PHYSICAL REVIEW D **92,** 106002 (2015)

Insulator/metal phase transition and colossal magnetoresistance in holographic model

Rong-Gen Cai[*] and Run-Qiu Yang[†]

State Key Laboratory of Theoretical Physics,Institute of Theoretical Physics, Chinese Academy of Sciences,Beijing 100190, China

(Received 28 July 2015; published 5 November 2015)

Within massive gravity, we construct a gravity dual for the insulator/metal phase transition and colossal magnetoresistance effect found in some manganese oxides materials. In the heavy graviton limit, a remarkable magnetic-field-sensitive DC resistivity peak appears at the Curie temperature, where an insulator/metal phase transition happens and the magnetoresistance is scaled with the square of field-induced magnetization. We find that metallic and insulating phases coexist below the Curie point and the relation with the electronic phase separation is discussed.

DOI: 10.1103/PhysRevD.92.106002 PACS numbers: 11.25.Tq, 04.70.Bw, 04.50.Kd

I. INTRODUCTION

In recent years, the holographic correspondence [1–4], relating a weak coupling gravitational theory in a $(d+1)$-dimensional asymptotically anti–de Sitter (AdS) spacetime to a d-dimensional strong coupling conformal field theory (CFT) in the AdS boundary, has been extensively investigated and some remarkable progresses have been made in condensed matter physics systems [5–9]. For a recent review on the holographic superconductor/superfluid models, please refer to Ref. [10]. Very recently, the present authors and their collaborators have realized the paramagnetism/ferromagnetism and paramagnetism/antiferromagnetism phase transitions in holographic models by introducing a massive 2-form field in an AdS black brane background and some interesting magnetic properties of the models have been investigated in a series of papers [11–17]. In this paper, we will provide a new application of the holographic AdS/CFT correspondence by implementing the metal/insulator phase transition and the colossal magnetoresistance (CMR) effect found in some manganese oxides materials in a holographic model.

Complex magnetic materials showing strong magnetoresistance have simultaneously been the focus of the attentions of the magnetic recording industry and the study of strongly correlated electron systems. Particularly, the study of the manganites such as $A_{1-x}B_xMnO_3$ ($A = La$, Pr, Sm, etc. and $B = Ca$, Sr, Ba, Pb), which exhibit the "colossal" magnetoresistance effect, is among the main areas of research in strongly correlated electron systems [18–24]. These materials show remarkable magnetoresistivity and an insulator (or semiconductor)/metal phase transition associated with a paramagnetic/ferromagnetic phase transition, which has a completely different physical origin from the "giant" magnetoresistance observed in layered and clustered compounds. These materials are currently being intensively investigated by a sizable fraction of the condensed matter community, and its popularity is reaching the level comparable to the high-temperature superconducting cuprates.

After great efforts in recent years, mainly through computational and mean-field studies for realistic models, considerable progress has been achieved in understanding the curious properties of those compounds. However, a fully quantitative understanding of the CMR effect is still a challenge; much work remains to be carried out and it is the subject of current active investigation [25]. The holographic duality provides an alternative method for this type of strong correlated phenomena. In this paper, we will make a first attempt to build a holographic model to understand the CMR effect.

II. HOLOGRAPHIC MODEL

Before presenting our holographic model, let us make a brief analysis about how to build a holographic description for such a phenomenon. First, before the ferromagnetic phase transition happens, CMR materials are in an insulating phase where DC resistivity is finite and increases with decreasing temperature. So the translation symmetry is broken; otherwise no scattering happens and DC resistivity is divergent. More important is that, more and more results from experiments show that the insulating phase in CMR is charge ordered, in which charges are localized and form inhomogeneous structures [21]. To realize the inhomogeneity for the CMR effect is a challenge both in condensed matter theory and the holographic description. Fortunately, just as pointed out recently in Ref. [26], momentum relaxation by breaking the translation invariance can be achieved by introducing a mass term of the graviton in the bulk so that the macroscopic DC resistance becomes finite. This provides us with a very simple holographic model to study macroscopic DC resistance in some inhomogeneous

[*]cairg@itp.ac.cn
[†]aqiu@itp.ac.cn

materials without involving some complicated computations. Second, in general, only breaking the translational invariance cannot lead to an insulating resistivity. Meffort and Horowitz [27] proposed a simple framework to have the insulating behavior in general relativity without breaking translational invariance, where a real scalar field is coupled to an U(1) gauge field. But the Meffort-Horowitz model is only valid in the case of zero charge density. Thus a natural choice to build a holographic insulator model with finite charge density is to consider the Meffort-Horowitz model in a massive gravity theory. Finally, motivated by our previous work about DC resistivity in the paramagnetism/ferromagnetism phase transition in the probe limit [17], the model with a massive 2-form field coupled with the Maxwell field shows a metallic ferromagnetic phase at low temperatures.

Based on these considerations, we present the model with the massive 2-form field-Maxwell-dilaton theory in a massive gravity with a negative cosmological constant. The action can be written as,

$$S = \int d^4x\sqrt{-g}\left[\mathcal{R} + \frac{6}{L^2} - (L_{\rm ins} + \lambda^2 L_{\rm ferr}) + L_{\rm mg}\right],$$
$$L_{\rm ins} = e^{-2g_0\psi}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\nabla\psi)^2 + \frac{m_2^2}{2}\psi^2,$$
$$L_{\rm ferr} = \frac{(dM)^2}{12} + \frac{m_1^2}{4}M_{\mu\nu}M^{\mu\nu} + \frac{M^{\mu\nu}F_{\mu\nu}}{2} + \frac{J}{8}V_M,$$
$$L_{\rm mg} = \alpha{\rm Tr}\mathcal{K} + \beta[({\rm Tr}\mathcal{K})^2 - {\rm Tr}\mathcal{K}^2]. \tag{1}$$

Here L is the AdS radius and G is the Newtonian gravitational constant. Without loss of generality, we can set $L = 1$. λ, J, α, β and g_0 are all model parameters. $\mathcal{R}$ is the scalar curvature, and A_μ is the U(1) gauge field with field strength $F_{\mu\nu} = 2\nabla_{[\mu}A_{\nu]}$. ψ is a dilaton field with squared mass m_2^2. $M_{\mu\nu}$ is a 2-form field with squared mass m_1^2 and a nonlinear potential V_M. dM is the exterior differential of the 2-form field $M_{\mu\nu}$ and $(dM)^2 = 9\nabla_{[\mu}M_{\nu\tau]}\nabla^\mu M^{\nu\tau}$. $L_{\rm mg}$ is the mass term of the graviton, where the matrix $\mathcal{K}$ is defined in terms of the dynamical metric $g_{\mu\nu}$ and a reference background metric $\mathfrak{f}_{\mu\nu}$ by

$$\mathcal{K}^\mu{}_\nu\mathcal{K}^\nu{}_\tau = g^{\mu\nu}\mathfrak{f}_{\nu\tau}. \tag{2}$$

This is a special case of the formulation of the massive gravity theory [28,29] presented in Ref. [30]. As shown in Refs. [26,31], there is a position-dependent mass of the gravitational perturbations, $m_g^2(r) = -2\beta - \alpha/r$. Reference [26] showed that, in the leading level of the lattice, the graviton mass term is proportional to the square of the modulating amplitude multiplied by the wave vector in the dual boundary lattice. This gives a physical meaning of the graviton mass in the holographic model, i.e., m_g^2 describes the strength of inhomogeneity in the dual boundary theory. This consequence plays a key role in our holographic model. In this paper, we will pay attention to two limits, i.e., the weak inhomogeneous and strong inhomogeneous cases, which correspond to the cases of $m_g^2 \ll 1$ and $m_g^2 \gg 1$, respectively. These two cases admit us to obtain DC resistivity in a simple manner. We will see later, that only the case with a large value of m_g^2 can give rise to the typical CMR effect, which agrees with the fact that there is a strong inhomogeneity with the CMR effect in the manganite.

The parameter λ can be understood as the coupling strength between the polarization field $M_{\mu\nu}$ and the background Maxwell field strength by rescaling the polarization field and the parameter J. In the effective action of string theory, the value of the dilaton coupling parameter is taken to be $g_0 = 1$. The case with $g_0 = \sqrt{3}$ corresponds to the four-dimensional action by dimensionally reducing from the five-dimensional Kaluza-Klein theory [32]. Nonetheless, it is helpful to set g_0 as an arbitrary positive constant here so that we can see what role the dilaton field plays in the model. In addition, the choice of the nonlinear potential V_M is not unique. What we need is that there is a critical temperature, below which the xy component of $M_{\mu\nu}$ can condense. In this paper, we take the potential as follows:

$$V_M = ({}^*M_{\mu\nu}M^{\mu\nu})^2 = [{}^*(M\wedge M)]^2. \tag{3}$$

Here * is the Hodge-star operator. We choose this form just for simplicity. To be stable for both the bulk and boundary theories when $\psi = 0$, we require $m_g^2 \geq 0$ for all r [33,34]. Following Ref. [34], we take the degenerate reference background with $\mathfrak{f}_{xx} = \mathfrak{f}_{yy} = 1$ and the other components vanish. Although the reference background is degenerate, Ref. [34] showed that this two-parameter massive theory is ghost free for the case with a β mass term, but it has not yet been proven for the case with an α mass term. Therefore in this paper, we will take $\alpha = 0$ in order to avoid possible problems with causality.

III. BACKGROUND EQUATIONS

Now we are in the position to calculate the background solution from our holographic model (1). From this action, we can get the equations of motions for matter fields and gravity,

$$\nabla^\tau(dM)_{\tau\mu\nu} - m_1^2M_{\mu\nu} - J({}^*M_{\tau\sigma}M^{\tau\sigma})({}^*M_{\mu\nu}) = F_{\mu\nu},$$
$$\nabla^\mu\left(e^{-2g_0\psi}F_{\mu\nu} + \frac{\lambda^2}{4}M_{\mu\nu}\right) = 0,$$
$$\nabla^2\psi - m_2^2\psi + 2g_0e^{-2g_0\psi}F_{\mu\nu}F^{\mu\nu} = 0,$$
$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} - \frac{3}{L^2}g_{\mu\nu} - \beta X_{\mu\nu} = T_{\mu\nu}. \tag{4}$$

The energy-momentum tensor $T_{\mu\nu}$ reads

$$T_{\mu\nu} = \lambda^2 \left[\frac{1}{4}(dM)_{\sigma\mu\tau}(dM)^{\sigma\alpha\tau} g_{\alpha\nu} + \frac{m_1^2}{2} M_{\tau\mu} M^{\tau}{}_{\nu} + M_{\tau(\mu} F^{\tau}{}_{\nu)} + \frac{J}{8} V_M g_{\mu\nu}\right] + 2e^{-2g_0\psi} F^{\tau}{}_{\mu} F_{\tau\nu} + \nabla_\mu\psi\nabla_\nu\psi + \frac{1}{2}(\lambda^2 L_{\text{ferr}} + L_{\text{ins}}) g_{\mu\nu} \quad (5)$$

and the tensor field $X_{\mu\nu}$ is,

$$X_{\mu\nu} = (\mathcal{K}^2)_{\mu\nu} - (\text{Tr}\mathcal{K})\mathcal{K}_{\mu\nu} + \frac{1}{2} g_{\mu\nu}[(\text{Tr}\mathcal{K})^2 - \text{Tr}(\mathcal{K}^2)]. \quad (6)$$

In order to solve Eq. (4), we assume that the metric has the following form:

$$ds^2 = -r^2 f e^{-\chi} dt^2 + \frac{dr^2}{r^2 f} + r^2(dx^2 + dy^2), \quad (7)$$

where f and χ are two functions of r. Suppose the solution has a horizon at r_h; the associated temperature then is $T = r_h^2 f' e^{-\chi/2}/4\pi$. We take the following self-consistent ansatz for the matter fields:

$$A_\mu = \phi(r)dt + Bxdy, \qquad \psi = \psi(r),$$
$$M_{\mu\nu} = -p(r)dt \wedge dr + \rho(r)dx \wedge dy. \quad (8)$$

Here B is a constant magnetic field and it will be viewed as the external magnetic field in the boundary field theory. Putting the ansatz and the metric in Eq. (7) into Eq. (4), we can get a set of ordinary differential equations. To solve these equations, we need a total of seven initial conditions and the position of the horizon r_h. We impose the regular conditions at the horizon, which means that all the functions have Taylor expansions near the horizon and $f(r_h) = \phi(r_h) = 0$. This means there are only five independent initial parameters, which are $\rho(r_h)$, $\psi(r_h)$, $\chi(r_h)$, $p(r_h)$ and r_h.

Near the boundary $r \to \infty$, the equations give the following asymptotic solutions for the matter fields:

$$\rho = \rho_+ \left(\frac{r}{r_h}\right)^{(1+\delta_1)/2} + \rho_- \left(\frac{r}{r_h}\right)^{(1-\delta_1)/2} + \cdots + \frac{B}{m_1^2},$$
$$p = \frac{\sigma r_h^2}{m_1^2 r^2} + \cdots, \qquad \phi = \mu - \frac{\sigma r_h}{r} + \cdots,$$
$$\psi = \psi_+ \left(\frac{r}{r_h}\right)^{(\delta_2-3)/2} + \psi_- \left(\frac{r}{r_h}\right)^{-(\delta_2+3)/2} + \cdots, \quad (9)$$

where $\delta_1 = \sqrt{1 + 4m_1^2}$ and $\delta_2 = \sqrt{9 + 4m_2^2}$, μ is the chemical potential, σ is the charge density and $\rho_\pm$ and $\psi_\pm$ are all constants. We impose the regular conditions at the horizon and the Dirichlet and source-free conditions for the matter fields at the boundary of $r \to \infty$, i.e., $\phi = \mu$, $\psi_+ r_h^{(3-\delta_2)/2} = \Delta$ and $\rho_+ = 0$. Without loss of generality, we can set $\Delta = 1$. Note that a nontrivial solution for ψ always exists when g_0 and $F_{\mu\nu}$ are both nonzero. Four boundary conditions give four constraints for five initial values of $\rho(r_h)$, $\psi(r_h)$, $\chi(r_h)$, $p(r_h)$and r_h. As a result, only one of them is free. We can choose $p(r_h)$ as the free initial parameter.

Following Ref. [17], we need $J < 0$ and $\delta_1 > 1$, $\delta_2 < 3$. Otherwise, the nonlinear terms of ρ and ψ will play more and more important roles when $r \to \infty$, which will break the asymptotic AdS_4 geometry of space-time and lead to the instability of the dual theory in the UV region.

IV. DC RESISTIVITY IN THE HEAVY GRAVITON LIMIT

A. Perturbations in the low-frequency limit

Now let us study how DC conductivity is influenced by temperature and the external magnetic field in this model. We need to investigate the properties of perturbation both in the matter sector and the gravitational sector. Because it is isotropic in the $x - y$ plane, the DC resistivity is also isotropic. Then we only need to compute the DC resistivity along the x direction or the y direction. To do so, we consider the perturbation $\delta A_x = \epsilon a_x(r) e^{-i\omega t}$. Then all the terms of A_μ, $g_{\mu\nu}$ and $M_{\mu\nu}$ are involved. However, if we only care about the DC resistivity in the low-frequency limit, i.e., $T \gg \omega \to 0$, then the problem can be simplified. In such a limit, we need only consider the components,

$$\delta A_x = \epsilon a_x(r) e^{-i\omega t}, \qquad \delta M_{ij} = \epsilon C_{ij}(r) e^{-i\omega t},$$
$$\delta g_{\mu\nu} dx^\mu dx^\nu = 2\epsilon \left[r^2 g_{tx}(t) dx dt + \frac{i\omega g_{rx}(r)}{f} dr dx\right] e^{-i\omega t}. \quad (10)$$

Here $(i,j) = (r,x)$ or (t,y). At the linear level of ϵ, the equations for the matter field perturbations are,

$$C_{ty}'' + \frac{\chi'}{2} C_{ty}' + \left[\left(\frac{f'}{f} - \chi'\right) g_{tx} - g_{tx}'\right]\rho' - \frac{m_1^2 C_{ty}}{r^2 f} - 4J\rho p \left(\frac{C_{rx}}{r^2} + \frac{e^\chi p g_{tx}}{r^2 f}\right) + O(\omega) = 0, \quad (11a)$$

$$m_1^2 C_{rx} - a_x' - \frac{4J p \rho e^\chi}{r^4 f}(C_{ty} - \rho g_{tx}) + O(\omega) = 0, \quad (11b)$$

$$[r^2 f e^{-\chi/2}(e^{-2g_0\psi} a_x' - \lambda^2 C_{rx}/4)]' + e^{\chi/2} g_{tx}' r^2 (\lambda^2 e^{2g_0\psi} p/4 - \phi') + O(\omega) = 0, \quad (11c)$$

and the equations for the metric perturbations are,

$$g'_{tx} + [\lambda^2(2B - m_2^2\rho)\rho - 4B^2e^{-2g_0\psi}]\frac{e^{-\chi}}{r^4}g_{rx} + \frac{a_x}{r^2}(\lambda^2 p - 4\phi' e^{-2g_0\psi}) + \mathcal{O}(\omega) = \frac{2\beta e^{-\chi}}{r^2}g_{rx},$$
$$\times (r^2 g_{rx}e^{-\chi/2})' + \mathcal{O}(\omega) = -g_{tx}e^{\chi/2}/f \quad (12)$$

where ψ, p, ρ, f and χ are determined by background solutions. $O(\omega)$ are the terms with order of ω, and the other equations of gravity parts and matter parts are of order $O(\omega)$, all of which can be neglected when $\omega \to 0$.

Here it is worth stressing how to get these perturbational equations. The equations (11) are obtained directly from the equations of motion for $M_{\mu\nu}$ and A_μ. However, the perturbation equations for the gravity part are not from the gravity field equations directly. The first equation in Eq. (12) is a combination of the equations of the rx and xx components at the first and zeroth order of ϵ, respectively, where we eliminated the f'' term. Note that the perturbational equations for gravity at the linear level of ϵ also give a tx-component equation, which is rather complicated. It is not easy to see how to combine it with other equations including the gravity and matter equations to obtain a simple equation. A very simple method to obtain the second equation in Eq. (12) is as follows. First, from the definition of the energy-momentum tensor, combined with the equations of the matter fields, we have $\nabla^\mu T_{\mu\nu} = 0$ (of course, this is a generic conclusion). On the other hand, the Einstein tensor $G_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu}$ is also divergence free. Then the equations of the gravity field imply $\nabla^\mu X_{\mu\nu} = 0$, which should hold at any order of ϵ. At the linear order of ϵ, it gives the second equation in Eq. (12).

B. Heavy graviton limit

In the case of weak inhomogeneity $m_g^2 \ll 1$, the main part of the DC resistivity is very simple. In that case, the graviton mass is very small, and gravitation fluctuations suffer from smaller scattering than others. So the DC conductivity is dominated by the background geometry. In other words, we can neglect the fluctuations of the matter fields when we compute the DC resistivity. Then following Refs. [26,31], we can find that the DC resistivity $R \propto m_g^2 \to 0$. The more interesting case is the strong inhomogeneous limit, i.e., $m_g^2 \gg 1$, which is the case that really interests us in this paper. In such a limit, the graviton has very heavy mass so that it is in fact very hard to be excited by fluctuations. The heavy mass term suppress the fluctuation of gravity so that we can fix the background geometry. In this case, the main part of the DC resistivity can be obtained by just considering the fluctuations of the matter fields. To see that, now let us consider the case when $|\beta| \gg 1$. We assume that ϕ, p, ρ, χ, f are of order $\mathcal{O}(1)$; then the first equation of Eq. (12) shows that g_{rx} is of order $\mathcal{O}(1/\beta)$ and similarly for g_{tx}. This means that in the heavy graviton limit, i.e., $\beta \gg 1$, we can neglect the fluctuations of the metric. This limit enables us to obtain the formula for the DC resistivity easily.

After neglecting the fluctuations of the metric, we have the following equations of perturbations in the low-frequency limit:

$$C''_{ty} + \frac{1}{2}\chi' C'_{ty} - \frac{m_1^2 C_{ty}}{r^2 f} - \frac{4Jp\rho C_{rx}}{r^2} + O(\omega) = 0, \quad (13a)$$

$$m_1^2 C_{rx} - a'_x - \frac{4Je^\chi p\rho C_{ty}}{r^4 f} + O(\omega) = 0, \quad (13b)$$

$$[r^2 f e^{-\chi/2}(e^{-2g_0\psi}a'_x - \lambda^2 C_{rx}/4)]' + O(\omega) = 0. \quad (13c)$$

At the boundary, we have the following asymptotic solutions:

$$C_{ty} = C_{ty+}r^{-(1-\delta_1)/2} + C_{ty-}r^{-(1+\delta_1)/2} + \cdots,$$
$$C_{rx} = -\frac{a_{x-}}{m_1^2 r^2} + \cdots, \qquad a_x = a_{x+} + \frac{a_{x-}}{r} + \cdots. \quad (14)$$

Since $\delta_1 > 1$, we need a boundary condition $C_{ty+} = 0$. According to the dictionary of AdS/CFT correspondence, the electric current is $\langle J\rangle = a_{x-}$ and the DC conductivity is given by $R = \lim_{\omega\to 0} i\omega a_{x+}/a_{x-}$.

From Eq. (13c), we see that in the limit $\omega \to 0$, there is a radial conserved quantity,

$$F = r^2 f e^{-\chi/2}(e^{-2g_0\psi}a'_x - \lambda^2 C_{rx}/4). \quad (15)$$

At the boundary $r \to \infty$, by taking the asymptotic solutions (14) and the fact that $\psi = \chi = 0$ and $f = 1$ into account we have,

$$F = -(1 - \lambda^2/4m_1^2)\langle J\rangle. \quad (16)$$

At the horizon, using Eqs. (13a) and (13b) with the fact that C_{ty} is regular at the horizon, we have,

$$F = \lim_{r\to r_h} r^2 f a'_x\left[e^{-2g_0\psi} - \frac{\lambda^2 m_1^2}{4(m_1^4 + 16J^2 e^\chi p^2\rho^2/r^4)}\right]. \quad (17)$$

The ingoing condition for a_x at the horizon implies,

$$r^2 f a'_x = \frac{d}{dr_*}a'_x = -i\omega a_x \quad (18)$$

at $r \to r_h$. Here $r_* = \int dr/(r^2 f)$ is the tortoise coordinate. So we get,

$$\langle J\rangle = \frac{i\omega a_x(r_h)}{1 - \frac{\lambda^2}{4m_1^2}}\left[e^{-2g_0\psi_0} - \frac{\lambda^2 m_1^2}{4(m_1^4 + 16J^2 e^{\chi_0}p_0^2\rho_0^2/r_h^4)}\right]. \quad (19)$$

Here ψ_0, p_0, χ_0 and ρ_0 are the initial values of ψ, p, χ and ρ at the horizon. Now our mission is to find the relation of $a(r_h)$ and a_+. In the-low frequency limit, Eq. (13c) implies that the electric field is constant, i.e., $\lim_{r=r_h} a_x(r) = a_{x+}$. So we obtain the DC resistivity in the heavy graviton limit as,

$$\frac{1}{R_{\text{heavy}}} = \left(1 - \frac{\lambda^2}{4m_1^2}\right)^{-1} \left[e^{-2g_0\psi_0} - \frac{\lambda^2 m_1^2}{4(m_1^4 + 16J^2 e^{\chi_0} p_0^2 \rho_0^2 / r_h^4)} \right] + \mathcal{O}(1/m_g^2). \tag{20}$$

We see that the DC resistivity is controlled by the values of the field at the horizon. This is in agreement with the statement proposed first by Iqbal and Liu in Ref. [35] using the "membrane paradigm" of a black hole.

As a self-consistency check, we can take $\lambda = 0$. In that case there are only the dilaton and the Maxwell field. Then we have $R_{\text{heavy}}^{-1} = e^{-2g_0\psi_0} + \mathcal{O}(1/m_g^2)$, which agrees with the exact result $R^{-1} = Z(\psi_0) + \mu^2/m_g^2$ given in Ref. [31] with the dilaton coupling $Z(\psi_0) = e^{-2g_0\psi_0}$. From the expression for the DC resistivity, we see that when $R_{\text{heavy}} \sim m_g^2$, the heavy graviton limit is broken. In that case, the fluctuations of the metric have to be taken into account.

V. METAL/INSULATOR PHASE TRANSITION AND MAGNETORESISTANCE IN STRONG INHOMOGENEITY

The physical phase at different temperatures depends on the model parameters. As a typical case, we fix $m_1^2 = 1/3$, $m_2^2 = -2$, $m_g^2 = 40$, $g_0 = 1$ and $\lambda = 3/4$ to compute the DC resistivity at different temperatures and small external magnetic fields numerically. All the results are shown in Fig. 1.

In Fig. 1(a), we plot the DC resistivity at zero magnetic field with different values of the chemical potential μ. There is a critical $\mu_c \simeq 9.71$. When $\mu < \mu_c$, there is an insulator/metal phase transition. The resistivity shows an insulator's behavior described by the dilaton field when $T > T_C$. When the temperature is lowered to the Curie temperature T_C, ρ begins to condense spontaneously and a ferromagnetic phase transition happens. Below and near T_C, the resistivity decreases when the temperature is lowered, which shows a metal's behavior. Though the behavior of the DC resistivity is transformed into metallic from insulating, the insulating phase described by the dilaton field coexists with the ferromagnetic metallic phase in the sample. Numerical results show that two different electronic phases can coexist below the Curie temperature. There is a distinct peak at the temperature where spontaneous magnetization begins to appear and an insulator/metal phase transition happens there. This is just one of the characteristic properties of CMR materials in manganese

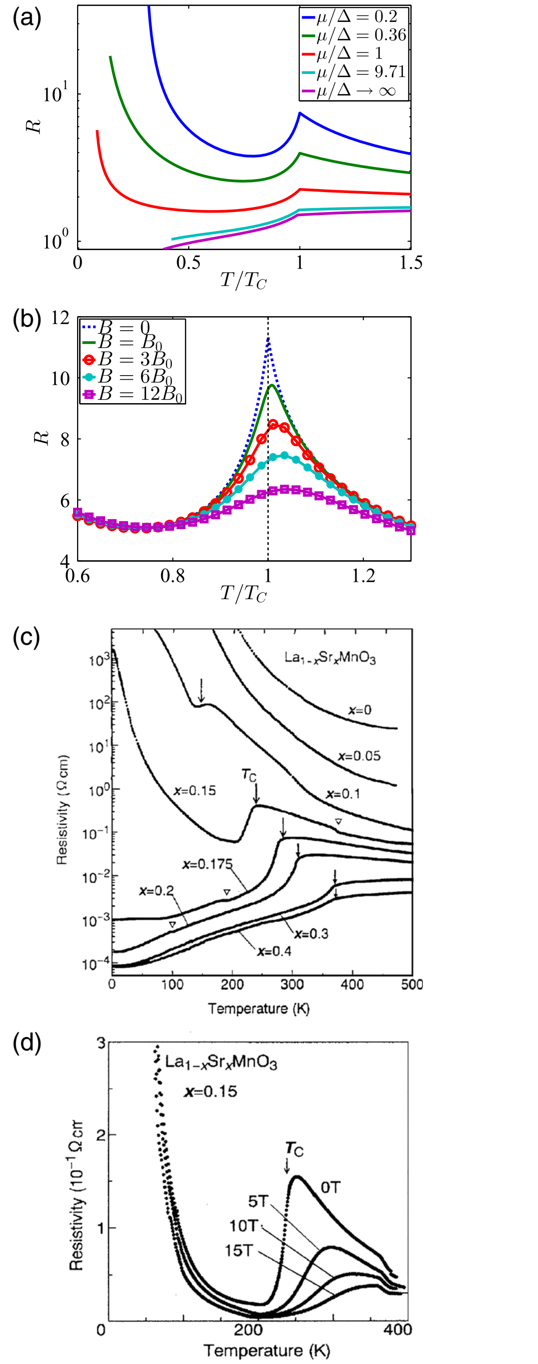

FIG. 1 (color online). (a): The behavior of the DC resistivity vs temperature for different μ with $B = 0$ and $J = -2$. (b): The DC resistivity vs temperature for different external magnetic fields. Here $B_0/T_C^2 \simeq 5.5 \times 10^{-4}$, and $J = -1/3$. (c) and (d): The DC resistivity for $La_{1-x}Sr_xMnO_3$ as a function of temperature for different doping x and magnetic fields. The experimental data are from Ref. [18].

oxides. When $\mu > \mu_c$, the DC resistivity shows a metallic behavior in the whole temperature range (up to the region where numerical computations can be done), though there is still a sudden drop at T_C. What is more, when a small magnetic field B is turned on, we find that the resistivity is very sensitive to the external magnetic field [see Fig. 1(b)].

Here we emphasize that the heavy graviton limit plays a very important role. Just as mentioned above, for the light graviton case, the behavior of the DC resistivity is dominated by the fluctuation of the graviton; then no such metal/insulator phase transition or CMR effects appear. This is in agreement with the fact that the CMR effect is due to the fact that two electronic phases mix with each other and form an inhomogeneity at the nm scale [19,21,36].

It is interesting to compare our holographic results with some experimental data of CMR materials. The resistivity of a typical CMR material $La_{1-x}Sr_xMnO_3$ is shown in Figs. 1(c) and 1(d). We see that our holographic model gives qualitatively similar results when $x \geq 0.1$, which is a powerful evidence to support that this holographic model is a suitable one to describe the CMR effect. Furthermore, in $La_{1-x}Sr_xMnO_3$, when $x > x_c \simeq 0.2$, the peak of the DC resistivity disappears, which is very similar to the case when the chemical potential $\mu > \mu_c$. This is consistent with the standard holographic dictionary that the chemical potential μ relates to the doping of materials. In addition, we can find from Eq. (20) the following scaling relation for magnetoresistance (MR) in the case of a weak magnetic field and $T \to T_C^+$:

$$\mathrm{MR}(B) = 1 - R(B)/R(B=0) \propto \rho_0^2 \propto B^2. \qquad (21)$$

Note that in the region of $T > T_C$, the system is in the paramagnetic phase. Therefore the magnetic moment N is proportional to B in the weak-field case. Then Eq. (21) tells us that $\mathrm{MR}(B) \propto N^2$. This result is in complete agreement with the experimental data of CMR materials [18].

Here it is worth making some additional comments on Fig. 1. In Fig. 1(b) we only plotted the DC resistivity with respect to temperature for different magnetic fields near the Curie point so that the different curves can be seen clearly. When the temperature is much less than T_C, the DC resistivity will grow similarly to Fig. 1(d). The characteristic phenomenon of the CMR effect is that the DC resistivity is sensitive to the magnetic field at the Curie temperature. Since the Curie point disappears when $x \to 0$, there is no CMR effect. The model can only cover the materials in the region where the CMR effect can happen, so it cannot reproduce the behavior of $La_{1-x}Sr_xMnO_3$ when $x \to 0$. The model here is a phenomenological one for the CMR effect rather than a model for the material $La_{1-x}Sr_xMnO_3$, so it can only cover the region where the CMR effect happens in $La_{1-x}Sr_xMnO_3$.

VI. DISCUSSION

In this paper we have presented a gravity dual to the metal/insulator phase transition and found that the model can describe the CMR effect in some manganese oxides materials. The behavior of DC resistivity is in qualitative agreement with experimental data. The model provides a new example to apply the AdS/CFT correspondence to condensed matter systems. As the first attempt to describe the CMR effect in a holographic setup, more aspects of the model should be further studied. The first one is to study the behavior of DC resistivity in the case with an arbitrary graviton mass. For this, we need to consider the perturbations of the gravitational background, which is under investigation. Note that the current model only realizes the macroscopic phenomenon at large scales. It is very interesting to consider whether one can directly realize such a local electronic phase separation in a holographic setup with Einstein's gravity theory rather than massive gravity. For example, we can set the chemical potential to be periodic in the spatial directions and take the lattice and impurity into account. In those cases, we can expect both ψ and ρ to be inhomogeneous, and electronic phase separation may be realized. For this, we have to deal with a set of partially differential equations and the involved numerical computation is extremely nontrivial. We expect it could be reported in the future.

ACKNOWLEDGMENTS

We thank helpful discussions with J. Erdmenger, S. Hartnool, F. Kunsmartsev, K. Schalm, J. Zaanen and many participants of the KITPC program "Holographic duality for condensed matter physics" (July 6-31,2015, Beijing). This work was supported in part by the National Natural Science Foundation of China (No. 11375247, and No. 11435006).

[1] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. **2**, 231 (1998) Int. J. Theor. Phys. **38**, 1113 (1999).

[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Gauge theory correlators from non-critical string theory, Phys. Lett. B **428**, 105 (1998).

[3] E. Witten, Anti–de Sitter space and holography, Adv. Theor. Math. Phys. **2**, 253 (1998).

[4] E. Witten, Anti–de Sitter space, thermal phase transition, and confinement in gauge theories, Adv. Theor. Math. Phys. **2**, 505 (1998).

[5] S. S. Gubser, Breaking an Abelian gauge symmetry near a black hole horizon, Phys. Rev. D **78**, 065034 (2008).

[6] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, Building a holographic superconductor, Phys. Rev. Lett. **101**, 031601 (2008).
[7] S. S. Lee, A non-Fermi liquid from a charged black hole: a critical Fermi ball, Phys. Rev. D **79**, 086006 (2009).
[8] H. Liu, J. McGreevy, and D. Vegh, Non-Fermi liquids from holography, Phys. Rev. D **83**, 065029 (2011).
[9] M. Cubrovic, J. Zaanen, and K. Schalm, String theory, quantum phase transitions and the emergent Fermi-liquid, Science **325**, 439 (2009).
[10] R. G. Cai, L. Li, L. F. Li, and R. Q. Yang, Introduction to holographic superconductor models, Sci. China Phys. Mech. Astron. **58**, 1 (2015).
[11] R.-G. Cai and R.-Q. Yang, Paramagnetism-ferromagnetism phase transition in a dyonic black hole, Phys. Rev. D **90**, 081901 (2014).
[12] R. G. Cai and R. Q. Yang, Holographic model for the paramagnetism/antiferromagnetism phase transition, Phys. Rev. D **91**, 086001 (2015).
[13] R. G. Cai and R. Q. Yang, Coexistence and competition of ferromagnetism and p-wave superconductivity in holographic model, Phys. Rev. D **91**, 026001 (2015).
[14] R. G. Cai and R. Q. Yang, Antisymmetric tensor field and spontaneous magnetization in holographic duality, Phys. Rev. D **92**, 046001 (2015).
[15] R. G. Cai, R. Q. Yang, and F. V. Kusmartsev, Holographic model for antiferromagnetic quantum phase transition induced by magnetic field, Phys. Rev. D **92**, 086001 (2015).
[16] R. G. Cai, R. Q. Yang, and F. V. Kusmartsev, Holographic antiferromagnetic quantum criticality and AdS_2 scaling limit, Phys. Rev. D **92**, 046005 (2015).
[17] R. G. Cai, R. Q. Yang, Y. B. Wu, and C. Y. Zhang, Massive 2-form field and holographic ferromagnetic phase transition, arXiv:1507.00546.
[18] A. Urushibara, Y. Moritomo, T. Arima, A. Asamitsu, G. Kido, and Y. Tokura, Insulator-metal transition and giant magnetoresistance in $La_{1-x}Sr_xMnO_3$, Phys. Rev. B **51**, 14103 (1995).
[19] M. Uehara, S. Mori, C. H. Chen, and S. W. Cheong, Percolative phase separation underlies colossal magnetoresistance in mixed-valent manganites, Nature (London) **399**, 560 (1999).
[20] M. B. Salamon, and M. Jaime, The physics of manganites: structure and transport, Rev. Mod. Phys. **73**, 583 (2001).
[21] E. Dagotto, T. Hotta, A. Moreo, Colossal magnetoresistant materials: the key role of phase separation, Phys. Rep. **344**, 1 (2001).
[22] E. L. Nagaev, Colossal-magnetoresistance materials: manganites and conventional ferromagnetic semiconductors, Phys. Rep. **346**, 387 (2001).
[23] C. Sen, G. Alvarez, and E. Dagotto, Unveiling first order CMR transitions in the two-orbital model for manganites, Phys. Rev. Lett. **105**, 097203 (2010).
[24] A. Mukherjee, W. S. Cole, P. Woodward, M. Randeria, and N. Trivedi, Theory of strain-controlled magnetotransport and stabilization of the ferromagnetic insulating phase in manganite thin films, Phys. Rev. Lett. **110**, 157201 (2013).
[25] E. Dagotto, Open questions in CMR manganites, relevance of clustered states and analogies with other compounds including the cuprates, New J. Phys. **7**, 67 (2005).
[26] M. Blake, D. Tong, and D. Vegh, Holographic lattices give the graviton an effective mass, Phys. Rev. Lett. **112**, 071602 (2014).
[27] E. Mefford and G. T. Horowitz, Simple holographic insulator, Phys. Rev. D **90**, 084042 (2014).
[28] C. de Rham and G. Gabadadze, Generalization of the Fierz-Pauli action, Phys. Rev. D **82**, 044020 (2010).
[29] C. de Rham, G. Gabadadze and A. J. Tolley, Resummation of massive gravity, Phys. Rev. Lett. **106**, 231101 (2011).
[30] S. F. Hassan and R. A. Rosen, On non-linear actions for massive gravity, J. High Energy Phys. 07 (2011) 009.
[31] M. Blake and D. Tong, Universal resistivity from holographic massive gravity, Phys. Rev. D **88**, 106004 (2013).
[32] D. L. Wiltshire, Dilaton black holes with a cosmological term, J. Aust. Math. Soc. Series B, Appl. Math. **41**, 198 (1999).
[33] R. A. Davison, Momentum relaxation in holographic massive gravity, Phys. Rev. D **88**, 086003 (2013).
[34] D. Vegh, Holography without translational symmetry, arXiv:1301.0537.
[35] N. Iqbal and H. Liu, Universality of the hydrodynamic limit in AdS/CFT and the membrane paradigm, Phys. Rev. D **79**, 025023 (2009).
[36] S.-W. Cheong, P. A. Sharma, N. Hur, Y. Horibe and C. H. Chen, Electronic phase separation in complex materials, Physica B (Amsterdam) **318**, 39 (2002).