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Particle collisions near a Kerr-like black hole in Brans-Dicke theory

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A recent discovery in 2009 by Bañados, Silk and West (BSW), which generated a lot of interest, involves the arbitrary high center-of-mass (c.m.) energies for free particle collisions at the horizon of an extreme kerr black hole when one of the free particles has a critical value of the angular momentum. In light of this we consider the rotating Kerr-like black hole solution in Brans-Dicke theory and study the motion of scalar test charges in the vicinity of the black hole horizon. We show that the interaction of the test scalar charges with the background scalar field in this spacetime suppresses the c.m. energy for collisions occurring near the event horizon, and the value of the c.m. energy there, is finite irrespective of whether the black hole is extreme or not and its value is also independent of the angular momenta of the colliding test charges.

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I. INTRODUCTION

Recently Bañados, Silk and West (BSW) [1] showed that when two free particles which are released from rest at infinity in the equatorial plane of an extreme Kerr black hole collide in the vicinity of the event horizon, the c.m energy of the collision can be arbitrary high provided that one of the colliding particles has a critical value of the angular momentum. At first glance this interesting phenomenon suggests that rapidly spinning black holes at the center of most galaxies can act as Planck energy scale particle accelerators, allowing us to explore the physics of ultrahigh energy particle collisions that cannot be achieved with the present day terrestrial accelerators. The calculation of the c.m. energy for particle collisions near the horizon of a Schwarzschild black hole was shown earlier [2] to be finite and not significantly larger than the combined rest mass of the colliding particles. So it immediately became evident that rotation is an essential requirement for achieving such high energies. The infinite c.m. energy in the BSW process for the Kerr black hole can be explained in terms of the kinematics of the particles near the horizon (see Ref. [3] for a detailed explanation). Basically this occurs due to the fact that the relative velocity between the colliding particles approaches the speed of light, meaning that the Lorentz factor becomes unbound, when one of these particles attains a critical value of the angular momentum.

Shortly after this discovery Berti *et al.* [4] and Jacobson and Sotiriou [5] criticized its practicality in astrophysical scenarios by pointing out a number of practical limitations that prevent these arbitrary high energies. First the BSW process requires very fine tuning and it takes an infinite amount of time to access the infinite collision energy [5]. Moreover on a more practical side there is the nonexistence of extremal black holes in nature, i.e. the spin parameter a of an astrophysical black hole of mass m cannot exceed $a/m = 0.998$ [6]. In any case by the third law of black hole thermodynamics [7] we know that the black hole spin cannot be increased to its extreme value $a = m$ in a finite amount of time. Added to this one should take into consideration gravitational radiation and the backreaction effects [4] in such a process. Despite these limitations the BSW effect has generated a lot of interest and in recent years it has been studied extensively both for the Kerr black hole [8–15] and other black hole systems [16–28], with or without a cosmological background and in alternative theories of gravity [28–32]. In particular the requirements of rotation and extremality of the black hole have been reexamined for the Kerr black hole as well as other black hole systems. So for example, by considering the scattering of particles in the gravitational field of a Kerr black hole, it was shown in Ref. [10] that very large c.m. energies for particle collisions at the horizon are still possible for nonextremal black holes. It was also shown in Ref. [16] that in the case of the static extreme Reissner-Nordström black hole, infinite c.m. energies can also be achieved at the horizon for collisions between radially moving charged particles with one of the particles having a critical value of the electric charge. Moreover recently it was shown (see Refs. [33,34]) that under some reasonable conditions the above mentioned limitations due to backreaction effects do not necessarily exclude the possibility of infinite c.m. energies for nongeodesic particle collisions. The BSW process has also been studied in cosmological spacetimes such as the Kerr-de Sitter black hole [21] and the Reissner-Nordström-de Sitter black

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hole [23]. In the latter case the authors claim that infinite c.m. energy for particle collisions at the cosmological horizons are possible without the requirement of extremality of the black hole. However this claim was refuted in Ref. [35], where it was shown that the infinite c.m. energy at the cosmological would be attained in a finite amount of time and would therefore make the process unphysical.

A study of the BSW effect in the presence of scalar fields was first carried out by Patil and Joshi [30] who obtained infinite c.m. energies for free particle collisions in the vicinity of the naked singularity of the static spherically symmetric Janis-Newman-Winicour (JNW) spacetime [36] which contains a massless minimally coupled scalar field that is also singular at the position of the naked singularity. This led them to conclude that instead of spinning up a black hole or charging it [as in the case of the Reissner-Nordström (RN) solution], one can also crank up the c.m. energy of the collision by "charging" the black hole with a static massless scalar field. However in our earlier study [32] we showed that the presence of a scalar field does not always lead to infinite c.m. energy in other black hole systems containing scalar fields. Examples of these systems include the static spherically symmetric asymptotically flat black hole solution with a conformally coupled massless scalar field found by Bocharova, Bronnikov and Melnikov [37] and independently by Bekenstein [38] (also called the BBMB solution) and its generalization to the case of a positive cosmological constant that was found by Martinez, Troncoso and Zanelli (MTZ) [39] which has a massless conformally coupled scalar field with a quadratic potential that depends on the cosmological constant. Unlike the JNW spacetime both of these solutions represent genuine black holes without naked singularities and in the former case the scalar field is again singular on the event horizon. In each case it was shown that the c.m. energy for free particle collisions in the vicinity of the event horizon is finite, and is given by the same expression obtained by Baushev [2] for the Schwarzschild solution. Moreover the c.m. energy for collisions involving scalar test charges that interact with the background scalar field in these spacetimes, was shown to be even smaller than in the case of free particle collisions, and therefore the presence of a scalar field (or better its interaction with the test scalar charges) suppresses the c.m. energy rather than enhancing it as had originally been claimed by Patil and Joshi [30]. This suggests that the infinite energy for free particle collisions seen in the JNW spacetime does not arise from the presence of the massless scalar field, but may be instead related to the naked singularity. In fact in our earlier study we had also considered a particular case in the class of Einstein-scalar solutions obtained by Anabalón and Cisterna [40] which also contains a naked singularity and showed that the c.m. energy for free particle collisions there, is also infinite.

Since so far the black hole Einstein-scalar solutions used to study the BSW process were all static, in this paper we consider a Kerr-like solution in Brans-Dicke theory having a scalar field which is singular on the event horizon (which from now on will be referred to as the Brans-Dicke-Kerr (BDK) black hole [41,42]), and we will obtain the c.m. energy for collisions between test scalar charges in the vicinity of the event horizon. We show that unlike the Kerr black hole the c.m. energy for collisions between test scalar charges in the vicinity of the event horizon is finite for $-5/2 \le \omega \le -3/2$, where ω is the Brans-Dicke parameter, and its value is equal to the sum of the particles' rest masses, irrespective of whether the black hole is extreme or not. For other values of ω the BDK spacetime has a naked singularity and in this case the c.m. energy becomes infinite in the vicinity of the singularity as in the JNW spacetime. If instead of test scalar charges, one uses free particles that do not interact with the background scalar field, then the situation is no different than the Kerr case, i.e. the c.m. energy near the horizon is again infinite. Therefore as we have seen in our previous study which was limited to static examples, the presence of the scalar field in rotating black hole spacetimes also suppresses the c.m. energy for test scalar charge collisions, such that its value on the horizon is reduced to the sum of the particles' rest masses. Hence in some sense the BSW process can distinguish between black hole solutions with and without scalar fields.

This paper is organized as follows. In Sec. II we present the BDK solution and discuss the cases corresponding to different values of ω. Then in Sec. III we consider the trajectories of test scalar charges coupled to the background scalar field and obtain the corresponding c.m. energy. Results are summarized and discussed in the Conclusion. Unless otherwise noted, in this paper we use geometric units, $G = c = 1$.

II. BRANS-DICKE-KERR (BDK) SOLUTION

The first well-known scalar tensor extension to Einstein's general relativity is Brans-Dicke (BD) theory [43] which was developed to accommodate both Mach's principle [44] and Dirac's large number hypothesis [45], and in which Newton's gravitational constant $G = 1/\psi$ is a variable written in terms of a scalar field ψ. The action of BD-theory in the so-called Jordan frame is given by

$$S^{(\mathrm{BD})} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\psi R - \frac{\omega}{\psi}g^{cd}\nabla_c\psi\nabla_d\psi - V(\psi)\right] + S^{(\mathrm{M})}, \tag{1}$$

where

$$S^{(\mathrm{M})} = \int d^4x\sqrt{-g}\mathcal{L}^{\mathrm{M}} \tag{2}$$

is the matter action and ω is the dimensionless Brans-Dicke parameter. In our case we take the scalar field potential $V(\psi)$ to be zero. Varying the action with respect to the metric tensor g_{ab} gives

$$G_{ab} = \frac{8\pi}{\psi} T^{(\mathrm{M})}_{ab} + \frac{\omega}{\psi^2}\left(\nabla_a\psi\nabla_b\psi - \frac{1}{2}g_{ab}\nabla^c\psi\nabla_c\psi\right) + \frac{1}{\psi}(\nabla_a\nabla_b\psi - g_{ab}\Box\psi), \tag{3}$$

where

$$T^{(\mathrm{M})}_{ab} = \frac{-2}{\sqrt{-g}}\frac{\delta}{\delta g^{ab}}(\sqrt{-g}\mathcal{L}^{(\mathrm{M})}). \tag{4}$$

Varying the action with respect to the scalar field gives

$$\Box\psi = \frac{8\pi}{2\omega+3}T^{(\mathrm{M})}, \tag{5}$$

where $T = T^\mu_\mu$ is the trace of the matter energy momentum tensor. The first exact static and spherically symmetric solution to (3) with a massless scalar field was obtained by Brans and Dicke [43] themselves. In the Einstein frame, which is obtained by taking the conformal transformation

$$g_{ab} \to \tilde{g}_{ab} = \Omega^2 g_{ab} \tag{6}$$

with $\Omega = \sqrt{G\psi}$ and the scalar field redefinition given by

$$\tilde{\psi} = \sqrt{\frac{2\omega+3}{16\pi G}}\ln\left(\frac{\psi}{\psi_0}\right), \tag{7}$$

where ψ_0 is the current value of the gravitational constant, the solution obtained by Brans and Dicke reduces to the JNW spacetime that was used by Patil and Joshi [30] in their earlier study of the BSW process in this spacetime.

Obtaining stationary axisymmetric solutions of BD theory is not such a straightforward job, and in most cases algorithms generating exact solutions from already known simpler solutions in BD theory or even Einstein's theory are used. One such algorithm that allows the generation of a stationary axisymmetric solution in vacuum BD theory from the known Kerr solution in vacuum Einstein's theory, was presented by Tiwari and Nayak [41]. In this case the metric which is a solution of (3) and which we will refer to as the BDK metric is given by (see also Refs. [42] and [46])

$$\begin{aligned} ds^2 = {} & \Delta^{-2/(2\omega+3)}\sin^{-4/(2\omega+3)}\theta\left[-\left(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right)dt^2 \right. \\ & - \frac{2a\sin^2\theta(r^2+a^2-\Delta)}{\Sigma}dtd\phi \\ & \left. + \left(\frac{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2\right] \\ & + \Delta^{2/(2\omega+3)}\sin^{4/(2\omega+3)}\theta\left[\frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2\right], \end{aligned} \tag{8}$$

where $\Sigma = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2mr + a^2$, with m and a denoting the Arnowitt-Deser-Misner (ADM) mass and angular momentum per units mass, respectively. The associated scalar field ψ which satisfies (5) is given by

$$\psi(r,\theta) = \Delta^{2/(2\omega+3)}\sin^{4/(2\omega+3)}\theta. \tag{9}$$

As expected the above metric reduces to the Kerr solution when $\omega \to \infty$. The spacetime has the same ring shaped curvature singularity $\Sigma = 0$ present in the Kerr metric. Moreover, the Killing field $\chi^\mu = \xi^\mu + \Omega_H\eta^\mu$, where $\xi^\mu = (\partial/\partial t)^\mu$ and $\eta^\mu = (\partial/\partial\phi)^\mu$ with Ω_H being the angular velocity of the horizon, becomes null on the surface $\Delta^{(2\omega+1)/(2\omega+3)} = 0$, meaning that for $\omega < -3/2$ or $\omega > -1/2$ the spacetime has also the same Killing horizons $r_\pm = m \pm (m^2 - a^2)^{1/2}$ of the Kerr solution. A computation of the Kretschmann scalar $\kappa = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ reveals that for $-5/2 \le \omega < -3/2$ this is finite and vanishes on the Killing horizons $r_\pm$ and the scalar field $\psi(r_\pm,\theta) \to \infty$ there, so that the effective Newtonian constant tends to zero on r_+. For $-1/2 < \omega < \infty$ and $\omega < -5/2$ the Kretschmann scalar becomes infinite on r_+ so in these cases the spacetime has a second curvature singularity at the horizon, while for $-3/2 < \omega \le 1/2$ the spacetime has no Killing horizons and so the curvature singularity $\Sigma = 0$ is naked. In other words for the range $-5/2 \le \omega < -3/2$, the BDK metric in (8) represents a nontrivial black hole in BD theory and has the same Killing horizons and ring singularity found in the Kerr spacetime. It should be noted that if we let $a = 0$ in the above metric, then we get a static axisymmetric metric which can be called the Brans-Dicke-Schwarzschild (BDS) that is not spherically symmetric and therefore different than the solution obtained by Brans and Dicke in Ref. [43].

At this point we should point out that the stationary axisymmetric vacuum solution of BD theory is not unique. So in the literature one can find other generating techniques for obtaining such solutions from known ones. For example Krori and Bhattacharjee [47] applied the method of Newman and Janis [48] which was originally used to derive the Kerr metric from the Schwarzschild metric via a complex coordinate transformation, and they obtained a Kerr-like metric in BD theory from the Brans and Dicke's metric obtained in Ref. [43]. However although this type of metric has been used in a number of later articles (see for example Refs. [49–51]) mainly in its Einstein frame form (where it can be interpreted as a rotating generalization of the JNW metric), one can check that the original metric derived by Krori and Bhattacharjee does not satisfy the field equations in (3). Other generating techniques for Kerr-like solutions associated with either minimally or conformally coupled scalar fields can be found in Refs. [52,53]. In most cases the generated solutions have naked curvature singularities and/or violate energy conditions in parts of the spacetime, and the closest Kerr-like metric in BD theory that resembles the Kerr solution is that given by (8) above, which we will therefore use in the next section to compute the c.m. energy for particle collisions in the vicinity of the event horizon.

In the Einstein frame, obtained by using the conformal transformation in (6), the metric reduces to

$$ds^2 = -\left(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right)dt^2 - \frac{2a\sin^2\theta(r^2+a^2-\Delta)}{\Sigma}dtd\phi + \left(\frac{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2 + \Delta^{4/(2\omega+3)}\sin^{8/(2\omega+3)}\theta\left[\frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2\right] \quad (10)$$

while the scalar field becomes

$$\tilde{\psi} = \frac{1}{2\sqrt{\pi}\sqrt{2\omega+3}}\ln(\Delta\sin^2\theta), \quad (11)$$

and the field equations satisfied by this metric take the simple form

$$R_{ab} = 8\pi\tilde{\psi}_a\tilde{\psi}_b$$
$$\Box\tilde{\psi} = 0.$$

However for the range $-5/2 \leq \omega < -3/2$ for which the metric represents a black hole as discussed above, the transformed scalar field $\tilde{\psi}$ is imaginary, and therefore in the next section we use the Jordan frame form of the metric as given by (8).

III. SCALAR CHARGES AND THE C.M. ENERGY

In order to study the effect of the massless scalar field on the c.m. energy for collisions near the event horizon in the BDK spacetime, it is necessary to consider the motion of test scalar charges which can interact with the background scalar field instead of taking simply the motion of free particles as has been done in earlier studies of the BSW effect, where most of the spacetimes used had no associated scalar fields. Assuming a simple linear coupling between the background scalar field ψ and the test scalar charges having mass μ, as we did in our earlier study [32] (see also Ref. [54]) the action takes the form

$$S = -\int(\mu + f\psi)\left(-g_{ab}\frac{dx^a}{d\lambda}\frac{dx^b}{d\lambda}\right)^{1/2}d\lambda, \quad (12)$$

where $f > 0$ is a constant representing the coupling strength between the scalar field and the test scalar charges and λ is the parameter along the particle trajectory $x^a(\lambda)$. Taking λ to be the proper time τ and varying S with respect to x^a we get the equation of motion in the form

$$(\mu + f\psi)\frac{D^2x^a}{d\tau^2} = -f\left(g^{ab}\psi_{,b} + \psi_{,b}\frac{dx^b}{d\tau}\frac{dx^a}{d\tau}\right), \quad (13)$$

where

$$\frac{D^2x^a}{d\tau^2} = u^b\nabla_b u^a = \frac{d^2x^a}{d\tau^2} + \Gamma^a_{bc}\frac{dx^b}{d\tau}\frac{dx^c}{d\tau}, \quad (14)$$

and $u^a = \frac{dx^a}{d\tau}$ represents the four velocity of the test charges, that satisfies the normalization condition $u_a u^a = -1$. Therefore it is clear from (13) that the motion of test scalar charges is not a geodesic, as in the case of a free particle for which $f = 0$. For the BDK metric Eq. (13) leads to the following constants of the motion [corresponding to $a = t$ and $a = \phi$ in (13), respectively]

$$E = -(1 + f\psi/\mu)g_{ab}u^a\left(\frac{\partial}{\partial t}\right)^b = -(1 + f\psi/\mu)(g_{tt}u^t + g_{t\phi}u^\phi)$$
$$L = (1 + f\psi/\mu)g_{ab}u^a\left(\frac{\partial}{\partial\phi}\right)^b = (1 + f\psi/\mu)(g_{t\phi}u^t + g_{\phi\phi}u^\phi). \quad (15)$$

Moreover from (13) (for $a = \theta$) one can easily check that if the test scalar charges are initially moving in the equatorial plane $\theta = \pi/2$, then the ensuing trajectories will remain on this plane, and therefore for simplicity we can take equatorial orbits such that $u^\theta = 0$. Solving (15) and using the normalization condition we get the following components for the four velocity

$$u^t = \frac{\mu[Er(a^2+r^2) + 2am(aE - L)]}{r(r^2 - 2mr + a^2)(f + \frac{\mu}{(r^2-2mr+a^2)^{2/(3+2\omega)}})}, \quad (16)$$

$$u^\phi = \frac{\mu[2m(aE - L) + Lr]}{r(r^2 - 2mr + a^2)(f + \frac{\mu}{(r^2-2mr+a^2)^{2/(3+2\omega)}})}, \quad (17)$$

while the radial component is given by

$$u^r = \left(\frac{1}{r^3}\right)^{1/2}\left[-r(r^2-2mr+a^2)^{1-2/(3+2\omega)} + \frac{\mu^2(E^2r^3 + (a^2E^2 - L^2)r + 2m(L - aE)^2)}{(\mu + f(r^2-2mr+a^2)^{2/(3+2\omega)})^2}\right]^{1/2}. \quad (18)$$

As expected these four velocity components reduce to those obtained by BSW for the Kerr solution in the limits $f \to 0$ and $\omega \to \infty$. Also in the vicinity of the horizon, where $\Delta = r^2 - 2mr + a^2 = (r - r_-)(r - r_+) \to 0$, the radial component of the velocity becomes

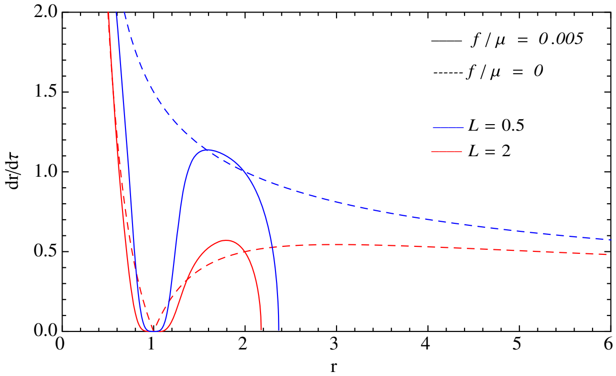

FIG. 1 (color online). The radial component of the velocity $dr/d\tau$ for free particles (dashed curves with $f/\mu = 0$, $\omega = \infty$) and coupled scalar test charges (solid curves with $f/\mu = 0.005$, $\omega = -2$) for angular momenta $L = 2$ (red curves) and $L = 0.5$ (blue curves). We take $m = a = 1$ and $E = 1$ in both cases. Note that in the coupled case there is a turning point at the event horizon for both values of L.

$$u^r = \left(\frac{(r^2 - 2mr + a^2)^{-2/(3+2\omega)}}{r^{3/2}}\right) \times [E^2 r^3 + (a^2E^2 - L^2)r + 2m(L - aE)^2]^{1/2} \times \left(\frac{f}{\mu} + (r^2 - 2mr + a^2)^{-2/(3+2\omega)}\right)^{-1}, \quad (19)$$

so that for $-5/2 \leq \omega < -3/2$ the radial velocity vanishes at the horizon $r = r_+$ irrespective of the values of the parameters a, m, E, and L, and therefore the test scalar charges fall on a trajectory that spirals asymptotically into an unstable circular orbit at the horizon radius, taking an infinite amount of proper time to do so. We know that this also happens for free particles released from rest at infinity in the Kerr solution as shown by BSW in Fig. 3 in Ref. [1]. However in the latter case this requires fine tuning with the black hole being extremal $(a = m)$ and with the particles having a critical value of the angular momentum. This is also shown in Fig. 1 where we take the two cases $f/\mu = 0$, $\omega = \infty$ representing the Kerr solution and $f/\mu = 0.005$, $\omega = -2$ for the BDK solution. For simplicity in both cases we take $a = m = 1$, $E = 1$ and consider two different values of L. As shown in Fig. 1 in the BDK case the radial velocity goes to zero on the horizon $r_+ = 1$ for both values of L. Note also that for the chosen values of the parameters, in this case the motion of the test charges is restricted to finite values of the radial coordinate r, unlike the Kerr case where the free particles are released from rest at infinity; hence the value $E = 1$.

Now the four momenta of two particles of equal mass μ moving along timelike orbits can be expressed in terms of their four-velocities u_i^a, $(i = 1, 2)$ by $p_i^a = \mu u_i^a$, such that the cm energy of their collision is given in terms of the total momentum $p_t^a = p_1^a + p_2^a$ by

$$E_{\rm cm}^2 = -p_t^a p_{ta} = -\mu^2 (u_1^a + u_2^a)(u_{1a} + u_{2a}), \quad (20)$$

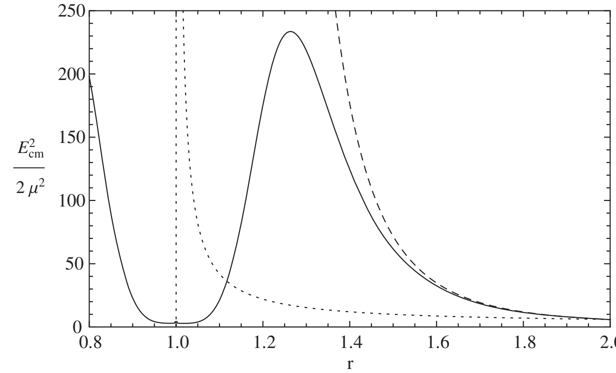

FIG. 2. Variation of $\frac{E_{\rm cm}^2}{2\mu^2}$ with radius for the BDK metric with $f = 0.005$, $\omega = -2$ (solid curve) and $f = 0$, $\omega = -2$ (dashed curve). The situation in the Kerr case $f = 0$, $\omega = \infty$ (dotted curve) is shown for comparison. We take $E_1 = 2$, $E_2 = 1$, $a = m = 1$, $L_1 = 2$, $L_2 = -2(1 + \sqrt{2})$ such that the horizon in all three cases occurs at $r = r_+ = 1$.

or as given in most cases [1] by

$$\frac{E_{\rm cm}^2}{2\mu^2} = (1 - g_{ab} u_1^a u_2^b). \quad (21)$$

So substituting (16)–(18) in the above expression and assuming that the test scalar charges have the same mass μ but different values of the constants E_i and L_i $(i = 1, 2)$, we get

$$\frac{E_{\rm cm}^2}{2\mu^2} = 1 + \frac{\mu^2(a^2E_1E_2 - L_1L_2)(r^2 - 2mr + a^2)^{-1+\frac{2}{3+2\omega}}}{(\mu + f(r^2 - 2mr + a^2)^{\frac{2}{3+2\omega}})^2} + [(r^2 - 2mr + a^2)^{-1+\frac{2}{3+2\omega}}(2(aE_1 - L_1) \times (aE_2 - L_2)m\mu^2 - r^3(-E_1E_2\mu^2 + (\mu + f(r^2 - 2mr + a^2)^{\frac{2}{3+2\omega}})^2 \times \sqrt{P_1(r)P_2(r)}))]/(r(\mu + f(r^2 - 2mr + a^2)^{\frac{2}{3+2\omega}})^2), \quad (22)$$

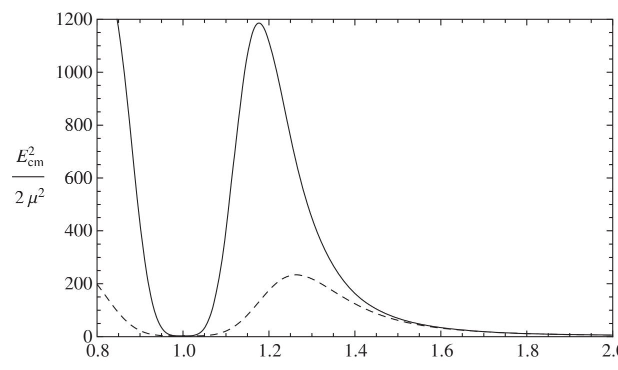

FIG. 3. Variation of $\frac{E_{\rm cm}^2}{2\mu^2}$ with radius for the BDK metric with $f = 0.005$ (dashed curve) and $f = 0.001$ (solid curve). In both cases we take $\omega = -2$, $E_1 = 2$, $E_2 = 1$, $a = m = 1$, $L_1 = 2$, and $L_2 = -2(1 + \sqrt{2})$ such that $r_+ = 1$.

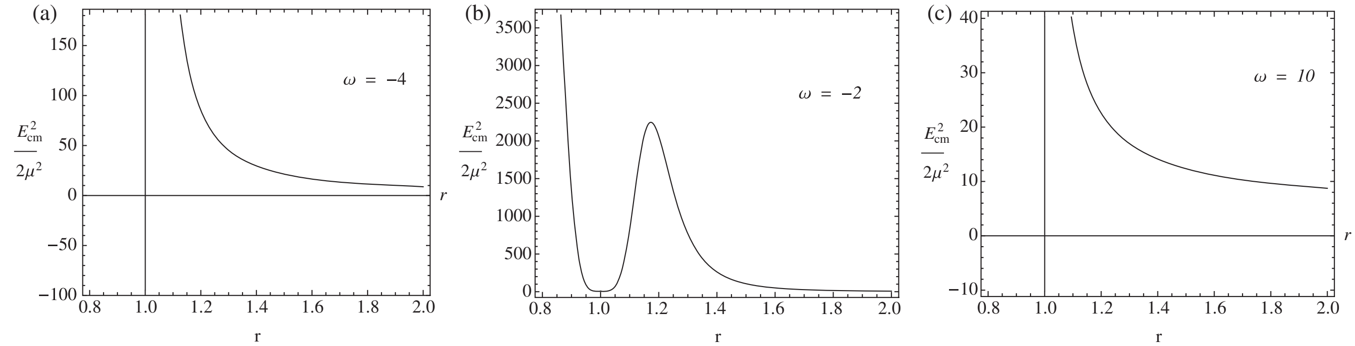

FIG. 4. Variation of $\frac{E_{cm}^2}{2\mu^2}$ with radius for the BDK metric for different values of ω. In all cases we take $f = 0.001$, $E_1 = 2$, $E_2 = 1$, $a = m = 1$, $L_1 = 4$, $L_2 = -5$ such that $r_+ = 1$.

where

$$P_i(r) = \frac{1}{r^3}\left[-r(r^2 - 2mr + a^2)^{1-\frac{2}{3+2\omega}} + \frac{\mu^2(2m(L_i - aE_i)^2 + (a^2E_i^2 - L_i^2)r + E_i^2 r^3)}{(\mu + f(r^2 - 2mr + a^2)^{\frac{2}{3+2\omega}})^2}\right] \tag{23}$$

and $i = 1, 2$. If one takes the assumptions used by BSW in their original study, i.e. $m = a = 1$ and $E_1 = E_2 = 1$, then as expected for $f = 0$ and $\omega \to \infty$ the above expression reduces to the much more simple expression obtained by BSW for free particles as given in Eq. (14) in their paper. Although the expression for the c.m. energy for test scalar charge collisions looks quite complicated, one can easily show that when $-5/2 \le \omega - 3/2$, for which the solution in (8) represents a genuine black hole, we get

$$\lim_{r \to r_+} E_{cm} = 2\mu, \tag{24}$$

such that the energy takes its minimum value, i.e. the rest mass of the test scalar charges and is therefore independent of the free parameters E_i, L_i, m, and a. The same thing was observed in our earlier study [32] for the static BBMB black hole, in which the scalar field is also infinite on the event horizon, and the c.m. energy also attains its minimum value there. On the other hand letting $f = 0$ we get $\lim_{r\to\infty} E_{cm} = \infty$, as in the case of the Kerr black hole, although in the latter case this only happens when the particle has a critical value of the angular momentum L and when the black hole is extremal. This is shown in Fig. 2 below, which shows the variation of the c.m. energy with radius for the two cases $f = 0$ and $f > 0$ for a fixed value of the Brans-Dicke parameter ω. The Kerr case $f = 0$, $\omega = \infty$ is included for comparison. We have taken $L_1 = 2$ and $L_2 = -2(1 + \sqrt{2})$ so that the Kerr case matches the situation shown in Fig. 3(b) of BSW in Ref. [1]. Hence we see that it is the interaction of the test scalar charges with the background scalar field which is what suppresses the energy of the collision rather than the mere presence of the background scalar field. In fact for higher values of the interaction parameter f, the suppression effect of the background scalar field on the c.m. energy of the collision is more pronounced at all radii as shown in Fig. 3. For other values of the Brans-Dicke parameter ω for which the spacetime has a curvature singularity at $r = r_+$ the value of the c.m. energy is arbitrarily high on r_+ as in the Kerr solution, irrespective of whether f is zero or not. This is seen in Fig. 4 which shows the behavior of the c.m. energy for three different values of ω, two of which lie outside the range $-5/2 \le \omega < -3/2$, and therefore represent naked singularities.

IV. CONCLUSION

In light of the recent discovery by BSW that Kerr black holes can act, at least theoretically, as particle accelerators with arbitrary high energy, we examine the case of a Kerr-like black hole surrounded by a scalar field that becomes singular on the event horizon, and study the motion of test scalar charges in the vicinity of the horizon. When the Brans-Dicke parameter ω takes values, $-5/2 \le \omega < -3/2$ for which the metric in (8) represents a black hole, we have seen from Eq. (19) and Fig. 1 that the radial velocity of the test charges vanishes at the event horizon $r = r_+$ such that their trajectories spiral asymptotically to unstable circular orbits on $r = r_+$ taking an infinite amount of time to do so. The same thing happens in the Kerr solution, although in our case as seen in Fig. 1 no fine tuning of the particles' angular momentum is required. Moreover, as shown in Fig. 2 the c.m. energy for collisions between scalar test charges remains finite in the vicinity of the horizon and decreases to its minimum possible value there, i.e. $E_{cm}(r_+) = 2\mu$. For free particles which are uncoupled ($f = 0$) to the background scalar field the energy becomes arbitrary high at the horizon, just like the case of the Kerr solution. Therefore it is clear that the decrease in the value of E_{cm} is due to the interaction between the test scalar charges and the background scalar field and as we can see from Fig. 3, this effect occurs at all radii and increases with the value of the coupling strength f. This behavior has also been observed in our earlier study [32] for a static

Schwarzschild-like black hole surrounded by a scalar field, although in that case when $f = 0$, $E_{\mathrm{cm}}(r_+) > 2\mu$ is still finite as in the case of the Schwarzschild solution.

As mentioned above, for $\infty > \omega > -1/2$ and $\omega < -5/2$ the metric still attains Killing horizons at $r = r_\pm$, but the Kretschmann scalar becomes infinite there, so that the surfaces $r = r_\pm$ are singular and therefore the metric cannot describe a black hole spacetime. In this case it was shown (see Fig. 4) that $\lim_{r\to r_+} E_{\mathrm{cm}} = \infty$ irrespective of the value of the interaction parameter f. This conforms well with the result obtained by Patil and Joshi [30] for the JNW spacetime which also has a naked singularity at the Killing horizon. However in that case the authors incorrectly attributed the arbitrary high energy at the horizon to the presence of the scalar field rather than the presence of a curvature singularity there. They had even claimed that the presence of a scalar field in the static JNW spacetime plays the role of rotation in the Kerr spacetime in the sense that it can be used to crank up the c.m. energy of the collision to arbitrary high values, just like the role played by the electric field in the extreme Reissner-Nordström black hole, which leads to arbitrary high c.m. energies for collisions between radially moving charges close to the event horizon [16]. However contrary to this claim we have now seen again here as we have already observed for the static case in our previous study, that even when rotation is involved, the interaction with the background scalar field suppresses the c.m. energy in the vicinity of the black hole horizon.

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