

Localization and quasilocalization of a spin-1/2 fermion field on a two-field thick braneworld

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Localization of a spin-1/2 fermion on the braneworld is an important and interesting problem. It is well known that a five-dimensional free massless fermion Ψ minimally coupled to gravity cannot be localized on the Randall-Sundrum braneworld. In order to trap such a fermion, the coupling between the fermion and bulk scalar fields should be introduced. In this paper, localization and quasilocalization of a bulk fermion on the thick braneworld generated by two scalar fields (a kink scalar ϕ and a dilaton scalar π) are investigated. Two types of couplings between the fermion and two scalars are considered. One coupling is the usual Yukawa coupling $-\eta\bar{\Psi}\phi\Psi$ between the fermion and kink scalar, another one is $\lambda\bar{\Psi}\Gamma^M\partial_M\pi\gamma^5\Psi$ between the fermion and dilaton scalar. The left-chiral fermion zero mode can be localized on the brane, and both the left- and right-chiral fermion massive Kaluza-Klein modes may be localized or quasilocalized. Hence the four-dimensional massless left-chiral fermion and massive Dirac fermions, whose lifetime is infinite or finite, can be obtained on the brane.

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I. INTRODUCTION

The braneworld scenarios that our observed four-dimensional universe can be considered as a brane embedded in a higher-dimensional spacetime can supply new insights for solving the gauge hierarchy problem [1,2] and the cosmological constant problem [3–5]. The effective four-dimensional gravity could be recovered even in the case of noncompact extra dimensions in the Randall-Sundrum (RS) braneworld model [6], where singularities are present at the position of the branes. The smooth braneworld solutions (thick brane scenario or domain wall scenario) are generally based on gravity coupled to one or several bulk scalar fields [7–14]. For some comprehensive reviews about thick branes, please see Refs. [15–20].

In braneworld scenarios, an interesting and important problem is whether various bulk fields can be confined to the brane by a natural mechanism. Generally, a free massless scalar field [21] can be localized on branes of different types. A free vector field can be localized on the Randall-Sundrum brane in some higher-dimensional cases [22] or on some thick de Sitter branes and Weyl thick branes [23,24].

Since the particles in our world are described by fermions, localization of a bulk fermion is very important for any braneworld model [21–53]. In general, if one does not introduce the coupling between the fermion and scalars which generate the thick braneworlds, a bulk fermion does not have a normalizable zero mode in five dimensions. So in order to localize a bulk fermion, the scalar-fermion coupling should be introduced. Usually, the Yukawa coupling $-\eta\bar{\Psi}\phi\Psi$ is a natural choice when the scalar has the configuration of a kink (i.e., ϕ is an odd function of the extra dimensional coordinate). However, if the background scalar has the shape of lump (or an even function of the extra dimensional coordinate, see Refs. [54–59]), the Yukawa coupling cannot keep the Z_2 reflection symmetry of Lagrange and hence this localization mechanism does not work anymore. Recently, Liu *et al.* presented a new localization mechanism of a bulk fermion on braneworlds generated by such a scalar with even parity [46]. The scalar-fermion coupling is given by $\lambda\bar{\Psi}\Gamma^M\partial_M\pi\gamma^5\Psi$, which ensures not only the Z_2 reflection symmetry of the Lagrange but also localization of the left- or right-chiral fermion zero mode on the branes.

In this paper, we will explore localization of a spin-1/2 fermion Ψ on the Minkowski brane generated by two scalar fields (a kink scalar field ϕ with odd parity and a dilaton scalar field π with even parity). We introduce both two types of couplings between the fermion and scalars: one is the usual Yukawa coupling $-\eta\bar{\Psi}\phi\Psi$ between the fermion

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and kink scalar, another is the coupling $\lambda\bar{\Psi}\Gamma^M\partial_M\pi\gamma^5\Psi$ between the fermion and dilaton scalar. If only introducing the Yukawa coupling, the effective potentials of the left- and right-chiral fermion Kaluza-Klein (KK) modes in the corresponding Schrödinger equations are modified volcano-type potentials. And the left- and right-chiral fermion zero modes tend to a constant and infinity, respectively, when far away from the brane. So none of the two zero modes cannot be localized on the brane. However, the massive resonant KK modes may exist for this type of potential. If only introducing the second coupling, the effective potentials of the KK modes are modified Pöschl-Teller potentials, which lead to localization of the left- or right-chiral fermion zero mode as well as to mass gaps in the mass spectra. When both two couplings are considered, the potentials will have a barrier at each side of the brane, and tend to a positive constant when far away from the brane. Then one of the left- and right-chiral fermions zero modes can be localized on the brane, and there exists a mass gap in the mass spectrum of the fermion KK modes. Then the massive bound and/or resonant KK modes can be localized and/or quasilocalized on the brane.

The organization of this paper is as follows: in Sec. II, we first review the Minkowski thick brane generated by two scalar fields. Then, in Sec. III, we investigate localization and quasilocalization of a spin-1/2 fermion field on this thick brane by introducing two kinds of scalar-fermion couplings. Finally, our conclusion is given in Sec. IV together with some discussion on the presented material.

II. REVIEW OF THE MINKOWSKI THICK BRANEWORLD GENERATED BY TWO SCALAR FIELDS

In this paper, we consider a Minkowski brane generated by two interacting scalar fields ϕ and π , embedded in a five-dimensional spacetime. The action of such a system is

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2\kappa_5^2} R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2} (\partial\pi)^2 - V(\phi, \pi) \right], \quad (1)$$

where R is the five-dimensional scalar curvature and $\kappa_5^2 = 8\pi G_5$ with G_5 the five-dimensional Newton constant. For simplicity, the constant κ_5 is set to $\kappa_5 = 1$. The line element of the Minkowski brane is assumed as

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (2)$$

which can also be transformed to the conformally flat one

$$ds^2 = e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2), \quad (3)$$

by introducing the coordinate transformation, $dz = e^{-A(y)} dy$. Here e^{2A} is the warp factor and $\eta_{\mu\nu}$ is the metric

of four-dimensional Minkowski spacetime on the brane. y or z denotes the extra dimensional coordinate. The background scalar fields ϕ and π are assumed to be only the functions of the extra dimensional coordinate.

By considering the metric (3), the equations of motion generated from action (1) read as

$$\frac{1}{2}\phi'^2 + \frac{1}{2}\pi'^2 - e^{2A}V = 6A'^2, \quad (4)$$

$$\frac{1}{2}\phi'^2 + \frac{1}{2}\pi'^2 + e^{2A}V = -3A'' - 3A'^2, \quad (5)$$

$$\phi'' + 3A'\phi' = e^{2A} \frac{\partial V}{\partial \phi}, \quad (6)$$

$$\pi'' + 3A'\pi' = e^{2A} \frac{\partial V}{\partial \pi}, \quad (7)$$

where the prime denotes the derivative with respect to z . The solutions of the thick brane can be found by following the superpotential method [7,12]. By supposing $V(\phi, \pi) = \frac{1}{2}e^{-\frac{2\pi}{\sqrt{3}}}\left[\left(\frac{\partial W}{\partial \phi}\right)^2 - W^2\right]$ and considering a specific superpotential function $W(\phi) = va\phi(1 - \frac{\phi^2}{3v^2})$ [54,55], one solution is obtained

$$A(z) = -\frac{v^2}{9} \left[\ln(\cosh^2(az)) + \frac{1}{2} \tanh^2(az) \right], \quad (8)$$

$$\phi(z) = v \tanh(az), \quad (9)$$

$$\pi(z) = \sqrt{3}A(z), \quad (10)$$

$$V(\phi, \pi) = \frac{a^2}{18v^2} e^{-\frac{2\pi}{\sqrt{3}}} \times [9v^4 - 9v^2(v^2 + 2)\phi^2 + 3(2v^2 + 3)\phi^4 - \phi^6], \quad (11)$$

where v and a are positive constants. It can be seen that the solution for ϕ is a kink and π is the dilaton field, which are odd and even functions of the extra dimensional coordinate z , respectively. The energy density of the thick brane is written as follows:

$$\begin{aligned} T_{00}(z) &= \frac{1}{2}(\phi'^2 + \pi'^2) + e^{2A}V(\phi, \pi) \\ &= \frac{a^2v^2}{27} [v^2 \text{sech}^6(az) + 3(v^2 + 9)\text{sech}^4(az) - 4v^2]. \end{aligned} \quad (12)$$

From the above expression (12), we can see that the energy density has a maximum value $T_{00}^{\max} = a^2v^2$ at $z = 0$ and tends to the minimum value $T_{00}^{\min} = -\frac{4}{27}a^2v^2$ when

$z \rightarrow \pm\infty$, so the Minkowski brane locates at $z=0$. Localization of gravity, scalar, vector, and Kalb-Ramond fields was investigated in previous papers [56,57]. So in the next section, localization of a spin-1/2 fermion with two types of couplings between the fermion and background scalar fields will be discussed.

III. LOCALIZATION OF A BULK FERMION ON THE BRANE

In this section, we will investigate localization of a bulk spin-1/2 fermion on the brane generated by the scalar fields. In five-dimensional spacetime, fermions are four-component spinors and their Dirac structure can be described by $\Gamma^M = e_M^{\bar{M}} \Gamma^{\bar{M}}$ with $e_M^{\bar{M}}$ being the vielbein and $\{\Gamma^{\bar{M}}, \Gamma^{\bar{N}}\} = 2g^{\bar{M}\bar{N}}$. In this section, $\bar{M}, \bar{N}, \dots = 0, 1, 2, 3, 5$ and $M, N, \dots = 0, 1, 2, 3, 5$ denote the five-dimensional local Lorentz indices and spacetime coordinates, respectively, and $\Gamma^{\bar{M}}$ are the gamma matrices in five-dimensional flat spacetime. Our action describing a massless Dirac fermion coupled with the background scalars ϕ and π in five-dimensional spacetime is assumed as follows:

$$S_{\frac{1}{2}} = \int d^5x \sqrt{-g} [\bar{\Psi} \Gamma^M (\partial_M + \omega_M) \Psi - \eta \bar{\Psi} f(\phi, \pi) \Psi + \lambda \bar{\Psi} \Gamma^M \partial_M g(\phi, \pi) \gamma^5 \Psi]. \quad (13)$$

Here ω_M is the spin connection defined as $\omega_M = \frac{1}{4} \omega_M^{\bar{M}\bar{N}} \Gamma_{\bar{M}} \Gamma_{\bar{N}}$ with

$$\omega_M^{\bar{M}\bar{N}} = \frac{1}{2} e^{N\bar{M}} (\partial_M e_N^{\bar{N}} - \partial_N e_M^{\bar{N}}) - \frac{1}{2} e^{N\bar{N}} (\partial_M e_N^{\bar{M}} - \partial_N e_M^{\bar{M}}) - \frac{1}{2} e^{P\bar{M}} e^{Q\bar{N}} (\partial_P e_{Q\bar{R}} - \partial_Q e_{P\bar{R}}) e_M^{\bar{R}}. \quad (14)$$

Both the usual Yukawa coupling $-\eta \bar{\Psi} f(\phi, \pi) \Psi$ and a new kind of coupling between the Dirac fermion and background scalar fields [46] $\lambda \bar{\Psi} \Gamma^M \partial_M g(\phi, \pi) \gamma^5 \Psi$ are introduced. In this paper, we assume that both coupling constants η and λ are positive.

Considering the conformally flat metric (3), the nonvanishing components of the spin connection are $\omega_\mu = \frac{1}{2} A'(z) \gamma_\mu \gamma_5$. Thus, the equation of motion corresponding to the action (13) can be written as

$$[\gamma^\mu \partial_\mu + \gamma^5 (\partial_z + 2A'(z)) - \eta e^A f(\phi, \pi) + \lambda g'(\phi, \pi)] \Psi = 0. \quad (15)$$

In order to investigate the above five-dimensional Dirac equation (15) and write the spinor in terms of four-dimensional effective fields, we make the general chiral decomposition:

$$\Psi = e^{-2A} \left(\sum_n \psi_{L_n}(x) L_n(z) + \sum_n \psi_{R_n}(x) R_n(z) \right), \quad (16)$$

where $\psi_{L_n}(x) = -\gamma^5 \psi_{L_n}(x)$ and $\psi_{R_n}(x) = \gamma^5 \psi_{R_n}(x)$ are the left- and right-chiral components of a four-dimensional Dirac field, respectively. Hence, we assume that $\psi_{L_n}(x)$ and $\psi_{R_n}(x)$ satisfy the four-dimensional Dirac equations

$$\gamma^\mu \partial_\mu \psi_{L_n}(x) = m_n \psi_{R_n}(x), \quad (17a)$$

$$\gamma^\mu \partial_\mu \psi_{R_n}(x) = m_n \psi_{L_n}(x). \quad (17b)$$

Here the four-dimensional chiral fields ψ_{L_n} and ψ_{R_n} are coupled by the mass m_n . If a four-dimensional fermion is massless, the two equations in Eqs. (17a) and (17b) are decoupled:

$$\gamma^\mu \partial_\mu \psi_{L_n}(x) = 0, \quad (18a)$$

$$\gamma^\mu \partial_\mu \psi_{R_n}(x) = 0. \quad (18b)$$

Then the KK modes $L_n(z)$ and $R_n(z)$ should satisfy the following coupled equations:

$$[\partial_z + \eta e^A f(\phi, \pi) - \lambda g'(\phi, \pi)] L_n(z) = m_n R_n(z), \quad (19a)$$

$$[\partial_z - \eta e^A f(\phi, \pi) + \lambda g'(\phi, \pi)] R_n(z) = -m_n L_n(z). \quad (19b)$$

The above coupled equations can be recast into

$$Q^\dagger Q L_n(z) = m_n^2 L_n(z) = m_{L_n}^2 L_n(z), \quad (20a)$$

$$Q Q^\dagger R_n(z) = m_n^2 R_n(z) = m_{R_n}^2 R_n(z), \quad (20b)$$

where the operator Q is defined as $Q = \partial_z + \eta e^A f(\phi, \pi) - \lambda g'(\phi, \pi)$ and m_{L_n, R_n} are the mass of the left- and right-chiral fermion KK modes, which are equal to the mass of the four-dimensional Dirac fermion. According to the supersymmetric quantum mechanics, there are no tachyon fermion KK modes with negative mass square m_{L_n, R_n}^2 . The above equations (20) can also be rewritten as the following Schrödinger-like equations for the left- and right-chiral KK modes of fermions:

$$(-\partial_z^2 + V_L(z)) L_n = m_{L_n}^2 L_n, \quad (21a)$$

$$(-\partial_z^2 + V_R(z)) R_n = m_{R_n}^2 R_n, \quad (21b)$$

where the effective potentials are given by

$$V_L(z) = \eta^2 e^{2A} f^2(\phi, \pi) + \lambda^2 g'^2(\phi, \pi) - \eta e^A [A' f(\phi, \pi) + f'(\phi, \pi)] - 2\eta \lambda A(z) f(\phi, \pi) g'(\phi, \pi) + \lambda g''(\phi, \pi), \quad (22a)$$

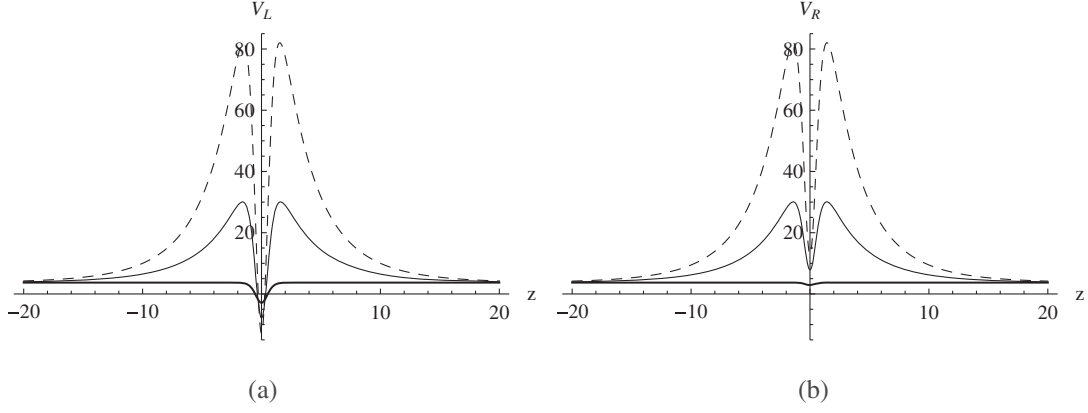


FIG. 1. The shapes of the effective potentials of the left- and right-chiral fermions $V_{L,R}(z)$ for different values of the parameter η . The parameter η is set to $\eta = 0$ for the thick line, $\eta = 5$ for the thin line, and $\eta = 10$ for the dashed line. Other parameters are set to $v = 1$, $a = 1$, and $\lambda = 5$. (a) $V_L(z)$, $\lambda = 5$ and (b) $V_R(z)$, $\lambda = 5$.

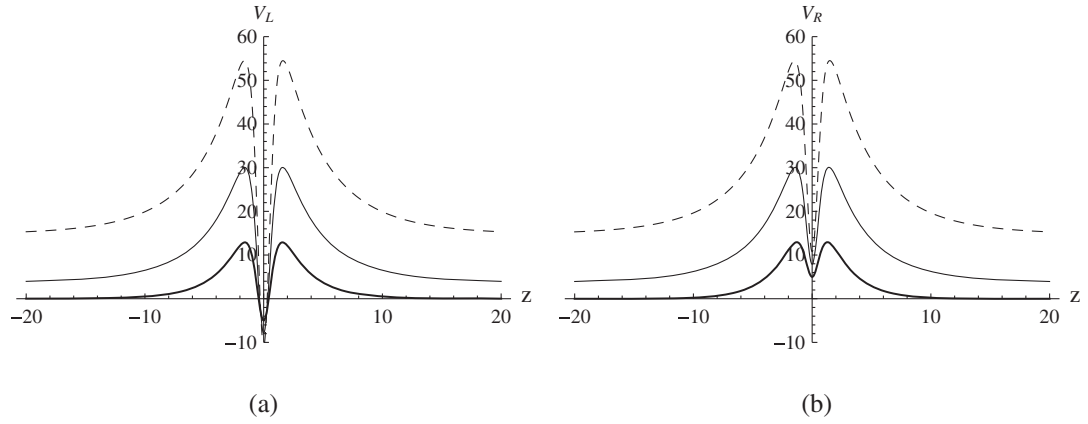


FIG. 2. The shapes of the effective potentials of the left- and right-chiral fermions $V_{L,R}(z)$ for different values of the parameter λ . The parameter λ is set to $\lambda = 0$ for the thick line, $\lambda = 5$ for the thin line, and $\lambda = 10$ for the dashed line. Other parameters are set to $v = 1$, $a = 1$, and $\eta = 5$. (a) $V_L(z)$, $\eta = 5$ and (b) $V_R(z)$, $\eta = 5$.

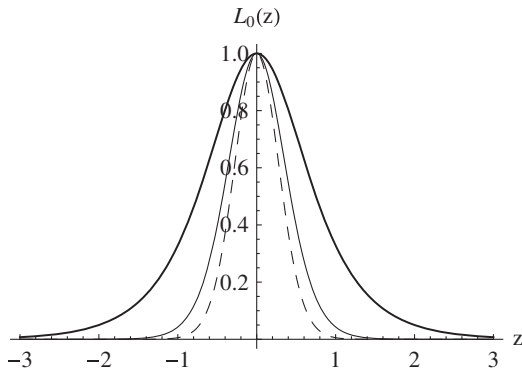


FIG. 3. The shapes of the zero mode of the left-chiral fermion $L_0(z)$ for different values of the parameter η . The coupling coefficient η is set to $\eta = 0$, $\eta = 5$, and $\eta = 10$ for thick, thin, and dashed lines, respectively. Other parameters are set to $v = 1$, $a = 1$, $C_L = 1$ and $\lambda = 5$.

$$\begin{aligned}
 V_R(z) = & \eta^2 e^{2A} f^2(\phi, \pi) + \lambda^2 g'^2(\phi, \pi) \\
 & + \eta e^A [A' f(\phi, \pi) + f'(\phi, \pi)] \\
 & - 2\eta\lambda A(z) f(\phi, \pi) g'(\phi, \pi) - \lambda g''(\phi, \pi). \quad (22b)
 \end{aligned}$$

For the purpose of getting the standard four-dimensional action for a massless chiral fermion and a series of massive fermions

$$\begin{aligned}
 S_{\frac{1}{2}} = & \int d^5x \sqrt{-g} \bar{\Psi} [\Gamma^M (\partial_M + \omega_M) - \eta f(\phi, \pi) \\
 & + \lambda \Gamma^M \partial_M g(\phi, \pi) \gamma^5] \Psi \\
 = & \sum_n \int d^4x [\bar{\psi}_{L_n} \gamma^\mu \partial_\mu \psi_{L_n} + \bar{\psi}_{R_n} \gamma^\mu \partial_\mu \psi_{R_n} \\
 & - m_n (\bar{\psi}_{L_n} \psi_{R_n} + \bar{\psi}_{R_n} \psi_{L_n})] \\
 = & \sum_n \int d^4x \bar{\psi}_n [\gamma^\mu \partial_\mu - m_n] \psi_n, \quad (23)
 \end{aligned}$$

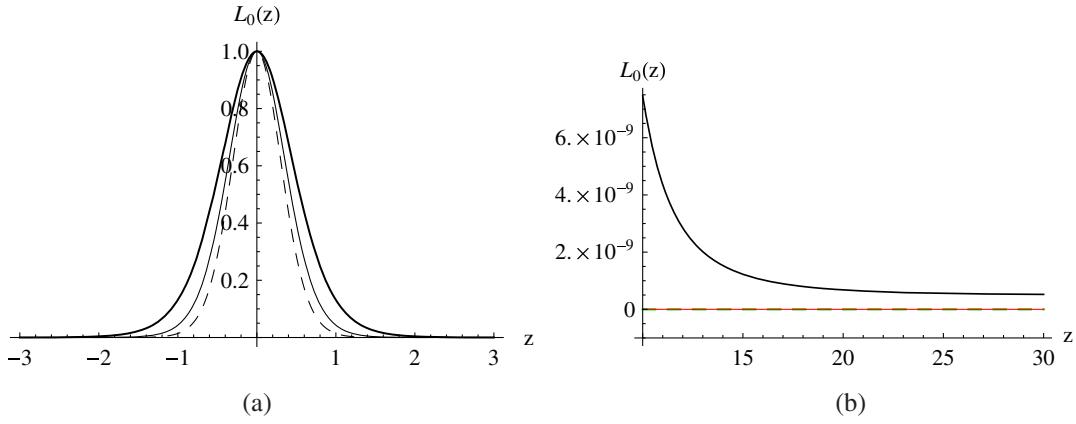


FIG. 4 (color online). The shapes of the zero mode of the left-chiral fermion $L_0(z)$ for different values of the parameter λ . The coupling coefficient λ is set to $\lambda = 0$, $\lambda = 5$, and $\lambda = 10$ for thick, thin, and dashed lines, respectively. Other parameters are set to $v = 1$, $a = 1$, $C_L = 1$, and $\eta = 5$. (a) $-3 \leq z \leq 3$ and (b) $10 \leq z \leq 30$.

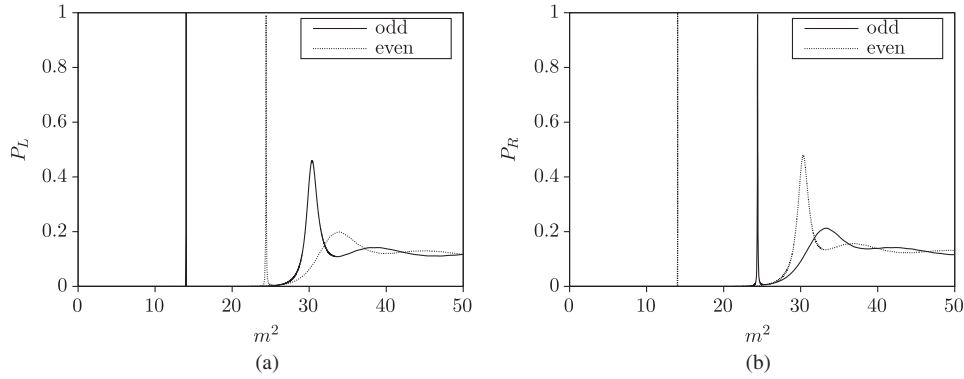


FIG. 5. The shapes of the relative probability $P_{L,R}(m^2)$ for the (a) left- and (b) right-chiral fermion resonant KK modes, with the parameters $a = 1$, $v = 1$, $\lambda = 5$, and $\eta = 5$.

the following orthonormalization conditions for L_n and R_n are needed:

$$\int_{-\infty}^{+\infty} L_m L_n dz = \delta_{mn}, \quad (24)$$

$$\int_{-\infty}^{+\infty} R_m R_n dz = \delta_{mn}, \quad (25)$$

$$\int_{-\infty}^{+\infty} L_m R_n dz = 0. \quad (26)$$

By setting $m_n = 0$ in Eq. (19), we can obtain the left- and right-chiral KK fermion zero modes

$$L_0 \propto e^{\lambda g(\phi, \pi) - \eta} \int e^A f(\phi, \pi) dz, \quad (27a)$$

$$R_0 \propto e^{-\lambda g(\phi, \pi) + \eta} \int e^A f(\phi, \pi) dz. \quad (27b)$$

It is impossible to make both massless left- and right-chiral KK fermion modes to be localized on the brane at the same time, since when one is normalizable, the other one is not.

From Eqs. (21) and (22), it is clear that, if we do not introduce the coupling term in the action (13), i.e., $\eta = 0$ and $\lambda = 0$, the effective potentials for left- and right-chiral KK modes $V_{L,R}(z)$ will vanish and both left- and right-chiral fermions cannot be localized on the thick brane. Moreover, if we demand $V_L(z)$ and $V_R(z)$ to be Z_2 even with respect to the extra dimension z , then the coupling functions $f(\phi, \pi)$ and $g(\phi, \pi)$ must be odd and even functions of z , respectively. The simplest choice is that $f(\phi, \pi) = \phi$ and $g(\phi, \pi) = \pi$, since the kink scalar field ϕ and dilaton field π are odd and even functions of z . In this paper, we want to compare the effect of localization of a bulk fermion with two types of couplings.

By considering the solution of the Minkowski brane (8), (9), and (10), the effective potentials (22) can be expressed as follows:

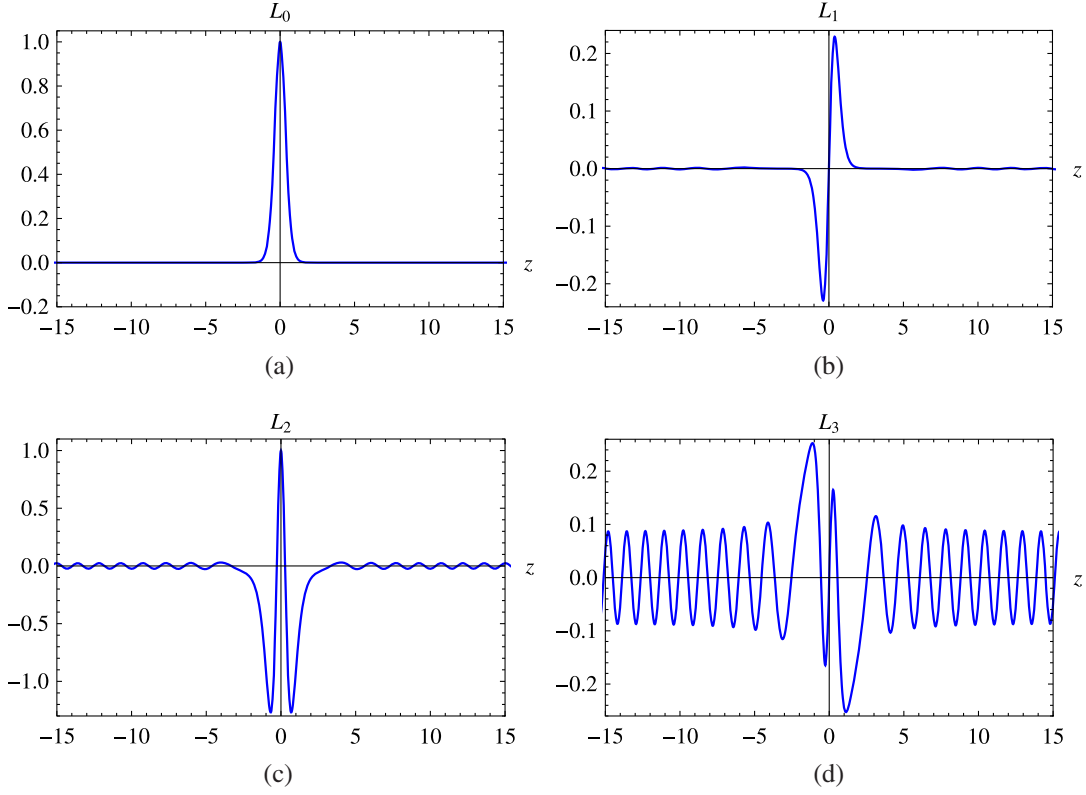


FIG. 6 (color online). The shapes of the wave functions for the left-chiral fermion KK modes L_n , with the parameters $a = 1$, $v = 1$, $\lambda = 5$, and $\eta = 5$. (a) $n = 0$, bound state; (b) $n = 1$, resonant state; (c) $n = 2$, resonant state; and (d) $n = 3$, resonant state.

$$V_L(z) = \frac{1}{27} v e^{-\frac{1}{9}v^2 \tanh^2 az} (\cosh az)^{-\frac{4v^2}{9}} [3a\eta e^{\frac{1}{18}v^2 \tanh^2 az} (\cosh az)^{2(\frac{v^2}{9}-1)} (v^2(2\sqrt{3}\lambda + 1)(2 + \cosh 2az) \tanh^2 az - 9) + a^2 v \lambda e^{\frac{1}{9}v^2 \tanh^2 az} (\cosh az)^{\frac{4v^2}{9}} \operatorname{sech}^4 az (v^2 \lambda (\sinh 2az + \tanh az)^2 - 9\sqrt{3}) + 27v\eta^2 \tanh^2 az], \quad (28)$$

$$V_R(z) = \frac{1}{27} v e^{-\frac{1}{9}v^2 \tanh^2 az} (\cosh az)^{-\frac{4v^2}{9}} [3a\eta e^{\frac{1}{18}v^2 \tanh^2 az} (\cosh az)^{2(\frac{v^2}{9}-1)} (v^2(2\sqrt{3}\lambda - 1)(2 + \cosh 2az) \tanh^2 az + 9) + a^2 v \lambda e^{\frac{1}{9}v^2 \tanh^2 az} (\cosh az)^{\frac{4v^2}{9}} \operatorname{sech}^4 az (v^2 \lambda (\sinh 2az + \tanh az)^2 + 9\sqrt{3}) + 27v\eta^2 \tanh^2 az]. \quad (29)$$

Here we need to analyze the behavior of the potentials near $z = 0$ and as $z \rightarrow \pm\infty$, respectively. From the above expressions (28) and (29), it is easy to obtain

$$V_L(z=0) = -\frac{1}{3} av(3\eta + \sqrt{3}av\lambda), \quad (30)$$

$$V_R(z=0) = \frac{1}{3} av(3\eta + \sqrt{3}av\lambda), \quad (31)$$

and
$$V_{L,R}'(z=0) = 0, \quad (32)$$

$$V_L''(z=0) = \frac{1}{3} a^2 v [2a^2 v^3 \lambda^2 + 4\sqrt{3}av(a + v\eta)\lambda + 6v\eta^2 + 3a(2 + v^2)\eta], \quad (33)$$

$$V_R''(z=0) = \frac{1}{3} a^2 v [2a^2 v^3 \lambda^2 - 4\sqrt{3}av(a - v\eta)\lambda + 6v\eta^2 - 3a(2 + v^2)\eta]. \quad (34)$$

Since the parameters a , v , η , and λ are all positive, $V_L''(z=0) > 0$ and $V_L(z)$ has a local minimum value (negative value) at $z = 0$. As $z \rightarrow \pm\infty$, both potentials tend to a constant only due to the coupling coefficient λ ,

$$V_{L,R}(z \rightarrow \pm\infty) = \frac{4}{27} a^2 v^4 \lambda^2. \quad (35)$$

The shapes of the effective potentials for the left- and right-chiral fermions are plotted in Figs. 1 and 2 for different values of coupling constants, η and λ .

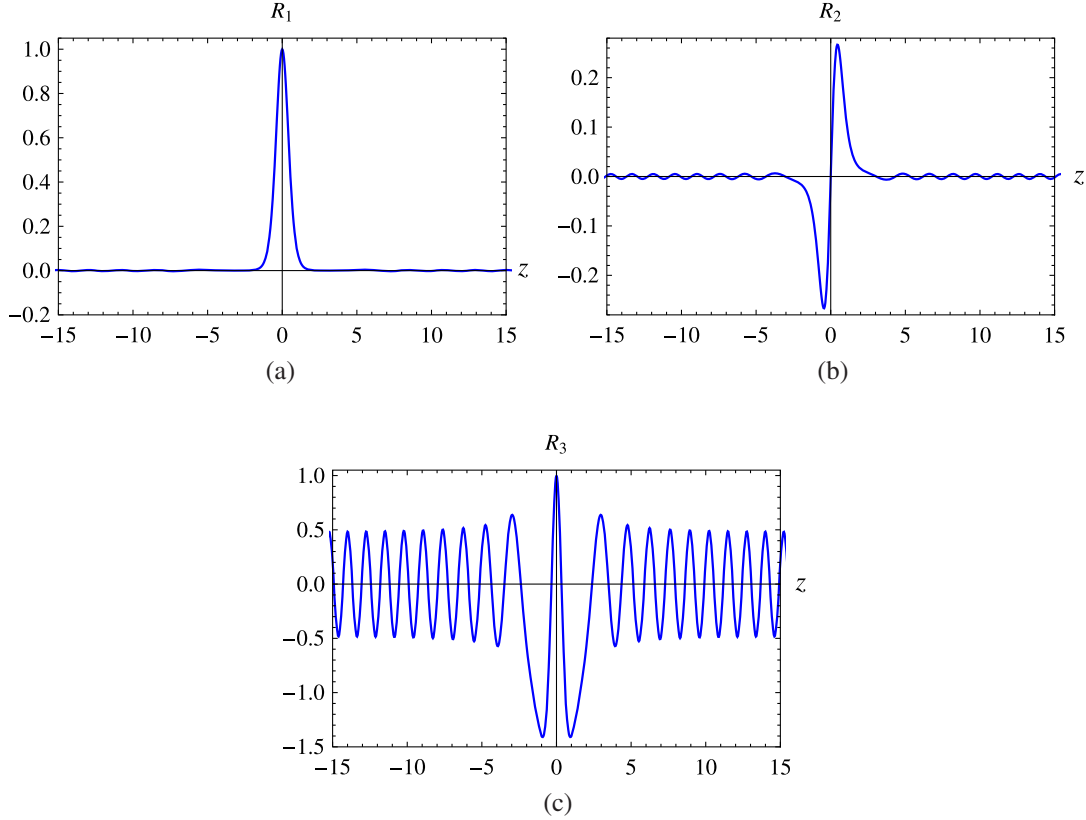


FIG. 7 (color online). The shapes of the wave functions for the right-chiral fermion KK modes R_n , with the parameters $a = 1$, $v = 1$, $\lambda = 5$, and $\eta = 5$. (a) $n = 1$, resonant state; (b) $n = 2$, resonant state; and (c) $n = 3$, resonant state.

It is clear that, with the positive coupling coefficients η and λ , only the potential for left-chiral fermion KK modes has negative part around $z = 0$. So only the left-chiral fermion zero mode may be localized on the brane. The expression of the zero mode can be written as

$$\begin{aligned}
 L_0(z) &= C_L e^{\lambda\pi - \eta} \int_0^z e^{A(z)} \phi(\bar{z}) d\bar{z} \\
 &= C_L (\cosh az)^{-\frac{2\sqrt{3}}{9}v^2\lambda} \\
 &\quad \times e^{-\frac{\sqrt{3}}{18}v^2\lambda \tanh^2 az - v\eta \int_0^z (\cosh a\bar{z})^{-\frac{2\sqrt{3}}{9}} (\tanh a\bar{z}) e^{-\frac{1}{18}v^2 \tanh^2 a\bar{z}} d\bar{z}}, \quad (36)
 \end{aligned}$$

where C_L is a constant. Here the explicit analytic expression of the left-chiral fermion zero mode cannot be obtained. However, from the asymptotic behavior of the warp factor $A(z)$, kink scalar field $\phi(z)$, and dilaton scalar field $\pi(z)$ as $z \rightarrow \pm\infty$, the asymptotic behavior of the zero mode can be analyzed

$$\begin{aligned}
 L_0(z \rightarrow \pm\infty) &\rightarrow C_L 2^{\frac{2\sqrt{3}}{9}v^2\lambda} \\
 &\quad \times \exp\left(-\frac{9\eta}{2av} 2^{\frac{2\sqrt{3}}{9}} e^{-\frac{v^2}{18}} - \frac{\sqrt{3}}{18} v^2\lambda - \frac{2\sqrt{3}}{9} v^2\lambda az\right). \quad (37)
 \end{aligned}$$

Then we can conclude from the above expression that the zero mode of the left-chiral fermion tends to a constant

$C_L e^{-\frac{9\eta}{2av} 2^{\frac{2\sqrt{3}}{9}} e^{-\frac{v^2}{18}}}$ when the coupling coefficient $\lambda = 0$, and it cannot be localized on the brane. For the purpose of localizing the zero mode of the left-chiral fermion, the coupling term $\lambda \bar{\Psi} \Gamma^M \partial_M \pi \gamma^5 \Psi$ in the action (13) must be introduced. We also show the shapes of the left-chiral fermion zero mode for different values of the parameters η and λ in Figs. 3 and 4, respectively. In Fig. 4(b), it is clear that, when $\lambda = 0$, the left-chiral fermion zero mode tends to a constant as $z \rightarrow \pm\infty$, and it cannot be localized on the brane. Thus, we set the coupling coefficient $\lambda > 0$. So $V_L(z \rightarrow \pm\infty) = \frac{4}{27} a^2 v^4 \lambda^2 > 0$, and there is a massless bound state (the left-chiral fermion zero mode) at least, and continuous spectra of the left- and right-chiral fermion KK modes starting from $m^2 > \frac{4}{27} a^2 v^4 \lambda^2$, which indicates the presence of a mass gap in the fermion KK modes spectra.

Next, the effect of the two types of couplings between the fermion and scalar fields for localization of the fermion KK modes will be investigated. In Fig. 2, when $\lambda = 0$ and $\eta > 0$, the potentials $V_{L,R}(z)$ are the modified volcano-type potential. The zero modes of the left- and right-chiral fermions cannot be localized on the brane, and there is no massive bound state, i.e., the massive KK modes of the

TABLE III. The mass, width, and lifetime of bound or resonant KK modes of the fermions. The parameters are set to $a = 1$, $v = 1$, $\lambda = 15$, and $\eta = 0, 5, 10$.

λ	η	Height of V_L	n	Bound or Resonant	m^2	m	Γ	τ
15	0	$V_L^{\text{PT}}(\infty) = 33.3333$	0	Bound state	0	0	0	∞
			1	Bound state	15.2586	3.9062	0	∞
			2	Bound state	26.2153	5.1201	0	∞
	5	$V_L^{\text{max}} = 86.2314$	0	Bound state	0	0	0	∞
			1	Bound state	25.4736	5.0471	0	∞
			2	Resonant state	47.1786	6.8687	7.42×10^{-13}	1.35×10^{12}
			3	Resonant state	64.9127	8.0568	4.38×10^{-6}	2.284×10^5
			4	Resonant state	78.2950	8.8485	4.52×10^{-3}	221.25
			5	Resonant state	86.3584	9.2929	0.11159	8.96161
			6	Resonant state	86.3584	9.2929	0.11159	8.96161
	10	$V_L^{\text{max}} = 165.24$	0	Bound state	0	0	0	∞
			1	Resonant state	35.5690	5.9640	8.85×10^{-16}	1.13×10^{15}
			2	Resonant state	67.5881	8.2212	1.78×10^{-15}	5.63×10^{14}
			3	Resonant state	95.9463	9.7952	3.27×10^{-11}	2.73×10^{10}
4			Resonant state	120.4832	10.9765	6.00×10^{-7}	1.67×10^6	
5			Resonant state	140.9340	11.8716	3.09×10^{-4}	3.232×10^3	
6			Resonant state	156.6930	12.5177	0.0162	61.55221	
7	Resonant state	166.8501	12.9171	0.1707	5.8578			

waves when far away from the brane, and cannot be normalized. As in Ref. [36], the relative probability function of a resonance on the thick brane is defined as follows:

$$P_{L,R}(m^2) = \frac{\int_{-z_b}^{z_b} |L(z), R(z)|^2 dz}{\int_{-z_{\text{max}}}^{z_{\text{max}}} |L(z), R(z)|^2 dz}, \quad (38)$$

where $2z_b$ is about the width of the thick brane, and z_{max} is set to $z_{\text{max}} = 10z_b$. It is clear that when $m^2 \gg V_{L,R}^{\text{max}}$, the fermion KK modes are approximately taken as plane waves and the value of $P_{L,R}(m^2)$ will tend to $1/10$. As in Ref. [60], the lifetime τ is estimated to $\tau \sim \Gamma^{-1}$, with $\Gamma = \delta m$ being the full width at half maximum of the peak.

Equation (21) can be solved by the numerical method, and we will set the coupling coefficients λ and η as different values, respectively, for comparing two types of couplings. When $\lambda = 5$ and $\eta = 0$, only the left-chiral fermion zero mode (bound state) can be localized on the brane. However, for the right-chiral fermion KK modes, there is no bound state or resonant state. When $\lambda = 5$ and $\eta = 5$, for the left-chiral fermion KK modes, there is only one bound zero mode and three resonant KK modes; but for right-chiral fermion KK modes, there are only three resonant KK modes. The profiles of the relative probability $P_{L,R}(m^2)$ corresponding to coupling coefficients $\lambda = 5$ and $\eta = 5$ are shown in Fig. 5. In these figures, each peak corresponds to a resonant state, and the left- and right-chiral fermion KK modes are shown in Figs. 6 and 7. For the left-chiral fermion KK modes, it can be seen that the first massive KK mode is an odd-parity wave function, and the second one has even parity. However, for the right-chiral fermion KK modes, the first massive KK mode has even parity and the

second one has odd parity. This is held for any n th fermion KK mode, namely, the parities of the n th left- and right-chiral KK modes are opposite. In fact, this conclusion is originated from the coupled equations of the left- and right-chiral fermions. For the case of $\lambda = 5$ and $\eta = 10$, the conclusion is similar to that of $\lambda = 5$, $\eta = 5$. The mass, width, and lifetime of the left- and right-chiral fermion KK modes with different values of η are listed in Table I. It can be seen that the mass and lifetime of the left- and right-chiral fermion resonances are almost the same; thus, the formation of the four-dimensional massive Dirac fermions can be realized [35]. So we can summarize that the four-dimensional massless left-chiral fermion can be localized on the brane, and the four-dimensional massive Dirac fermions can also be obtained, which consist of the pairs of coupled left- and right-chiral KK modes with different parities. On the other hand, the total number of resonant KK modes increases with the coupling coefficient η .

Next, we turn to investigate the effect of the coupling coefficient λ on localization of the fermion KK mode. The left- and right-chiral fermion KK modes and the corresponding resonant mass spectrum are solved and calculated for different values of λ . We only list the mass, width, and lifetime of the left-chiral fermion KK modes in Tables II and III, since the result of the right-chiral fermion KK modes is the same except the zero mode. From Tables I, II, and III, it can be concluded that for the same coupling coefficient λ , with the increase of the coupling coefficient η , the number of fermion resonances also increases, and the number of fermion bound KK modes decreases. The reason is that, for the same λ , as increasing η the potential barriers at both sides of the origin of the extra dimension become higher, and the potential well of the left-chiral fermion

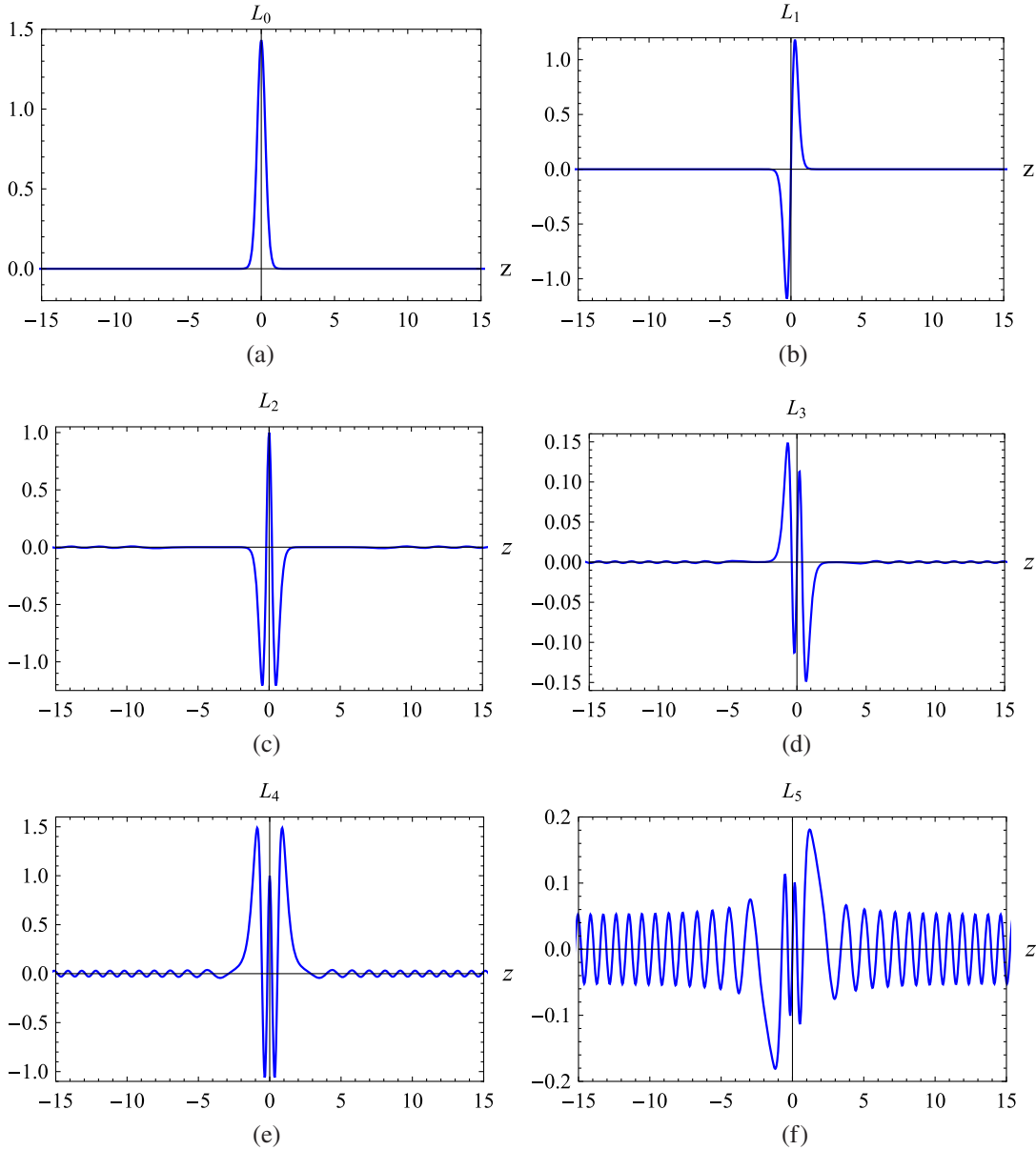


FIG. 8 (color online). The shapes of the wave functions for the left-chiral fermion KK modes L_n , with the parameters $a = 1$, $v = 1$, $\lambda = 15$, and $\eta = 5$. (a) $n = 0$, bound state; (b) $n = 1$, bound state; (c) $n = 2$, resonant state; (d) $n = 3$, resonant state; (e) $n = 4$, resonant state; and (f) $n = 5$, resonant state.

KK modes at $z \approx 0$ tends to be deeper and narrower. The mass of the first massive KK mode also increases with η . On the other hand, for the potential of the right-chiral fermion KK modes, when $\eta \geq \frac{1}{27}av\lambda(4v^2\lambda - 9\sqrt{3})$, we have $V_R(0) \geq V_R(\infty)$, and there is no right-chiral fermion KK bound state and no mass gap in the mass spectrum of the right-chiral fermion KK modes.

For the same λ and η , the lifetime of a resonant KK mode decreases with its mass. As an example, when $\lambda = 15$ and $\eta = 5$, all localization and quasilocalization of the left-chiral fermion KK modes are shown in Fig. 8. The zero mode and first massive KK mode are bound states, which are localized on the brane. Some other massive KK modes

are resonances, which are quasilocalized on the brane. The mass spectra of the left- and right-chiral fermion KK modes are also plotted in Fig. 9. Only the left-chiral fermion zero mode can be localized on the brane, so there exists only the four-dimensional massless left-chiral fermion. The first massive left- and right-chiral fermion KK modes can also be localized on the brane, so a four-dimensional massive Dirac fermion consisting of a pair of coupled left- and right-chiral KK modes can be localized on the brane. When $m_{L,R}^2 \geq V_{L,R}(\infty) = \frac{4}{27}a^2v^4\lambda^2$, there are four pairs of coupled left- and right-chiral resonant KK modes, so the four-dimensional massive Dirac fermions with finite lifetimes can be quasilocalized on the brane. When

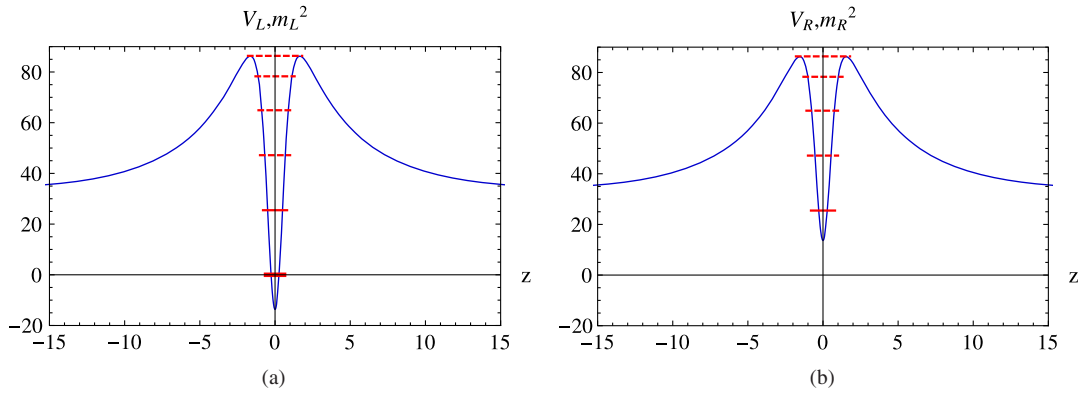


FIG. 9 (color online). The mass spectra and effective potentials for the (a) left- and (b) right-chiral fermion KK modes. The blue lines represent the potentials V_L and V_R , the red lines represent the square of the mass for bound KK modes, and the red dashed lines represent the square of the mass for resonant KK modes. The parameters are set to $a = 1$, $v = 1$, $\lambda = 15$, and $\eta = 5$.

$m_{L,R}^2 \gg V_{L,R}^{\max}$, both the left- and right-chiral fermion KK modes cannot be confined to the brane and will be excited into the bulk.

IV. CONCLUSION AND DISCUSSION

In this paper, localization and mass spectra of a bulk spin-1/2 fermion field on a thick brane generated by two scalar fields (a kink scalar and a dilaton scalar) in five dimensions have been investigated.

For localization of the fermion zero mode, the coupling between the fermion and scalar fields must be introduced in the five-dimensional action. In this paper, we introduce two types of the couplings: the usual Yukawa coupling $-\eta\bar{\Psi}\phi\Psi$ between the fermion and kink scalar and the new coupling $\lambda\bar{\Psi}\Gamma^M(\partial_M\pi)\gamma^5\Psi$ between the fermion and dilaton scalar.

If only introducing the Yukawa coupling, both of the left- and right-chiral fermion zero modes cannot be localized on the brane. So the new coupling term $\lambda\bar{\Psi}\Gamma^M(\partial_M\pi)\gamma^5\Psi$ must be introduced. If only introducing the new coupling, the left-chiral fermion zero mode is localized on the brane, and a finite number of bound massive KK modes of left- and right-chiral fermions may be localized on the brane. The number of bound states increases with the coupling coefficient λ . If we further introduce the Yukawa coupling, there exists a finite number of resonant massive KK modes of left- and right-chiral fermions, and the number of resonances also increases with the Yukawa coupling coefficient η . Hence, the massless fermion localized on the brane consists of just the left-chiral KK mode, while the massive fermions localized or quasilocalized on the brane consist of the left- and right-chiral fermion KK modes and, hence, represent the four-dimensional Dirac massive fermions. The lifetime of the fermion KK resonant modes decreases with their masses.

Finally, we note that if by making the replacement, the Dirac coupling term into the Majorana coupling term in the action (13), we can obtain the action of a Majorana fermion

coupling with the background scalars ϕ and π in five-dimensional spacetime is written as follows:

$$S_M = \int d^5x \sqrt{-g} \left[\bar{\Psi}\Gamma^N(\partial_N + \omega_N)\Psi - \frac{1}{2}\eta(\bar{\Psi}f(\phi, \pi)\Psi^c + \text{H.c.}) + \frac{1}{2}\lambda(\bar{\Psi}\Gamma^N\partial_N g(\phi, \pi)\gamma^5\Psi^c + \text{H.c.}) \right], \quad (39)$$

where Ψ^c is defined as $\Psi^c \equiv C_5\Psi^*$ with the five-dimensional charge conjugation, $C_5 = \Gamma^5\Gamma^2$. Thus, the equation of motion corresponding to the action (39) can be written as

$$[\gamma^\mu\partial_\mu + \gamma^5(\partial_z + 2A'(z))]\Psi - \eta e^A f(\phi, \pi)\Psi^c + \lambda g'(\phi, \pi)\Psi^c = 0. \quad (40)$$

Since the Majorana fermion describes a neutral fermion, we can conclude the Majorana condition

$$\Psi = \Psi^c, \quad (41)$$

which implies the equality of particle and antiparticle. We choose to decompose the five-dimensional fields in terms of left- and right-chiral components to keep our derivation easily comparable with the Dirac case. The general chiral decomposition is written as follows:

$$\Psi = e^{-2A} \left(\sum_n \psi_{L_n}(x) L_n(z) + \sum_n \psi_{R_n}(x) R_n(z) \right), \quad (42a)$$

$$\Psi^c = e^{-2A} \left(\sum_n \psi_{L_n}^c(x) L_n^*(z) + \sum_n \psi_{R_n}^c(x) R_n^*(z) \right), \quad (42b)$$

and the left- and right-chiral four-dimensional fields $\psi_{L_n}(x)$ and $\psi_{R_n}(x)$ satisfy the four-dimensional Majorana equations and Majorana conditions

$$\gamma^\mu \partial_\mu \psi_{L_n}(x) = m_n \psi_{R_n}(x) = m_n \psi_{L_n}^c(x), \quad (43a)$$

$$\gamma^\mu \partial_\mu \psi_{R_n}(x) = m_n \psi_{L_n}(x) = m_n \psi_{R_n}^c(x). \quad (43b)$$

Then the KK modes $L_n(z)$ and $R_n(z)$ also satisfy the following coupled equations:

$$\partial_z L_n(z) + \eta e^A f(\phi, \pi) R_n^*(z) - \lambda g'(\phi, \pi) R_n^*(z) = m_n R_n(z), \quad (44a)$$

$$\begin{aligned} \partial_z R_n(z) + \eta e^A f(\phi, \pi) L_n^*(z) - \lambda g'(\phi, \pi) L_n^*(z) \\ = -m_n L_n(z). \end{aligned} \quad (44b)$$

From the five- and four-dimensional Majorana conditions, it is easily conclude $L_n(z) = R_n^*(z)$ and $R_n(z) = L_n^*(z)$, so Eq. (44) and Eq. (19) are same. Hence the four-dimensional left- and right-chiral fermions are coupled by the same

mass, and the situation of localization for the Majorana fermion is similar to the Dirac fermion. We plan to return to this problem on another occasion.

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