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Scalar glueballs: Constraints from the decays into η or η'

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We study the mixing of the scalar glueball into the isosinglet mesons $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ to describe the two-body decays to pseudoscalars. We use an effective Hamiltonian and employ the two-angle mixing scheme for η and η'. In this framework, we analyze existing data and look forward to new data into η and η' channels. For now, the $f_0(1710)$ has the largest glueball component and a sizable branching ratio into $\eta\eta'$, testable at BESIII.

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I. INTRODUCTION

Glueballs are arguably the most important unconfirmed prediction of quantum chromodynamics (QCD) (see Refs. [1–3] for reviews). Lattice QCD calculations predict the lowest-lying glueball around 1.6–1.7 GeV, with quantum numbers $J^{PC} = 0^{++}$. It is expected to mix with $q\bar{q}$ states in the same mass region, resulting in more 0^{++} states than naively expected from $q\bar{q}$ spectroscopy. There seems to be some consensus that the states $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ are the relevant mass eigenstates made up from $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$, $s\bar{s}$, and the 0^{++} glueball G (see Table I for masses and widths). We note however that the lightest state $f_0(1370)$ is very poorly known, due to overlap with nearby states. To find the correct mixing structure one uses data on f_0 both from production (seeking "glue-rich" channels) and decay; for this it is important to identify processes that are particularly sensitive to the glueball couplings.

Gluons have a strong coupling to η and η' through the axial anomaly, which neatly describes the ratio $\Gamma(J/\psi \to \eta'\gamma)/\Gamma(J/\psi \to \eta\gamma) \sim 5$ [5–8]. For glueballs G—produced for example in $J/\psi \to G\gamma$—we expect $\Gamma(G \to \eta\eta) > \Gamma(G \to \pi\pi, KK)$ (expect the same from lattice QCD [9,10]). See Table II for experimental decay rates of the potential glueball candidates $f_0(1370, 1500, 1710)$ into $\eta\eta$ and $\eta\eta'$ from WA102 (soon to be updated with BESIII data).[1] It is hard to see a coherent picture emerging in the data. In particular, all $f_0(1370)$ data are inconclusive/questionable to say the least. $f_0(1500)$ is probably the best-studied state; only the $\eta\eta'$ channel is murky due to it being at the kinematic threshold. $f_0(1710)$ data come mainly from WA102 and desperately require confirmation by BESIII. Our goal in this paper is to provide a framework to interpret upcoming data, which will hopefully clarify the picture.

The assignments of the three f_0 states to quark and glueball states are for the moment unclear, with different proposals in the literature [18]. Namely, Refs. [19–21] suggested $f_0(1500)$ to be dominantly glueball in nature, which does not fare that well anymore [10]. References [10,22] propose the mixing matrix

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} \equiv \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 0.78 & 0.51 & -0.36 \\ -0.54 & 0.84 & -0.03 \\ 0.32 & 0.18 & 0.93 \end{pmatrix} \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix}, \quad (1)$$

so $f_0(1710)$ is mostly a glueball (same qualitative picture found e.g. in Ref. [23]). The decay $f_0 \to \eta\eta'$ is however not discussed, because it is close to threshold for the $f_0(1500)$—and hence subject to systematic errors—and inconsistently measured for the $f_0(1700)$. Since the glueball is expected to have a strong connection to the η–η' system, we will pay particular attention to these final states in this paper (see also Ref. [24,25]).

A strong argument for the glueball nature of $f_0(1710)$ comes from the J/ψ decay rates [10,26,27]

$$\frac{\Gamma(J/\psi \to \gamma f_0(1710))}{\Gamma(J/\psi \to \gamma f_0(1500))} = 10.5 \pm 6.5 \quad (2)$$

(where we used the $\pi\pi$ channel [27]), which is expected to be enhanced for glueballs due to $J/\psi \to \gamma gg \to \gamma G$. One also finds [28]

$$\frac{\Gamma(J/\psi \to \gamma f_0(1370))}{\Gamma(J/\psi \to \gamma f_0(1710))} = 0.51 \pm 0.41, \quad (3)$$

which fits well into that picture.

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[1] See Refs. [11,12] for early experimental results on the η channels and Ref. [13] for π channels.

TABLE I. Some relevant masses and widths [4].

State	Mass (MeV)	Width (MeV)
$a_0(1450)$	1474 ± 19	265 ± 13
$f_0(1370)$	1350 ± 150	350 ± 150
$f_0(1500)$	1505 ± 6	109 ± 7
$f_0(1710)$	1722 ± 6	135 ± 7
η	547.86 ± 0.02	$(1.31 \pm 0.05) \times 10^{-3}$
η'	957.78 ± 0.06	0.23 ± 0.02

TABLE II. Data without a reference have been obtained by combining referenced data. Data with an asterisk are fitted, everything else are predictions.

Decay ratio	Data	Fit 1	Fit 2
$\frac{\Gamma(a_0(1450)\to KK)}{\Gamma(a_0(1450)\to\pi\eta)}$	0.88 ± 0.23 [4]*	$0.75^{+0.08}_{-0.06}$	$0.81^{+0.08}_{-0.07}$
$\frac{\Gamma(a_0(1450)\to\pi\eta')}{\Gamma(a_0(1450)\to\pi\eta)}$	0.35 ± 0.16 [4]	0.44	0.44
$\frac{\Gamma(f_0(1370)\to\pi\pi)}{\Gamma(f_0(1370)\to KK)}$	12.5 ± 12.5 [14]*	11^{+99}_{-8}	$10.1^{+41.9}_{-6.4}$
$\frac{\Gamma(f_0(1500)\to\pi\pi)}{\Gamma(f_0(1500)\to KK)}$	4.07 ± 0.43 [4]*	$3.99^{+0.94}_{-0.65}$	$4.04^{+1.48}_{-0.89}$
$\frac{\Gamma(f_0(1710)\to\pi\pi)}{\Gamma(f_0(1710)\to KK)}$	$0.41^{+0.11}_{-0.17}$ [4]*	$0.46^{+0.04}_{-0.05}$	$0.20^{+0.03}_{-0.03}$
$\frac{\Gamma(f_0(1370)\to\eta\eta)}{\Gamma(f_0(1370)\to\pi\pi)}$	$0.03^{+0.04}_{-0.03}$	$0.03^{+0.08}_{-0.03}$	$0.04^{+0.08}_{-0.04}$
$\frac{\Gamma(f_0(1500)\to\eta\eta)}{\Gamma(f_0(1500)\to\pi\pi)}$	0.145 ± 0.027 [4]*	$0.14^{+0.02}_{-0.02}$	$0.14^{+0.03}_{-0.03}$
$\frac{\Gamma(f_0(1710)\to\eta\eta)}{\Gamma(f_0(1710)\to\pi\pi)}$	$1.17^{+0.61}_{-0.48}$	$0.44^{+0.10}_{-0.08}$	$0.96^{+0.32}_{-0.22}$
$\frac{\Gamma(f_0(1370)\to\eta\eta)}{\Gamma(f_0(1370)\to KK)}$	0.35 ± 0.30 [15,16]*	$0.36^{+9.9}_{-0.36}$	$0.39^{+4.4}_{-0.37}$
$\frac{\Gamma(f_0(1500)\to\eta\eta)}{\Gamma(f_0(1500)\to KK)}$	0.59 ± 0.12 [4]	$0.56^{+0.19}_{-0.14}$	$0.58^{+0.26}_{-0.16}$
$\frac{\Gamma(f_0(1710)\to\eta\eta)}{\Gamma(f_0(1710)\to KK)}$	0.48 ± 0.15 [4,15]*	$0.20^{+0.03}_{-0.03}$	$0.19^{+0.03}_{-0.03}$
$\frac{\Gamma(f_0(1500)\to\eta\eta')}{\Gamma(f_0(1500)\to\eta\eta)}$	0.38 ± 0.16 [4]*	$0.53^{+0.13}_{-0.11}$	$0.47^{+0.14}_{-0.12}$
$\frac{\Gamma(f_0(1710)\to\eta\eta')}{\Gamma(f_0(1710)\to\eta\eta)}$	<0.08 [15]	$0.64^{+0.22}_{-0.22}$	$0.32^{+0.20}_{-0.18}$
$\mathrm{Br}(f_0(1500)\to\pi\pi)$	0.349 ± 0.023 [4]*	$0.36^{+0.08}_{-0.07}$	$0.35^{+0.07}_{-0.06}$
$\mathrm{Br}(f_0(1710)\to KK)$	0.36 ± 0.12 [17]*	$0.37^{+0.06}_{-0.06}$	$0.44^{+0.07}_{-0.07}$
$\chi^2_{\mathrm{min}}/\mathrm{d.o.f.}$		4.9/3	6.4/3

One more guideline for the identification of (predominantly) glueball states may be the total width, with a direct decay to quarks expected to be somewhat suppressed [by the Okubo-Zweig-Iizuka (OZI) rule or large-N calculations]. This argument is however difficult to implement at the level of two-meson decays, as the total cross section may very well be dominated by more complicated final states (in particular $f_0 \to 4\pi$). Furthermore, some studies suggest rather broad glueballs [29], so we will not impose theoretical conditions on the glueball width in this study.

II. EFFECTIVE HAMILTONIAN

To calculate the f_0 decays, we employ the effective Hamiltonian [30–32]

$$\mathcal{H} = h_1 \mathrm{tr}[X_F PP] + h_2 G \mathrm{tr}[PP] + h_3 G \mathrm{tr}[P]\mathrm{tr}[P], \quad (4)$$

with the pseudoscalar $SU(3)_f$ nonet

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} \end{pmatrix}, \quad (5)$$

and the scalar $q\bar{q}$ states

$$X_F = \begin{pmatrix} u\bar{u} & & \\ & d\bar{d} & \\ & & s\bar{s} \end{pmatrix}. \quad (6)$$

See Fig. 1 for a diagrammatic representation of these operators (see also Ref. [18]). (Note that Ref. [33] proposes an entirely different effective Hamiltonian of chiral perturbation theory; for the time being, we prefer not to rely on chiral perturbation in the energy range considered—more on this later about chiral suppression.)

Assuming isospin invariance, we will adopt the standard mixing scheme

$$\begin{pmatrix} a_0(1450) \\ f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 1 & \\ & U \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & & \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} u\bar{u} \\ d\bar{d} \\ s\bar{s} \\ G \end{pmatrix}, \quad (7)$$

where U is a real orthogonal 3×3 matrix describing the mixing of $n\bar{n}$, $s\bar{s}$, and G into the f_0 states, further abbreviated by $(F_1,F_2,F_3) \equiv (f_0(1370),f_0(1500),f_0(1710))$.

With the Hamiltonian from Eq. (4) one can easily calculate all the $f_0 \to \pi\pi$, KK, $\eta\eta$ decays [22], as well as the $f_0 \to \eta\eta'$ decays of interest to us (which has been done already in Ref. [34], but only using $h_{1,2}$). The question is now how to include $SU(3)_f$-breaking effects in Eq. (4)—which should exist due to $m_s \gg m_{u,d}$. Some remarks are in order:

(i) The coupling h_1 [Fig. 1(a)] describes the OZI-allowed decay that should dominate the $q\bar{q}$ decays. The creation of an $s\bar{s}$ from the vacuum is modified relative to $u\bar{u}$ by a factor $r_a \sim 1$ [22], which we include explicitly in the amplitudes of h_1.

(ii) h_2 corresponds to the dominant glueball couplings to the nonet [Fig. 1(b)]. A flavor-democratic coupling of glueballs to quarks would lead to the ratios

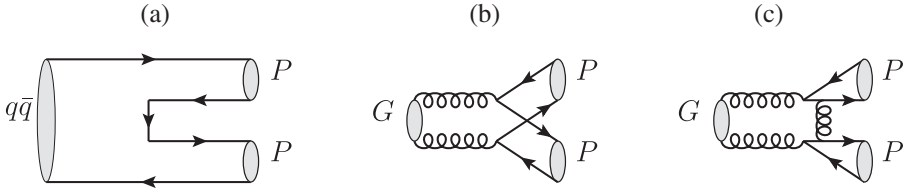

FIG. 1. Diagrammatic representation of the effective couplings h_1 (a), h_2 (b), and h_3 (c).

$$\mathcal{M}(G\to\pi\pi):\ \mathcal{M}(G\to KK):\ \mathcal{M}(G\to\eta\eta) : \mathcal{M}(G\to\eta'\eta'):\ \mathcal{M}(G\to\eta\eta') = 1:1:1:1:0,$$

and hence decay rates $3:4:1:1:0$ modulo phase space [3]. Some of the relevant diagrams, however, contain the subprocess $gg\to q\bar{q}$ [differently attached gluons in Fig. 1(b)] and it has been argued that this implies a chiral suppression $\sigma(gg\to q\bar{q})\propto m_q$ [35,36]. How this suppression propagates to the hadronized $G\to PP$ rate is unfortunately unknown. A proportionality of $\mathcal{M}(G\to PP)$ to current-quark masses is highly improbable for several reasons: (i) even accepting a Feynman-diagram picture of these strong interactions, the gluons may land on different quark lines,[2] (ii) even accepting the chiral suppression, we know that chiral symmetry is broken by confinement itself, leading to constituent quark masses, (iii) alternate calculations based on perturbative QCD (equally out of their domain of application as chiral perturbation) suggest instead the decay constants f_π, f_K, etc., as suppression [22,37], and (iv) completely different arguments would use the overlap of wave functions at the origin as a factor governing the flavor nonuniversality.[3]

(Note also that the chiral suppression argument of the coupling of quarks to two gluons does not apply to the case of pseudoscalars through the quantum anomaly.) We thus leave the extent of the suppression (if any) free for now, and introduce different couplings for the PP states to take the chiral dependence into account, following again Ref. [22]:

$$h_2 G\mathrm{tr}[PP]\to G[h_{2,\pi}((\pi^0)^2+2\pi^+\pi^-) + 2h_{2,K}(K^0\bar{K}^0+K^+K^-) + h_{2,0}(\eta_0)^2+h_{2,8}(\eta_8)^2]. \quad (8)$$

[2]Meaning $\sigma(gg\to q\bar{q})\propto m_q$ is in any case only one possible subprocess.

[3]In Ref. [38], a coupling of the form $M_P^2P^2G$ is motivated within the Witten-Sakai-Sugimoto model for the singlet pseudoscalar η_0 and subsequently extended to all P.

(iii) h_3 [Fig. 1(c)] describes the $G\eta_0\eta_0$ coupling that can be associated with the anomaly [30,31] and is of particular interest to our study of $\eta\eta'$ final states. (In Ref. [39] this is taken one step further and promoted to a glueball-glueball-glueball coupling followed by a mixing of η and η' with the 0^{-+} glueball [40].) The full coupling is $\mathcal{H}\supset(h_{2,0}+3h_3)G(\eta_0)^2$, but we will fix $h_{2,0}$ instead of fitting it, so h_3 is indeed a free parameter for us.

(iv) Couplings $\mathrm{tr}[X_F]\mathrm{tr}[PP]$, $\mathrm{tr}[X_FP]\mathrm{tr}[P]$, and $\mathrm{tr}[X_F]\mathrm{tr}[P]\mathrm{tr}[P]$ would correspond to OZI-suppressed diagrams (see Ref. [41]) and are neglected here as usual.

(v) The free mixing matrix U of Eq. (7) describes in itself $SU(3)_f$ violation coming from the nondiagonal mass matrix of the $q\bar{q}$ and G states. We will not assume a structure in this mass matrix (see e.g. Refs. [42,43]) but rather determine U directly from the f_0 decays, keeping the mass eigenvalues and widths fixed to their Particle Data Group (PDG) values (Table I).

A. η–η' mixing

Most studies of f_0 decays assume the η–η' system (Table I) to be an orthogonal transformation of the $SU(3)_f$ states,

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix}=\begin{pmatrix}\cos\theta_P & -\sin\theta_P\\ \sin\theta_P & \cos\theta_P\end{pmatrix}\begin{pmatrix}\eta_8\\ \eta_0\end{pmatrix}, \quad (9)$$

with flavor eigenstates $\eta_0=(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}$, $\eta_8=(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$, and mixing angle $\theta_P=-11.4°$ [4] (see also Ref. [44]).

It has been shown that this approach is a little too naive, because the large hierarchy $m_\eta\ll m_{\eta'}$, together with renormalization effects, complicate matters [45–47]. In effect, one should rather use a general (invertible) transformation matrix, which has four real parameters,

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix}=\frac{1}{F}\begin{pmatrix}F_8\cos\theta_8 & -F_0\sin\theta_0\\ F_8\sin\theta_8 & F_0\cos\theta_0\end{pmatrix}\begin{pmatrix}\eta_8\\ \eta_0\end{pmatrix}, \quad (10)$$

and is called the two-mixing-angle scheme.[4] (It reduces to the one-angle scheme in the limit $F_0 = F_8 = F$, $\theta_0 = \theta_8 = \theta_P$.) Here, $F = 92.2$ MeV is a normalization constant, and a fit gives [48]

$$F_8/F = 1.26 \pm 0.04, \qquad \theta_8 = (-21.2 \pm 1.6)^\circ, \quad (11)$$

$$F_0/F = 1.17 \pm 0.03, \qquad \theta_0 = (-9.2 \pm 1.7)^\circ. \quad (12)$$

(Other methods of determining these parameters give similar values; see e.g. Refs [49–52].)

Since this two-angle scheme is rather successful and stable in describing η–η' data, we will keep the above parameters fixed to the best-fit values. For $a_0(1450)$, we then immediately arrive at the ratios

$$\frac{\Gamma(a_0(1450) \to \pi\eta')}{\Gamma(a_0(1450) \to \pi\eta)} = \left(\frac{2F_8\cos\theta_8 + \sqrt{2}F_0\sin\theta_0}{2F_8\sin\theta_8 - \sqrt{2}F_0\cos\theta_0}\right)^2 \frac{p_{\eta'}}{p_\eta} \simeq 0.44, \quad (13)$$

and

$$\frac{\Gamma(a_0(1450) \to KK)}{\Gamma(a_0(1450) \to \pi\eta)} = \left(\frac{\sqrt{3}r_a F_0 F_8 \cos(\theta_0 - \theta_8)/F}{\sqrt{2}F_0\cos\theta_0 - 2F_8\sin\theta_8}\right)^2 \frac{p_K}{p_\eta} \simeq 0.84 r_a^2, \quad (14)$$

perfectly compatible with experimental results for $r_a \simeq 1$ (Table II).

The f_0 decays are more involved due to the mixing matrix U, which we will determine by a numerical fit in the next section. The formulas for the decay rates are collected in the Appendix.

B. Fit

Having defined our Hamiltonian, we can perform our own fit to the decay data in Table II, using a simple χ^2 function. Note that we will keep the masses fixed (Table I) and do not impose a structure on the f_0 mass matrix in flavor basis [given by $U^T \mathrm{diag}(M_{F_1}, M_{F_2}, M_{F_3})U$]. Since there is unfortunately not enough reliable data to obtain a statistically significant result, our best-fit values have to be taken with a grain of salt.

In order to calculate the rate $f_0(1500) \to \eta\eta'$, which is at threshold because $M_\eta + M_{\eta'} = 1505$ MeV $\gtrsim M_{F_2}$ (see Table I), we integrate the partial widths with a Breit-Wigner distribution:

$$\frac{\Gamma(F_2 \to \eta\eta')}{\Gamma(F_2 \to \eta\eta)} = \frac{|\mathcal{M}(F_2 \to \eta\eta')|^2}{|\mathcal{M}(F_2 \to \eta\eta)|^2} \frac{\int_{M_\eta + M_{\eta'}}^{M_{F_2}+\Gamma_{F_2}} \mathrm{d}E\, p(E, M_\eta, M_{\eta'})/[(E^2 - M_{F_2}^2)^2 + M_{F_2}^2\Gamma_{F_2}^2]}{\int_{M_{F_2}-\Gamma_{F_2}}^{M_{F_2}+\Gamma_{F_2}} \mathrm{d}E\, p(E, M_\eta, M_\eta)/[(E^2 - M_{F_2}^2)^2 + M_{F_2}^2\Gamma_{F_2}^2]}. \quad (15)$$

Here,

$$p(E, M_1, M_2) \equiv \frac{\sqrt{[E^2 - (M_1 + M_2)^2][E^2 - (M_1 - M_2)^2]}}{2E} \quad (16)$$

is the momentum of a daughter particle in a two-body decay, where the initial particle has "mass" E. Since the $F_2 \to \eta\eta'$ decay relies on a slightly off-shell F_2, we integrated the $\eta\eta'$ and $\eta\eta$ decay rates with the highly peaked Breit-Wigner distribution over $[M_{F_2} - \Gamma_{F_2}, M_{F_2} + \Gamma_{F_2}]$, which reduces to $[M_\eta + M_{\eta'}, M_{F_2} + \Gamma_{F_2}]$ for $\eta\eta'$ due to kinematics. Numerically, one finds

$$\frac{\Gamma(F_2 \to \eta\eta')}{\Gamma(F_2 \to \eta\eta)} = 0.14 \times \frac{|\mathcal{M}(F_2 \to \eta\eta')|^2}{|\mathcal{M}(F_2 \to \eta\eta)|^2}, \quad (17)$$

fairly stable against small changes in the domain of integration.

For the decay ratios of J/ψ we make the simple assumption that the decay is dominated by $J/\psi \to gg\gamma \to G\gamma$, leading to the simple formula

$$\frac{\Gamma(J/\psi \to \gamma F_i)}{\Gamma(J/\psi \to \gamma F_j)} = \frac{|U_{i3}|^2}{|U_{j3}|^2} \frac{p_{F_i}}{p_{F_j}}, \quad (18)$$

but we will not include these ratios in our fits. See e.g. Refs. [53,54] for dedicated studies.

1. Fit 1: Flavor blind

Taking for simplicity a flavor-blind G coupling $h_2 \equiv h_{2,\pi} = h_{2,K} = h_{2,0} = h_{2,8}$ and the two-angle η–η' values from above we can fit $h_{1,2,3}$ and the mixing matrix U

[4] Using the quark basis $\eta_{q,s}$ instead of the flavor basis $\eta_{8,0}$ in the two-angle scheme leads to two mixing angles $\theta_{q,s}$ that are accidentally close to each other, allowing (at current precision) for a one-angle description $\theta_q = \theta_s$. Since this is obviously a basis-dependent effect, we will keep the two-angle formalism here.

parametrized via three angles θ_{ij} [see Eq. (A2)]. We find[5]

$$r_a = 0.94^{+0.05}_{-0.04}, \qquad h_1 = (698 \pm 30)\ \text{MeV}, \tag{19}$$

$$h_2/h_1 = 1.12^{+0.03}_{-0.04}, \qquad h_3/h_1 = 1.5^{+0.2}_{-0.3}, \tag{20}$$

for the coupling constants and

$$\theta_{23} = 0.67 \pm 0.03, \qquad \theta_{12} = 3.56 \pm 0.04,$$
$$\theta_{13} = 0.38^{+0.01}_{-0.02}, \tag{21}$$

for the mixing angles, resulting in the mixing structure

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} -0.85 & -0.38 & 0.37 \\ 0.53 & -0.63 & 0.57 \\ 0.02 & 0.68 & 0.73 \end{pmatrix} \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix} \tag{22}$$

$$= \begin{pmatrix} -0.91 & -0.18 & 0.37 \\ 0.07 & 0.82 & 0.57 \\ 0.41 & -0.55 & 0.73 \end{pmatrix} \begin{pmatrix} \text{singlet} \\ \text{octet} \\ G \end{pmatrix}, \tag{23}$$

meaning that the largest glueball component is in $f_0(1710)$. The best-fit decay widths and ratios are given in Table II.

We also gave the mixing matrix for the singlet-octet-G basis in Eq. (23), which seems to be a better starting point than $n\bar{n}$–$s\bar{s}$–G, since the mixing angles in this basis are smaller. $f_0(1500)$ is dominantly a flavor octet, similar to Refs. [10,22]. While we have made no considerations on the mass mixing matrix, and have determined the mixing matrix U from decay properties; it is an interesting consistency check to verify to which original mass matrix $M = U^T M_{\text{diag}} U$ in the flavor basis $n\bar{n}$, $s\bar{s}$, G our fit corresponds:

$$M = \begin{pmatrix} 1393.09 & -46.93 & 51.41 \\ -46.93 & 1584.15 & 129.72 \\ 51.41 & 129.72 & 1599.76 \end{pmatrix} \text{MeV}, \tag{24}$$

or

$$\begin{pmatrix} 1412.53 & -74.43 & 116.87 \\ -74.43 & 1564.71 & -76.24 \\ 116.87 & -76.24 & 1599.76 \end{pmatrix} \text{MeV}, \tag{25}$$

in the singlet-octet-G basis. It is reassuring to see that the $SU(3)$-breaking elements are indeed smaller than the conserving (diagonal) ones and that no fine-tuned cancellations occur. However, the "octet mass" 1565 MeV comes out surprisingly large compared to the states $K^*(1430)$ and $a_0(1450)$.

The ratios for J/ψ decays into γF_j show the qualitative behavior we expected from our initial considerations,

$$\frac{\Gamma(J/\psi \to \gamma f_0(1370))}{\Gamma(J/\psi \to \gamma f_0(1710))} = 0.3,$$
$$\frac{\Gamma(J/\psi \to \gamma f_0(1710))}{\Gamma(J/\psi \to \gamma f_0(1500))} = 1.5.$$

We find, however, a very small $f_0(1370)$ width into PP,

$$\Gamma(f_0(1370) \to PP) \simeq 2\ \text{MeV}, \tag{26}$$

dominated by the $\pi\pi$ final state. From Eq. (A3) we see that this small rate is due to an interference of the $n\bar{n}$ and G amplitudes, reducing the rate by an order of magnitude. While we expect this rate to be subdominant to $f_0(1370) \to 4\pi$ [4], and the full width is known only very imprecisely, this might still be the most worrying result of our fit. (In Ref. [55] it is argued that the 4π channel is subdominant to the 2π channel.)

2. Fit 2: Chiral suppression

Since we expect a chiral suppression in the $G \to PP$ decays, we perform a second fit with the ansatz

$$h_{2,\pi} = h_2, \qquad h_{2,K} = (f_K/f_\pi)^2 h_2, \tag{27}$$

$$h_{2,0} = (F_0/F)^2 h_2, \qquad h_{2,8} = (F_8/F)^2 h_2 \tag{28}$$

(motivated partly by Refs. [22,37]), which gives

$$r_a = 0.98^{+0.05}_{-0.04}, \qquad h_1 = (910 \pm 40)\ \text{MeV}, \tag{29}$$

$$h_2/h_1 = -0.59 \pm 0.02, \qquad h_3/h_1 = -1.0 \pm 0.2, \tag{30}$$

for the coupling constants and

$$\theta_{23} = 0.60 \pm 0.02, \qquad \theta_{12} = 0.57 \pm 0.04,$$
$$\theta_{13} = 0.51^{+0.01}_{-0.02}, \tag{31}$$

for the mixing angles, resulting in the mixing structure

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 0.74 & 0.47 & 0.49 \\ -0.68 & 0.55 & 0.49 \\ -0.04 & -0.69 & 0.72 \end{pmatrix} \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix} \tag{32}$$

$$= \begin{pmatrix} 0.87 & 0.04 & 0.49 \\ -0.24 & -0.84 & 0.49 \\ -0.43 & 0.54 & 0.72 \end{pmatrix} \begin{pmatrix} \text{singlet} \\ \text{octet} \\ G, \end{pmatrix} \tag{33}$$

with only minor quantitative differences to the flavor-blind fit. The mass matrix in the $n\bar{n}$–$s\bar{s}$–G basis is given by

[5]The 1σ range for parameter x has been obtained by fixing all other parameters to their best-fit values and then solving $\chi^2(x) = \chi^2_{\text{min}} + 1$ for x.

$$\begin{pmatrix} 1421.53 & -48.29 & -61.19 \\ -48.29 & 1574.06 & -143.85 \\ -61.19 & -143.85 & 1581.41 \end{pmatrix} \text{MeV}, \quad (34)$$

and

$$\begin{pmatrix} 1426.85 & -55.81 & -133.02 \\ -55.81 & 1568.74 & 82.13 \\ -133.02 & 82.13 & 1581.41 \end{pmatrix} \text{MeV}, \quad (35)$$

in the singlet-octet-G basis. In addition, we obtain the partial width

$$\Gamma(f_0(1370) \to PP) = 4 \text{ MeV}, \quad (36)$$

and the ratios

$$\frac{\Gamma(J/\psi \to \gamma f_0(1370))}{\Gamma(J/\psi \to \gamma f_0(1710))} = 0.5,$$
$$\frac{\Gamma(J/\psi \to \gamma f_0(1710))}{\Gamma(J/\psi \to \gamma f_0(1500))} = 2.$$

The same comments as above apply regarding the width of $f_0(1370)$.

III. DISCUSSION AND CONCLUSION

With the present state of data, the above fits can obviously not convincingly decide on the main issues of glueball spectroscopy, namely the composition of the f_0 states and their decay properties. We consider the present attempt mainly as a preparation for the interpretation of future data. Nevertheless, in both fits we find that $f_0(1710)$ has the largest admixture of the 0^{++} glueball, whereas $f_0(1500)$ comes out close to the octet flavor structure. The very small rate of $f_0(1370) \to PP$ is particularly concerning, even though $f_0(1370) \to 4\pi$ is expected to make up most of the width. The coupling of the glueball to η_0 plays an important role in both fits and is numerically large, making η and η' final states crucial testing grounds. We certainly expect $f_0(1710) \to \eta\eta'$ to be visible in BESIII, which will hopefully clarify some of the issues. It may also be surprising to some that we obtain a better fit with flavor-blind glueball couplings.

We stress that the use of the two-angle scheme for η–η' mixing is important when discussing the $f_0 \to \eta\eta(')$ decay modes, as the usually employed one-angle scheme is inconsistent.

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APPENDIX: DECAY RATES AND RATIOS

The 3×3 orthogonal mixing matrix U that describes the mixing of the f_0 states and the $q\bar{q}$ and glueball states is defined by

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} \equiv \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = U \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix}, \quad (A1)$$

and for the numerical fit parametrized by three mixing angles θ_{12}, θ_{23}, and θ_{13} using the common PDG notation

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12} & -s_{23}c_{12} - c_{23}s_{13}s_{12} & c_{23}c_{13} \end{pmatrix} \quad (A2)$$

with $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. For the decay rates, we define $\rho_{2,X} \equiv h_{2,X}/h_1$ and $\rho_3 \equiv h_3/h_1$, leading to

$$\Gamma(F_i \to \pi\pi) = \frac{1}{8\pi}\frac{3h_1^2}{2}(\sqrt{2}U_{i1} + 2\rho_{2,\pi}U_{i3})^2 \frac{p_\pi}{m_i^2}. \quad (A3)$$

Here and below, $p_X = |\boldsymbol{p}_X|$, where $\boldsymbol{p}_X$ denotes the three-momentum of X in the two-body decay $F_i \to XY$. Introducing

$$A_i \equiv \sqrt{2}U_{i1} + 4r_aU_{i2} + 6\rho_{2,8}U_{i3}, \quad (A4)$$

$$B_i \equiv U_{i1} - \sqrt{2}r_aU_{i2}, \quad (A5)$$

$$C_i \equiv 2[\sqrt{2}U_{i1} + r_aU_{i2} + 3(\rho_{2,0} + 3\rho_3)U_{i3}], \quad (A6)$$

we find the ratios

$$\frac{\Gamma(F_i \to \pi\pi)}{\Gamma(F_i \to KK)} = \frac{3}{2}\left(\frac{\sqrt{2}U_{i1} + 2\rho_{2,\pi}U_{i3}}{r_aU_{i1} + \sqrt{2}U_{i2} + 2\sqrt{2}\rho_{2,K}U_{i3}}\right)^2 \frac{p_\pi}{p_K}, \quad (A7)$$

$$\frac{\Gamma(F_i \to \eta\eta)}{\Gamma(F_i \to \pi\pi)} = \frac{1}{27}\left(\frac{F^2[F_0^2A_i\cos^2\theta_0 - 4F_0F_8B_i\cos\theta_0\sin\theta_8 + F_8^2C_i\sin^2\theta_8]}{F_0^2F_8^2\cos^2(\theta_0 - \theta_8)[\sqrt{2}U_{i1} + 2\rho_{2,\pi}U_{i3}]}\right)^2 \frac{p_\eta}{p_\pi}, \quad (A8)$$

$$\frac{\Gamma(F_i \to \eta\eta')}{\Gamma(F_i \to \eta\eta)} = \frac{1}{2}\left(\frac{F_0^2 A_i \sin 2\theta_0 + 4F_0F_8B_i\cos(\theta_0+\theta_8) - F_8^2C_i\sin 2\theta_8}{F_0^2A_i\cos^2\theta_0 - 4F_0F_8B_i\cos\theta_0\sin\theta_8 + F_8^2C_i\sin^2\theta_8}\right)^2 \frac{p_{\eta'}}{p_\eta}. \tag{A9}$$

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