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Symplectic gauge fields and dark matter

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The dynamics of symplectic gauge fields provides a consistent framework for fundamental interactions based on spin-3 gauge fields. One remarkable property is that symplectic gauge fields only have minimal couplings with gravitational fields and not with any other field of the Standard Model. Interactions with ordinary matter and radiation can only arise from radiative corrections. In spite of the gauge nature of symplectic fields they acquire a mass by the Coleman-Weinberg mechanism which generates Higgs-like mass terms where the gravitational field is playing the role of a Higgs field. Massive symplectic gauge fields weakly interacting with ordinary matter are natural candidates for the dark matter component of the Universe.

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I. INTRODUCTION

In the Standard Model all fundamental interactions are described by gauge theories. In the Einstein theory of general relativity (GR) the gravitational interaction is also formulated in terms of a gauge field. Although there are significant differences between both theories, mainly due to the strong connection of GR with the structure of space-time, the fact that both theories are gauge theories helped to consolidate the gauge paradigm where all fundamental interactions are described by gauge fields.

The search of new physics beyond the Standard Model is supported by astrophysical and cosmological evidence of the existence of a new type of invisible matter with unknown interacting properties. The search for new types of interactions following the gauge principle suggest exploring the possibility of gauge theories with higher spin [1,2]. The pathologies associated to interactions based on massless particles with helicities higher than 2 [3–5] provided an argument to explain why these kinds of interaction are not observed in Nature. Nevertheless, the challenge is so interesting that there have been numerous attempts to give a physical meaning to gauge theories of higher helicity fields. Free massless fields with arbitrary helicity (or its generalizations) do exist in any dimension. In fact, Wigner's theory of covariant representations of the Poincaré group provides a general theory of free massless gauge fields [6]. Massless fields with integer helicity are described by transverse, symmetric traceless tensor fields with some equivalence relations which are reminiscent of gauge transformations [1,2]. The application of Becchi-Rouet-Stora-Tyutin (BRST) methods to the consistency analysis of generalized gauge theories boosted the attempts to extend the analysis of free massless gauge fields to interacting theories from a new viewpoint [7–9]. The consistency of the BRST approach requires an infinite tower of higher helicities [7,9–12] in close analogy with string theory. However, even in that case it was impossible to show the consistency of the interacting theory [7–9,13,14].

The appearance of new string dualities introduced new approaches based on five-dimensional theories on anti–de Sitter backgrounds [15–17]. In such a scheme the approach to higher spin fields acquired a new perspective [18–20].

In this paper we explore a different approach to higher spin gauge fields based on a gauge theory of symplectic fields [21]. In this approach gauge fields are symplectic connections and since their covariant derivatives are non-trivial only for fields of spin higher than 2, they are minimally decoupled from the Standard Model physics and only interact with gravitational fields. This special characteristic promoted these fields as excellent candidates for the invisible dark matter component of the Universe.

II. SYMPLECTIC GAUGE FIELDS

Let us consider a symplectic form ω in a four-dimensional space-time,[1] i.e., a regular antisymmetric tensor field $\omega_{\mu\nu} = -\omega_{\mu\nu}$ which is closed $d\omega = 0$. The symplectic form ω can be considered as the antisymmetric component of a generalized space-time metric in the sense

[1]The theory can be generalized for arbitrary even-dimensional space-times.

first considered by Forster (formerly known as Bach) and developed by Schrödinger and Einstein in the context of unified field theories. It can also be considered as a background electromagnetic field $\omega_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ with nontrivial topological density $\epsilon^{\mu_1\mu_2\mu_3\mu_4}\omega_{\mu_1\mu_2}\omega_{\mu_3\mu_4}(x) \neq 0$.

A symplectic gauge field is by definition a linear connection which preserves the symplectic form; i.e., the covariant derivative of ω,

$$D_\mu \omega = 0, \tag{1}$$

vanishes. In local coordinates

$$\partial_\mu \omega_{\nu\sigma} - \Gamma^\alpha_{\mu\sigma}\omega_{\alpha\nu} + \Gamma^\alpha_{\mu\nu}\omega_{\alpha\sigma} = 0, \tag{2}$$

where $\Gamma^\alpha_{\mu\sigma}$ are the local components of the symplectic gauge field. Although gravitational fields are also defined in a similar manner as the linear connections that preserve the space-time metric symmetric g, the contrast between both types of fields is very important as we will see below.

The gauge symmetry is given by space-time transformations which leave the symplectic form invariant (*symplectomorphisms*). They are canonical transformations whose infinitesimal generators are given in local coordinates by vector fields of the form

$$\xi_\mu = \partial_\mu \phi, \tag{3}$$

where ϕ is any scalar field. By using canonical transformations it is always possible to find local coordinates, Darboux coordinates, where ω becomes a constant form

$$\omega = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}.$$

In those coordinates, $\partial_\mu \omega = 0$ and

$$\Gamma^\alpha_{\mu\sigma}\omega_{\alpha\nu} = \Gamma^\alpha_{\mu\nu}\omega_{\alpha\sigma}. \tag{4}$$

If we impose the vanishing of the torsion as in the case of a Levi-Cività metric connection, we have

$$\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}. \tag{5}$$

The components of a torsionless symplectic gauge field in Darboux coordinates

$$T_{\nu\mu\sigma} = \Gamma^\alpha_{\mu\nu}\omega_{\alpha\sigma} \tag{6}$$

define a 3-covariant symmetric tensor

$$T_{\nu\mu\sigma} = T_{\mu\nu\sigma} = T_{\nu\sigma\mu} = T_{\mu\sigma\nu} = T_{\sigma\nu\mu} = T_{\sigma\mu\nu}. \tag{7}$$

Thus, the space of torsionless symplectic gauge fields [22–25] can be identified with the space of 3-covariant symmetric tensors. This space of symplectic gauge fields is infinite dimensional in contrast with the space of Riemannian gauge fields where the Levi-Cività connection is unique for any Riemannian metric.

The curvature tensor $R^\alpha_{\beta\mu\nu}$,

$$R^\alpha_{\beta\mu\nu} = \partial_\mu \Gamma^\alpha_{\beta\nu} - \partial_\nu \Gamma^\alpha_{\beta\mu} + \Gamma^\sigma_{\nu\beta}\Gamma^\alpha_{\mu\sigma} - \Gamma^\sigma_{\mu\beta}\Gamma^\alpha_{\nu\sigma}, \tag{8}$$

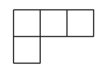
defines by contraction with ω a (0,4)-tensor

$$R_{\alpha\beta\mu\nu} = \omega_{\alpha\sigma} R^\alpha{}_{\beta\mu\nu}, \tag{9}$$

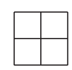
with interesting symmetry properties:

$$R_{\alpha\beta\mu\nu} = -R_{\alpha\beta\nu\mu} = R_{\beta\alpha\mu\nu},$$
$$R_{(\alpha\beta\mu\nu)} = R_{\alpha\beta\mu\nu} + R_{\nu\alpha\beta\mu} + R_{\mu\nu\alpha\beta} + R_{\beta\mu\nu\alpha} = 0.$$

The permutation symmetries of this tensor are characterized by the Young tableau

,

which is in contrast with that of the standard Riemannian tensor

.

A symplectic Ricci tensor can also defined by

$$R_{\beta\nu} = \omega^{\mu\alpha} R_{\alpha\beta\mu\nu}, \tag{10}$$

and is symmetric,

$$R_{\nu\mu} = R_{\mu\nu}, \tag{11}$$

like the Riemannian Ricci tensor. However, there is no scalar symplectic curvature because the contraction of the Ricci tensor with the symplectic form vanishes.

III. SYMPLECTIC FIELD THEORY

The simplest dynamics for symplectic gauge fields is defined by the action

$$S(\Gamma,\omega) = \frac{1}{2\alpha_0{}^2}\int d^4x R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} + \frac{\theta}{32\pi^2}\int d^4x (R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 2R_{\mu\nu}R^{\mu\nu}), \tag{12}$$

which only involves the curvature tensors

$$R^{\alpha\beta\mu\nu} = \omega^{\alpha\alpha'}\omega^{\beta\beta'}\omega^{\mu\mu'}\omega^{\nu\nu'}R_{\alpha'\beta'\mu'\nu'} \quad R^{\mu\nu} = \omega^{\mu\mu'}\omega^{\nu\nu'}R_{\mu'\nu'} \tag{13}$$

and the symplectic form ω. The second term of (12) is proportional to the Pontryagin class of the manifold which has a topological meaning and does not contribute to the classical dynamics.

The metric independence of (12) implies that the dynamics of the symplectic fields is completely decoupled from gravity.

The action (12) is the most general metric independent action of symplectic fields with quadratic dependence in the curvature tensor [26]. Although one could add an extra term proportional to the square of the Ricci tensor (10), it turns out that such a term is not independent of the other two terms of the action (12). Thus, the extra term can be absorbed by shifting the couplings α_0 and θ.

The theory is invariant under symplectomorphisms, i.e., canonical transformations. Symplectic gauge fields, however, transform as

$$T'_{\mu\nu\sigma} = T_{\mu\nu\sigma} + D_\mu D_\nu \partial_\sigma \phi \tag{14}$$

under symplectomorphisms, where $D_\mu = \partial_\mu + \Gamma^\sigma_{\mu\nu}$. The invariance of the action (12) under these transformations implies the existence of an infinity of zero modes.

The field theory governed by (5) is very interesting from a geometrical viewpoint [26], but from a quantum field theory perspective it presents many pathologies. The Cauchy problem is highly degenerated as indicated by the existence of many zero modes in quadratic terms which are not associated to any known gauge symmetry. Apart from the zero modes associated to the symplectic gauge symmetry (14), there are nine extra zero modes. The remaining non-null modes of the quadratic variation of the action on a trivial $T = 0$ background are of the form

$$-\frac{1}{3}p^2, \qquad \pm\frac{\sqrt{2}}{3}p^2 \qquad \text{(double degenerated)}$$
$$\frac{1}{3}p^2, \qquad -\frac{2}{3}p^2, \qquad \pm\frac{1}{\sqrt{3}}p^2 \qquad \text{(non degenerated)},$$

where p_μ are the momentum of Fourier modes in Darboux coordinates. Although the eigenvalues of the quadratic terms of the action are $SO(4)$ rotation invariant, the corresponding eigenfunctions $T_{\mu\nu\sigma}$ are not invariant under Euclidean or Poincaré transformations. This is due to the background symplectic form $\omega_{\mu\nu}$ which introduces a phase space structure in the space-time which is not compatible with Euclidean or Poincaré symmetries. Moreover, the quadratic terms of the action are not definite positive as a consequence of the symplectic structure. This implies that the Gaussian projection defines a theory with ghost fields which is not unitary quantum field theory.

Poincaré symmetry can be recovered if we consider a generalization of the action where the symplectic form ω becomes a full-fledged dynamical field. A natural choice is to introduce a kinetic term for the symplectic form

$$\frac{1}{2e^2}\int d^4x\omega^{\mu\nu}\omega_{\mu\nu},$$

with $\omega_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. But, because of the identity $\omega^{\mu\nu}\omega_{\mu\nu} = 4$ the integrand is constant and there is no dynamical content as the trivial motion equations indicate.

The only nontrivial possibility is to include terms with tensorial contractions which involve the space-time metric (i.e., coupling to gravity). In this framework it is possible to recover Poincaré invariance in a Minkowskian metric background.

IV. INTERACTION WITH GRAVITY

Let us consider a different theory of the symplectic gauge fields interacting with the space-time metric g,

$$S_0(\Gamma, \omega, g) = \frac{1}{2}\int d^4x\sqrt{g}g^{\mu\mu'}g^{\nu\nu'}\omega_{\mu'\nu'}\omega_{\mu\nu}. \tag{15}$$

Instead of imposing the restriction to the symplectic gauge fields that preserve the symplectic form ω (2), we introduce the constraint in a softer way via a Lagrange multiplier term in the action:

$$S'_0(\Gamma, \omega, g) = \frac{1}{2{\alpha_0}^2}\int d^4x\sqrt{g}g^{\gamma\gamma'}g^{\mu\mu'}g^{\nu\nu'}D_{\gamma'}\omega_{\mu'\nu'}D_\gamma\omega_{\mu\nu}. \tag{16}$$

The strong symplectic condition, $D_\gamma\omega_{\mu\nu} = 0$, is recovered in the weak coupling limit $\alpha_0 \to 0$.

The main interaction of symplectic fields with gravity can be introduced by contracting indices of the curvature tensor with the space-time metric instead of only using the symplectic form, e.g.,

$$S_1(\Gamma, \omega, g) = \alpha^2\int d^4x\sqrt{g}g^{\alpha\alpha'}g^{\beta\beta'}g^{\mu\mu'}g^{\nu\nu'}R_{\alpha'\beta'\mu'\nu'}R_{\alpha\beta\mu\nu} + \cdots \tag{17}$$

However, integration over symplectic forms can generate new local terms in the effective action and the renormalizability condition requires us to consider all possible relevant couplings which do not violate any fundamental gauge symmetry. Since the symplectic gauge fields generically do not preserve the space-time metric

$$D_\sigma g_{\mu\nu} \neq 0, \tag{18}$$

marginally relevant terms of the form

$${S_1}'(\Gamma, \omega, g) = {\alpha_1}^2\int d^4x\sqrt{g}|D_\sigma D_\delta g_{\mu\nu}|^2 + \cdots \tag{19}$$

should also be considered because there is no symmetry preventing its appearance as radiative corrections.

In summary, one has to include all renormalizable possible independent couplings between gravitational field and the symplectic gauge field. There are only six independent types of renormalizable interaction terms

$$DDgDDg, \qquad DgDgDDg, \qquad DgDgDgDg,$$
$$RR, \qquad RDDg, \qquad RDgDg, \tag{20}$$

because all others can be expressed as linear combinations of these terms [21]. However, the different contraction of the Lorentz indices gives rise to 78 different interaction terms involving 78 independent dimensionless couplings $\alpha_1, \ldots, \alpha_{78}$: twenty-two ($\alpha_1 \ldots \alpha_{22}$) of the type DDg, DDg, six ($\alpha_{23} \ldots \alpha_{28}$) of the type $Dg\ Dg\ Dg\ Dg$, and fifty ($\alpha_{29} \ldots \alpha_{78}$) of the type $Dg\ Dg\ DDg$. The complete list of these terms is given by Eqs. (A1)–(A3) in the Appendix.

The corresponding theory is renormalizable. In particular, the effective action generated by integrating out the symplectic form ω in the action S_1 has nontrivial contributions to all $78\,\alpha$ couplings of symplectic fields with gravity. In fact, these corrections are logarithmically divergent and the coefficients of the corresponding contributions to the beta functions are displayed in Table I of the Appendix.

We remark that some of the beta functions are positive and some others are negative. This means that not all of them will be relevant in the full-fledged quantum theory. However, the above calculations have not taken into account the radiative corrections due to symplectic gauge field fluctuations. This calculation is beyond the scope of this paper, but it is crucial to elucidate which couplings of the theory are finally relevant.

The above calculations show that the symplectic field theory is a renormalizable quantum field theory; however, the appearance of fourth-order derivative terms in the action introduces some ghost components in the symplectic gauge theory. The absence of a larger gauge symmetry means that unitary is not guaranteed.

V. SYMPLECTIC FIELDS AND DARK MATTER

Symplectic gauge fields as linear connections cannot interact by minimal couplings with scalar fields, because the minimal coupling in this case reduces to $D_\mu \phi = \partial_\mu \phi$. A similar effect arises in the interaction with fermions. The gauge group of symplectic connections is $GL(4, \mathbb{R})$ and only the trivial representation of this group acts on spinors; i.e., there is no analogue of spin connection for symplectic gauge fields, so $D\psi = \partial\psi$.

Thus, the minimal coupling of symplectic gauge fields to Standard Model particles can only be possible with gauge particles: the photon or the intermediate gauge bosons $W^\pm$ and Z. However, due to the intrinsic gauge character of these particles this coupling is not possible. The torsionless character of symplectic gauge fields is responsible for the decoupling also of vector potentials. Indeed,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \Gamma^\sigma_{\mu\nu} A_\sigma - \Gamma^\sigma_{\nu\mu} A_\sigma = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{21}$$

Thus, symplectic gauge fields cannot minimally interact with any particle of the Standard Model. They can only minimally couple to gravitation, whenever $D_\gamma g_{\mu\nu} \neq 0$. If the corresponding quanta were massive particles, they will be natural candidates for the dark matter component of the Universe and indeed, this is what happens. In the standard ΛCDM cosmological model, dark matter is usually assumed to be fermionic matter. However, a bosonic component could solve some dark matter puzzles as we shall discuss below.

However, some nonminimal couplings of symplectic gauge fields with ordinary matter like $\phi^\dagger \partial_\nu \phi D_\mu g^{\mu\nu}$, $|\phi|^2 D_\mu D_\nu g^{\mu\nu}$, or $\bar{\psi}\gamma_\nu \psi D_\mu g^{\mu\nu}$ can arise as radiative corrections. However, the genuine interacting terms of symplectic

TABLE I. Beta function coefficients of gravitational α couplings of symplectic gauge fields.

$\beta_1 = -\frac{11}{320}$	$\beta_2 = -\frac{9019}{15360}$	$\beta_3 = \frac{1103}{6144}$	$\beta_4 = -\frac{569}{3840}$	$\beta_5 = -\frac{221}{640}$
$\beta_6 = \frac{151}{1536}$	$\beta_7 = -\frac{811}{7680}$	$\beta_8 = -\frac{2173}{7680}$	$\beta_9 = \frac{569}{3840}$	$\beta_{10} = -\frac{481}{3840}$
$\beta_{11} = -\frac{1}{24}$	$\beta_{12} = \frac{509}{3840}$	$\beta_{13} = \frac{1733}{2560}$	$\beta_{14} = -\frac{1}{32}$	$\beta_{15} = -\frac{959}{10240}$
$\beta_{16} = \frac{5}{96}$	$\beta_{17} = -\frac{5}{96}$	$\beta_{18} = -\frac{125}{512}$	$\beta_{19} = \frac{983}{1920}$	$\beta_{20} = \frac{811}{7680}$
$\beta_{21} = \frac{161}{3840}$	$\beta_{22} = -\frac{143}{1280}$	$\beta_{23} = \frac{353}{1024}$	$\beta_{24} = -\frac{77}{7680}$	$\beta_{25} = -\frac{6691}{30720}$
$\beta_{26} = -\frac{1}{80}$	$\beta_{27} = -\frac{601}{1280}$	$\beta_{28} = \frac{1981}{1920}$	$\beta_{29} = \frac{3299}{10240}$	$\beta_{30} = \frac{121}{480}$
$\beta_{31} = \frac{8977}{15360}$	$\beta_{32} = \frac{151}{1280}$	$\beta_{33} = \frac{2283}{5120}$	$\beta_{34} = \frac{1083}{2560}$	$\beta_{35} = \frac{293}{3840}$
$\beta_{36} = -\frac{13}{960}$	$\beta_{37} = -\frac{1447}{3840}$	$\beta_{38} = \frac{4909}{15360}$	$\beta_{39} = -\frac{15437}{30720}$	$\beta_{40} = -\frac{169}{960}$
$\beta_{41} = -\frac{1807}{15360}$	$\beta_{42} = -\frac{95}{256}$	$\beta_{43} = \frac{187}{7680}$	$\beta_{44} = \frac{121}{160}$	$\beta_{45} = -\frac{8459}{7680}$
$\beta_{46} = -\frac{101}{384}$	$\beta_{47} = -\frac{89}{96}$	$\beta_{48} = -\frac{769}{7680}$	$\beta_{49} = \frac{167}{15360}$	$\beta_{50} = \frac{8647}{30720}$
$\beta_{51} = \frac{349}{3840}$	$\beta_{52} = \frac{2449}{3840}$	$\beta_{53} = \frac{3323}{15360}$	$\beta_{54} = -\frac{6377}{15360}$	$\beta_{55} = \frac{271}{320}$
$\beta_{56} = \frac{1921}{3072}$	$\beta_{57} = -\frac{3407}{15360}$	$\beta_{58} = -\frac{457}{768}$	$\beta_{59} = \frac{629}{15360}$	$\beta_{60} = -\frac{57}{512}$
$\beta_{61} = \frac{61}{640}$	$\beta_{62} = \frac{1453}{7680}$	$\beta_{63} = -\frac{695}{3072}$	$\beta_{64} = -\frac{55}{64}$	$\beta_{65} = -\frac{33}{2560}$
$\beta_{66} = -\frac{513}{5120}$	$\beta_{67} = \frac{73}{960}$	$\beta_{68} = -\frac{351}{2560}$	$\beta_{69} = \frac{1}{2}$	$\beta_{70} = \frac{203}{960}$
$\beta_{71} = \frac{7253}{15360}$	$\beta_{72} = \frac{1753}{15360}$	$\beta_{73} = -\frac{15}{64}$	$\beta_{74} = -\frac{6607}{15360}$	$\beta_{75} = -\frac{1417}{1920}$
$\beta_{76} = \frac{2603}{7680}$	$\beta_{77} = -\frac{329}{640}$	$\beta_{78} = \frac{1309}{2560}$		

gauge fields with gravitation (20) are invariant under the signature flip transformation,

$$g_{\mu\nu} \to -g_{\mu\nu}, \tag{22}$$

which changes the signature of the metric tensor $g_{\mu\nu}$ from (1,3) to (3,1). This symmetry acts as a custodial symmetry which prevents the appearance of nonminimal coupling between ordinary matter and symplectic gauge fields.

Although the coupling of symplectic gauge fields to the symplectic field ω (16) breaks the signature flip symmetry, the effects of such a symmetry breaking only affect the couplings between ordinary matter and symplectic gauge fields via radiative corrections at the two-loop level.

The breaking of signature flip symmetry also affects the couplings of gravity to symplectic gauge fields via one-loop corrections. We have assumed until now that these couplings are dimensionless; however, radiative corrections generate terms of the form

$$S_1''(\Gamma,\omega,g) = \frac{1}{2\alpha_m{}^2}\int d^4x\sqrt{g}|D_\sigma g_{\mu\nu}|^2 + \cdots. \tag{23}$$

Since the metric g is not preserved by symplectic gauge fields, nothing prevents the appearance of these terms with mass square dimension. Indeed, such radiative corrections appear in the form

$$S''_{\rm Higgs} = \int d^4x D_{\gamma_1} g_{\mu_1\nu_1} D_{\gamma_2} g_{\mu_2\nu_2}\bigg(\frac{1}{48} g^{\gamma_1\mu_1} g^{\nu_1\mu_2} g^{\gamma_2\nu_2} + \frac{5}{16} g^{\gamma_1\gamma_2} g^{\mu_1\mu_2} g^{\nu_1\nu_2} - \frac{3}{16} g^{\gamma_1\mu_2} g^{\mu_1\gamma_2} g^{\nu_1\nu_2}\bigg) I_2, \tag{24}$$

of quadratic divergent terms, with

$$I_2 = \int \frac{1}{(2\pi)^4}\frac{d^4r}{r^2}. \tag{25}$$

Thus, such terms must be included in the bare action to ensure the renormalizability of the theory. Now, in a Minkowski background (i.e., $g_{\mu\nu} = \eta_{\mu\nu}$) these terms provide real mass terms for the spin-3 gauge fields because then

$$S''_{\rm Higgs} \approx \frac{1}{2\alpha_m{}^2}\int d^4x \tilde{T}^{\mu\nu\sigma} T_{\mu\nu\sigma}, \tag{26}$$

i.e., although symplectic gauge fields were in principle related to massless particles, they acquire a mass from quantum radiative corrections in Minkowski space-time metric backgrounds. The phenomenon is reminiscent of the Coleman-Weinberg mechanism of generation of mass for conformal scalar electrodynamics.

The way symplectic gauge fields $T_{\mu\nu\sigma}$ acquire a mass is also reminiscent of the Higgs mechanism with the gravitational field playing the role of the Higgs field.

Conversely, the alternative mechanism where symplectic fields condensate into a nontrivial value and provide a mass term for the graviton is also possible but not physically realistic, because a nontrivial expectation value of such a field will break Lorentz invariance, which is quite unlikely to happen. As a consequence, the graviton remains massless but the symplectic fields become massive.

In a similar manner, radiative corrections generate new interacting terms at the two-loop level involving symplectic fields and Higgs fields of the form

$$S'''_{\rm Higgs}(\Gamma,\phi) = \frac{1}{2\alpha_h{}^2}\int d^4x\sqrt{g}|\phi|^2|D_\sigma g_{\mu\nu}|^2 + \cdots, \tag{27}$$

which in a Minkowski background provide real mass terms for the symplectic gauge fields like in Eq. (26):

$$S'''_{\rm Higgs} \approx \frac{|v|^2}{2\alpha_h{}^2}\int d^4x \tilde{T}^{\mu\nu\sigma} T_{\mu\nu\sigma}, \tag{28}$$

where $v = \langle\phi\rangle$ is the vacuum expectation value of the Higgs field. The Higgs contribution to the mass of the symplectic gauge fields (28) is similar to the mass terms of the other particles of the Standard Model. The only difference is that the mass term of symplectic gauge fields has an extra mass contribution due to radiative corrections of symplectic fields.

VI. DISCUSSION

The Standard Model sector of the Universe contains a large variety of particles. It is then imaginable that the dark matter sector is also made of more than one type of particle. The characteristics of spin-3 massive gauge particles associated to symplectic gauge fields suggest that they are natural candidates as components of dark matter. The mass of these gauge particles is only dictated by the coupling to gravitation, which means that generically it can be large enough to provide a relevant component of cold dark matter. On the other hand, the bosonic character of the new particles could explain the smooth behavior of the central dark matter density in galaxy halos [27–30], and it could give rise to bosonic condensates which provide interesting scenarios for dwarf galaxies [31–34].

Since the only primary interaction of symplectic gauge fields involves gravitational fields, the effect of the new interaction can be mimicked by a effective theory of gravitation. The results obtained via integration of symplectic gauge fields yield an effective action which is highly nonlocal and it will only become local in the infinite mass limit of symplectic gauge fields. In that case one gets back the standard gravitational action with extra R^2 terms. However, the physical interpretation of the effective theory is very subtle because the calculation is highly dependent on the background space-time metric. There are metric backgrounds where the Higgs mechanism provides a mass to symplectic gauge fields, and metric backgrounds without such a mass generating mechanism. In the latter case, the symplectic gauge fields contain massless particles. Thus, the theory provides scenarios which interpolate between hot and cold dark matter scenarios depending on the gravitational background. This chameleonic property of symplectic gauge fields is very attractive and deserves further exploration.

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APPENDIX: RENORMALIZATION

The 78 independent dimensionless couplings of the symplectic gauge fields to gravity can be obtained by using the Tensorial and FeynCalc packages of Mathematica.

There are three different types of terms: 22 of the type $DDg\ DDg$,

$$\begin{aligned} S'_{22} = \int d^4x\sqrt{g}(D_{\tau_1}D_{\gamma_1}g_{\mu_1\nu_1})(D_{\tau_2}D_{\gamma_2}g_{\mu_2\nu_2})[&\alpha_1 g^{\mu_1\nu_1}g^{\mu_2\nu_2}g^{\tau_1\gamma_1}g^{\tau_2\gamma_2} + \alpha_2 g^{\mu_1\nu_1}g^{\tau_1\gamma_1}g^{\gamma_2\nu_2}g^{\tau_2\mu_2} + \alpha_3 g^{\mu_1\tau_2}g^{\mu_2\nu_2}g^{\nu_1\gamma_2}g^{\tau_1\gamma_1} \\ &+ \alpha_4 g^{\mu_1\tau_2}g^{\nu_1\mu_2}g^{\tau_1\gamma_1}g^{\gamma_2\nu_2} + \alpha_5 g^{\mu_1\gamma_2}g^{\nu_1\mu_2}g^{\tau_1\gamma_1}g^{\tau_2\nu_2} + \alpha_6 g^{\mu_1\mu_2}g^{\nu_1\nu_2}g^{\tau_1\gamma_1}g^{\tau_2\gamma_2} + \alpha_7 g^{\gamma_1\nu_1}g^{\tau_1\mu_1}g^{\gamma_2\nu_2}g^{\tau_2\mu_2} + \alpha_8 g^{\mu_2\nu_2}g^{\nu_1\gamma_2}g^{\gamma_1\tau_2}g^{\tau_1\mu_1} \\ &+ \alpha_9 g^{\nu_1\mu_2}g^{\gamma_1\tau_2}g^{\tau_1\mu_1}g^{\gamma_2\nu_2} + \alpha_{10} g^{\nu_1\mu_2}g^{\gamma_1\gamma_2}g^{\tau_1\mu_1}g^{\tau_2\nu_2} + \alpha_{11} g^{\nu_1\tau_2}g^{\gamma_1\mu_2}g^{\tau_1\mu_1}g^{\gamma_2\nu_2} + \alpha_{12} g^{\nu_1\gamma_2}g^{\gamma_1\mu_2}g^{\tau_1\mu_1}g^{\tau_2\nu_2} + \alpha_{13} g^{\mu_2\nu_2}g^{\nu_1\gamma_2}g^{\gamma_1\mu_1}g^{\tau_1\tau_2} \\ &+ \alpha_{14} g^{\nu_1\mu_2}g^{\gamma_1\mu_1}g^{\tau_1\tau_2}g^{\gamma_2\nu_2} + \alpha_{15} g^{\mu_1\nu_1}g^{\mu_2\nu_2}g^{\gamma_1\gamma_2}g^{\tau_1\tau_2} + \alpha_{16} g^{\mu_1\mu_2}g^{\nu_1\nu_2}g^{\gamma_1\gamma_2}g^{\tau_1\tau_2} + \alpha_{17} g^{\mu_1\gamma_2}g^{\nu_1\nu_2}g^{\gamma_1\mu_2}g^{\tau_1\tau_2} + \alpha_{18} g^{\mu_1\mu_2}g^{\nu_1\nu_2}g^{\gamma_1\tau_2}g^{\tau_1\gamma_2} \\ &+ \alpha_{19} g^{\mu_1\tau_2}g^{\nu_1\nu_2}g^{\gamma_1\mu_2}g^{\tau_1\gamma_2} + \alpha_{20} g^{\nu_1\tau_2}g^{\gamma_1\mu_1}g^{\tau_1\mu_2}g^{\gamma_2\nu_2} + \alpha_{21} g^{\mu_1\tau_2}g^{\nu_1\nu_2}g^{\gamma_1\gamma_2}g^{\tau_1\mu_2} + a_{22} g^{\mu_1\tau_2}g^{\nu_1\gamma_2}g^{\gamma_1\nu_2}g^{\tau_1\mu_2}], \end{aligned} \tag{A1}$$

6 of the type $Dg\ Dg\ Dg\ Dg$,

$$\begin{aligned} S'_6 = \int d^4x\sqrt{g}&(D_{\gamma_1}g_{\mu_1\nu_1})(D_{\gamma_2}g_{\mu_2\nu_2})(D_{\gamma_3}g_{\mu_3\nu_3})(D_{\gamma_4}g_{\mu_4\nu_4}) \\ &\times [\alpha_{23} g^{\mu_2\gamma_3}g^{\nu_1\gamma_2}g^{\nu_2\mu_3}g^{\nu_3\mu_4}g^{\gamma_1\mu_1}g^{\gamma_4\nu_4} + \alpha_{24} g^{\mu_2\gamma_3}g^{\mu_3\gamma_4}g^{\nu_1\gamma_2}g^{\nu_2\mu_4}g^{\nu_3\nu_4}g^{\gamma_1\mu_1} + \alpha_{25} g^{\mu_2\mu_3}g^{\nu_1\gamma_2}g^{\nu_2\mu_4}g^{\gamma_1\mu_1}g^{\gamma_3\nu_3}g^{\gamma_4\nu_4} \\ &+ \alpha_{26} g^{\nu_1\mu_2}g^{\nu_3\mu_4}g^{\gamma_1\mu_1}g^{\gamma_2\nu_2}g^{\gamma_3\mu_3}g^{\gamma_4\nu_4} + \alpha_{27} g^{\nu_1\mu_2}g^{\nu_2\mu_3}g^{\nu_3\mu_4}g^{\gamma_1\mu_1}g^{\gamma_2\gamma_3}g^{\gamma_4\nu_4} + \alpha_{28} g^{\nu_1\mu_2}g^{\nu_2\mu_4}g^{\nu_3\nu_4}g^{\gamma_1\mu_1}g^{\gamma_2\mu_3}g^{\gamma_3\gamma_4}], \end{aligned} \tag{A2}$$

and 50 of the type $Dg\ Dg\ DDg$,

$$\begin{aligned} S'_{50} = \int d^4x\sqrt{g}(D_{\gamma_1}g_{\mu_1\nu_1})(D_{\gamma_2}g_{\mu_2\nu_2})(D_{\tau_3}D_{\gamma_3}g_{\mu_3\nu_3})[&\alpha_{29} g^{\mu_2\tau_3}g^{\mu_3\nu_3}g^{\nu_1\gamma_2}g^{\nu_2\gamma_3}g^{\gamma_1\mu_1} + \alpha_{30} g^{\mu_2\tau_3}g^{\nu_1\gamma_2}g^{\nu_2\mu_3}g^{\gamma_1\mu_1}g^{\gamma_3\nu_3} \\ &+ \alpha_{31} g^{\mu_2\gamma_3}g^{\nu_1\gamma_2}g^{\nu_2\mu_3}g^{\gamma_1\mu_1}g^{\tau_3\nu_3} + \alpha_{32} g^{\mu_2\mu_3}g^{\nu_1\gamma_2}g^{\nu_2\nu_3}g^{\gamma_1\mu_1}g^{\tau_3\gamma_3} + \alpha_{33} g^{\mu_3\nu_3}g^{\nu_1\mu_2}g^{\gamma_1\mu_1}g^{\gamma_2\nu_2}g^{\tau_3\gamma_3} + \alpha_{34} g^{\nu_1\mu_2}g^{\gamma_1\mu_1}g^{\gamma_2\nu_2}g^{\gamma_3\nu_3}g^{\tau_3\mu_3} \\ &+ \alpha_{35} g^{\mu_3\nu_3}g^{\nu_1\mu_2}g^{\nu_2\gamma_3}g^{\gamma_1\mu_1}g^{\gamma_2\tau_3} + \alpha_{36} g^{\nu_1\mu_2}g^{\nu_2\mu_3}g^{\gamma_1\mu_1}g^{\gamma_2\tau_3}g^{\gamma_3\nu_3} + \alpha_{37} g^{\nu_1\mu_2}g^{\nu_2\mu_3}g^{\gamma_1\mu_1}g^{\gamma_2\gamma_3}g^{\tau_3\nu_3} + \alpha_{38} g^{\nu_1\mu_2}g^{\nu_2\tau_3}g^{\gamma_1\mu_1}g^{\gamma_2\mu_3}g^{\gamma_3\nu_3} \\ &+ \alpha_{39} g^{\nu_1\mu_2}g^{\nu_2\gamma_3}g^{\gamma_1\mu_1}g^{\gamma_2\mu_3}g^{\tau_3\nu_3} + \alpha_{40} g^{\nu_1\mu_2}g^{\nu_2\nu_3}g^{\gamma_1\mu_1}g^{\gamma_2\mu_3}g^{\tau_3\gamma_3} + \alpha_{41} g^{\mu_3\nu_3}g^{\nu_1\tau_3}g^{\nu_2\gamma_3}g^{\gamma_1\mu_1}g^{\gamma_2\mu_2} + \alpha_{42} g^{\nu_1\tau_3}g^{\nu_2\mu_3}g^{\gamma_1\mu_1}g^{\gamma_2\mu_2}g^{\gamma_3\nu_3} \\ &+ \alpha_{43} g^{\mu_2\mu_3}g^{\nu_1\tau_3}g^{\nu_2\nu_3}g^{\gamma_1\mu_1}g^{\gamma_2\gamma_3} + \alpha_{44} g^{\mu_2\gamma_3}g^{\nu_1\tau_3}g^{\nu_2\nu_3}g^{\gamma_1\mu_1}g^{\gamma_2\mu_3} + \alpha_{45} g^{\nu_1\gamma_3}g^{\nu_2\mu_3}g^{\gamma_1\mu_1}g^{\gamma_2\mu_2}g^{\tau_3\nu_3} + \alpha_{46} g^{\mu_2\mu_3}g^{\nu_1\gamma_3}g^{\nu_2\nu_3}g^{\gamma_1\mu_1}g^{\gamma_2\tau_3} \\ &+ \alpha_{47} g^{\mu_2\tau_3}g^{\nu_1\gamma_3}g^{\nu_2\nu_3}g^{\gamma_1\mu_1}g^{\gamma_2\mu_3} + \alpha_{48} g^{\nu_1\mu_3}g^{\nu_2\nu_3}g^{\gamma_1\mu_1}g^{\gamma_2\mu_2}g^{\tau_3\gamma_3} + \alpha_{49} g^{\mu_2\gamma_3}g^{\nu_1\mu_3}g^{\nu_2\nu_3}g^{\gamma_1\mu_1}g^{\gamma_2\tau_3} + \alpha_{50} g^{\mu_2\tau_3}g^{\nu_1\mu_3}g^{\nu_2\nu_3}g^{\gamma_1\mu_1}g^{\gamma_2\gamma_3} \\ &+ \alpha_{51} g^{\mu_2\tau_3}g^{\nu_1\mu_3}g^{\nu_2\gamma_3}g^{\gamma_1\mu_1}g^{\gamma_2\nu_3} + \alpha_{52} g^{\mu_1\mu_2}g^{\mu_3\nu_3}g^{\nu_1\tau_3}g^{\nu_2\gamma_3}g^{\gamma_1\gamma_2} + \alpha_{53} g^{\mu_1\mu_2}g^{\nu_1\tau_3}g^{\nu_2\mu_3}g^{\gamma_1\gamma_2}g^{\gamma_3\nu_3} + \alpha_{54} g^{\mu_1\mu_2}g^{\nu_1\gamma_3}g^{\nu_2\mu_3}g^{\gamma_1\gamma_2}g^{\tau_3\nu_3} \\ &+ \alpha_{55} g^{\mu_1\mu_2}g^{\nu_1\mu_3}g^{\nu_2\nu_3}g^{\gamma_1\gamma_2}g^{\tau_3\gamma_3} + \alpha_{56} g^{\mu_1\tau_3}g^{\mu_2\mu_3}g^{\nu_1\gamma_3}g^{\nu_2\nu_3}g^{\gamma_1\gamma_2} + \alpha_{57} g^{\mu_1\tau_3}g^{\mu_2\gamma_3}g^{\nu_1\mu_3}g^{\nu_2\nu_3}g^{\gamma_1\gamma_2} + \alpha_{58} g^{\mu_1\gamma_2}g^{\mu_3\nu_3}g^{\nu_1\nu_2}g^{\gamma_1\mu_2}g^{\tau_3\gamma_3} \\ &+ \alpha_{59} g^{\mu_1\gamma_2}g^{\nu_1\nu_2}g^{\gamma_1\mu_2}g^{\gamma_3\nu_3}g^{\tau_3\mu_3} + \alpha_{60} g^{\mu_1\gamma_2}g^{\mu_3\nu_3}g^{\nu_1\tau_3}g^{\nu_2\gamma_3}g^{\gamma_1\mu_2} + \alpha_{61} g^{\mu_1\gamma_2}g^{\nu_1\tau_3}g^{\nu_2\mu_3}g^{\gamma_1\mu_2}g^{\gamma_3\nu_3} + \alpha_{62} g^{\mu_1\gamma_2}g^{\nu_1\gamma_3}g^{\nu_2\mu_3}g^{\gamma_1\mu_2}g^{\tau_3\nu_3} \\ &+ \alpha_{63} g^{\mu_1\gamma_2}g^{\nu_1\mu_3}g^{\nu_2\nu_3}g^{\gamma_1\mu_2}g^{\tau_3\gamma_3} + \alpha_{64} g^{\mu_1\nu_2}g^{\nu_1\tau_3}g^{\gamma_1\mu_2}g^{\gamma_2\mu_3}g^{\gamma_3\nu_3} + \alpha_{65} g^{\mu_1\nu_2}g^{\nu_1\gamma_3}g^{\gamma_1\mu_2}g^{\gamma_2\mu_3}g^{\tau_3\nu_3} + \alpha_{66} g^{\mu_1\nu_2}g^{\nu_1\mu_3}g^{\gamma_1\mu_2}g^{\gamma_2\tau_3}g^{\gamma_3\nu_3} \\ &+ \alpha_{67} g^{\mu_1\nu_2}g^{\nu_1\mu_3}g^{\gamma_1\mu_2}g^{\gamma_2\gamma_3}g^{\tau_3\nu_3} + \alpha_{68} g^{\mu_1\nu_2}g^{\nu_1\mu_3}g^{\gamma_1\mu_2}g^{\gamma_2\nu_3}g^{\tau_3\gamma_3} + \alpha_{69} g^{\mu_1\tau_3}g^{\nu_1\mu_3}g^{\nu_2\nu_3}g^{\gamma_1\mu_2}g^{\gamma_2\gamma_3} + \alpha_{70} g^{\mu_1\tau_3}g^{\nu_1\mu_3}g^{\nu_2\gamma_3}g^{\gamma_1\mu_2}g^{\gamma_2\nu_3} \\ &+ \alpha_{71} g^{\mu_1\gamma_3}g^{\nu_1\mu_3}g^{\nu_2\nu_3}g^{\gamma_1\mu_2}g^{\gamma_2\tau_3} + \alpha_{72} g^{\mu_1\gamma_3}g^{\nu_1\mu_3}g^{\nu_2\tau_3}g^{\gamma_1\mu_2}g^{\gamma_2\nu_3} + \alpha_{73} g^{\mu_1\mu_3}g^{\nu_1\nu_3}g^{\nu_2\gamma_3}g^{\gamma_1\mu_2}g^{\gamma_2\tau_3} + \alpha_{74} g^{\mu_1\tau_3}g^{\mu_2\nu_3}g^{\nu_1\gamma_3}g^{\gamma_1\nu_2}g^{\gamma_2\mu_3} \\ &+ \alpha_{75} g^{\mu_1\mu_2}g^{\nu_1\gamma_3}g^{\nu_2\nu_3}g^{\gamma_1\tau_3}g^{\gamma_2\mu_3} + \alpha_{76} g^{\mu_1\mu_2}g^{\nu_1\mu_3}g^{\nu_2\nu_3}g^{\gamma_1\tau_3}g^{\gamma_2\gamma_3} + \alpha_{77} g^{\mu_1\mu_2}g^{\nu_1\mu_3}g^{\nu_2\gamma_3}g^{\gamma_1\tau_3}g^{\gamma_2\nu_3} + \alpha_{78} g^{\mu_1\mu_2}g^{\nu_1\tau_3}g^{\nu_2\gamma_3}g^{\gamma_1\mu_3}g^{\gamma_2\nu_3}]. \end{aligned} \tag{A3}$$

Integration over the symplectic fields ω in the action S_1 generates logarithmically divergent contributions to all α couplings. The coefficients of these divergent terms can be identified with the coefficients of the beta functions of α couplings displayed in Table I.

The fact that no new couplings are generated by one-loop diagrams points to the renormalizable character of the theory. The couplings whose beta function coefficients are listed in Table I are the only dimensionless renormalized couplings of the theory.

[1] C. Fronsdal, Phys. Rev. D **18**, 3624 (1978).
[2] M. Asorey, J. F. Cariñena, and L. J. Boya, Rep. Math. Phys. **21**, 391 (1985).
[3] S. R. Coleman and J. Mandula, Phys. Rev. **159**, 1251 (1967).
[4] S. Weinberg, Phys. Rev. **135B**, B1049 (1964); Phys. Rev. **181**, 1893 (1969).
[5] R. Haag, J. T. Lopuszanski, and M. Sohnius, Nucl. Phys. **B88**, 257 (1975).
[6] E. P. Wigner, Ann. Math. **40**, 149 (1939).
[7] A. K. H. Bengtsson, Phys. Rev. D **32**, 2031 (1985).
[8] A. K. H. Bengtsson and I. Bengtsson, Classical Quantum Gravity **3**, 927 (1986).
[9] F. A. Berends, G. J. H. Burgers, and H. van Dam, Z. Phys. C **24**, 247 (1984); Nucl. Phys. **B260**, 295 (1985).
[10] B. de Wit and D. Z. Freedman, Phys. Rev. D **21**, 358 (1980).
[11] J. M. F. Labastida, Nucl. Phys. **B322**, 185 (1989).
[12] E. S. Fradkin and M. A. Vasiliev, Nucl. Phys. **B291**, 141 (1987); Phys. Lett. B **189**, 89 (1987).
[13] C. Aragone and S. Deser, Phys. Lett. **86B**, 161 (1979).
[14] F. Berends, J. van Holten, P. van Nieuwenhuizen, and B. de Wit, J. Phys. A **13**, 1643 (1980).
[15] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
[16] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998).
[17] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[18] M. A. Vasiliev, Int. J. Mod. Phys. D **05**, 763 (1996).
[19] V. E. Lopatin and M. A. Vasiliev, Mod. Phys. Lett. A **03**, 257 (1988).
[20] M. A. Vasiliev, Nucl. Phys. **B301**, 26 (1988).
[21] M. Asorey and D. García-Alvarez, AIP Conf. Proc. **1241**, 1192 (2010).
[22] V. G. Lemlein, Dokl. Akad. Nauk SSSR **115**, 655 (1957).
[23] Ph. Tondeur, Commentarii mathematici Helvetici **36**, 234 (1962).
[24] F. Bayen, M. Flato, C. Fronsdal, A. Lichnerowicz, and D. Sternheimer, Ann. Phys. (N.Y.) **111**, 61 (1978).
[25] I. Gelfand, V. Retakh, and M. Shubin, Adv. Math. **136**, 104 (1998).
[26] P. Bourgeois and M. Cahen, J. Geom. Phys. **30**, 233 (1999).
[27] P. J. E. Peebles and A. Vilenkin, Phys. Rev. D **60**, 103506 (1999).
[28] P. J. E. Peebles, Phys. Rev. D **62**, 023502 (2000); P. J. E. Peebles, Astrophys. J. **534**, L127 (2000).
[29] J. Lesgourgues, A. Arbey, and P. Salati, New Astron. Rev. **46**, 791 (2002).
[30] A. Arbey, J. Lesgourgues, and P. Salati, Phys. Rev. D **68**, 023511 (2003).
[31] S. U. Ji and S. J. Sin, Phys. Rev. D **50**, 3655 (1994).
[32] J. W. Lee and I. G. Koh, Phys. Rev. D **53**, 2236 (1996).
[33] J. W. Lee, J. Korean Phys. Soc. **54**, 2622 (2009).
[34] A. Arbey, Phys. Rev. D **74**, 043516 (2006).