1}{2}$	0
$\hat{\omega}_1^+$	ω_1^+	$\tilde{\omega}_1^+$	**1**	1	−2
$\hat{\omega}_2^-$	ω_2^-	$\tilde{\omega}_2^-$	**1**	−1	2

II. THE MODEL

Superfields of the SUSY Zee model are partially listed in Table I. The transformation property under the R-parity is given by $(-1)^{3(B-L)+2s}$, where B (L) is the baryon (lepton) number and s denotes the spin. Component fields with a tilde are odd under the R-parity. The relevant part of the superpotential is constructed as

$$\mathcal{W} = y_\ell \hat{\ell}^c \hat{L}_\ell^T(-i\sigma_2)\hat{\Phi}_d + (Y_A^{(0)})_{\ell\ell'}\hat{L}_\ell^T(i\sigma_2)\hat{L}_{\ell'}\hat{\omega}_1^+ + \mu_\Phi \hat{\Phi}_d^T(i\sigma_2)\hat{\Phi}_u + \mu_\omega \hat{\omega}_1^+\hat{\omega}_2^-, \tag{1}$$

where $(Y_A^{(0)})^T = -Y_A^{(0)}$, and σ_i $(i = 1\text{-}3)$ are the Pauli matrices. The relevant part of the soft-SUSY breaking terms is given by

$$\begin{aligned}\mathcal{L}_{\rm soft} = &-m_{\Phi_u}^2\Phi_u^\dagger\Phi_u - m_{\Phi_d}^2\Phi_d^\dagger\Phi_d - \{B_\Phi\mu_\Phi\Phi_d^T(i\sigma_2)\Phi_u + \text{H.c.}\} - m_{\omega_1}^2\omega_1^+\omega_1^- - m_{\omega_2}^2\omega_2^+\omega_2^- - \{B_\omega\mu_\omega\omega_1^+\omega_2^- + \text{H.c.}\}\\ &- (m_{\tilde{L}}^2)_{\ell\ell'}\tilde{L}_\ell^\dagger\tilde{L}_{\ell'} - (m_{\tilde{\ell}}^2)_{\ell\ell'}\tilde{\ell}^*\tilde{\ell}' - \{y_\ell(A_E)_{\ell\ell'}\tilde{\ell}^*\tilde{L}_{\ell'}^T(i\sigma_2)\Phi_d + \text{H.c.}\} - \{(A_\omega)_{\ell\ell'}\tilde{L}_\ell(i\sigma_2)\tilde{L}_{\ell'}\omega_1^+ + \text{H.c.}\}\\ &- \{C_1\Phi_u^\dagger\Phi_d\omega_1^+ + C_2\Phi_d^\dagger\Phi_u\omega_2^- + \text{H.c.}\},\end{aligned} \tag{2}$$

where $A_\omega^T = -A_\omega$. Notice that the term of C_1 gives the important interaction required in the non-SUSY Zee model as the source of the lepton number violation by two units.[3]

In order to generate the neutrino mass matrix at the one-loop level, the following Yukawa interactions are used:

$$\begin{aligned}\mathcal{L}_{\rm Yukawa} = &\, y_\ell\overline{\ell_R}L_{\ell'}^T(-i\sigma_2)\Phi_d + (Y_2)_{\ell\ell'}\overline{\ell_R}L_{\ell'}^T\Phi_u^*\\ &+ (Y_A^{(0)})_{\ell\ell'}L_\ell^T(i\sigma_2)L_{\ell'}\omega_1^+\\ &+ (Y_{2A})_{\ell\ell'}L_\ell^T(i\sigma_2)L_{\ell'}\omega_2^+,\end{aligned} \tag{3}$$

where the first and the third terms are obtained from the superpotential $\mathcal{W}$ in Eq. (1). The second and the fourth terms are generated at the one-loop level [32–37]. In Eq. (3), Y_{A2} can be ignored, when ω_2^+ is very heavy. We here consider such a case. The Yukawa matrix Y_2 is generated through the slepton mixing as follows [32–37]:

$$(Y_2)_{\ell\ell'} = y_\ell\{(\epsilon_1)_\ell\delta_{\ell\ell'} + (\epsilon_2)_{\ell\ell'}\}, \tag{4}$$

[3] Such three-scalar terms with both fields and conjugate fields are the so-called C-terms [38], which are usually ignored because most SUSY breaking scenarios do not generate the C-terms. However, the C-terms can be generated in some SUSY breaking scenarios, e.g., in an intersecting D-brane model with the flux compactification [39]. Some detailed discussion about the SUSY breaking effects for the radiative neutrino mass can be found, e.g., in Ref. [40].

$$(\epsilon_1)_\ell = -\frac{\alpha_1}{8\pi}\mu_\Phi M_1[2I_3(M_1^2, m^2_{\tilde{\ell}_L}, m^2_{\tilde{\ell}_R}) + I_3(M_1^2, \mu_\Phi^2, m^2_{\tilde{\ell}_L}) - 2I_3(M_1^2, \mu_\Phi^2, m^2_{\tilde{\ell}_R})] + \frac{\alpha_2}{8\pi}\mu_\Phi M_2[I_3(M_2^2, \mu_\Phi^2, m^2_{\tilde{\ell}_L}) + 2I_3(M_2^2, \mu_\Phi^2, m^2_{\tilde{\nu}_{\ell L}})], \tag{5}$$

$$\begin{aligned}(\epsilon_2)_{\ell\ell'} = &-\frac{\alpha_1}{8\pi}\mu_\Phi M_1(\Delta m^2_{\tilde{L}})_{\ell\ell'}[2I_4(M_1^2, m^2_{\tilde{\ell}_L}, m^2_{\tilde{\ell}_R}, m^2_{\tilde{\ell}'_L}) + I_4(M_1^2, \mu_\Phi^2, m^2_{\tilde{\ell}_L}, m^2_{\tilde{\ell}'_L})] \\ &-\frac{\alpha_1}{8\pi}\mu_\Phi M_1(\Delta m^2_{\tilde{\ell}})_{\ell\ell'}[2I_4(M_1^2, m^2_{\tilde{\ell}_L}, m^2_{\tilde{\ell}_R}, m^2_{\tilde{\ell}'_R}) + 2I_4(M_1^2, \mu_\Phi^2, m^2_{\tilde{\ell}_R}, m^2_{\tilde{\ell}'_R})] \\ &+\frac{\alpha_2}{8\pi}\mu_\Phi M_2(\Delta m^2_{\tilde{L}})_{\ell\ell'}[I_4(M_2^2, \mu_\Phi^2, m^2_{\tilde{\ell}_L}, m^2_{\tilde{\ell}'_L}) + 2I_4(M_2^2, \mu_\Phi^2, m^2_{\tilde{\nu}_{\ell L}}, m^2_{\tilde{\nu}_{\ell' L}})],\end{aligned} \tag{6}$$

where M_1 (M_2) is the soft SUSY breaking mass of $\mathrm{U}(1)_Y$ [$\mathrm{SU}(2)_L$] gauginos. The matrices $\Delta m^2_{\tilde{L}}$ and $\Delta m^2_{\tilde{\ell}}$ denote off-diagonal parts of $(m^2_{\tilde{L}})_{\ell\ell'}$ and $(m^2_{\tilde{\ell}})_{\ell\ell'}$, respectively. Thus, $(\epsilon_2)_{\ell\ell} = 0$. Loop functions $I_3(x, y, z)$ and $I_4(x, y, z, w)$ are defined as

$$I_3(x, y, z) \equiv -\frac{xy\ln(x/y) + yz\ln(y/z) + zx\ln(z/x)}{(x-y)(y-z)(z-x)}, \tag{7}$$

$$I_4(x, y, z, w) \equiv -\frac{x\ln x}{(y-x)(z-x)(w-x)} - \frac{y\ln y}{(x-y)(z-y)(w-y)} - \frac{z\ln z}{(x-z)(y-z)(w-z)} - \frac{w\ln w}{(x-w)(y-w)(z-w)}. \tag{8}$$

Although all terms in Eq. (6) [in Eq. (5)] are proportional to μ_Φ, the first and the third terms in Eq. (6) [the first term in Eq. (5)] do not contain Higgsinos in the loop. Therefore, sizable ϵ_2 (and ϵ_1 also) can be generated by these terms if we take much larger μ_Φ than the other mass scales (e.g., M_1, $m_{\tilde{L}}$). Then, ϵ_1 and ϵ_2 are almost independent of the value of M_2. Yukawa interactions in Eq. (3) can be rewritten as

$$\mathcal{L}_{\mathrm{Yukawa}} = \frac{\sqrt{2}m_\ell}{v}\bar{\ell}_R L_\ell^T \Phi_v^* + \frac{\sqrt{2}m_\ell}{v}X_{\ell\ell'}\bar{\ell}_R L_{\ell'}^T\Phi_0^* + (Y_A)_{\ell\ell'}L_\ell^T(i\sigma_2)L_{\ell'}\omega_1^+, \tag{9}$$

where $v^2 \equiv v_u^2 + v_d^2 = (246\ \mathrm{GeV})^2$, $v_u \equiv \sqrt{2}\langle\phi_u^0\rangle$, and $v_d \equiv \sqrt{2}\langle\phi_d^0\rangle$. The matrix Y_A is an arbitrary antisymmetric matrix. Two Higgs doublet fields Φ_0 and Φ_v are defined as

$$\begin{pmatrix}\Phi_0 \\ \Phi_v\end{pmatrix} \equiv \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta\end{pmatrix}\begin{pmatrix}\Phi_u \\ (-i\sigma_2)\Phi_d^*\end{pmatrix}, \tag{10}$$

where $s_\beta \equiv \sin\beta$ and $c_\beta \equiv \cos\beta$ for $\tan\beta = v_u/v_d$. Interactions in Eq. (9) are expressed in terms of mass eigenstates of charged leptons,[4] which are given by diagonalizing the mass matrix $c_\beta y_\ell + s_\beta Y_2$. By keeping ϵ_1 and ϵ_2 up to linear terms (keeping $\epsilon_1 \tan\beta$ for all order), the matrix X is given by [32]

$$X_{\ell\ell'} = -\tan\beta\delta_{\ell\ell'} + \frac{1+\tan^2\beta}{(1+\tan\beta(\epsilon_1)_\ell)^2}\{(\epsilon_1)_\ell\delta_{\ell\ell'} + (\epsilon_2)_{\ell\ell'}\}. \tag{11}$$

The off-diagonal elements of X provide FCNC, and they are important to obtain the appropriate structure of the neutrino mass matrix.

Since Nambu-Goldstone modes are contained in Φ_v, mass eigenstates of the three charged bosons are given by linear combinations of ω_1^+, ω_2^+, and a charged component ϕ_0^+ of Φ_0. The matrix of squared masses of charged bosons is given in a basis of $(\phi_0^+, \omega_1^+, \omega_2^+)$ by

[4]Charged leptons in Eq. (3) are not mass eigenstates although we used the same notation ℓ.

$$M^2_{H^+} = \begin{pmatrix} m_W^2 + m_A^2 & \frac{C_1 v}{\sqrt{2}} & \frac{C_2 v}{\sqrt{2}} \\ \frac{C_1 v}{\sqrt{2}} & (M^2_{H^+})_{22} & B_\omega \mu_\omega \\ \frac{C_2 v}{\sqrt{2}} & B_\omega \mu_\omega & (M^2_{H^+})_{33} \end{pmatrix}, \tag{12}$$

$$(M^2_{H^+})_{22} = -m_W^2 \tan^2\theta_W \cos(2\beta) + m^2_{\omega 1} + \mu^2_\omega, \tag{13}$$

$$(M^2_{H^+})_{33} = m_W^2 \tan^2\theta_W \cos(2\beta) + m^2_{\omega 2} + \mu^2_\omega, \tag{14}$$

where m_A is the mass of the CP-odd Higgs boson A^0. The matrix $M^2_{H^+}$ is diagonalized as $M^2_{H^+} = U_{H^+} \mathrm{diag}(m^2_{H_1^+}, m^2_{H_2^+}, m^2_{H_3^+}) U^\dagger_{H^+}$ with a unitary matrix U_{H^+}. We here assume that $(M^2_{H^+})_{33}$ is much larger than the other elements for simplicity, so that mixing effects via C_2 and B_ω can be ignored. The mass eigenvalues of charged scalar bosons are given by

$$m^2_{H_1^+} = \frac{1}{2}\Big\{ (M^2_{H^+})_{22} + m_W^2 + m_A^2 - \sqrt{((M^2_{H^+})_{22} - m_W^2 - m_A^2)^2 + 2C_1^2 v^2} \Big\}, \tag{15}$$

$$m^2_{H_2^+} = \frac{1}{2}\Big\{ (M^2_{H^+})_{22} + m_W^2 + m_A^2 + \sqrt{((M^2_{H^+})_{22} - m_W^2 - m_A^2)^2 + 2C_1^2 v^2} \Big\}, \tag{16}$$

$$m^2_{H_3^+} = (M^2_{H^+})_{33}. \tag{17}$$

The mixing matrix is given by

$$U_{H^+} = \begin{pmatrix} \cos\theta_+ & -\sin\theta_+ & 0 \\ \sin\theta_+ & \cos\theta_+ & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{18}$$

$$\sin^2 2\theta_+ = \frac{2C_1^2 v^2}{(m^2_{H_2^+} - m^2_{H_1^+})^2} = \frac{4(m^2_{H_2^+} - m_W^2 - m_A^2)(m_W^2 + m_A^2 - m^2_{H_1^+})}{(m^2_{H_2^+} - m^2_{H_1^+})^2}. \tag{19}$$

For $\theta_+ \simeq 0$, the charged scalar boson H_1^+ is the singletlike one ($H_1^+ \simeq \omega_1^+$) while H_2^+ is almost the same as the charged Higgs boson of the MSSM.

III. NEUTRINO MASS AND BENCHMARK SCENARIO

The neutrino mass matrix $(m_\nu)_{\ell\ell'}$ in the flavor basis, $(1/2)(m_\nu)_{\ell\ell'} \overline{(\nu_{\ell L})^c} \nu_{\ell' L} + \mathrm{H.c.}$, is generated by the diagram in Fig. 1 and its transpose diagram. Keeping the leading terms of the charged lepton masses, the neutrino mass matrix is expressed as

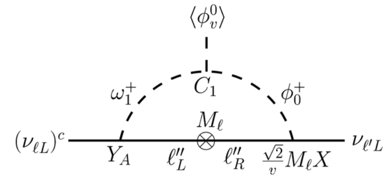

FIG. 1. The one-loop diagram for light Majorana neutrino masses in the SUSY Zee model.

$$m_\nu = \frac{\sqrt{2}}{v} C_{\mathrm{loop}} [Y_A M_\ell^2 X + (Y_A M_\ell^2 X)^T], \tag{20}$$

where $M_\ell \equiv \mathrm{diag}(m_e, m_\mu, m_\tau)$ and C_{loop} is a flavor-independent factor calculated by the loop integration. The explicit form of C_{loop} is given by

$$C_{\mathrm{loop}} = \frac{\sin 2\theta_+}{32\pi^2} \ln \frac{m^2_{H_1^+}}{m^2_{H_2^+}}. \tag{21}$$

The case with $X = 1$ corresponds to the Zee-Wolfenstein model. Since we assume rather heavy SUSY particles in order to satisfy constraints from LFV decays of charged leptons, we can ignore a contribution from the $\tilde{\omega}_1^+$-$\tilde{\phi}_0^+$-$\tilde{\ell}$ loop.[5]

A mass matrix $(m_\nu)_{\ell\ell'}$ for Majorana neutrinos in the flavor basis can be diagonalized by the Maki-Nakagawa-Sakata (MNS) matrix [41] U_{MNS} as

$$m_\nu = U^*_{\mathrm{MNS}} \mathrm{diag}(m_1 e^{i\alpha_{12}}, m_2, m_3 e^{i\alpha_{32}}) U^\dagger_{\mathrm{MNS}}, \tag{22}$$

where α_{12} and α_{32} are Majorana phases [42]. The MNS matrix can be parametrized as

$$U_{\mathrm{MNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{23}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The measurement of the ν_μ disappearance at the T2K experiment [6] shows

$$\sin^2\theta_{23} = 0.514^{+0.055}_{-0.056}, \quad \Delta m^2_{32} = (2.51 \pm 0.10) \times 10^{-3}\ \mathrm{eV}^2, \tag{24}$$

for the normal mass ordering ($m_1 < m_3$) and

[5]The interaction $\Phi_0^\dagger \Phi_v \omega_1^+$ in Fig. 1 is replaced with a non-holomorphic Yukawa interaction $\tilde{\Phi}_0^\dagger \Phi_v \tilde{\omega}_1^+$, which is generated via a one-loop diagram with Φ_0, ω_1^+, and the bino.

$$\sin^2\theta_{23} = 0.511^{+0.055}_{-0.055},$$
$$\Delta m^2_{23} = (2.48 \pm 0.10) \times 10^{-3}\ \mathrm{eV}^2, \qquad (25)$$

for the inverted mass ordering ($m_3 < m_1$). A combined analysis [2] of the solar neutrino measurements and the KamLAND data results in

$$\tan^2\theta_{12} = 0.427^{+0.027}_{-0.024},$$
$$\Delta m^2_{21} = 7.46^{+0.20}_{-0.19} \times 10^{-5}\ \mathrm{eV}^2. \qquad (26)$$

The reactor $\overline{\nu_e}$ measurement at the DayaBay experiment [8] gives

$$\sin^2 2\theta_{13} = 0.084 \pm 0.005. \qquad (27)$$

Let us define $(Y_A)_{e\mu} \equiv (m_\tau/m_\mu)^2 (Y'_A)_{e\mu}$ and keep only terms of m_τ^2 in Eq. (20). We then obtain $(m_\nu)_{\tau\tau} = 0$, which can be satisfied for the inverted mass ordering with the following values:

$$\sin^2\theta_{23} = 0.511, \quad \sin^2 2\theta_{13} = 0.09, \quad \tan^2\theta_{12} = 0.427, \qquad (28)$$

$$\alpha_{12} = \alpha_{32} = \pi, \qquad \delta = 0, \qquad (29)$$

$$\Delta m^2_{23} = 2.48 \times 10^{-3}\ \mathrm{eV}^2,$$
$$\Delta m^2_{21} = 7.46 \times 10^{-5}\ \mathrm{eV}^2,$$
$$m_3 = 5.05 \times 10^{-2}\ \mathrm{eV}. \qquad (30)$$

For these values, the structure of the neutrino mass matrix is given by

$$m_\nu = \begin{pmatrix} -2.86 & 4.23 & -4.81 \\ 4.23 & -2.14 & -3.91 \\ -4.81 & -3.91 & 0 \end{pmatrix} \times 10^{-2}\ \mathrm{eV}. \qquad (31)$$

When we search for a set of model parameters which gives the structure of m_ν in Eq. (31), constraints from LFV decays of charged leptons have to be taken into account. In Table II, we summarize current data from various LFV experiments.

By assuming $X_{\mu e} = 0$, $X_{\mu\mu} = X_{\tau\tau}$, and $X_{\mu\tau} = X_{\tau\mu}$, the neutrino mass matrix in Eq. (20) is determined by five combinations of model parameters: $X_{\tau e}/X_{\tau\tau}$, $X_{\tau\mu}/X_{\tau\tau}$, $(Y_A)_{e\tau}/(Y_A)_{\mu\tau}$, $(Y'_A)_{e\mu}/(Y_A)_{\mu\tau}$, and $C_{\mathrm{loop}} m_\tau^2 (Y_A)_{\mu\tau} X_{\tau\tau}$. They are constrained as

$$\frac{(Y_A)_{e\tau}}{(Y_A)_{\mu\tau}} \frac{X_{\tau e}}{X_{\tau\tau}} = \frac{(m_\nu)_{ee}}{2(m_\nu)_{\mu\tau}} = 0.366, \qquad (32)$$

$$\frac{(Y'_A)_{e\mu}}{(Y_A)_{\mu\tau}} \frac{X_{\tau\mu}}{X_{\tau\tau}} + \frac{(Y_A)_{e\tau}}{(Y_A)_{\mu\tau}} = \frac{(m_\nu)_{e\tau}}{(m_\nu)_{\mu\tau}} = 1.23, \qquad (33)$$

$$\frac{(Y'_A)_{e\mu}}{(Y_A)_{\mu\tau}} + \frac{(Y_A)_{e\tau}}{(Y_A)_{\mu\tau}} \frac{X_{\tau\mu}}{X_{\tau\tau}} + \frac{X_{\tau e}}{X_{\tau\tau}} = \frac{(m_\nu)_{e\mu}}{(m_\nu)_{\mu\tau}} = -1.08, \qquad (34)$$

$$\frac{X_{\tau\mu}}{X_{\tau\tau}} = \frac{(m_\nu)_{\mu\mu}}{2(m_\nu)_{\mu\tau}} = 0.273, \qquad (35)$$

TABLE II. Constraints and future sensitivities for LFV decays of charged leptons at 90% confidence level (C.L.).

	Current bound	Future sensitivity
BR($\mu \to e\gamma$)	$<5.7 \times 10^{-13}$ (MEG) [43]	6×10^{-14} (MEG upgrade) [44]
BR($\mu \to \bar{e}ee$)	$<1.0 \times 10^{-12}$ (SINDRUM) [45]	$\sim 10^{-16}$ (Mu3e) [46]
BR($\tau \to e\gamma$)	$<1.2 \times 10^{-7}$ (Belle) [47]	
	$<3.3 \times 10^{-8}$ (*BABAR*) [48]	
BR($\tau \to \mu\gamma$)	$<4.5 \times 10^{-8}$ (Belle) [47]	5×10^{-9} (Belle II) [49]
	$<4.4 \times 10^{-8}$ (*BABAR*) [48]	
BR($\tau \to \bar{e}ee$)	$<2.7 \times 10^{-8}$ (Belle) [50]	
	$<2.9 \times 10^{-8}$ (*BABAR*) [51]	
BR($\tau \to \bar{e}e\mu$)	$<1.8 \times 10^{-8}$ (Belle) [50]	
	$<2.2 \times 10^{-8}$ (*BABAR*) [51]	
BR($\tau \to \bar{e}\mu\mu$)	$<1.7 \times 10^{-8}$ (Belle) [50]	
	$<2.6 \times 10^{-8}$ (*BABAR*) [51]	
BR($\tau \to \bar{\mu}ee$)	$<1.5 \times 10^{-8}$ (Belle) [50]	
	$<1.8 \times 10^{-8}$ (*BABAR*) [51]	
BR($\tau \to \bar{\mu}e\mu$)	$<2.7 \times 10^{-8}$ (Belle) [50]	
	$<3.2 \times 10^{-8}$ (*BABAR*) [51]	
BR($\tau \to \bar{\mu}\mu\mu$)	$<2.1 \times 10^{-8}$ (Belle) [50]	1×10^{-9} (Belle II) [49]
	$<3.3 \times 10^{-8}$ (*BABAR*) [51]	
	$<4.6 \times 10^{-8}$ (LHCb) [52]	
BR($\tau \to \mu\eta$)	$<6.5 \times 10^{-8}$ (Belle) [53]	

$$\frac{\sqrt{2}}{v}C_{\rm loop}m_\tau^2(Y_A)_{\mu\tau}X_{\tau\tau} = (m_\nu)_{\mu\tau} = -3.91\times 10^{-2}\ {\rm eV}. \tag{36}$$

These constraints are satisfied with the following benchmark set for model parameters[6]:

$$M_0 \equiv m_{\tilde{L}} = m_{\tilde{\ell}} = M_1 = 10\ {\rm TeV},\quad M_2 = 3\ {\rm TeV}, \tag{37}$$

$$(\Delta m^2_{\tilde{L}})_{\tau\mu} = (\Delta m^2_{\tilde{\ell}})_{\tau\mu} = -(0.8M_0)^2, \tag{38}$$

$$(\Delta m^2_{\tilde{L}})_{\tau e} = (\Delta m^2_{\tilde{\ell}})_{\tau e} = -(0.708M_0)^2, \tag{39}$$

$$(\Delta m^2_{\tilde{L}})_{\mu e} = (\Delta m^2_{\tilde{\ell}})_{\mu e} = 0, \tag{40}$$

$$\mu_\Phi = 762M_0,\qquad \tan\beta = 2, \tag{41}$$

$$m_A = 380\ {\rm GeV},\quad m_{H_1^+} = 350\ {\rm GeV},\quad \sin^2\theta_+ = 10^{-5}, \tag{42}$$

$$(Y_A)_{\mu\tau} = -1.31\times 10^{-4},\quad (Y_A)_{e\tau} = -2.24\times 10^{-4}, \tag{43}$$

$$(Y_A)_{e\mu} = \frac{m_\tau^2}{m_\mu^2}(Y'_A)_{e\mu} = 6.50\times 10^{-1}, \tag{44}$$

where $m^2_{H_2^+} \simeq m_A^2 + m_W^2 = (388\ {\rm GeV})^2$ at the tree level. For the mixing angle α of CP-even neutral Higgs bosons, we obtain $\cos(\beta-\alpha) = -0.027$ from $\tan(2\alpha)/\tan(2\beta) = (m_A^2+m_Z^2)/(m_A^2-m_Z^2)$. Notice that a low $\tan\beta$ value and large scales of μ_Φ and soft breaking parameters in the benchmark set are chosen in order to satisfy constraints from neutrino oscillation data and searches for the LFV decays of charged leptons. Nevertheless, even taking such a low $\tan\beta$ value, it can also be compatible with $m_h = 125$ GeV if we take $M_S^2 \equiv m_{\tilde{t}_1}m_{\tilde{t}_2} > (O(10)\ {\rm TeV})^2$ [54], where $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ are masses of two top squarks, and such a large value of M_S is consistent with large breaking scales in the benchmark set.

The value $m_A = 380$ GeV satisfies the constraint $m_A \gtrsim 350$ GeV for $\tan\beta = 2$ [55] which comes from the $A^0 \to Zh^0$ search at the CMS experiment with the 19.7 $\rm fb^{-1}$ data at $\sqrt{s} = 8$ TeV [56]. The values at the benchmark point for m_A and $\tan\beta$ also satisfy the constraint at the ATLAS experiment with the 20.3 $\rm fb^{-1}$ data at $\sqrt{s} = 8$ TeV [57]. The value $m_{H_1^+} = 350$ GeV for $H_1^+ \simeq \omega_1^+$ is consistent with a constraint on $m_{\tilde{\ell}}$ for a massless neutralino, which is $m_{\tilde{\ell}} \gtrsim 250$ GeV obtained at the ATLAS experiment for 20.3 $\rm fb^{-1}$ of the integrated luminosity with $\sqrt{s} = 8$ TeV [58]. The CMS experiment gives $m_{\tilde{\ell}} \gtrsim 200$ GeV for 19.5 $\rm fb^{-1}$ of the integrated luminosity with $\sqrt{s} = 8$ TeV [59].

The Yukawa matrix for Φ_0^* in Eq. (9) with the benchmark set is calculated as

$$X' \equiv \frac{\sqrt{2}}{v}M_\ell X = \begin{pmatrix} -1.63\times 10^{-5} & 0 & -3.49\times 10^{-6} \\ 0 & -3.39\times 10^{-3} & 9.25\times 10^{-4} \\ -1.22\times 10^{-2} & -1.55\times 10^{-2} & -5.69\times 10^{-2} \end{pmatrix}. \tag{45}$$

This matrix is the source of FCNC.

IV. PHENOMENOLOGY

A. Lepton flavor violating decays of charged leptons

For $\ell \to \ell'\gamma$, contributions from one-loop diagrams with charged scalars $H_i^\pm$ are negligible because $(Y_A)_{\ell\ell''}(Y_A)_{\ell''\ell'}$ are small enough. Since slepton masses are $O(10)$ TeV, one-loop diagrams involving sleptons have only negligible contributions to $\ell \to \ell'\gamma$ although $(\Delta m^2_{\tilde{L}})_{\tau e}$ and $(\Delta m^2_{\tilde{L}})_{\tau\mu}$ have a similar size to $m^2_{\tilde{L}}$ (also for $m^2_{\tilde{\ell}}$). The condition in Eq. (40) forbids not only the one-loop contribution of sleptons to $\mu \to e\gamma$ but also the Barr-Zee type two-loop contributions [60,61] with $X'_{\mu e}$. The dominant contribution to $\mu \to e\gamma$ comes from a one-loop diagram involving τ with $X'_{\tau\mu}X'_{\tau e}$, which results in ${\rm BR}(\mu \to e\gamma) = 1.5\times 10^{-13}$ with $m_H \simeq m_A$. This value satisfies the current constraint ${\rm BR}(\mu \to e\gamma) < 5.7\times 10^{-13}$ (90% C.L.) at the MEG experiment [43], and it could be observed at a planned MEG experiment upgrade [44] where a sensitivity for ${\rm BR}(\mu \to e\gamma) \simeq 6\times 10^{-14}$ is expected. If we take a larger value of $\tan\beta$ (e.g., $\tan\beta = 3$), $X'_{\tau\tau}$ is enhanced, and then it is required that $X'_{\tau\mu}$ (and also $X'_{\tau e}$ in fact) becomes larger due to the condition in Eq. (35); thus, a low $\tan\beta$ value is required to satisfy the constraint on ${\rm BR}(\mu \to e\gamma)$ with $m_A = 380$ GeV which is experimentally accessible. On the other hand, Barr-Zee diagrams involving the top quark in a loop give dominant contributions to $\tau \to \ell\gamma$ because $(\Delta m^2_{\tilde{L}})_{\tau e}$, $(\Delta m^2_{\tilde{\ell}})_{\tau e}$, $(\Delta m^2_{\tilde{L}})_{\tau\mu}$, and $(\Delta m^2_{\tilde{\ell}})_{\tau\mu}$ are not zero at our benchmark set. By using formulas in e.g. Ref. [61] with ${\rm BR}(\tau \to e\nu_\tau\bar{\nu}_e) = 0.17$, we have ${\rm BR}(\tau \to e\gamma) = 2.7\times 10^{-9}$ and ${\rm BR}(\tau \to \mu\gamma) = 4.4\times 10^{-9}$ which satisfy ${\rm BR}(\tau \to e\gamma) < 3.3\times 10^{-8}$ (90% C.L.) and ${\rm BR}(\tau \to \mu\gamma) < 4.4\times 10^{-8}$ (90% C.L.) obtained at the *BABAR* experiment [48]. These values are comparable to expected sensitivity ${\rm BR}(\tau \to \ell\gamma) \sim 10^{-9}$ at the Belle II experiment [49].

The Yukawa matrix X' can cause $\mu \to \bar{e}ee$ and $\tau \to \bar{\ell}\ell'\ell''$ at the tree level. Although there is the stringent experimental constraint ${\rm BR}(\mu \to \bar{e}ee) < 1.0\times 10^{-12}$

[6]If the contribution from $(Y_A)_{e\mu}$ to m_ν is naively ignored, larger off-diagonal elements of X are necessary. We do not take this option in order to suppress LFV decays of charged leptons as much as possible.

(90% C.L.) at the SINDRUM experiment [45], our benchmark set trivially satisfies it because $X'_{\mu e} = 0$ in Eq. (45) gives $\mathrm{BR}(\mu \to \bar{e}ee) = 0$ at the tree level. For $\tau \to \bar{\ell}\ell'\ell''$, the tree level contributions at the benchmark set result in $\mathrm{BR}(\tau \to \bar{e}\mu\mu) = \mathrm{BR}(\tau \to \bar{\mu}ee) = 0$, $\mathrm{BR}(\tau \to \bar{e}ee) \sim \mathrm{BR}(\tau \to \bar{e}e\mu) \sim 10^{-16}$, and $\mathrm{BR}(\tau \to \bar{\mu}e\mu) \sim \mathrm{BR}(\tau \to \bar{\mu}\mu\mu) \sim 10^{-11}$ where experimental bounds are $\mathrm{BR}(\tau \to \bar{\ell}\ell'\ell'') \lesssim 10^{-8}$ (90% C.L.) [50–52]. The constraint $\mathrm{BR}(\tau \to \mu\eta) < 6.5 \times 10^{-8}$ (90% C.L.) [53] is also satisfied even if we use the relation $\mathrm{BR}(\tau \to \mu\eta) \simeq 8.4 \times \mathrm{BR}(\tau \to \bar{\mu}\mu\mu)$ [62].

The LFV coupling $X'_{\tau\mu} = -1.55 \times 10^{-2}$ in Eq. (45) is comparable to $\sqrt{2}m_\tau/v = 10^{-2}$. However, the branching ratio $\mathrm{BR}(h \to \mu\tau)$ is suppressed to about 10^{-4} by $\cos^2(\alpha - \beta) \simeq 10^{-3}$. Therefore, the benchmark point satisfies the current upper limit on $\mathrm{BR}(h \to \mu\tau)$ [63,64], although the 2.4σ excess which is currently reported by the CMS [63] is not explained by our benchmark point.

B. Dark matter

Large values of μ_Φ and M_1 are preferred in order to obtain sizable off-diagonal elements of X (namely, ϵ_2) which are required for the appropriate structure of the neutrino mass matrix. On the other hand, the value of M_2 is not required to be very large. Therefore, there is a possibility of the wino dark matter in the SUSY Zee model. In our benchmark set, we take $M_2 = 3$ TeV for which the relic abundance of dark matter can be explained [65]. The spin-independent cross section of the wino scattering on a proton is evaluated as $\sim 10^{-47}$ cm^2 (see, e.g., Ref. [66]), which is greater than the neutrino background [67].

C. Phenomenology of charged scalar bosons

Decay branching ratios of $H_1^\pm$ and $H_2^\pm$ are shown in Fig. 2 as a function of $\sin^2\theta_+$ where $H_1^\pm = \omega_1^\pm$ if $\sin^2\theta_+ = 0$. The red thick and the blue thin solid lines show $\mathrm{BR}(H^\pm \to e\nu)$ [$=\mathrm{BR}(H^\pm \to \mu\nu)$] and $\mathrm{BR}(H^\pm \to \tau\nu)$, respectively. The magenta dashed line is for $\mathrm{BR}(H^\pm \to tb)$. Except for $\sin^2\theta_+$ and Y_A, parameters are set to the benchmark point. Values of elements of Y_A depend on $\sin^2\theta_+$ through a condition Eq. (36). For $\sin^2\theta_+ \gtrsim 10^{-2}$, H_1^- is the doubletlike Higgs boson for which the decay $H_1^- \to tb$ is the dominant channel. In this case, since the dominant decay channel of H_2^- is also tb (for any $\sin^2\theta_+$), existence of an SU(2)-singlet component is hidden. Such doubletlike charged Higgs bosons (the same as the one in the type-II two Higgs doublet model) with $m_{H_1^\pm} = 350$ GeV and $m_{H_2^\pm} \simeq 388$ GeV for $\tan\beta = 2$ may be observed at the LHC with $\sqrt{s} = 14$ TeV and 300 fb^{-1} of the integrated luminosity via the production process $gb \to tH^\pm$ followed by the decay $H^\pm \to tb$ [68,69].

On the other hand, $H_1^\pm$ dominantly decays into leptons via Y_A for $\sin^2\theta_+ \lesssim 10^{-3}$. The hierarchical structure of Y_A in Eqs. (43) and (44) gives a characteristic prediction

$$\mathrm{BR}(H_1^\pm \to e\nu) : \mathrm{BR}(H_1^\pm \to \mu\nu) : \mathrm{BR}(H_1^\pm \to \tau\nu) \simeq 1:1:0, \tag{46}$$

where neutrinos in the final states are summed because experiments are not sensitive to their flavors. If the coincidence $\mathrm{BR}(H_1^\pm \to e\nu) = \mathrm{BR}(H_1^\pm \to \mu\nu)$ is observed, it would be regarded as a nonaccidental one but a natural consequence of the $\mathrm{SU}(2)_L$-singlet charged scalar with a $(Y_A)_{e\mu}$-dominated Yukawa matrix. Nonobservation of the signal of $H_1^\pm \to \tau\nu$ would also suggest that $H_1^\pm$ does not come from an $\mathrm{SU}(2)_L$-doublet Higgs which has a vacuum expectation value. For the singletlike charged scalar, a region of $m_{H_1^\pm} \lesssim 430$ GeV can be probed at the LHC with $\sqrt{s} = 14$ TeV and 100 fb^{-1} of the integrated luminosity [70]. At the ILC with $\sqrt{s} = 1$ TeV, the cross section is about 10 fb [21] which would be enough to observe $H_1^\pm$. The singletlike $H_1^\pm$ would be distinguished from right-handed sleptons if the lightest neutralino is sufficiently heavy such that it does not mimic a neutrino.

The behavior of $m^2_{H_2^\pm}$ at the tree level with respect to $\sin^2\theta_+$ is shown in Fig. 3. In the region of small $\sin^2\theta_+$, the relation $m^2_{H_2^\pm} = m_A^2 + m_W^2$ approximately holds with an appropriate radiative correction [71]. On the other hand, the relation between m_A and the mass of the charged Higgs

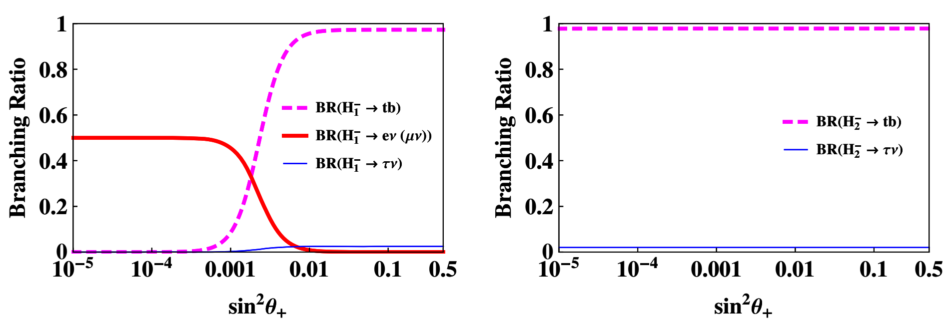

FIG. 2 (color online). Decay branching ratios of $H_1^\pm$ (left) and $H_2^\pm$ (right) with respect to $\sin^2\theta_+$. The red thick and the blue thin solid lines show $\mathrm{BR}(H^\pm \to e\nu)$ [$=\mathrm{BR}(H^\pm \to \mu\nu)$] and $\mathrm{BR}(H^- \to \tau\nu)$, respectively. The magenta dashed line is for $\mathrm{BR}(H^\pm \to tb)$.

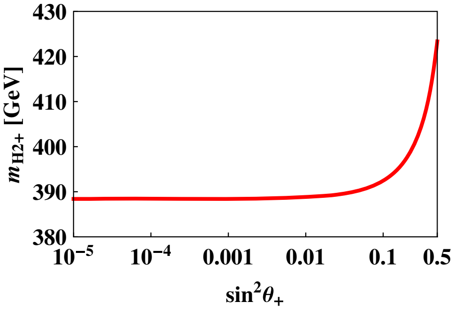

FIG. 3 (color online). The mass of $H_2^\pm$ at the tree level with respect to $\sin^2\theta_+$ for $m_A = 380$ GeV and $m_{H_1^\pm} = 350$ GeV.

boson in the MSSM at the one-loop level is expressed as [72]

$$m_{H^+}^2 = m_A^2 + m_W^2 + \Pi_{AA}(m_A^2) - \Pi_{H^+H^-}(m_A^2 + m_W^2) + \Pi_{WW}(m_W^2), \quad (47)$$

where formulas of self-energies $\Pi_{AA}(q^2)$, $\Pi_{H^+H^-}(q^2)$, and $\Pi_{WW}(q^2)$ can be found in Ref. [72]. This relation at the one-loop level also approximately holds for $m_{H_2^+}^2$ in the SUSY Zee model for a small value of $\sin\theta_+$, where H_2^+ is almost the same as the charged Higgs boson in the MSSM. The benchmark set gives $\delta_{H_2^\pm} \simeq 0.1$, where the $\delta_{H_2^\pm} \simeq 0.1$ is defined as $m_{H_2^+} = \sqrt{m_A^2 + m_W^2}(1 + \delta_{H_2^\pm} \simeq 0.1)$. If the mass relation is experimentally confirmed in addition to Eq. (46) which is a consequence of m_ν in Eq. (20), the SUSY Zee model can be highly supported.

The detection prospect of A^0 with $m_A = 380$ GeV seems marginal for $A^0 \to \tau\tau$ at the LHC with $\sqrt{s} = 14$ TeV [55,68,73] but sufficient for $A^0 \to Zh$ [55]. Discovery of the A^0 can be expected also via $A^0 \to tt$ [55]. The cross section for a process $e^+e^- \to Z^* \to H^0A^0 \to ttbb$ for the A^0 at the ILC with $\sqrt{s} = 1$ TeV is greater than 0.1 fb [68] which would be sufficient to detect the signal.

V. CONCLUSION

We have extended the Zee model to a SUSY model with the conserved R-parity. The MSSM has been extended by introducing $SU(2)_L$-singlet complex superfields $\hat{\omega}_1^+$ and $\hat{\omega}_2^-$. In order to generate three nonzero neutrino masses, the extension gives the simplest alternative to the SUSY seesaw model where three right-handed neutrino superfields are introduced. Tiny neutrino masses are obtained at the one-loop level. We have shown that the mass matrix can be consistent with the current neutrino oscillation data thanks to the nonholomorphic Yukawa interaction, which is dominantly generated by one-loop diagrams involving sleptons and the bino. We have obtained a benchmark point which satisfies not only neutrino oscillation data but also constraints from LFV decays of charged leptons and the current LHC results. The parameter set is also consistent with $m_h = 125$ GeV. The dark matter is stabilized thanks to the R-parity conservation in the SUSY Zee model, and we have found that the wino can be dark matter.

While SUSY particles are rather heavy at the benchmark point, additional scalar bosons (H^0, A^0, $H_1^\pm$, and $H_2^\pm$) are light enough to be discovered at the LHC and future lepton colliders. If the lightest charged scalar bosons $H_1^\pm$ are almost singletlike due to a small mixing, their decay branching ratios have been predicted as $\mathrm{BR}(H_1^\pm \to e\nu)$: $\mathrm{BR}(H_1^\pm \to \mu\nu)$:$\mathrm{BR}(H_1^\pm \to \tau\nu) = 1:1:0$. On the other hand, the heavier charged scalar bosons $H_2^\pm$ decay into tb. For such a small mixing case, a relation between $m_{H_2^\pm}$ and m_A in the MSSM remains because $m_{H_2^\pm}$ is almost the same as the charged Higgs boson mass in the MSSM. Therefore, our model can be tested by measuring the specific decay patterns of $H_1^\pm$ and $H_2^\pm$ and the discriminative mass spectrum of additional scalar bosons.

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