

Quark and lepton mass matrix model with only six family-independent parameters

Yoshio Koide^{1,*} and Hiroyuki Nishiura^{2,†}

¹*Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*

²*Faculty of Information Science and Technology, Osaka Institute of Technology, Hirakata, Osaka 573-0196, Japan*

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We propose a unified mass matrix model for quarks and leptons, in which 16 observables of mass ratios and mixings of the quarks and neutrinos are described by using no family-number-dependent parameters except for the charged lepton masses and only six family-number-independent free parameters. The model is constructed by extending the so-called ‘‘Yukawaon’’ model to a seesaw-type model. As a result, once the six parameters are fixed by the quark mixing and the mass ratios of quarks and neutrinos, no free parameters are left in the lepton mixing matrix. The results are in excellent agreement with the neutrino mixing data. We predict $\delta_{CP}^{\ell} = -68^{\circ}$ for the leptonic CP violating phase and $\langle m \rangle \approx 21$ meV for the effective Majorana neutrino mass.

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I. INTRODUCTION

It is a big concern in flavor physics to investigate the origin of the observed hierarchical structures of masses and mixings of quarks and leptons. In the present paper, we attempt to describe the quark and neutrino mass matrices in terms of the charged lepton masses as only family-number-dependent parameters with the help of the smallest number of possible family-number-independent parameters. We report in this paper that such an ambitious attempt has succeeded, and what is more surprising, the number of family-number-independent free parameters of the model is only six for 16 observables. It should be noted that even in the latest Yukawaon model [1] we needed ten family-number-independent parameters. The success suggests that hierarchical structures in all the quark and lepton mass matrices are caused by one common origin. This result will bring new light to help us understand the origin of flavors.

In the so-called Yukawaon model [2], the Yukawa coupling constants are considered to be effective coupling constants Y_f^{eff} which are given by vacuum expectation values (VEVs) of scalars (‘‘Yukawaons’’) Y_f with 3×3 components for each flavor f :

$$\langle Y_f^{\text{eff}} \rangle_i^j = (y_f/\Lambda) \langle Y_f \rangle_i^j \quad (f = u, d, \nu, e), \quad (1)$$

where Λ is an energy scale of the effective theory. Although the Yukawaon model is a kind of flavon model [3], all the flavons in the Yukawaon model are expressed by 3×3 components. The Yukawaon model is based on the basic concepts that (i) the fundamental flavor basis is a basis in which charged lepton mass matrix M_e is diagonal and (ii) fundamental parameters in the quarks and leptons are

$(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ [not (m_e, m_μ, m_τ)]. These concepts have been motivated by a phenomenological success of the charged lepton mass relation [4] $m_e + m_\mu + m_\tau = (2/3)(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$.

II. VEV RELATIONS

At first, we propose a model in the present paper, in which VEVs of the Yukawaons Y_f (would-be Dirac mass matrices) take a universal seesaw form as given by

$$\langle \hat{Y}_f \rangle_i^j = k_f \langle \Phi_{0f} \rangle_i^\alpha \langle (S_f)^{-1} \rangle_\alpha^\beta \langle \bar{\Phi}_{0f}^T \rangle_\beta^j \quad (f = u, d, \nu, e), \quad (2)$$

where VEVs of the fields Φ_{0f} , S_f , $\bar{\Phi}_0$, P_f , and so on are defined by

$$\langle \Phi_{0f} \rangle_i^\alpha = (1/\Lambda) \langle \bar{\Phi}_0 \rangle_{ik} \langle \bar{P}_f \rangle^{k\alpha}, \quad (3)$$

$$\langle P_u \rangle = v_P \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}), \quad \langle P_d \rangle = v_P \mathbf{1},$$

$$\langle P_\nu \rangle = v_P \mathbf{1}, \quad \langle P_e \rangle = v_P \mathbf{1}, \quad (4)$$

$$\langle \bar{\Phi}_0 \rangle = v_0 \text{diag}(z_1, z_2, z_3) \propto \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}), \quad (5)$$

$$\langle S_f \rangle = v_{Sf} (\mathbf{1} + b_f e^{i\beta_f} X_3). \quad (6)$$

Here $(m_{e1}, m_{e2}, m_{e3}) \equiv (m_e, m_\nu, m_\tau)$. $\mathbf{1}$ and X_3 are defined by

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (7)$$

Here, we have assumed $U(3) \times U(3)'$ family symmetries, and indexes i, j, \dots and α, β, \dots denote those of $U(3)$

*koide@kuno-g.phys.sci.osaka-u.ac.jp
†hiroyuki.nishiura@oit.ac.jp

and $U(3)'$, respectively. The VEV form of S_f , Eq. (6), breaks the symmetry $U(3)'$ into a discrete symmetry S_3 . The factor S_f^{-1} in Eq. (2) comes from a seesaw scenario discussed in detail in Sec. III. In particular, $U(3)'$ plays an essential role in considering a family gauge boson model [5], in which masses of family gauge bosons A_i^j are given by a VEV of $(\Phi_0)_i^\alpha$, so that the model can avoid a severe constraint from the observed $K^0-\bar{K}^0$ mixing.

In this model, a VEV form which explicitly breaks $U(3)$ family symmetry is only the form $\langle\Phi_0\rangle$, Eq. (5). In this paper, we do not discuss the origin of the values (z_1, z_2, z_3) given by Eq. (5), which is a basic assumption in the Yukawaon model.

A neutrino mass matrix is assumed, by adopting the conventional seesaw mechanism, as

$$(M_\nu^{\text{Majorana}})_{ij} = \langle\hat{Y}_\nu\rangle_i^k \langle\bar{Y}_R^{-1}\rangle_{kl} \langle\hat{Y}_\nu^T\rangle_l^j. \quad (8)$$

Here, the following VEV structure of the Y_R (the Majorana mass matrix of the right-handed neutrinos ν_R) is assumed, according to the previous Yukawaon model [1]:

$$\begin{aligned} \langle\bar{Y}_R\rangle^{ij} = & k_R \frac{1}{\Lambda} \left[(\langle\bar{\Phi}_0\rangle^{ik} \langle\hat{Y}_u\rangle_k^j + \langle\hat{Y}_u^T\rangle_i^k \langle\bar{\Phi}_0\rangle^{kj}) \right. \\ & \left. + \frac{\xi_R}{\Lambda} \langle\hat{Y}_e^T\rangle_i^k \langle\bar{E}\rangle^{kl} \langle\hat{Y}_e\rangle_l^j \right], \quad (9) \end{aligned}$$

where $\langle E\rangle = v_E \mathbf{1}$. Here, the last term (ξ_R term) has been introduced in order to give a reasonable value for a neutrino mass squared difference ratio $R_\nu = (m_{\nu 2}^2 - m_{\nu 1}^2)/(m_{\nu 3}^2 - m_{\nu 2}^2)$. Exactly speaking, three terms $\bar{E}^{ik} \langle\hat{Y}_e\rangle_k^l \langle\hat{Y}_e\rangle_l^j$, $\langle\hat{Y}_e^T\rangle_i^k \bar{E}^{kl} \langle\hat{Y}_e\rangle_l^j$, and $\langle\hat{Y}_e^T\rangle_i^k \langle\hat{Y}_e^T\rangle_k^l \bar{E}^{lj}$ are possible as the ξ_R term. However, since we have considered $\langle E\rangle = v_E \mathbf{1}$, we have denoted only one term of the three terms in Eq. (9) for convenience.

Since we deal with mass ratios and mixings only, the common coefficients k_f , v_{Sf} , and so on do not affect the numerical results, so that hereafter we omit such coefficients even if those have dimensions.

Let us state some remarks on our new Yukawaon model in order:

- The VEV form (5) is a fundamental postulation in the Yukawaon model. We assume that the VEV form of Φ_0 is diagonal in the flavor basis in which $\langle S_f\rangle$ takes a form ‘‘unit matrix plus democratic matrix.’’ We do not ask the origin of the values of z_i for the moment.
- The structures of the quark and Dirac neutrino mass matrices $\langle\hat{Y}_f\rangle$ are essentially determined by the parameters $\tilde{b}_f e^{i\beta_f}$. Since we take a superpotential

$$W_S = \mu_{1S} \text{Tr}[S_e S_\nu] + \mu_{2S} \text{Tr}[S_e] \text{Tr}[S_\nu], \quad (10)$$

which leads to $\langle S_e\rangle = v_S \mathbf{1}$ and $\langle S_\nu\rangle = v_S \mathbf{1}$, we obtain $b_e = b_\nu = 0$ in the lepton sector, so that

$\langle\hat{Y}_e\rangle$ and $\langle\hat{Y}_\nu\rangle$ are given by a common form $\langle\Phi_0\rangle\langle\bar{\Phi}_0\rangle$. (However, this does not mean that \hat{Y}_e and \hat{Y}_ν are the same flavon.) Of course, here, we assume R charges, $R(S_\nu) + R(S_e) = 2$.

- VEV relations are derived from the supersymmetric vacuum conditions. The possible combinations among those flavons are selected by R charges in the SUSY scenario. See, for instance, Ref. [6]. For example, the forms (4) are derived superpotential terms

$$\begin{aligned} W_{Pq} = & (1/\Lambda) (\lambda_{1Pq} \text{Tr}[P_u \bar{P}_u P_d \bar{P}_d] \\ & + \lambda_{2Pq} \text{Tr}[P_u \bar{P}_u] \text{Tr}[P_d \bar{P}_d]), \\ W_{P\ell} = & (1/\Lambda) (\lambda_{1P\ell} \text{Tr}[P_\nu \bar{P}_\nu P_e \bar{P}_e] \\ & + \lambda_{2P\ell} \text{Tr}[P_\nu \bar{P}_\nu] \text{Tr}[P_e \bar{P}_e]), \quad (11) \end{aligned}$$

which lead to VEV relations $\langle P_f\rangle\langle\bar{P}_f\rangle = v_P^2 \mathbf{1}$ ($f = u, d, \nu, e$). We regard the VEV form (4) as one of the special solutions in the general relation $\langle P_f\rangle\langle\bar{P}_f\rangle = v_P^2 \mathbf{1}$. [Here, we have taken R charge relations $R(P_u) + R(P_d) = 1$ and $R(P_\nu) + R(P_e) = 1$.]

- The parameters ϕ_i ($i = 1, 2, 3$) in Eq. (4) look like typical family-number-dependent parameters. However, in the previous Yukawaon model [1], we have proposed a mechanism that the parameters ϕ_i can always be expressed in terms of the charged lepton mass parameters m_{ei} with the help of two family-number-independent parameters. The reason is as follows: when we put $(\phi_1, \phi_2, \phi_3) = (\phi_0 + \tilde{\phi}_1, \phi_0 + \tilde{\phi}_2, \phi_0)$, the phase values $(\tilde{\phi}_1, \tilde{\phi}_2)$ are observables in fitting of the Cabibbo-Kobayashi-Maskawa (CKM) [7] mixing parameters, but ϕ_0 is not observable. Therefore, we can always relate the values (ϕ_1, ϕ_2, ϕ_3) to the values (m_e, m_μ, m_τ) by adjusting ϕ_0 . Therefore, for convenience, we count the parameters $(\tilde{\phi}_1, \tilde{\phi}_2)$ as family-number-independent parameters. (For details, see Ref. [1].)
- In the previous model [1], we have discussed the VEV form $P_f \Phi_0 S_f \Phi_0 P_f$ with $S_f = \mathbf{1} + a_f e^{i\alpha_f} X_3$ (not $\Phi_0 S_f^{-1} \Phi_0$), in which we have taken the cases that $(\alpha_u = 0, P_u \neq \mathbf{1})$ and $(\alpha_d \neq 0, P_d = \mathbf{1})$. The result comes from a rule that all VEV matrices of flavons satisfy $\langle\bar{A}\rangle = \langle A\rangle$ except for $\langle\bar{P}\rangle = \langle P\rangle^*$ with $\phi_i \neq 0$. However, in the present model, the corresponding VEV form is $\bar{P}_f \Phi_0 S_f \bar{\Phi}_0 P_f$, not $\bar{P}_f \Phi_0 S_f \Phi_0 P_f$, so that we cannot obtain a similar result as in Ref. [1]. Therefore, in the present paper, the relations $(\beta_u = 0, P_u \neq \mathbf{1})$ and $(\beta_d \neq 0, P_d = \mathbf{1})$ are only assumptions. This is a task for the future.

Finally, we summarize the parameters in the present model. We have only six free parameters $b_u, b_d, \beta_d, (\tilde{\phi}_1, \tilde{\phi}_2), \xi_R$ for 16 observable quantities [four quark mass ratios, two neutrino mass ratios, four CKM mixing parameters and four plus two Pontecorvo-Maki-Nakagawa-Sakata

(PMNS) [8] mixing parameters]. [Hereafter, for convenience, we denote (ϕ_1, ϕ_2) as (ϕ_1, ϕ_2) simply.] Note that the number of parameters is surprisingly small. The parameter b_u gives the up-quark mass ratios m_u/m_c and m_c/m_t , so that we have one prediction. The parameters b_d and β_d are fixed by the observed down-quark mass ratios. Therefore, four parameters in the CKM mixing matrix are described only by two parameters (ϕ_1, ϕ_2) . The parameter ξ_R adjusts the neutrino mass ratio R_ν . If the value ξ_R is fixed, four plus two lepton mixing parameters are predicted with no free parameters. However, since the neutrino mass ratio has not been so precisely measured at present, we will give predictions of two neutrino mass ratios and four PMNS mixing parameters (and with two Majorana phase) by adjusting one parameter ξ_R as a practical matter. (See Fig. 2.)

III. SEESAW-TYPE MASS MATRIX MODEL

Let us discuss the origin of the VEV structure in Eq. (2). The mass matrix model given in Eq. (2) is a sort of seesaw-type mass matrix model for all the flavors. The form (2) has been suggested by a block diagonalization of a 6×6 mass matrix term in a universal seesaw model given by

$$(\bar{f}_L^i \quad \bar{F}_L^\alpha) \begin{pmatrix} (0)_i^j & \langle \Phi_{0f} \rangle_i^\beta \\ \langle \bar{\Phi}_{0f}^T \rangle_\alpha^j & -\langle S_f \rangle_\alpha^\beta \end{pmatrix} \begin{pmatrix} f_{Rj} \\ F_{R\beta} \end{pmatrix}. \quad (12)$$

Here $f_{L(R)}$ and $F_{L(R)}$ are, respectively, left-handed (right-handed) light and heavy fermion fields. Exactly speaking, we have to read \bar{f}_L in Eq. (13) as $\bar{f}_L H_{u/d}/\Lambda$. However, for convenience, we have denoted those as \bar{f}_L simply. The mass matrix model with the heavy fermion mass matrix $M_F = -S_f$ with the form (6) is known as the so-called ‘‘democratic seesaw’’ model [9]. The authors in Ref. [9] have considered that the observed top quark mass enhancement originates in a condition $\det M_F = 0$ in the seesaw mass matrix (12). In the seesaw approximation form (2), the condition $\det M_F \rightarrow 0$ gives $m_3 \rightarrow \infty$ for one of the mass eigenvalues (m_1, m_2, m_3) . However, they found that the exact diagonalization of 6×6 mass matrix (12) gives $m_3 \sim \Lambda_{\text{weak}}$, for one of the mass eigenvalues $(m_1, m_2, m_3, m_4, m_5, m_6)$, where Λ_{weak} is a breaking scale of the electroweak symmetry. The reason is quite simple: The matrix (6) with $f = u$ takes $S_u = \text{diag}(1, 1, 0)$ in the limit of $b_u \rightarrow -1$ (i.e. $\det S_u \rightarrow 0$), so that the seesaw suppression affects only the first and second generations of up-quarks, so that the third generation quark t takes mass of the order of Λ_{weak} without the seesaw suppression. Furthermore, the model can give $m_u \sim m_d \sim m_e$ insensitively to the values of the parameters b_u and b_d as seen in Sec. IV later. In spite of such a successful description of quark masses and mixing, the authors in Ref. [9] failed to give reasonable neutrino masses and mixing. On the other hand, the Yukawaon model have succeeded in giving not only reasonable quark mass ratios and mixing but also

neutrino masses and mixing. However, the Yukawaon model needs a lot of parameters. In the present paper, we have applied this seesaw model to our Yukawaon model.

However, note that the 3×3 mass matrix between \bar{f}_L and f_R is absent in the form (12). If we assume the seesaw mechanism plus the Yukawaon model,

$$(\bar{f}_L^i \quad \bar{F}_L^\alpha) \begin{pmatrix} (\hat{Y}_f)_i^j & (\Phi_{0f})_i^\beta \\ (\bar{\Phi}_{0f}^T)_\alpha^j & -(S_f)_\alpha^\beta \end{pmatrix} \begin{pmatrix} f_{Rj} \\ F_{R\beta} \end{pmatrix}, \quad (13)$$

then we obtain a 3×3 mass matrix between \bar{f}_L' and f_R' ,

$$M_f \simeq \hat{Y}_f + \Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}, \quad (14)$$

after the block diagonalization. (Here and hereafter, for convenience, we sometimes omit the notations ‘‘ $\langle \rangle$ ’’ and ‘‘ $\langle \rangle$ ’’ which denote VEV matrices.) However, note that, in Eq. (14), the first term \hat{Y}_f is independent of the second term $\Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}$. In order to obtain the relation (2), we make the following two assumptions:

[Assumption 1] The VEV value \hat{Y}_f and the VEV value $M_F = -S_f$ take the same scale transformation (we denote the scale transformation as a parameter ζ_f):

$$M_f = \zeta_f \hat{Y}_f + (1/\zeta_f) \Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}. \quad (15)$$

[Assumption 2] The VEV value \hat{Y}_f is taken so that M_f takes a locally minimum value under the ζ_f transformation:

$$\frac{\partial M_f}{\partial \zeta_f} = \hat{Y}_f - \frac{1}{\zeta_f^2} \Phi_{0f} S_f^{-1} \bar{\Phi}_{0f} = 0. \quad (16)$$

Then, we obtain

$$\begin{aligned} \hat{Y}_f &= (1/\zeta_f^2) \Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}, \\ \text{i.e. } M_f &= (2/\zeta_f) \Phi_{0f} S_f^{-1} \bar{\Phi}_{0f} = 2\zeta_f \hat{Y}_f. \end{aligned} \quad (17)$$

In Ref. [9], the up-quark masses m_{ui} have been estimated by diagonalizing 6×6 mass matrix (12) with the input value $b_u = -1$ with $\beta_u = 0$. However, in this paper, for convenience, we use the approximate expression (17), although the seesaw approximation (17) is not valid for $b_u = -1$. Therefore, instead of $b_u = -1$, the parameter value b_u is fixed by the observed value of the up-quark mass ratio m_c/m_t . (We will obtain $b_u = -1.011$ as seen in Sec. IV.) We use the relation (17) for Dirac masses of all quarks and leptons. (Hereafter, we put simply $\zeta_f = 1$.)

IV. NUMERICAL PREDICTIONS

We summarize our mass matrices for the numerical analysis as follows:

YOSHIO KOIDE and HIROYUKI NISHIURA

$$\hat{Y}_u = \Phi_0 \bar{P}_u (\mathbf{1} + b_u X_3)^{-1} P_u \bar{\Phi}_0,$$

$$\hat{Y}_d = \Phi_0 (\mathbf{1} + b_d e^{i\beta_d} X_3)^{-1} \bar{\Phi}_0, \quad \hat{Y}_e = \Phi_0 \bar{\Phi}_0, \quad (18)$$

$$M_\nu^{\text{Majorana}} = \hat{Y}_\nu \bar{Y}_R^{-1} \hat{Y}_\nu^T,$$

$$\bar{Y}_R = \bar{\Phi}_0 \hat{Y}_u + \hat{Y}_u^T \bar{\Phi}_0 + \xi_R \hat{Y}_e^T \hat{Y}_e,$$

$$\hat{Y}_\nu = \Phi_0 \bar{\Phi}_0. \quad (19)$$

For convenience of numerical fitting, we redefine all VEV matrices of flavons as dimensionless matrices, i.e. $\bar{P}_u = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$, $\bar{\Phi}_0 = \text{diag}(z_1, z_2, z_3)$, and so on. For the input values m_{ei} , we use the values at $\mu = m_Z$. For the parameter ξ_R defined in \bar{Y}_R , we redefine ξ_R/Λ as ξ_R .

The parameter value of b_u can be determined from the observed up-quark mass ratio m_c/m_t at $\mu = m_Z$. We determine $b_u = -1.011$, which leads to the up-quark mass ratios

$$r_{12}^u \equiv \sqrt{m_u/m_c} = 0.061, \quad r_{23}^u \equiv \sqrt{m_u/m_c} = 0.060, \quad (20)$$

which are in good agreement with the observed up-quark mass ratios at $\mu = m_Z$ [10], $r_{12}^u = 0.045_{-0.010}^{+0.013}$ and $r_{23}^u = 0.060 \pm 0.005$. Here, we have used the value of r_{23}^u as an input value in determining b_u because the light quark masses have large errors. Although the predicted value r_{12}^u in Eq. (20) is somewhat large compared with the observed value, we consider that this discrepancy is acceptable, since the purpose of the present paper is to give an overview of quark and lepton masses and mixings with as few free parameters as possible.

For down-quark mass ratios, we take parameter values $b_d = -3.3522$ and $\beta_d = 17.7^\circ$ which give down-quark mass ratios

$$r_{12}^d \equiv m_d/m_s = 0.049, \quad r_{23}^d \equiv m_s/m_b = 0.027. \quad (21)$$

The observed down-quark mass ratios at $\mu = m_Z$ are a little controversial: $r_{12}^d = 0.053_{-0.003}^{+0.005}$ and $r_{23}^d = 0.019 \pm 0.006$ (Xing *et al.* [10]) and also $r_{12}^d = 0.050 \pm 0.010$ and $r_{23}^d = 0.031 \pm 0.004$ (Fusaoka-Koide [11]). Our model cannot give the observed values by Xing *et al.* Our best fit parameter values are close to the values given in Ref. [11]. The fittings of b_d and β_d have been done while keeping in mind that they also lead to consistent CKM mixings, since the CKM mixings also depend on b_d and β_d .

The explicit mass eigenvalues are as follows: $(m_u, m_c, m_t) = (0.000398, 0.1064, 29.74)m_0$ and $(m_d, m_s, m_b) = (0.000725, 0.01467, 0.5365)m_0$, where $m_0 = (v_0 v_P/\Lambda)^2/v_S$ and $v_{Su} = v_{Sd} \equiv v_S$, so that we can obtain a reasonable m_d/m_u ratio, $m_d/m_u = 1.8$, which well agrees with the observed ratio [11] $m_d/m_u = 2.01_{-0.46}^{+0.47}$. Though we obtain $(m_e, m_\mu, m_\tau) = (0.000263, 0.0555, 0.9442)m_0$, the ratio $m_e/m_u \sim 0.66$ is not in agreement with the observed value

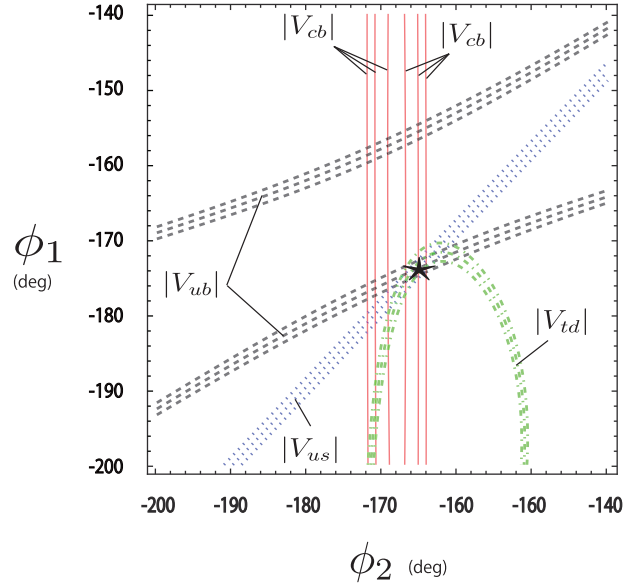
PHYSICAL REVIEW D **92**, 111301(R) (2015)

FIG. 1 (color online). Contour curves in the (ϕ_1, ϕ_2) parameter plane of the observed CKM mixing matrix elements of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and $|V_{td}|$. We draw the three contour curves, which correspond to the center, upper, and lower values of the observed constraints for each of the CKM mixing matrix elements, taking $b_u = -1.011$, and $b_d = -3.3522$, $\beta_d = 17.7^\circ$. We find that the parameter set around $(\phi_1, \phi_2) = (-176.05^\circ, -167.91^\circ)$ indicated by a star (\star) is consistent with all the observed values.

$(m_e/m_u)^{\text{obs}} \sim 0.38$. We think that the common coefficient m_0 should be distinguished between $(m_0)_{\text{quark}}$ and $(m_0)_{\text{lepton}}$, and the difference is originated in the difference of $v_{Sq} \equiv v_S(\text{quark})$ and $v_{S\ell} \equiv v_S(\text{lepton})$ in Eq. (6). It is interesting that this discrepancy suggests $v_{Sq} \approx v_{S\ell}/\sqrt{3}$.

Next, let us try to fit CKM mixing parameters. Since the parameters b_u , b_d , and β_d have been fixed by the observed quark mass ratios, the CKM mixing matrix elements $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and $|V_{td}|$ are functions of the remaining two free parameters ϕ_1 and ϕ_2 . In Fig. 1, we draw contour curves of the CKM mixing matrix elements in the (ϕ_1, ϕ_2) parameter plane which are obtained from the observed constraints of the CKM mixing matrix elements, taking $b_u = -1.011$, and $b_d = -3.3522$, $\beta_d = 17.7^\circ$. As shown in Fig. 1, all the experimental constraints on CKM parameters are satisfied by fine-tuning of the parameters ϕ_1 and ϕ_2 as $(\phi_1, \phi_2) = (-176.05^\circ, -167.91^\circ)$, which predicts

$$\begin{aligned} |V_{us}| &= 0.2257, & |V_{cb}| &= 0.03996, \\ |V_{ub}| &= 0.003701, & |V_{td}| &= 0.009173, \\ \delta_{\text{CP}}^q &= 80.99. \end{aligned} \quad (22)$$

Now let us present the result for the neutrino sector. Substantial differences between the present and previous papers appear in the parameter fitting of the PMNS lepton

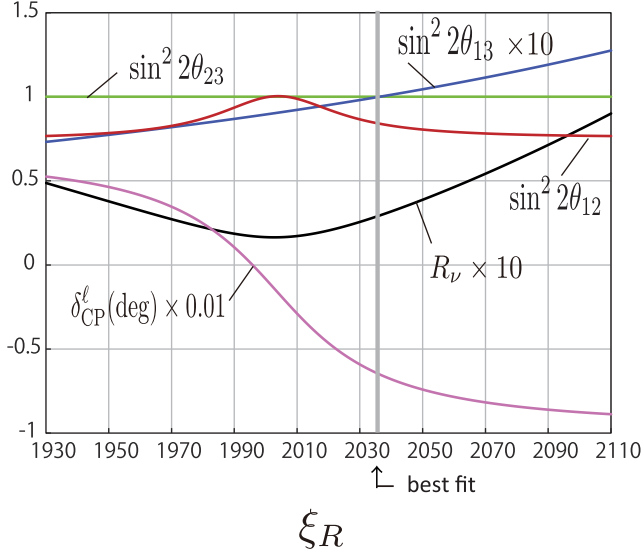


FIG. 2 (color online). ξ_R dependence of the lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared difference ratio R_ν . We draw curves of those as functions of ξ_R for the case of $b_u = -1.011$ and $(\phi_1, \phi_2) = (-176.05^\circ, -167.91^\circ)$.

mixing. We have already fixed the five parameters b_u , b_d , β_d , ϕ_1 , and ϕ_2 from the quark mass ratios and CKM mixing. Therefore, the PMNS mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$; CP violating Dirac phase parameter δ_{CP}^l ; and the neutrino mass squared difference ratio $R_\nu \equiv \Delta m_{21}^2 / \Delta m_{32}^2$ turn out to be functions of the only remaining parameter ξ_R . In Fig. 2, we draw the curves as functions of

ξ_R taking $b_u = -1.011$, $b_d = -3.3522$, $\beta_d = 17.7^\circ$, and $(\phi_1, \phi_2) = (-176.05^\circ, -167.91^\circ)$. As seen in Fig. 2, we find that the predicted value of the $\sin^2 2\theta_{23}$ is almost constant as $\sin^2 2\theta_{23} \approx 1$ and the value $\sin^2 2\theta_{12}$ is not so sensitive to the parameter ξ_R as $\sin^2 2\theta_{12} = 0.8\text{--}1.0$. By using the observed value of R_ν as input, we determine the value $\xi_R = 2039.6$ in the unit of $v_e^2 v_E / v_o v_u \Lambda$, which gives the predictions

$$\begin{aligned} \sin^2 2\theta_{12} &\approx 0.8254, & \sin^2 2\theta_{23} &\approx 0.9967, \\ \sin^2 2\theta_{13} &\approx 0.1007, & \delta_{CP}^l &\approx -68.1^\circ, \\ R_\nu &\approx 0.03118. \end{aligned} \quad (23)$$

These predictions are in good agreement with the observed values [12] given in Table I.

Now, we predict neutrino masses with a normal hierarchy, which are consistent with the observed oscillation data, as

$$\begin{aligned} m_{\nu 1} &\approx 0.038 \text{ eV}, & m_{\nu 2} &\approx 0.039 \text{ eV}, \\ m_{\nu 3} &\approx 0.063 \text{ eV}, \end{aligned} \quad (24)$$

by using the input value [12] $\Delta m_{32}^2 \approx 0.00244 \text{ eV}^2$. We also predict the effective Majorana neutrino mass [13] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\begin{aligned} \langle m \rangle &= |m_{\nu 1}(U_{e1})^2 + m_{\nu 2}(U_{e2})^2 + m_{\nu 3}(U_{e3})^2| \\ &\approx 21 \times 10^{-3} \text{ eV}. \end{aligned} \quad (25)$$

The predictions of our model are listed in Table I. The process for fitting parameters is summarized in Table II.

TABLE I. Predicted values vs observed values.

	$ V_{us} $	$ V_{cb} $	$ V_{ub} $	$ V_{td} $	$\delta_{CP}^q (^\circ)$	r_{12}^u	r_{23}^u	r_{12}^d	r_{23}^d
Predicted	0.2257	0.03996	0.00370	0.00917	81.0	0.061	0.060	0.049	0.027
Observed	0.22536	0.0414	0.00355	0.00886	69.4	0.045	0.060	0.053	0.019
	± 0.00061	± 0.0012	± 0.00015	± 0.00033 -0.00032	± 3.4	$+0.013$ -0.010	± 0.005	-0.003 $+0.005$	$+0.006$ -0.006
	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$	$R_\nu (10^{-2})$	$\delta_{CP}^l (^\circ)$	$m_{\nu 1} \text{ (eV)}$	$m_{\nu 2} \text{ (eV)}$	$m_{\nu 3} \text{ (eV)}$	$\langle m \rangle \text{ (eV)}$
Predicted	0.8254	0.9967	0.1007	3.118	-68.1	0.038	0.039	0.063	0.021
Observed	0.846	0.999	0.093	3.09	no data	no data	no data	no data	$< O(10^{-1})$
	± 0.021	-0.018 $+0.001$	± 0.008	± 0.15					

TABLE II. Process for fitting parameters.

Step	Inputs	N_{input}	Parameters	$N_{\text{parameter}}$	Predictions
1st	m_c/m_t	1	b_u	1	m_u/m_c
	$m_d/m_s, m_s/m_b$	2	a_d, β_d	2	m_d/m_u
2rd	$ V_{us} , V_{cb} $	2	(ϕ_1, ϕ_2)	2	$ V_{ub} , V_{td} , \delta_{CP}^q$
3rd	R_ν	1	ξ_R	1	$\sin^2 2\theta_{12}, \sin^2 2\theta_{23}, \sin^2 2\theta_{13}, \delta_{CP}^l$
Option	Δm_{32}^2		$m_{\nu 3}$		2 Majorana phases, $\frac{m_{\nu 1}}{m_{\nu 2}}, \frac{m_{\nu 2}}{m_{\nu 3}}$
$\sum N_{\text{total}}$		6		6	$(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}), \langle m \rangle$

V. CONCLUDING REMARKS

We have proposed a model which combines the Yukawaon model [1] with the democratic seesaw scenario [9], and have demonstrated that the observed masses and mixings of quarks and neutrinos can be described only by the observed charged lepton mass values and six family-number-independent parameters. The model provides an interesting picture that the observed hierarchical structures of the masses and the mixings of quarks and neutrinos are brought by a

common origin which comes from the charged lepton mass spectrum (m_e, m_μ, m_τ) . The model also predicts $\delta_{CP}^\ell = -68^\circ$ for the leptonic CP violating Dirac phase, which will be checked by neutrino oscillation experiments in the near future. The prediction $\langle m \rangle \simeq 21$ meV is also within the reach of neutrinoless double beta decay experiments in the near future.

For a full table of our flavons together with their R charges and full expressions of all the superpotential terms, we will give them elsewhere.

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- [1] Y. Koide and H. Nishiura, *Phys. Rev. D* **91**, 116002 (2015).
 [2] Y. Koide, *Phys. Rev. D* **79**, 033009 (2009); *Phys. Lett. B* **680**, 76 (2009).
 [3] C. D. Froggatt and H. B. Nelsen, *Nucl. Phys.* **B147**, 277 (1979); For recent works, for instance, see R. N. Mohapatra, *AIP Conf. Proc.* **1467**, 7 (2012); A. J. Buras, M. V. Carlucci, L. Merlo, and E. Stamou, *J. High Energy Phys.* **03** (2012) 088.
 [4] Y. Koide, *Lett. Nuovo Cimento* **34**, 201 (1982); *Phys. Lett.* **120B**, 161 (1983); *Phys. Rev. D* **28**, 252 (1983).
 [5] Y. Sumino, *Phys. Lett. B* **671**, 477 (2009); Y. Koide and T. Yamashita, *Phys. Lett. B* **711**, 384 (2012); Y. Koide, *Phys. Lett. B* **736**, 499 (2014).
 [6] Y. Koide and H. Nishiura, *Phys. Rev. D* **90**, 016009 (2014).
 [7] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963); M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
 [8] B. Pontecorvo, *Zh. Eksp. Teor. Fiz.* **33**, 549 (1957); **34**, 247 (1957); Z. Maki, M. Nakagawa, and S. Sakata, *Prog. Theor. Phys.* **28**, 870 (1962).
 [9] Y. Koide and H. Fusaoka, *Z. Phys. C* **71**, 459 (1996).
 [10] Z.-z. Xing, H. Zhang, and S. Zhou, *Phys. Rev. D* **77**, 113016 (2008).
 [11] H. Fusaoka and Y. Koide, *Phys. Rev. D* **57**, 3986 (1998).
 [12] K. A. Olive *et al.* (Particle Data Group), *Chin. Phys. C* **38**, 090001 (2014).
 [13] M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, *Phys. Lett.* **103B**, 219 (1981); **113B**, 513 (1982).