F-wave heavy-light meson spectroscopy in QCD sum rules and heavy quark effective theory

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We study the F-wave $\bar{c}s$ heavy meson doublets $(2^+, 3^+)$ and $(3^+, 4^+)$. They have large orbital excitations $L = 3$, and may be good challenges (tests) for theoretical studies. To study them we use the method of QCD sum rule in the framework of heavy quark effective theory. Their masses are predicted to be $m_{(2^+,3^+)} = (3.45 \pm 0.25, 3.50 \pm 0.26)$ GeV and $m_{(3^+,4^+)} = (3.20 \pm 0.22, 3.26 \pm 0.23)$ GeV, with mass splittings $\Delta m_{(2^+,3^+)} = m_{3^+} - m_{2^+} = 0.046 \pm 0.030$ GeV and $\Delta m_{(3^+,4^+)} = 0.053 \pm 0.044$ GeV, respectively. We note that this is a pioneering work and these results are provisional.

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I. INTRODUCTION

Since 2003, big progress on the observations of heavy-light mesons has been made. When checking the 2014 edition of Particle Data Group (PDG) [1], we notice that charmed meson, charmed-strange meson, bottom meson, and bottom-strange meson families have become more and more abundant, which is due to these observed candidates of higher radial and orbital excitations of heavy-light mesons. In the following years, theorists and experimentalists will pay more attention to the study of heavy-light mesons with higher radial and orbital quantum numbers, especially with the running of LHCb and forthcoming BelleII.

With the charmed-strange meson family as an example, we introduce the research status of higher excitations of the heavy-light meson. There are two $1S$ states [$D_s(1968)$ and $D_s^*(2112)$] and four $1P$ states [$D_{s0}^*(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, and $D_{s2}^*(2573)$] established in PDG [1]. The observed $D_{s1}^*(2700)$ [2], $D_{s1}^*(2860)$ [3,4], and $D_{s3}^*(2860)$ [3,4] stimulated theorist's interest in studying the properties of $2S$ and $1D$ states [5–7], while the observation of $D_{sJ}(3040)$ [8] made us focus on the $2P$ states [9]. The research status of charmed-strange mesons can be found by

a minireview [10] and two recent systematical theoretical works [6,7]. The theoretical and experimental situation of charmed meson is similar to that of the charmed-strange meson, which can be found in Ref. [11].

Considering the above research status of the heavy-light meson, it is a suitable time to carry out the study of F-wave heavy-light mesons, since these $1F$ states will be reported in future experiments. The calculation of the mass spectrum of F-wave heavy-light mesons can provide valuable information to experimental search for them. Before the present paper, there were several quark model calculations of the mass spectrum of F-wave heavy-light mesons. For example, Ebert *et al.* adopted the relativistic quark model to get the heavy-light meson spectroscopy [12], which includes the $1F$ states. In Ref. [13], a relative quark model including the leading order correction in $1/m_{c,b}$ was applied to study heavy-light meson masses and light hadronic transition rates, where this study also contains $1F$ states. Recently, in Refs. [6,11], the masses of $1F$ states in charmed meson and charmed-strange meson families were obtained through the modified Godfrey-Isgur (GI) model, where the screening effect is considered in the introduced potential. For bottom and bottom-strange mesons, the masses of the $1F$ states were estimated by the GI model in Ref. [14].

Although there were quark model calculations of $1F$ states of heavy-light mesons, we notice that a QCD sum rule (QSR) study of mass spectrum of F-wave heavy-light

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mesons is still absent at present, which inspires our interest in performing the calculation of QSR of the mass spectrum of F-wave heavy-light mesons. In Refs. [15,16] Shifman wrote about QSR that: "One failure is quite obvious: the large-spin hadrons. Indeed, the latter have parametrically large sizes and a 'sausage-like shape' (growing with spin) and, therefore, it is quite clear that the basic idea of the method—extrapolation from short to intermediate distances—is not applicable. Practically, we have to stop at $S = 2$." However, it is still worth a try to apply QSR to study F-wave heavy mesons, because (a) we have used the same method to best study D-wave heavy mesons [17] and P-wave heavy baryons [18]; and (b) the LHCb experiments have just observed D-wave heavy mesons [3,4], and F-wave heavy mesons are expected in the following experiments. Hence, the present pioneering study not only provides an important hint to experimental exploration of F-wave heavy-light mesons, but also tests the applicability of QSR when applying QSR to study such higher radial excitations. This can be useful for quantifying potential overextensions of QSR in order to inspire ideas for its improvement, especially with future experimental data on F-wave heavy mesons.

The F-wave $\bar{Q}s$ ($Q = c$, b) heavy mesons have large orbital excitations $L = 3$, and may be good challenges (tests) for theoretical studies. Based on the heavy quark effective theory (HQET) [19–21], we can classify them into two doublets, $(2^+, 3^+)$ and $(3^+, 4^+)$, the light components of which have $j_l^{P_l} = 5/2^-$ and $j_l^{P_l} = 7/2^-$, respectively. In this paper we shall use the method of QCD sum rule [22,23] to study them, which has been successfully applied to study the ground state (S-wave) heavy meson doublet $(0^-, 1^-)$ [24–31], the P-wave heavy meson doublets $(0^+, 1^+)$ and $(1^+, 2^+)$ [32–36], and the D-wave heavy meson doublets $(1^-, 2^-)$ and $(2^-, 3^-)$ [17]. In this paper we shall follow the procedures used in these references, and study the F-wave $\bar{c}s$ heavy meson doublets $(2^+, 3^+)$ and $(3^+, 4^+)$. In the calculations we shall take into account the $\mathcal{O}(1/m_Q)$ corrections, where m_Q is the heavy quark mass. We note that the convergence of this $1/m_Q$ expansion can be problematic because F-wave heavy mesons (probably) have masses significantly larger than the heavy quark mass. However, we still hope that the leading terms and the $\mathcal{O}(1/m_Q)$ corrections could capture sufficiently much of the most important qualitative physics. We shall also carefully check this convergence in Sec. IV.

This paper is organized as follows. After this Introduction, we construct the F-wave $\bar{c}s$ interpolating currents for the heavy meson doublets $(2^+, 3^+)$ and $(3^+, 4^+)$ in Sec. II. These currents are then used to perform QCD sum rule analyses in the framework of HQET both at the leading order and at the $\mathcal{O}(1/m_Q)$ order. The calculations are done in Secs. II and III, and the results are summarized in Sec. IV.

II. THE SUM RULES AT THE LEADING ORDER (IN THE $m_Q \to \infty$ LIMIT)

The heavy meson interpolating currents have been systematically constructed in Refs. [32–34]. Here we follow Ref. [17] and briefly show how we construct the F-wave interpolating currents. We denote them as $J_{j,P,j_l}^{\alpha_1 \dots \alpha_j}$, where j and P are the total angular momentum and parity of the heavy meson, and j_l is the total angular momentum of the light components (containing three orbital excitations). We have the following relation:

$$\vec{j} = \vec{j}_l \otimes \vec{s}_Q, \tag{1}$$

where $s_Q = 1/2$ is the spin of the heavy quark.

To construct the F-wave interpolating currents, we just need to add three derivatives to the pseudoscalar current $\bar{h}_v \gamma_5 q$ of $J^P = 0^-$ and the vector current $\bar{h}_v \gamma_\mu q$ of $J^P = 1^-$. By doing this, the three orbital excitations can be explicitly written up. We act them on the light (strange) quark, and the obtained field has either $j_l^{P_l} = 5/2^-$:

$$\mathcal{D}_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} \mathcal{D}_t^{\beta} \times \gamma_\beta \gamma_5 q, \tag{2}$$

or $j_l^{P_l} = 7/2^-$:

$$\mathcal{D}_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} \mathcal{D}_t^{\alpha_3} \times q, \tag{3}$$

where $D_t^\mu = D^\mu - (D \cdot v)v^\mu$ with $D^\mu = \partial^\mu - igA^\mu$. Some other notations are: $\gamma_t^\mu = \gamma^\mu - \rlap{/}{v}v^\mu$, h_v denotes the heavy quark field in HQET, v is the velocity of the heavy quark, and $g_t^{\alpha_1\alpha_2} = g^{\alpha_1\alpha_2} - v^{\alpha_1}v^{\alpha_2}$ denotes the transverse metric tensor.

We use Eq. (2) of $j_l^{P_l} = 5/2^-$ to construct the interpolating currents coupling to the F-wave $(2^+, 3^+)$ spin doublet, based on $\bar{h}_v \gamma_5 q$ and $\bar{h}_v \gamma_\mu q$:

$$J_{x,+,5/2}^{\dagger \alpha_1 \alpha_2} = \bar{h}_v \gamma_5 \times \mathcal{D}_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} \mathcal{D}_t^{\beta} \times \gamma_\beta \gamma_5 q, \tag{4}$$

$$J_{y,+,5/2}^{\dagger \alpha_1 \alpha_2 \alpha_3} = \bar{h}_v \gamma_t^{\alpha_3} \times \mathcal{D}_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} \mathcal{D}_t^{\beta} \times \gamma_\beta \gamma_5 q. \tag{5}$$

Here x and y mean that these two currents are not pure 2^+ nor 3^+, while we can project out the two pure ones:

$$J_{2,+,5/2}^{\dagger \alpha_1 \alpha_2} = \sqrt{\frac{5}{6}} \bar{h}_v (-i)^3 \mathcal{S}_2 \left[\mathcal{D}_t^{\alpha_2} \left(\mathcal{D}_t^{\alpha_1} - \frac{2}{5} \gamma_t^{\alpha_1} \rlap{/}{D}_t \right) \rlap{/}{D}_t \right] q, \tag{6}$$

$$J_{3,+,5/2}^{\dagger \alpha_1 \alpha_2 \alpha_3} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma^5 (-i)^3 \mathcal{S}_3 [\gamma_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} \mathcal{D}_t^{\alpha_3} \rlap{/}{D}_t] q, \tag{7}$$

where $\mathcal{S}_j$ denotes symmetrization and subtracting the trace terms in the sets $(\alpha_1 \dots \alpha_j)$. We note that the expressions of

these currents have been modified to be consistent with Refs. [32–34].

Similarly, we use Eq. (3) of $j_l^{P_l} = 7/2^-$ to construct the interpolating currents coupling to the F-wave $(3^+, 4^+)$ spin doublet:

$$J_{3,+,7/2}^{\dagger\alpha_1\alpha_2\alpha_3} = \sqrt{\frac{7}{8}}\bar{h}_v\gamma_5(-i)^3\mathcal{S}_3\left[\mathcal{D}_t^{\alpha_2}\mathcal{D}_t^{\alpha_3}\left(\mathcal{D}_t^{\alpha_1} - \frac{3}{7}\gamma_t^{\alpha_1}\mathcal{D}_t\right)\right]q, \tag{8}$$

$$J_{4,+,7/2}^{\dagger\alpha_1\alpha_2\alpha_3\alpha_4} = \sqrt{\frac{1}{2}}\bar{h}_v(-i)^3\mathcal{S}_4[\gamma_t^{\alpha_1}\mathcal{D}_t^{\alpha_2}\mathcal{D}_t^{\alpha_3}\mathcal{D}_t^{\alpha_4}]q. \tag{9}$$

These interpolating currents are then used to perform QCD sum rule analyses. As discussed in Refs. [32–34], we do not need to investigate all of them, but just choose $J_{3,+,5/2}^{\dagger\alpha_1\alpha_2\alpha_3}$ and $J_{4,+,7/2}^{\dagger\alpha_1\alpha_2\alpha_3\alpha_4}$, because the calculation using these two currents are a bit simpler [to be technically precise, we use non-symmetrized currents to calculate the operator product expansion (OPE) and then do the "symmetrization and subtracting the trace terms"]. Moreover, we shall fix q to be the strange quark in the following, because we are mainly studying $\bar{c}s$ heavy mesons in this paper.

We follow the procedures used in Ref. [17], and assume $|j, P, j_l\rangle$ to be the heavy meson state with the quantum numbers j, P and j_l in the $m_Q \to \infty$ limit. The relevant interpolating field couples to it through

$$\langle 0|J_{j,P,j_l}^{\alpha_1...\alpha_j}|j', P', j_l'\rangle = f_{P,j_l}\delta_{jj'}\delta_{PP'}\delta_{j_lj_l'}\eta_t^{\alpha_1...\alpha_j}, \tag{10}$$

where f_{P,j_l} denotes the decay constant, and $\eta_t^{\alpha_1...\alpha_j}$ denotes the transverse, traceless, and symmetric polarization tensor, satisfying

$$\eta_t^{\alpha_1...\alpha_j}\eta_t^{*\beta_1...\beta_j} = \mathcal{S}_j'[\tilde{g}_t^{\alpha_1\beta_1}...\tilde{g}_t^{\alpha_j\beta_j}]. \tag{11}$$

In this expression $\tilde{g}_t^{\mu\nu} = g_t^{\mu\nu} - q_t^\mu q_t^\nu/m^2$, and $\mathcal{S}_j'$ denotes symmetrization and subtracting the trace terms in the sets $(\alpha_1...\alpha_j)$ and $(\beta_1...\beta_j)$. Based on Eq. (10), we can construct the two-point correlation function

$$\Pi_{j,P,j_l}^{\alpha_1...\alpha_j,\beta_1...\beta_j}(\omega)$$
$$= i\int d^4x e^{ikx}\langle 0|T[J_{j,P,j_l}^{\alpha_1...\alpha_j}(x)J_{j,P,j_l}^{\dagger\beta_1...\beta_j}(0)]|0\rangle$$
$$= (-1)^j\mathcal{S}_j'[\tilde{g}_t^{\alpha_1\beta_1}...\tilde{g}_t^{\alpha_j\beta_j}]\Pi_{j,P,j_l}(\omega), \tag{12}$$

and calculate it at the hadron level:

$$\Pi_{j,P,j_l}(\omega) = \frac{f_{P,j_l}^2}{2\bar{\Lambda}_{P,j_l} - \omega} + \text{higher states}, \tag{13}$$

where $\omega = 2v \cdot k$ denotes twice the external off-shell energy. $\bar{\Lambda}_{P,j_l} = \bar{\Lambda}_{j_l-1/2,P,j_l} = \bar{\Lambda}_{j_l+1/2,P,j_l}$ is defined to be

$$\bar{\Lambda}_{P,j_l} \equiv \lim_{m_Q\to\infty}(m_{j,P,j_l} - m_Q), \tag{14}$$

where m_{j,P,j_l} is the mass of the lowest-lying state which $J_{j,P,j_l}^{\alpha_1...\alpha_j}(x)$ couples to.

We can also calculate Eq. (12) at the quark and gluon level using the method of QCD sum rule in the framework of the heavy quark effective theory, i.e., we insert Eqs. (7) and (9) into Eq. (12), perform the Borel transformation, and then obtain (see Refs. [17,32–35] for details)

$$\Pi_{3,+,5/2}(\omega_c, T)$$
$$= f_{+,5/2}^2 e^{-2\bar{\Lambda}_{+,5/2}/T}$$
$$= \int_{2m_s}^{\omega_c}\left[\frac{3}{17920\pi^2}\omega^8 + \frac{3m_s}{8960\pi^2}\omega^7 - \frac{3m_s^2}{1280\pi^2}\omega^6\right.$$
$$\left. - \frac{\langle g_s^2 GG\rangle}{144\pi^2}\omega^4\right]e^{-\omega/T}d\omega, \tag{15}$$

$$\Pi_{4,+,7/2}(\omega_c, T)$$
$$= f_{+,7/2}^2 e^{-2\bar{\Lambda}_{+,7/2}/T}$$
$$= \int_{2m_s}^{\omega_c}\left[\frac{3}{17920\pi^2}\omega^8 + \frac{3m_s}{8960\pi^2}\omega^7 - \frac{3m_s^2}{1280\pi^2}\omega^6\right.$$
$$\left. - \frac{19\langle g_s^2 GG\rangle}{3072\pi^2}\omega^4\right]e^{-\omega/T}d\omega. \tag{16}$$

These two sum rules for $(2^+, 3^+)$ and $(3^+, 4^+)$ are similar. Similarly to Ref. [17], the quark condensate $\langle \bar{q}q\rangle$ and the mixed condensate $\langle g_s\bar{q}\sigma Gq\rangle$ both vanish, making the convergence of Eqs. (15) and (16) quite good. This can be easily verified because we need to apply as many as six covariant derivatives to the light quark propagator,

$$iS_q^{ab}(y, x)$$
$$\equiv \langle 0|T[q^a(y)\bar{q}^b(x)]|0\rangle$$
$$= \frac{i\delta^{ab}(\hat{y}-\hat{x})}{2\pi^2(y-x)^4} - \frac{\delta^{ab}}{12}\langle \bar{q}q\rangle - \frac{\delta^{ab}(y-x)^2}{192}\langle g_c\bar{q}\sigma Gq\rangle$$
$$- \frac{\delta^{ab}m_q}{4\pi^2(y-x)^2} + \frac{i\delta^{ab}m_q}{48}\langle \bar{q}q\rangle(\hat{y}-\hat{x}) + \frac{i\delta^{ab}(\hat{y}-\hat{x})m_q^2}{8\pi^2(y-x)^2}$$
$$- \frac{i}{32\pi^2}\frac{\lambda_{ab}^n}{2}g_c G_{\mu\nu}^n\frac{1}{(y-x)^2}(\sigma^{\mu\nu}(\hat{y}-\hat{x}) + (\hat{y}-\hat{x})\sigma^{\mu\nu})$$
$$+ \frac{1}{4\pi^2}\frac{\lambda_{ab}^n}{2}g_c G_{\mu\nu}^n\frac{1}{(y-x)^4}(\hat{y}-\hat{x})y_\mu x_\nu. \tag{17}$$

Differently, we need to carefully deal with the gluon terms contained in these covariant derivatives in order to evaluate the gluon condensate, which gives a significant contribution. The gluon condensate and the strange quark mass take the following values [32–35,37]:

$$\left\langle \frac{\alpha_s}{\pi} GG \right\rangle = 0.005 \pm 0.004 \text{ GeV}^4, \qquad (18)$$

$$m_s = 0.15 \text{ GeV}. \qquad (19)$$

We note that the radiative corrections are not taken into account in our calculations, which can be important but not easy to evaluate, because the six covariant derivatives also contribute to them (see discussions related to f_B in Ref. [38] and related references). However, we expect that they would lead an uncertainty significantly smaller than the gluon condensate and the charm quark mass. Hence, we shall discuss the change of the latter two parameters in Sec. IV, but do not discuss the radiative corrections any more.

To obtain $\bar{\Lambda}_{P,j_l}$, we just need to differentiate Log [Eq. (15)] and Log [Eq. (16)] with respect to $[-2/T]$:

$$\bar{\Lambda}_{P,j_l}(\omega_c, T) = \frac{\partial[\text{Log}\Pi_{j,P,j_l}(\omega_c, T)]}{\partial[-2/T]}. \qquad (20)$$

Then we can use it to further evaluate f_{P,j_l}:

$$f_{P,j_l}(\omega_c, T) = \sqrt{e^{2\bar{\Lambda}_{P,j_l}(\omega_c,T)/T} \times \Pi_{j,P,j_l}(\omega_c, T)}. \qquad (21)$$

There are two free parameters in these equations: the Borel mass T and the threshold value ω_c. We need to fix these two parameters to evaluate $\bar{\Lambda}_{P,j_l}(\omega_c, T)$ and $f_{P,j_l}(\omega_c, T)$.

We use two criteria to fix the Borel mass T. One criterion is to require that the high-order power corrections be less than 30% to determine its lower limit $T_{\min}$:

$$\text{Convergence (CVG)} \equiv \left| \frac{\Pi_{j,P,j_l}^{\text{high-order}}(\infty, T)}{\Pi_{j,P,j_l}(\infty, T)} \right| \leq 30\%, \qquad (22)$$

where $\Pi_{j,P,j_l}^{\text{high-order}}(\omega_c, T)$ denotes the high-order power corrections, for example,

$$\Pi_{3,+,5/2}^{\text{high-order}}(\omega_c, T) = \int_{2m_s}^{\omega_c} \left[-\frac{\langle g_s^2 GG \rangle}{144\pi^2} \omega^4 \right] e^{-\omega/T} d\omega. \qquad (23)$$

The other criterion is to require that the pole contribution (PC) be larger than 10% to determine its upper limit $T_{\max}$:

$$\text{PC} \equiv \frac{\Pi_{j,P,j_l}(\omega_c, T)}{\Pi_{j,P,j_l}(\infty, T)} \geq 10\%. \qquad (24)$$

Altogether we obtain a Borel window $T_{\min} < T < T_{\max}$ for a fixed threshold value ω_c. This ω_c is the other free parameter, which will be fixed in Sec. IV. Here we proceed using the sum rule (15) and taking $\omega_c = 3.0$ GeV as an example. Using this value of ω_c, we obtain a Borel window 0.376 GeV $< T < 0.513$ GeV for the sum rule (15): the
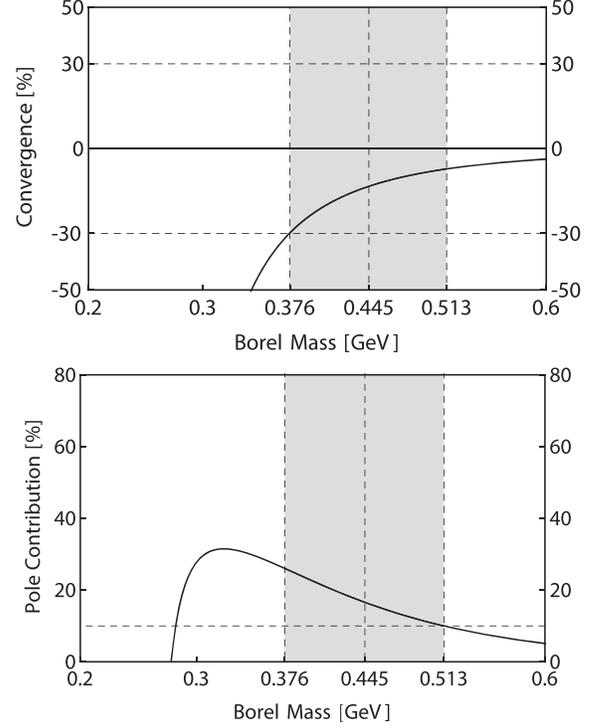
FIG. 1.   The variations of CVG and PC with respect to the Borel mass T. The sum rule (15) for the current $J_{3,+,5/2}^{\dagger \alpha_1 \alpha_2 \alpha_3}$ is used in both figures.
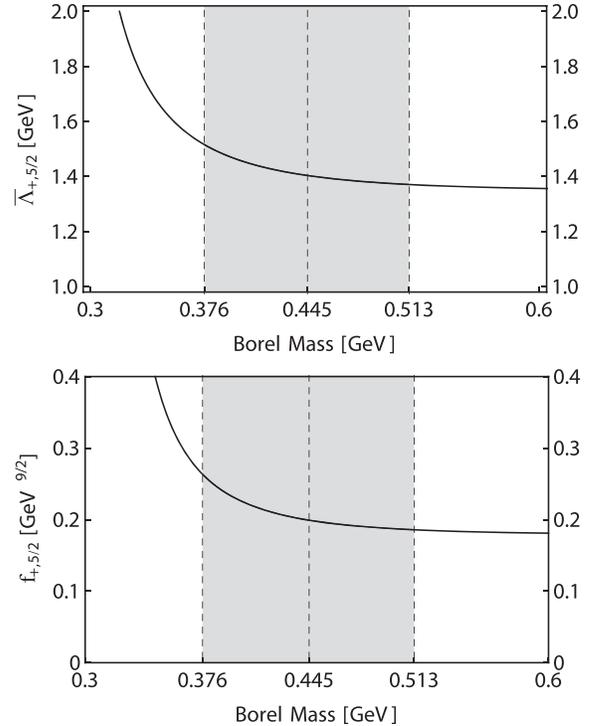
FIG. 2.   The variations of $\bar{\Lambda}_{+,5/2}$ (top) and $f_{+,5/2}$ (bottom) with respect to the Borel mass T. In both figures we take $\omega_c = 3.0$ GeV and the Borel window is 0.376 GeV $< T < 0.513$ GeV.

TABLE I. The mass of the heavy mesons belonging to the $(2^+, 3^+)$ spin doublet $m_{D_{s2}^*}$ and $m_{D_{s3}}$, and their differences $\Delta m_{+,5/2} = m_{D_{s3}} - m_{D_{s2}^*}$, for various threshold values ω_c. We also list Borel windows, $\bar{\Lambda}_{+,5/2}$, $f_{+,5/2}$, $K_{+,5/2}$, and $\Sigma_{+,5/2}$ for completeness.

ω_c [GeV]	Borel window [GeV]	$\bar{\Lambda}$ [GeV]	f[GeV$^{9/2}$]	K[GeV^2]	Σ[GeV^2]	$m_{D_{s2}^*}$ [GeV]	$m_{D_{s3}}$ [GeV]	Δm [GeV]
2.7	[0.376, 0.426]	1.4835	0.1748	−3.9886	0.02557	3.5055	3.5656	0.0601
2.8	[0.376, 0.459]	1.4210	0.1701	−3.9779	0.01961	3.4491	3.4953	0.0462
2.9	[0.376, 0.488]	1.4028	0.1812	−4.0794	0.01613	3.4556	3.4935	0.0379
3.0	[0.376, 0.513]	1.4036	0.1992	−4.2317	0.01375	3.4895	3.5218	0.0323
3.1	[0.376, 0.537]	1.4151	0.2221	−4.4156	0.01203	3.5394	3.5677	0.0283
3.2	[0.376, 0.560]	1.4333	0.2491	−4.6213	0.01071	3.5997	3.6250	0.0253
3.3	[0.376, 0.582]	1.4555	0.2800	−4.8432	0.009658	3.6669	3.6897	0.0228
3.4	[0.376, 0.603]	1.4807	0.3146	−5.0789	0.008789	3.7395	3.7602	0.0207
3.5	[0.376, 0.624]	1.5081	0.3532	−5.3259	0.008076	3.8163	3.8353	0.0190

lower limit is determined by using the first criterion of CVG, as shown in the top panel of Fig. 1, and the upper limit is determined by using the second criterion of PC, as shown in the bottom panel of Fig. 1.

Finally, we show the variations of $\bar{\Lambda}_{+,5/2}$ and $f_{+,5/2}$ with respect to the Borel mass T in Fig. 2. We show them in a broader region $0.3 \text{ GeV} < T < 0.6 \text{ GeV}$, while these curves are more stable in the Borel window $0.376 \text{ GeV} < T < 0.513 \text{ GeV}$. We obtain the following numerical results:

$$\bar{\Lambda}_{+,5/2} = 1.40 \text{ GeV}, \qquad f_{+,5/2} = 0.20 \text{ GeV}^{9/2}, \quad (25)$$

where the central value corresponds to $T = 0.445 \text{ GeV}$ and $\omega_c = 3.0 \text{ GeV}$.

The procedures are the same for different values of ω_c. We give it a large range $2.5 \text{ GeV} < \omega_c < 3.5 \text{ GeV}$, but find that there are Borel windows as long as $s_0 \geq 2.7 \text{ GeV}^2$. The corresponding Borel windows and the numerical results of $\bar{\Lambda}_{+,5/2}$ and $f_{+,5/2}$ are listed in Table I. We note that this table is shown in Sec. IV, where we shall fix ω_c to evaluate $m_{2,+,5/2}$ and $m_{3,+,5/2}$.

Similarly, we use the sum rule (16) to perform QCD sum rule analyses. The Borel windows and the numerical results of $\bar{\Lambda}_{+,7/2}$ and $f_{+,7/2}$ for various values of ω_c are listed in Table II, also shown in Sec. IV. Here we show the variations of $\bar{\Lambda}_{+,7/2}$ and $f_{+,7/2}$ with respect to the Borel mass T in Fig. 3, when we take $\omega_c = 3.0 \text{ GeV}$ and the Borel window is obtained to be $0.365 \text{ GeV} < T < 0.518 \text{ GeV}$. Again these curves are more stable inside this window. We obtain the following numerical results:

$$\bar{\Lambda}_{+,7/2} = 1.37 \text{ GeV}, \qquad f_{+,7/2} = 0.19 \text{ GeV}^{9/2}, \quad (26)$$

where the central value corresponds to $T = 0.442 \text{ GeV}$ and $\omega_c = 3.0 \text{ GeV}$.

III. THE SUM RULES AT THE $\mathcal{O}(1/m_Q)$ ORDER

In the previous section we have calculated $\bar{\Lambda}_{P,j_l} \equiv \lim_{m_Q \to \infty}(m_{j,P,j_l} - m_Q)$, the value of which is the same for both $\bar{\Lambda}_{j_l-1/2,P,j_l}$ and $\bar{\Lambda}_{j_l+1/2,P,j_l}$. To differentiate the masses within the same doublet, i.e., between $m_{j_l-1/2,P,j_l}$ and $m_{j_l+1/2,P,j_l}$, we need to work at the $O(1/m_Q)$ order, which will be done in this section.

TABLE II. The mass of the heavy mesons belonging to the $(3^+, 4^+)$ spin doublet $m_{D_{s3}'}$ and $m_{D_{s4}^*}$, and their differences $\Delta m_{+,7/2} = m_{D_{s4}^*} - m_{D_{s3}'}$, for various threshold values ω_c. We also list Borel windows, $\bar{\Lambda}_{+,7/2}$, $f_{+,7/2}$, $K_{+,7/2}$, and $\Sigma_{+,7/2}$ for completeness.

ω_c [GeV]	Borel window [GeV]	$\bar{\Lambda}$ [GeV]	f[GeV$^{9/2}$]	K[GeV^2]	Σ[GeV^2]	$m_{D_{s3}'}$ [GeV]	$m_{D_{s4}^*}$ [GeV]	Δm [GeV]
2.6	[0.365, 0.404]	1.4690	0.1594	−3.0566	0.02796	3.2940	3.3817	0.0877
2.7	[0.365, 0.440]	1.3884	0.1486	−2.9815	0.02076	3.2114	3.2765	0.0651
2.8	[0.365, 0.469]	1.3649	0.1571	−3.0294	0.01682	3.2043	3.2570	0.0527
2.9	[0.365, 0.495]	1.3632	0.1725	−3.1262	0.01420	3.2262	3.2707	0.0445
3.0	[0.365, 0.518]	1.3735	0.1926	−3.2523	0.01234	3.2645	3.3032	0.0387
3.1	[0.365, 0.542]	1.3907	0.2165	−3.3976	0.01090	3.3126	3.3469	0.0343
3.2	[0.365, 0.564]	1.4127	0.2442	−3.5579	0.009792	3.3680	3.3988	0.0308
3.3	[0.365, 0.585]	1.4379	0.2754	−3.7294	0.008901	3.4284	3.4563	0.0279
3.4	[0.365, 0.606]	1.4651	0.3102	−3.9120	0.008126	3.4928	3.5183	0.0255
3.5	[0.365, 0.627]	1.4941	0.3488	−4.1013	0.007510	3.5600	3.5836	0.0236
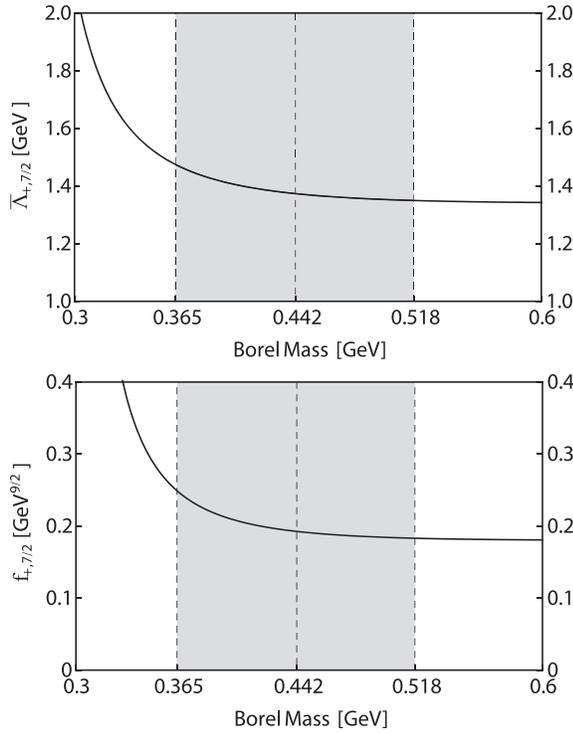
FIG. 3.   The variations of $\bar{\Lambda}_{+,7/2}$ (top) and $f_{+,7/2}$ (bottom) with respect to the Borel mass T. In both figures we take $\omega_c = 3.0$ GeV and the Borel window is 0.365 GeV $< T < 0.518$ GeV.

Again we follow the procedures used in Ref. [17] (see Refs. [17,32–35] for details), and write the pole term on the hadron side, Eq. (13), as

$$
\Pi(\omega)_{\text{pole}} = \frac{(f + \delta f)^2}{2(\bar{\Lambda} + \delta m) - \omega}
$$

$$
= \frac{f^2}{2\bar{\Lambda} - \omega} - \frac{2\delta m f^2}{(2\bar{\Lambda} - \omega)^2} + \frac{2f\delta f}{2\bar{\Lambda} - \omega}, \quad (27)
$$

where we have omitted the subscripts j, P, j_l for simplicity. The corrections to the mass m_{j,P,j_l} can be evaluated through

$$
\delta m_{j,P,j_l} = -\frac{1}{4m_Q}(K_{P,j_l} + d_{j,j_l}C_{\text{mag}}\Sigma_{P,j_l}), \quad (28)
$$

where $d_{j_l-1/2,j_l} = 2j_l + 2$, $d_{j_l+1/2,j_l} = -2j_l$, and $C_{\text{mag}}(m_Q/\mu) = [\alpha_s(m_Q)/\alpha_s(\mu)]^{3/\beta_0}$ with $\beta_0 = 11 - 2n_f/3$. The two corrections K_{P,j_l} and Σ_{P,j_l} come from the non-relativistic kinetic energy and the chromomagnetic inter-action, respectively. We can calculate them using the method of the QCD sum rule in the framework of HQET. We obtain the following two equations for $K_{+,5/2}$ and $\Sigma_{+,5/2}$:

$$
f^2_{+,5/2}K_{+,5/2}e^{-2\bar{\Lambda}_{+,5/2}/T}
$$

$$
= \int_{2m_s}^{\omega_c}\left[-\frac{1}{9216\pi^2}\omega^{10} + \frac{161\langle g_s^2 GG\rangle}{30720\pi^2}w^6\right]e^{-\omega/T}d\omega,
$$

$$(29)$$

$$
f^2_{+,5/2}\Sigma_{+,5/2}e^{-2\bar{\Lambda}_{+,5/2}/T} = \int_{2m_s}^{\omega_c}\left[\frac{\langle g_s^2 GG\rangle}{25600\pi^2}w^6\right]e^{-\omega/T}d\omega,
$$

$$(30)$$

and the following two equations for $K_{+,7/2}$ and $\Sigma_{+,7/2}$:

$$
f^2_{+,7/2}K_{+,7/2}e^{-2\bar{\Lambda}_{+,7/2}/T}
$$

$$
= \int_{2m_s}^{\omega_c}\left[-\frac{3}{35840\pi^2}\omega^{10} + \frac{211\langle g_s^2 GG\rangle}{61440\pi^2}w^6\right]e^{-\omega/T}d\omega,
$$

$$(31)$$

$$
f^2_{+,7/2}\Sigma_{+,7/2}e^{-2\bar{\Lambda}_{+,7/2}/T} = \int_{2m_s}^{\omega_c}\left[\frac{\langle g_s^2 GG\rangle}{26800\pi^2}w^6\right]e^{-\omega/T}d\omega.
$$

$$(32)$$

Again, these sum rules for $(2^+, 3^+)$ and $(3^+, 4^+)$ are similar. Then $K_{+,5/2}$, $\Sigma_{+,5/2}$, $K_{+,7/2}$, and $\Sigma_{+,7/2}$ can be simply obtained by dividing these equations with respect to the sum rules (15) and (16). We evaluate their numerical
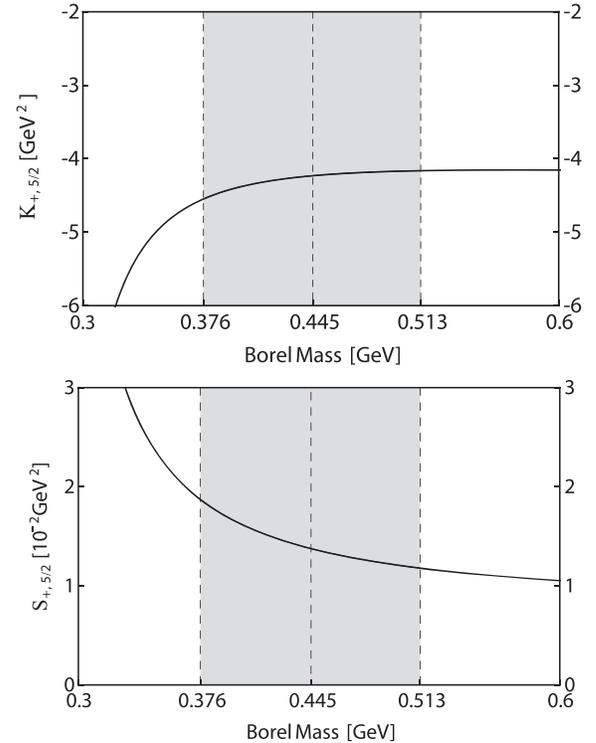
FIG. 4.   The variation of $K_{+,5/2}$ (top) and $\Sigma_{+,5/2}$ (bottom) with respect to the Borel mass T. In both figures we take $\omega_c = 3.0$ GeV and the Borel window is 0.376 GeV $< T < 0.513$ GeV.
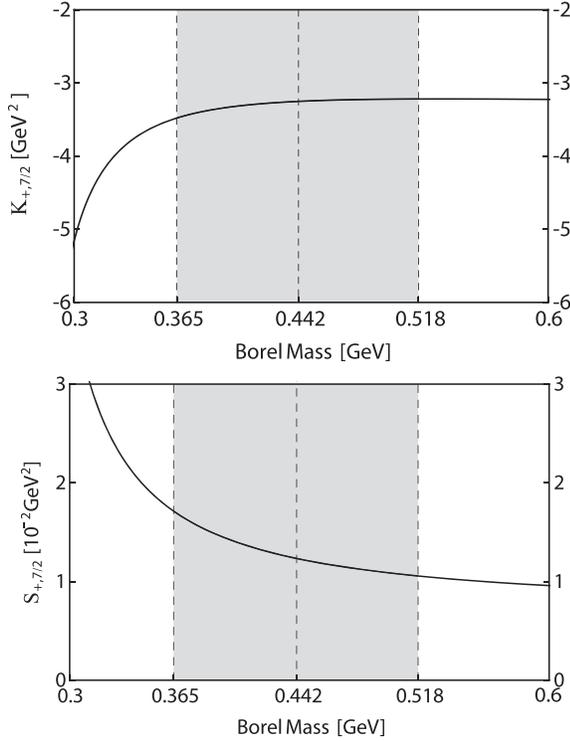
FIG. 5. The variation of $K_{+,7/2}$ (top) and $\Sigma_{+,7/2}$ (bottom) with respect to the Borel mass T. In both figures we take $\omega_c = 3.0$ GeV and the Borel window is 0.365 GeV $< T < 0.518$ GeV.

results in the Borel windows derived in the previous section, and list them for various values of ω_c in Tables I and II.

Here we take $\omega_c = 3.0$ GeV as an example, and show their variations with respect to the Borel mass T in Figs. 4 and 5. We use the Borel windows 0.376 GeV $< T < 0.513$ GeV for $K_{+,5/2}$ and $\Sigma_{+,5/2}$, and obtain the following numerical results:

$$K_{+,5/2} = -4.23 \text{ GeV}^2, \qquad \Sigma_{+,5/2} = 0.014 \text{ GeV}^2, \quad (33)$$

where the central value corresponds to $T = 0.445$ GeV and $\omega_c = 3.0$ GeV. We use the same Borel window 0.365 GeV $< T < 0.518$ GeV for $K_{+,7/2}$ and $\Sigma_{+,7/2}$, and obtain the following numerical results:

$$K_{+,7/2} = -3.25 \text{ GeV}^2, \qquad \Sigma_{+,7/2} = 0.012 \text{ GeV}^2, \quad (34)$$

where the central value corresponds to $T = 0.442$ GeV and $\omega_c = 3.0$ GeV.

IV. NUMERICAL RESULTS AND DISCUSSIONS

The mass of the F-wave $\bar{c}s$ heavy mesons can be obtained using Eqs. (14) and (28). We use $(m_{D_{s2}^*}, m_{D_{s3}})$ to denote the mass of the heavy mesons belonging to the $(2^+, 3^+)$ spin doublet, and they satisfy

$$\frac{1}{12}\left(5m_{D_{s2}^*} + 7m_{D_{s3}}\right) = m_c + \bar{\Lambda}_{+,5/2} - \frac{1}{4m_c}K_{+,5/2}, \quad (35)$$

$$m_{D_{s3}} - m_{D_{s2}^*} = \frac{3}{m_c}\Sigma_{+,5/2}. \quad (36)$$

We use $(m_{D_{s3}'}, m_{D_{s4}^*})$ to denote the mass of the heavy mesons belonging to the $(3^+, 4^+)$ spin doublet, and they satisfy

$$\frac{1}{16}\left(7m_{D_{s3}'} + 9m_{D_{s4}^*}\right) = m_c + \bar{\Lambda}_{+,7/2} - \frac{1}{4m_c}K_{+,7/2}, \quad (37)$$

$$m_{D_{s4}^*} - m_{D_{s3}'} = \frac{4}{m_c}\Sigma_{+,7/2}. \quad (38)$$

In this paper we use the charm quark mass $m_c = 1.275 \pm 0.025$ GeV, which is evaluated in the $\overline{\text{MS}}$ scheme [1]. Using the above equations, we calculate $(m_{D_{s2}^*}, m_{D_{s3}})$, $(m_{D_{s3}'}, m_{D_{s4}^*})$, and their differences for various threshold values ω_c. The results are listed in Tables I and II. For completeness, we also list Borel windows, $\bar{\Lambda}_{P,j_l}$, f_{P,j_l}, K_{P,j_l}, and Σ_{P,j_l} for various ω_c.

Now we can fix the threshold value ω_c. Our criterion is to require that the ω_c dependence of the mass prediction be the weakest. We show variations of $m_{D_{s2}^*}$ and $m_{D_{s4}^*}$ with respect to the threshold value ω_c in the top panels of Figs. 6 and 7, and quickly notice that this dependence is the weakest around $\omega_c \sim 2.8$ GeV for both cases. Accordingly, we choose the region 2.7 GeV $< \omega_c < 3.0$ GeV as our working region. We obtain the following numerical results for the $(2^+, 3^+)$ spin doublet:

$$m_{D_{s2}^*} = 3.45 \pm 0.25 \text{ GeV},$$

$$m_{D_{s3}} = 3.50 \pm 0.26 \text{ GeV},$$

$$\Delta m_{+,5/2} = 0.046 \pm 0.030 \text{ GeV}, \quad (39)$$

where the central value corresponds to $T = 0.418$ GeV and $\omega_c = 2.8$ GeV. Here the uncertainties are due to the Borel mass T, the threshold value ω_c, and the uncertainty of the gluon condensate $\langle \frac{\alpha_s}{\pi} GG \rangle = 0.005 \pm 0.001$ GeV^4. We obtain the following numerical results for the $(3^+, 4^+)$ spin doublet:

$$m_{D_{s3}'} = 3.20 \pm 0.22 \text{ GeV},$$

$$m_{D_{s4}^*} = 3.26 \pm 0.23 \text{ GeV},$$

$$\Delta m_{+,7/2} = 0.053 \pm 0.044 \text{ GeV}, \quad (40)$$

where the central value corresponds to $T = 0.417$ GeV and $\omega_c = 2.8$ GeV. However, we note that the mass differences within the same doublets, $\Delta m_{+,5/2} = m_{D_{s3}} - m_{D_{s2}^*}$ and $\Delta m_{+,7/2} = m_{D_{s4}^*} - m_{D_{s3}'}$, do depend on the threshold value ω_c, as shown in the bottom panels of Figs. 6 and 7.
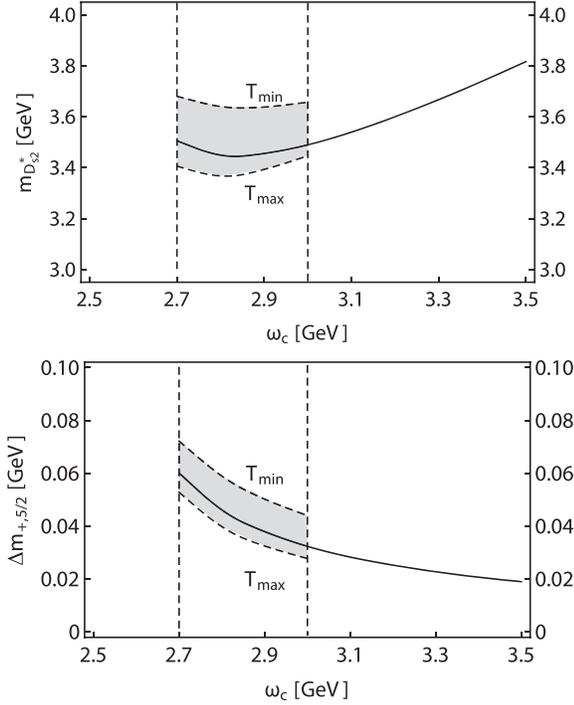
FIG. 6.   The variations of $m_{D_{s2}^*}$ (top) and $\Delta m_{+,5/2}$ (bottom) with respect to the threshold values ω_c. The upper and lower bands are obtained by using $T_{\min}$ and $T_{\max}$, respectively.
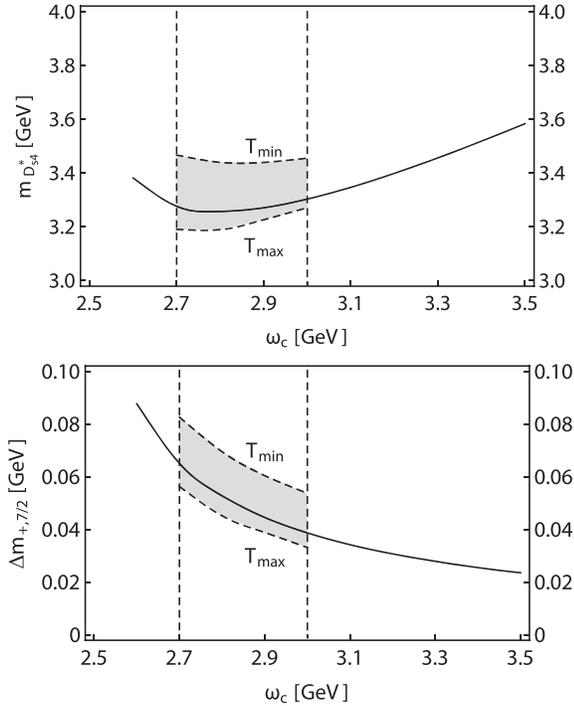
FIG. 7.   The variations of $m_{D_{s4}^*}$ (top) and $\Delta m_{+,7/2}$ (bottom) with respect to the threshold values ω_c. The upper and lower bands are obtained by using $T_{\min}$ and $T_{\max}$, respectively.

TABLE III.   Masses of F-wave charmed-strange ($\bar{c}s$) mesons (in GeV).

State	This work	State	Ref. [6]	Ref. [12]	Ref. [13]
2^+ in $(2^+, 3^+)$	3.45 ± 0.25	1^3F_2	3.159	3.230	3.224
3^+ in $(2^+, 3^+)$	3.50 ± 0.26	1^1F_3	3.151	3.266	3.247
3^+ in $(3^+, 4^+)$	3.20 ± 0.22	1^3F_3	3.157	3.254	3.203
4^+ in $(3^+, 4^+)$	3.26 ± 0.23	1^3F_4	3.143	3.300	3.220

The above analyses suggest that there is a heavy meson spin doublet $(2^+, 3^+)$ whose masses are around 3.45 and 3.50 GeV, and a spin doublet $(3^+, 4^+)$ whose masses are around 3.20 and 3.26 GeV. The latter is consistent with recent theoretical studies [6,12,13], while the former is larger but still within uncertainties, as shown in Table III. We note that the two sum rules for $(2^+, 3^+)$ and $(3^+, 4^+)$ are similar, see Eqs. (15), (16), (29), (31), (30), and (32), so the mass difference between $(2^+, 3^+)$ and $(3^+, 4^+)$ may be (partly) due to the theoretical uncertainty of the numerical analysis. Moreover, the expansion on the charm quark mass for the $(2^+, 3^+)$ spin doublet is

$$\text{Eq. (35)} \sim m_c + 1.4 \text{ GeV} + 1.0 \text{ GeV}, \qquad (41)$$

while the expansion for the $(3^+, 4^+)$ spin doublet has better convergence

$$\text{Eq. (37)} \sim m_c + 1.4 \text{ GeV} + 0.8 \text{ GeV}. \qquad (42)$$

This suggests that our results for the latter doublet are more reliable.

To make our analyses complete, we try to change the values of the parameters used in the previous analyses and redo the calculations:

(1) As shown in sum rules (15) and (16), the gluon condensate is important. Besides the value listed in Eq. (18), $\langle \frac{\alpha_s}{\pi} GG \rangle = 0.005 \pm 0.004$ GeV^4 [37,39], the value $\langle \frac{\alpha_s}{\pi} GG \rangle = 0.012 \pm 0.004$ GeV^4 is also widely used in QCD sum rule studies [22] (see Refs. [37,39] for detailed discussions). We use this value and redo the numerical analyses. The mass of the 2^+ heavy meson, $m_{D_{s2}^*}$, is shown in Fig. 8 with respect to the threshold value ω_c, using short-dashed curves. The obtained result is even larger than 4.0 GeV, which is not very reliable/reasonable.

(2) We change the charm quark mass from the $\overline{\text{MS}}$ value $m_c = 1.275 \pm 0.025$ GeV to its pole mass $m_c = 1.67 \pm 0.07$ [1], and redo the numerical analyses. The result is shown in Fig. 8 with respect to ω_c using a long-dashed curve. The obtained result is about 200 MeV larger than our previous result, suggesting that our results for the masses of the heavy mesons can have significant theoretical uncertainties (see also discussions in Ref. [17]).
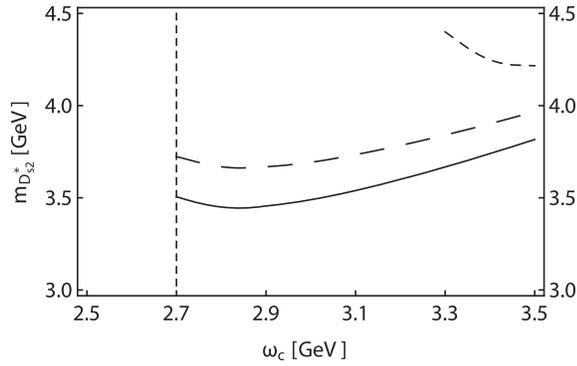
FIG. 8. The variations of $m_{D_{s2}^*}$ with respect to the threshold values ω_c. The solid curve is our previous result which has been plotted in the top panel of Fig. 6; the short-dashed curve is obtained by using the gluon condensate $\langle \frac{\alpha_s}{\pi} GG \rangle = 0.012$ GeV⁴ [22]; the long-dashed curve is obtained by using the pole mass of the charm quark $m_c = 1.67$ GeV [1].

We can similarly replace the charm quark by bottom quark and study the $\bar{b}s$ system [the factor C_{mag} in Eq. (28) is taken to be 0.8 [34,35]]. Again, these masses depend much on the bottom quark mass m_b, whose value has large uncertainties. When we use the 1S mass value $m_b = 4.66$ GeV [1], we can obtain the mass of the F-wave $\bar{b}s$ heavy-light mesons to be around 6.3 GeV, consistent with the results obtained in Refs. [12,14]. Their mass differences are $\Delta m_{+,5/2}^{[\bar{b}s]} \sim 0.010$ GeV and $\Delta m_{+,7/2}^{[\bar{b}s]} \sim 0.014$ GeV. However, if we replace the strange quark by up and down quarks, the sum rules (15) and (16) would become too simple to investigate the nonstrange D-wave heavy mesons.

In summary, in this paper we adopt the QSR approach to study the mass spectrum of F-wave heavy-light mesons in the framework of HQET. We obtain two similar sum rules for $(2^+, 3^+)$ and $(3^+, 4^+)$, see Eqs. (15) and (16), Eqs. (29) and (31), and Eqs. (30) and (32). Our results suggest that there is a $\bar{c}s$ heavy meson spin doublet $(2^+, 3^+)$ whose masses are $m_{(2^+,3^+)}^{[\bar{c}s]} = (3.45 \pm 0.25, 3.50 \pm 0.26)$ GeV,

with mass difference $\Delta m_{+,5/2}^{[\bar{c}s]} = 0.046 \pm 0.030$ GeV, and a spin doublet $(3^+, 4^+)$ whose masses are $m_{(3^+,4^+)}^{[\bar{c}s]} = (3.20 \pm 0.22, 3.26 \pm 0.23)$ GeV, with mass difference $\Delta m_{+,7/2}^{[\bar{c}s]} = 0.053 \pm 0.044$ GeV. We note that this is a pioneering work and these results are provisional.

Finally, we would like to note that this is a pioneering study applying HQET-based QSR to study F-wave heavy-light mesons (see also discussions in Sec. I). They have large orbital excitations $L = 3$, which can be explicitly written as three covariant derivatives, and are not easy to deal with. However, because the LHCb experiments have just observed D-wave heavy mesons [3,4], the theoretical analyses on F-wave heavy mesons, including our study in current paper, become helpful to the further experimental exploration of them. Moreover, they are also good challenges (tests) for theoretical studies. In the following experiments such as LHCb and BelleII, searching for higher excitations of heavy-light mesons will be an important task. We expect more experimental and theoretical progress on higher excitations of heavy-light mesons, which will make our knowledge of the heavy-light meson family become more and more abundant. This will improve our understanding to the nonperturbative behavior of QCD, and inspire ideas for the improvement of the QCD sum rule itself.

ACKNOWLEDGMENTS

[1] K. A. Olive *et al.* (Particle Data Group Collaboration), Review of particle physics, Chin. Phys. C **38**, 090001 (2014).

[2] B. Aubert *et al.* (BABAR Collaboration), Observation of a New D_s Meson Decaying to DK at a Mass of 2.86 GeV/c^2, Phys. Rev. Lett. **97**, 222001 (2006).

[3] R. Aaij *et al.* (LHCb Collaboration), Observation of Overlapping Spin-1 and Spin-3 $\bar{D}^0 K^-$ Resonances at Mass 2.86 GeV/c^2, Phys. Rev. Lett. **113**, 162001 (2014).

[4] R. Aaij *et al.* (LHCb Collaboration), Dalitz plot analysis of $B_s^0 \to \bar{D}^0 K^- \pi^+$ decays, Phys. Rev. D **90**, 072003 (2014).

[5] B. Zhang, X. Liu, W. Z. Deng, and S. L. Zhu, $D_{sJ}(2860)$ and $D_{sJ}(2715)$, Eur. Phys. J. C **50**, 617 (2007).

[6] Q. T. Song, D. Y. Chen, X. Liu, and T. Matsuki, Charmed-strange mesons revisited: Mass spectra and strong decays, Phys. Rev. D **91**, 054031 (2015).

[7] J. Segovia, D. R. Entem, and F. Fernandez, Charmed-strange meson spectrum: Old and new problems, Phys. Rev. D **91**, 094020 (2015).

[8] B. Aubert *et al.* (BABAR Collaboration), Study of D_{sJ} decays to $D^* K$ in inclusive $e^+ e^-$ interactions, Phys. Rev. D **80**, 092003 (2009).

[9] Z. F. Sun and X. Liu, Newly observed $D_{sJ}(3040)$ and the radial excitations of P-wave charmed-strange mesons, Phys. Rev. D **80**, 074037 (2009).

[10] X. Liu, The theoretical review of excited D/D_s mesons, Int. J. Mod. Phys. Conf. Ser. **02**, 147 (2011).

[11] Q. T. Song, D. Y. Chen, X. Liu, and T. Matsuki, Higher radial and orbital excitations in the charmed meson family, Phys. Rev. D **92**, 074011 (2015).

[12] D. Ebert, R. N. Faustov, and V. O. Galkin, Heavy-light meson spectroscopy and Regge trajectories in the relativistic quark model, Eur. Phys. J. C **66**, 197 (2010).

[13] M. Di Pierro and E. Eichten, Excited heavy-light systems and hadronic transitions, Phys. Rev. D **64**, 114004 (2001).

[14] Y. Sun, Q. T. Song, D. Y. Chen, X. Liu, and S. L. Zhu, Higher bottom and bottom-strange mesons, Phys. Rev. D **89**, 054026 (2014).

[15] M. A. Shifman, On quark-hadron duality for orbital excitations, Sov. J. Nucl. Phys. **36**, 749 (1982) [Yad. Fiz. **36**, 1290 (1982)].

[16] M. A. Shifman, QCD sum rules: The second decade, arXiv: hep-ph/9304253.

[17] D. Zhou, E. L. Cui, H. X. Chen, L. S. Geng, X. Liu, and S. L. Zhu, D-wave heavy-light mesons from QCD sum rules, Phys. Rev. D **90**, 114035 (2014).

[18] H. X. Chen, W. Chen, Q. Mao, A. Hosaka, X. Liu, and S. L. Zhu, P-wave charmed baryons from QCD sum rules, Phys. Rev. D **91**, 054034 (2015).

[19] B. Grinstein, The static quark effective theory, Nucl. Phys. **B339**, 253 (1990).

[20] E. Eichten and B. R. Hill, An effective field theory for the calculation of matrix elements involving heavy quarks, Phys. Lett. B **234**, 511 (1990).

[21] A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Heavy meson form-factors from QCD, Nucl. Phys. **B343**, 1 (1990).

[22] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, QCD and resonance physics. Sum rules, Nucl. Phys. **B147**, 385 (1979).

[23] L. J. Reinders, H. Rubinstein, and S. Yazaki, Hadron properties from QCD sum rules, Phys. Rep. **127**, 1 (1985).

[24] E. Bagan, P. Ball, V. M. Braun, and H. G. Dosch, QCD sum rules in the effective heavy quark theory, Phys. Lett. B **278**, 457 (1992).

[25] M. Neubert, Heavy meson form-factors from QCD sum rules, Phys. Rev. D **45**, 2451 (1992).

[26] M. Neubert, Heavy quark symmetry, Phys. Rep. **245**, 259 (1994).

[27] D. J. Broadhurst and A. G. Grozin, Operator product expansion in static quark effective field theory: Large perturbative correction, Phys. Lett. B **274**, 421 (1992).

[28] P. Colangelo, G. Nardulli, A. A. Ovchinnikov, and N. Paver, Semileptonic B decays into positive parity charmed mesons, Phys. Lett. B **269**, 201 (1991).

[29] P. Colangelo, G. Nardulli, and N. Paver, Semileptonic B decays into charmed p wave mesons and the heavy quark symmetry, Phys. Lett. B **293**, 207 (1992).

[30] P. Ball and V. M. Braun, Next-to-leading order corrections to meson masses in the heavy quark effective theory, Phys. Rev. D **49**, 2472 (1994).

[31] T. Huang and C. W. Luo, Light quark dependence of the Isgur-Wise function from QCD sum rules, Phys. Rev. D **50**, 5775 (1994).

[32] Y. B. Dai, C. S. Huang, M. Q. Huang, and C. Liu, QCD sum rules for masses of excited heavy mesons, Phys. Lett. B **390**, 350 (1997).

[33] Y. B. Dai, C. S. Huang, and H. Y. Jin, Bethe-Salpeter wave functions and transition amplitudes for heavy mesons, Z. Phys. C **60**, 527 (1993).

[34] Y. B. Dai, C. S. Huang, and M. Q. Huang, $O(1/m_Q)$ order corrections to masses of excited heavy mesons from QCD sum rules, Phys. Rev. D **55**, 5719 (1997).

[35] Y. B. Dai, C. S. Huang, C. Liu, and S. L. Zhu, Understanding the $D_{sJ}^+(2317)$ and $D_{sJ}^+(2460)$ with sum rules in HQET, Phys. Rev. D **68**, 114011 (2003).

[36] P. Colangelo, F. De Fazio, and N. Paver, Universal $\tau_{1/2}(y)$ Isgur-Wise function at the next-to-leading order in QCD sum rules, Phys. Rev. D **58**, 116005 (1998).

[37] B. L. Ioffe, QCD at low energies, Prog. Part. Nucl. Phys. **56**, 232 (2006).

[38] S. Narison, Decay constants of the B and D mesons from QCD duality sum rules, Phys. Lett. B **198**, 104 (1987).

[39] S. Narison, QCD as a Theory of Hadrons from Partons to Confinement (Cambridge University Press, 2007).