Hyperon forward spin polarizability γ_0 in baryon chiral perturbation theory

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We present the calculation of the hyperon forward spin polarizability γ_0 using manifestly Lorentz-covariant baryon chiral perturbation theory including the intermediate contribution of the spin-$3/2$ states. As at the considered order the extraction of γ_0 is a pure prediction of chiral perturbation theory, the obtained values are a good test for this theory. After including explicitly the decuplet states, our SU(2) results have a very good agreement with the experimental data and we extend our framework to SU(3) to give predictions for the hyperons' γ_0 values. Prominent are the Σ^- and Ξ^- baryons as their photon transition to the decuplet is forbidden in SU(3) symmetry and therefore they are not sensitive to the explicit inclusion of the decuplet in the theory.

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I. INTRODUCTION

From the experimental study of Compton scattering on a baryon target one can extract relevant information about the inner structure of baryons. With the help of the sum rules for integral characteristics of the cross sections, very important observables like polarizabilities can be assessed. The focus of this work is the forward spin polarizability γ_0, which represents the deformation of a hadron relative to its spin axis when scattering photons in the extreme forward direction. It is related to the photoabsorption γN cross sections $\sigma_{3/2,1/2}$ with total helicities $3/2$ (for parallel photon and target helicities) and $1/2$ (for antiparallel photon and target helicities) via the sum rule in Ref. [1],

$$\gamma_0 = -\frac{1}{4\pi^2}\int_{\omega_0}^{\infty} \mathrm{d}\omega \frac{\sigma_{3/2}(\omega) - \sigma_{1/2}(\omega)}{\omega^3}, \qquad (1)$$

originally found in Ref. [2]. The energy ω_0 is the threshold for an associated neutral pion in the intermediate state. Experimental results for the proton γ_0 were obtained in Refs. [3,4], and, more recently, in Ref. [5]. Furthermore, dispersion relation studies have been performed in Refs. [6–9] for both nucleon spin polarizabilities.

Concerning the theoretical approach, the nucleon's structure has been thoroughly studied with the help of effective field theories on Compton scattering data in Refs. [10–13]. The spin-dependent piece of the amplitude $\epsilon^{\mu}\mathcal{M}^{\mathrm{SD}}_{\mu\nu}\epsilon^{*\nu}$ attracted particular interest. The term proportional to ω^3 (ω is the photon's energy) contains the whole information about γ_0, via the master formula

$$\gamma_0[\vec{\sigma}\cdot(\vec{\epsilon}\times\vec{\epsilon}^*)] = -\frac{\mathrm{i}}{4\pi}\frac{\partial}{\partial\omega^2}\frac{\epsilon^{\mu}\mathcal{M}^{\mathrm{SD}}_{\mu\nu}\epsilon^{*\nu}}{\omega}\bigg|_{\omega=0}, \qquad (2)$$

as described in Refs. [1,14,15]. Here $\vec{\sigma}$ is the vector of Pauli matrices, and $\vec{\epsilon}$ and $\vec{\epsilon}^*$ are the polarizations of incoming and outgoing photons, respectively.

Early calculations in models of chiral perturbation theory (ChPT) that include only nucleonic intermediate states have been performed both in a heavy-baryon approach as well as in fully covariant calculations as in Ref. [16]. In Refs. [17–19] the theory was extended to include isospin-$3/2$ intermediate states, namely the $\Delta(1232)$ resonance. It was found that the inclusion of the latter state greatly improved the convergence between theory and empirical evidence.

When considering ChPT in SU(3) models, valuable predictions about the hyperons' polarizabilities can be calculated, where there are no experimental data available yet. First results with the help of heavy-baryon ChPT were obtained in Ref. [15] and later improved in our work, Ref. [20]. The predictions are expected to be more reliable when using a fully covariant model and introducing the $\Delta(1232)$.

Along this line, in this work we perform a calculation of the amplitude $\epsilon^{\mu}\mathcal{M}^{\mathrm{SD}}_{\mu\nu}\epsilon^{*\nu}$ including corrections induced both by a fully covariant version and intermediate spin-$3/2$ states in a full extension to SU(3) flavor. The leading ChPT order for the quantity γ_0 is a p^3 calculation. When including the $\Delta(1232)$ resonance, the study of Ref. [17] used the so-called small scale expansion introduced in

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Refs. [21,22]. We opt for a different power-counting scheme, following Ref. [23] and, furthermore, we introduce the couplings in a consistent dynamics, using the full $\Delta(1232)$ propagator as in Refs. [24–27]. A study comparing different effective field theoretical models has been performed in Ref. [28].

The outline of this paper is as follows. In Sec. II we give a theoretical introduction to the appropriate chiral Lagrangians and the power counting used. The kinematical considerations and assumptions for the calculation of γ_0 are presented in Sec. III, and the results are discussed in Sec. IV. Finally, we briefly summarize in Sec. V.

II. CHPT INVOLVING PSEUDOSCALAR MESONS, BARYONS, AND PHOTONS

For the description of hyperon polarizabilities we use a manifestly Lorentz-covariant SU(3) version of chiral perturbation theory involving pseudoscalar mesons, baryons, and photons (see details in Refs. [29,30]). The lowest-order chiral Lagrangian involving pseudoscalar mesons ϕ, baryons B, and photons A_μ reads

$$\mathcal{L} = \mathcal{L}_{\phi\phi}^{(2)} + \mathcal{L}_{\phi B}^{(1)}, \tag{3}$$

where

$$\mathcal{L}_{\phi\phi}^{(2)} = \frac{F_0^2}{4}\mathrm{Tr}(u_\mu u^\mu + \chi_+) \tag{4}$$

is the $O(p^2)$ meson Lagrangian and

$$\mathcal{L}_{\phi B}^{(1)} = \mathrm{Tr}(\bar{B}(\mathrm{i}\,\not{D} - m)B) + \frac{D}{2}\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}) + \frac{F}{2}\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5[u_\mu, B]) \tag{5}$$

is the $O(p^1)$ Lagrangian including baryons. The symbols [] and {} occurring in Eq. (5) and in the following denote the commutator and anticommutator in flavor space, respectively. The vielbein u_μ is given by $\mathrm{i}\{u^\dagger, \nabla_\mu u\}$, with $u^2 = U = \exp(\frac{\mathrm{i}\phi}{F_0})$, where $\nabla_\mu u = \partial_\mu u - \mathrm{i}(v_\mu + a_\mu)u + \mathrm{i}u(v_\mu - a_\mu)$ and $\mathrm{D}_\mu B = \partial_\mu B + [\Gamma_\mu, B]$ are the covariant derivatives acting on meson and baryon octet fields, respectively, and m denotes the baryon octet mass in the chiral limit. The chiral connection is given by $\Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{\mathrm{i}}{2}u^\dagger(v_\mu + a_\mu)u - \frac{\mathrm{i}}{2}u(v_\mu - a_\mu)u^\dagger$. Since we are working with photon fields, we set v_μ to $e\epsilon_\mu Q$ and a_μ to 0, e the negative elementary charge. The constant F_0 is the meson decay constant in the chiral limit and the low-energy constants D and F are determined from hyperon β decays, where the combination $F + D$ corresponds to the low-energy constant g_A in the SU(2) limit. The explicit forms of the 3×3 charge matrix Q, meson ϕ, and baryon B matrices are given in Appendix A. The term $\mathrm{Tr}(\chi_+)$ is responsible for the explicit breaking of the chiral symmetry due to the finite quark masses,

$$\mathrm{Tr}(\chi_+) = \mathrm{Tr}(\chi U^\dagger + U\chi^\dagger), \tag{6}$$

where in our case $\chi = M^2$ and M is the meson mass. The power-counting scheme followed here gives the order

$$N = 4N_L + \sum_{d=1}^{\infty} dN_d - 2P_\phi - P_B \tag{7}$$

to a diagram, where N_L stands for the number of loops, N_d for the number of vertices from Lagrangians of order d, and P_ϕ and P_B for the number of meson and baryon propagators, respectively (see also Ref. [31]).

In this work we also include isospin-3/2 resonances, which give significant corrections to the full amplitude. The relevant terms of the Lagrangians that couple these decuplet fields to the octets of baryons and mesons were partly given in Refs. [32–34], where the kinetic term reads

$$\mathcal{L}_\Delta^{(1)} = \bar{\Delta}_\mu^{abc}[\gamma^{\mu\nu\alpha}\mathrm{i}\partial_\alpha - M_\Delta\gamma^{\mu\nu}]\Delta_\nu^{abc}. \tag{8}$$

We added the missing couplings by extending the known vertices from SU(2) (see Refs. [27,35]) to SU(3):

$$\mathcal{L}_{\Delta\phi B}^{(1)} = \frac{-\mathrm{i}\sqrt{2}\mathcal{C}}{F_0 M_\Delta}\bar{B}^{ab}\varepsilon^{cda}\gamma^{\mu\nu\lambda}(\partial_\mu\Delta_\nu)^{dbe}(\mathrm{D}_\lambda\phi)^{ce} + \mathrm{H.c.}, \tag{9}$$

$$\mathcal{L}_{\Delta B}^{(2)} = -\frac{3\mathrm{i}eg_M}{\sqrt{2}m(m + M_\Delta)}\bar{B}^{ab}\varepsilon^{cda}Q^{ce}(\partial_\mu\Delta_\nu)^{dbe}\tilde{F}^{\mu\nu} + \mathrm{H.c.}, \tag{10}$$

where Δ^{ijk} are the decuplet states (see details in Appendix A), $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}\partial_\alpha A_\beta$ is the self-dual stress tensor of the electromagnetic field, and the Dirac tensors $\gamma^{\mu\nu}$ and $\gamma^{\mu\nu\lambda}$ are specified in Appendix A.

The couplings $\mathcal{C}$ and g_M are low-energy constants and M_Δ corresponds to the decuplet mass in the chiral limit. Note that the constant $\mathcal{C}$ corresponds to the low-energy constant h_A of SU(2) in Refs. [27,35] by the conversion $\mathcal{C} = -\frac{h_A}{2\sqrt{2}}$. This definition of h_A differs by a factor of 2 from the definition found in Ref. [17]. This low-energy constant is extracted from the strong decay of the decuplet into the baryon octet and has been determined to be $h_A = 2.85$ in Ref. [36] for SU(2) and $\mathcal{C} = -0.85$ in Ref. [34] for SU(3). It is also important to mention that the numerical value for the coupling constant g_M has not yet been studied when extending the model to SU(3). Therefore the quality of the predictions very much depends on its correct value. We follow the method of Ref. [17] and estimate the value of g_M by calculating the width of the electromagnetic decay of the $\Delta(1232)$:

$$\Gamma_\Delta^{\mathrm{EM}} = -2\mathrm{Im}(\Sigma_\Delta^{\mathrm{EM}}) = \frac{e^2 g_M^2(M_\Delta - m)^3(M_\Delta + m)^3}{4M_\Delta^3 m^2\pi}, \tag{11}$$

TABLE I. Numerical values for the hadron masses and decay constants used in the γ_0 calculations. All the dimensionful values are given in units of MeV. The physical choice values for SU(2) were taken as in Ref. [17] (notice the difference by a factor of 2 in the Lagrangian definitions of h_A), whereas for the chiral limit and SU(3) we followed Ref. [34]. The value for the coupling g_M is calculated through Eq. (11). The SU(3) physical choice for the baryon masses is given by the average over all the masses appearing in the octet or decuplet, respectively.

		m	M_Δ	M_π	M_K	M_η	F_0	g_A	D	F	$\mathcal{C}$	h_A	g_M
SU(2)	Chiral limit choice	880	1152	140	−−	−−	87	1.27	−−	−−	−−	2.85	3.16
	Physical choice	938.9	1232	138.04	−−	−−	92.21	1.27	−−	−−	−−	2.85	3.16
SU(3)	Chiral limit choice	880	1152	140	496	547	87	−−	0.623	0.441	$-D$	−−	3.16
	Physical choice	1149	1381	140	496	547	108	−−	0.8	0.47	−−	−−	3.16

where $\mathrm{Im}(\Sigma_\Delta^{\mathrm{EM}})$ is the imaginary part of the electromagnetic $\Delta(1232)$ self-energy amplitude. Therefore, using the relation $\Gamma_\Delta^{\mathrm{EM}}/(\Gamma_\Delta^{\mathrm{EM}}+\Gamma_\Delta^{\mathrm{Strong}}) = 0.55\%\ldots0.65\%$ and the strong decay width $\Gamma_\Delta^{\mathrm{Strong}} = (118\pm2)$ MeV, we get the value $g_M = 3.16\pm0.16$. Since data on the electromagnetic decays of the full decuplet are sparse and contain large errors, a determination of g_M in the SU(3) version is not viable. We therefore also fix g_M in SU(3) to the $\Delta\to\gamma N$ decay, i.e., we also use the value of $g_M = 3.16\pm0.16$ here. We should keep in mind that the central value of g_M will change when going from SU(2) to SU(3), but we expect that with the present sizable error on g_M this value is included.

For the covariant derivative we use

$$(\mathrm{D}_\lambda\phi)^{ab} = \frac{1}{\sqrt{2}}(\partial_\lambda\phi^{ab} - \mathrm{i}eA_\lambda[Q,\phi]^{ab}). \tag{12}$$

The factor $\frac{1}{\sqrt{2}}$ comes from the definition of the meson-octet matrix. When introducing the decuplet fields into the chiral theory, an additional small parameter of the ChPT expansion appears, $\delta = M_\Delta – m \sim 300$ MeV. Therefore, the counting scheme has to be revised. Here we follow the δ-counting scheme from Ref. [23], where δ^2 is counted as $\mathcal{O}(p)$. It is adequate for the low-energy range close to the pion-production threshold. Hence, one obtains the rule

$$N = 4N_L + \sum_{d=1}^{\infty} dN_d - 2P_\phi - P_B - \frac{1}{2}P_\Delta, \tag{13}$$

where now P_Δ is the number of Δ propagators. Since the $\Delta(1232)$ is a spin-3/2 resonance, it does not have the normal Dirac-propagator form for spin-1/2 particles, but rather takes the Rarita-Schwinger form given by

$$S_\Delta^{\alpha\beta}(p) = \frac{\not{p}+M_\Delta}{p^2-M_\Delta^2+\mathrm{i}\varepsilon}\left[-g^{\alpha\beta}+\frac{1}{d-1}\gamma^\alpha\gamma^\beta + \frac{1}{(d-1)M_\Delta}(\gamma^\alpha p^\beta - \gamma^\beta p^\alpha) + \frac{d-2}{(d-1)M_\Delta^2}p^\alpha p^\beta\right], \tag{14}$$

where d is the number of dimensions of the Minkowski space, which after dimensional regularization is set to $d=4$.

The calculations in this work are done up to order p^3 for the isospin-1/2 counting scheme, which is the leading-order calculation for the γ_0 observable, and up to order $p^{7/2}$ in the isospin-3/2 counting scheme. This choice is due to the fact that in the isospin-1/2 sector the first contributions appear at loop level, which corresponds to the order p^3 if all the coupling vertices are extracted from the lowest-order Lagrangian. Instead of going to higher-order couplings and therefore obtaining loops of order p^4, we included the isospin-3/2 sector, which due to its lower order (the leading-order diagrams are at order $p^{7/2}$) is expected to dominate over those contributions. In addition, there is the advantage that fewer additional low-energy constants are needed than for the case of higher orders. All the constants' values chosen for this work are given in Table I. In order to obtain the SU(2) results, we simply put all the channels with nonvanishing strangeness to 0 and keep only those channels involving nucleons, pions, and the isospin-3/2 quadruplet.

III. THE FORWARD SPIN POLARIZABILITY γ_0

The diagrams contributing to the forward spin polarizability γ_0 are shown in Figs. 1 and 2. The calculation of the amplitudes corresponding to each of these diagrams is performed in the rest frame of the baryon. To calculate the forward spin polarizability one needs to assume conservation of the photon energy $\omega=\omega'$; incoming and outgoing photons have the same momenta $\vec{q}=\vec{q}'$. In the following, the Minkowski-space vectors used are

$$\begin{aligned} p^\mu = p'^\mu &= (m,0,0,0),\\ \epsilon^\mu &= (0,\vec{\epsilon}),\\ \epsilon^{*\mu} &= (0,\vec{\epsilon}^*),\\ q^\mu = q'^\mu &= (\omega,\vec{q}), \end{aligned} \tag{15}$$

where q and q' are the 4-momenta of the incoming and outgoing photons, ϵ and ϵ^* are their respective polarizations, and p and p' are the momenta of the incoming

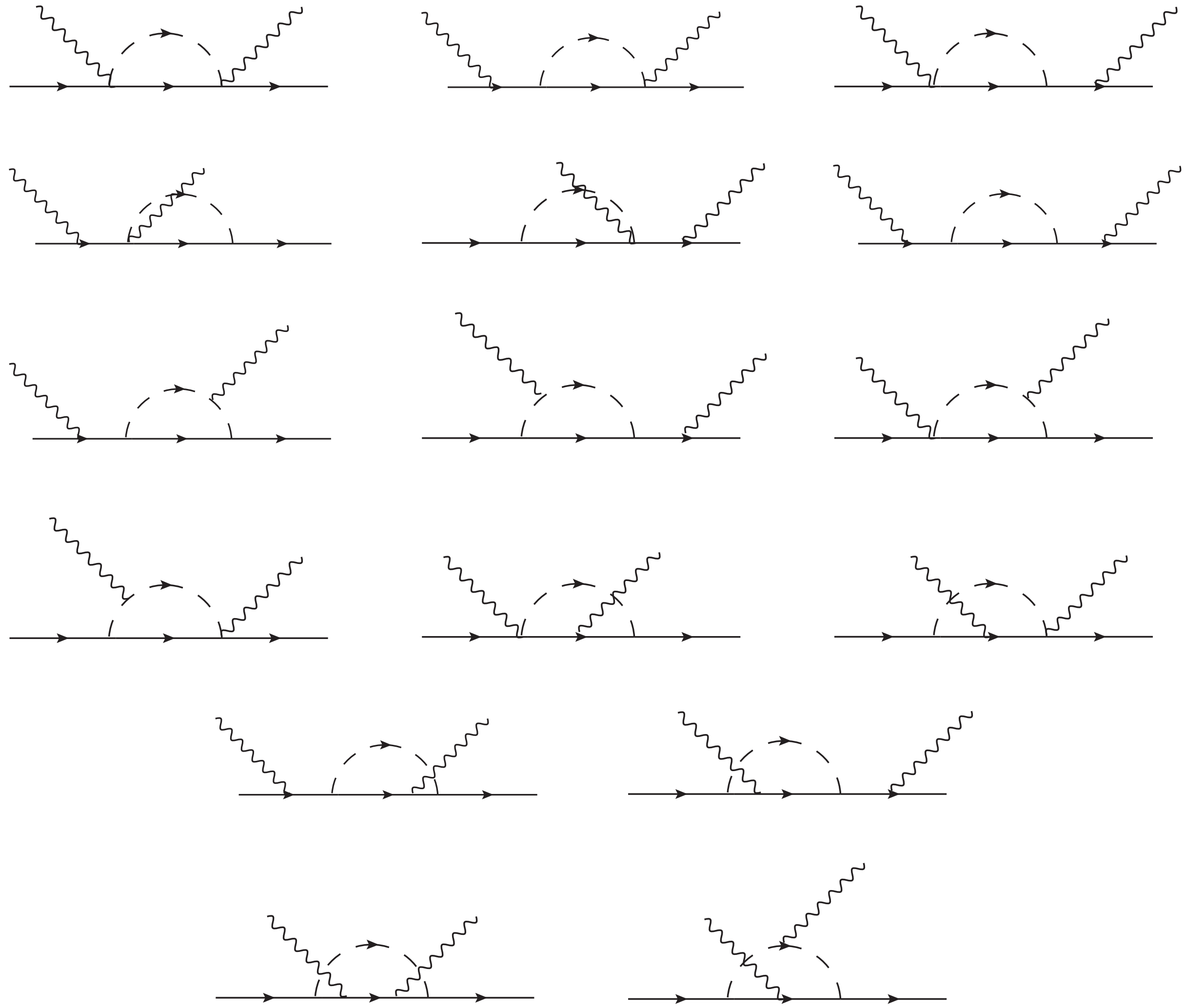

FIG. 1. Diagrams contributing to γ_0 with isospin-$1/2$ intermediate states. The crossed diagrams are obtained by the substitutions $\omega \leftrightarrow -\omega$ and $\not{\epsilon} \leftrightarrow \not{\epsilon}^*$.

and outgoing baryons, respectively. We work in the Weyl gauge, which leads to the condition $p \cdot \epsilon = 0$.

All terms containing the expression $\not{\epsilon}^* \not{\epsilon}$ contribute to γ_0, as can be seen when comparing Eq. (2) with

$$\not{\epsilon}^* \not{\epsilon} = \mathrm{i}\vec{\sigma}(\vec{\epsilon} \times \vec{\epsilon}^*) - (\vec{\epsilon}\vec{\epsilon}^*). \tag{16}$$

Terms like $\not{\epsilon}^* \not{q} \not{\epsilon}$ yield a contribution of $-\mathrm{i}\omega\vec{\sigma}(\vec{\epsilon} \times \vec{\epsilon}^*)$ when projected onto the baryon states. All the other expressions that arise can be reduced to this simple case.

While the full set of diagrams in the spin-$1/2$ sector is gauge invariant, special care has to be taken when including the spin-$3/2$ states. The diagrams which include a minimal coupling of the photon to the $\Delta(1232)$ would need terms of higher order to fully restore gauge invariance. In order to be fully gauge invariant the Lagrangian of Eq. (9) should include the covariant derivative $D_\mu \Delta_\nu$ as opposed to the partial derivative $\partial_\mu \Delta_\nu$ only. The difference between the two derivatives are higher-order terms. To solve this discrepancy, we follow the solution of Ref. [11], where this problem has already been addressed for the case of the proton. In fact, for the neutral octet baryons the diagrams of Fig. 2 are fully gauge invariant. As for the charged octet baryons, this is only the case for the diagrams with charged mesons. Therefore, for these baryons, the strategy is to study two sets of diagrams separately: on the one hand, we have the one-particle-irreducible diagrams of Figs. 2(b), 2(h), and 2(i), which are calculated summing over all isospin channels; on the other hand, the missing one-particle-reducible loop diagrams of Figs. 2(c) to 2(g) are first calculated only for the charged meson channels. For the other channels the isospin factor is chosen such that the ratio between the isospins of the one-particle-reducible and one-particle-irreducible diagrams is the same as for the charged meson channels. When doing so, gauge invariance is ensured and the restoration procedure involves higher-order terms.

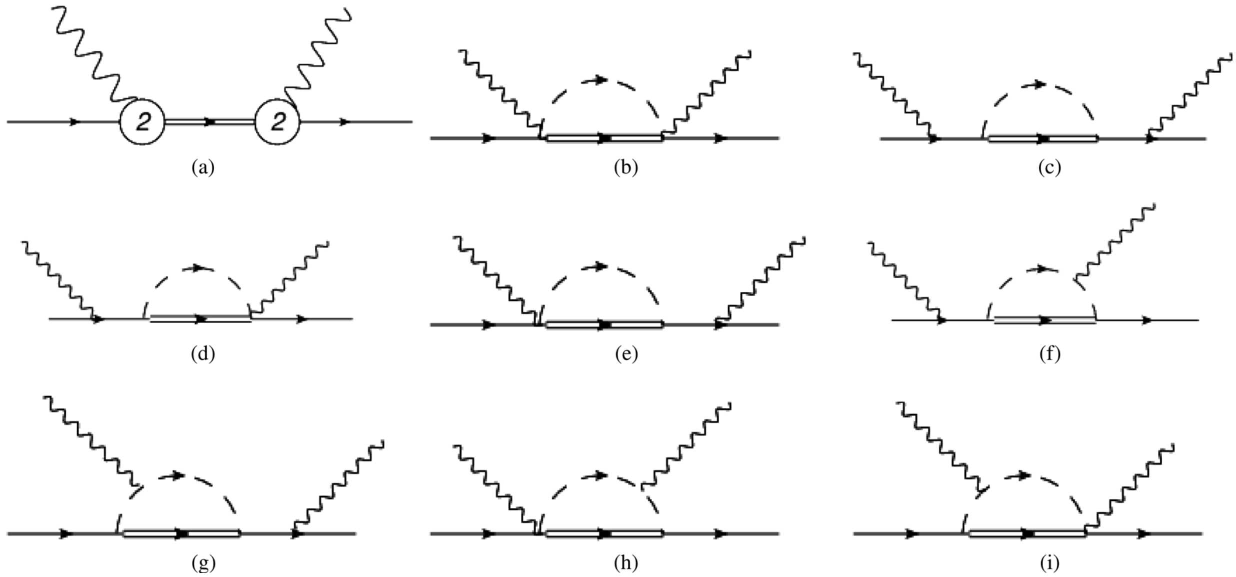

FIG. 2. Diagrams contributing to γ_0 with isospin-3/2 intermediate states. The crossed diagrams are obtained by the substitutions $\omega \leftrightarrow -\omega$ and $\epsilon \leftrightarrow \epsilon^*$. Except for the tree diagram (a), which has vertices of a second-order Lagrangian, all the vertices that appear are couplings of lowest-order Lagrangians. Furthermore, (b), (h) and (i) are one-particle irreducible, and (c)–(g) are one-particle reducible diagrams.

IV. RESULTS AND DISCUSSION

The numerical results for our calculations, when including nucleons, pions, and Δ resonances only (hadrons with no strangeness) are given in Table II, where a comparison with the numerical values found by other groups is also given. Our calculation for the isospin-1/2 sector completely agrees with the results of Ref. [17]. For completion, we also included how the γ_0 values vary when taking the chiral limit, where the masses were set to the best-fit chiral masses. We compare our results with the heavy baryon chiral perturbation theory (HBChPT) results from Refs. [16,18]. The discrepancy between the results does not lie in the parameter choice but in the heavy-mass expansion one assumes for HBChPT.

As for the isospin-3/2 sector, our results differ from those of Ref. [17]. The reason for this is that we use a different counting scheme, and therefore a different set of diagrams. We also have a different Lagrangian, which directly sorts out the spurious spin-1/2 contributions of the Rarita-Schwinger spin-3/2 spinor. In Ref. [13] the $\Delta(1232)$ was introduced in the same way as in the present work. One should remark that there a tree-level diagram of order $p^{9/2}$ was included, which we left out here for consistency. Without this diagram, the numerical results in Ref. [13] are in perfect agreement with ours. The decomposition of our results for the nucleon polarizabilities of Table II into their individual parts is listed in Table III. The main correction to the polarizability results comes from the tree-level diagrams with virtual spin-3/2 baryons, while their loop diagrams give only a small contribution.

We extended the calculations to the SU(3) sector, again distinguishing between the case where the isospin-3/2 resonances were included and where only octet baryons were taken into account as intermediate states. Here we also considered both cases: when taking the physical average values and when choosing the chiral limit; see Table I. We

TABLE II. Numerical values for γ_0 obtained in the SU(2) sector here and in other works in units of 10^{-4} fm^4. The choice of the numerical values for the constants for our own results can be found in Table I. The error in our results when including the $\Delta(1232)$ resonance arises from the uncertainty in the value of the low-energy constant g_M.

Model		Proton					Neutron				
		This work	[17]	[16]	[18]	[13]	This work	[17]	[16]	[18]	[13]
Without Δ	HBChPT			4.4					4.4		
	Covariant chiral limit	2.15					3.24				
	Covariant physical values	2.07	2.07				3.06	3.06			
With Δ	HBChPT				1.7					1.7	
	Covariant chiral limit	−1.59(38)					−0.59(38)				
	Covariant physical values	−0.76(28)	−1.74			−1.00	0.15(28)	−0.77			
Experiment [5]		$-0.90 \pm 0.08(\text{stat}) \pm 0.11(\text{syst})$									
Dispersion relations [9]		−1.1					−0.5				

TABLE III. Decomposition of the proton and neutron polarizability results in units of 10^{-4} fm^4 into the contributions coming from the different sets of diagrams, when using the chiral limit for the masses and low-energy constants. The difference in results when using physical values or the chiral limit can be seen as systematical uncertainty.

		Virtual spin-1/2 baryons	Virtual spin-3/2 baryons—tree level	Virtual spin-3/2 baryons—loops	Total
γ_0^p	SU(2)	2.15	−3.62	−0.13	−1.59
	SU(3)	1.53	−3.62	−0.05	−2.14
γ_0^n	SU(2)	3.24	−3.62	−0.21	−0.59
	SU(3)	2.28	−3.62	−0.08	−1.43

also obtain predictions for the hyperon forward spin polarizabilities. A full listing of the results for the octet baryons is given in Table IV and the decomposition of the results for the nucleons in Table III. When extending the model from SU(2) to SU(3), one takes into account additional virtual states and different values for the parameters, whose impact we discuss in more detail below. Another interesting feature in SU(3) is the appearance of the SU(3)-flavor forbidden photon transitions of the negatively charged octet baryons to those of the decuplet. The results of Table IV are also compared to HBChPT results (for preliminary results see Ref. [15] and for a complete and improved analysis see Ref. [20]). For HBChPT the nucleon values for γ_0 change only slightly when going from SU(2) to SU(3); the results remain large and positive. When changing to the covariant version (without the decuplet contribution) the SU(3) case leads to a reduction of the γ_0 results which still are positive. An additional inclusion of the decuplet leads in both the SU(2) and SU(3) cases to negative γ_0 values closer to the empirical value for the proton of $(-0.90 \pm 0.08(\mathrm{stat}) \pm 0.11(\mathrm{syst})) \cdot 10^{-4}$ fm^4 presented in Ref. [5].

It is also interesting to compare the γ_0 results for the nucleons to those from dispersion relation studies found in Ref. [9] to be $\gamma_0^p = -1.1 \times 10^{-4}$ fm^4 and $\gamma_0^n = -0.5 \times 10^{-4}$ fm^4. The inclusion of isospin-3/2 states, while already having an important effect in HBChPT, leads to an even better agreement with the empirical values in the case of fully covariant calculations, both when taking the chiral limit as well as when taking the average of the physical values for the constants. In fact, the difference between these two parameter sets is of higher chiral order for the polarizabilities. The main source of uncertainty of our results is the constant g_M, whose variation leads to an error estimate as shown in Table IV. We would like to stress that the results obtained here are not subject to uncertainties related to renormalization schemes; for the considered order there are no divergences or power-counting breaking terms entering into the value of γ_0.

As already discussed above, the inclusion of virtual decuplet states is crucial for an agreement of the nucleon baryon chiral perturbation theory (BχPT) polarizabilities with phenomenological values, which is dominantly because of the tree-level diagrams, as shown in Table III. However, this is not the case for the Σ^- and Ξ^- baryons since the photon transitions to the corresponding decuplet states Σ^{*-} and Ξ^{*-} are forbidden in SU(3) symmetry, and the tree-level diagrams do not appear. Hence, their values for the polarizabilities change only slightly by the small loop contributions with virtual decuplet baryons. To study the polarizabilities in BχPT, these two baryons might therefore be better suited than the proton and neutron since nearly all of the uncertainties coming from the inclusion of the decuplet drop out. Experimentally, it will be very hard to measure their polarizabilities, but it is feasible in lattice QCD. Furthermore, we want to emphasize that the main differences in numerical values for the proton and neutron polarizabilities in SU(2) and SU(3) come from the choice of the parameters in Table I. All contributions coming from K or η mesons are negligible. Choosing the masses and constants in SU(3) as in SU(2), which are equivalent parameter sets in terms of chiral counting up to the order p^3, will give nearly the same results.

The addition of p^4 contributions would be the next step to further refine the calculation.

V. SUMMARY

We have presented an extended calculation for the spin polarizability γ_0 of the baryon octet. The framework we chose is based on manifestly Lorentz-covariant baryon chiral perturbation theory, both in the SU(2) as well as SU(3) versions. Furthermore, we explicitly included

TABLE IV. Numerical values for γ_0 obtained in our calculations, in units of 10^{-4} fm^4 in the SU(3) sector. The choice of the numerical values for the constants in the covariant case can be found in Table I, both for the chiral limit and for the physical average case. As for the HBChPT limit, we cite the results in Ref. [15], which were later corrected in our work in Ref. [20]. The errors in the results with the decuplet arise from the uncertainty in the value of the low-energy constant g_M.

Model	Used values	p	n	Σ^+	Σ^-	Σ^0	Λ	Ξ^-	Ξ^0
Without decuplet, HBChPT	[15,20]	4.69	4.53	2.77	2.54	2.44	2.62	0.52	0.68
Without decuplet, covariant	Chiral limit	1.53	2.28	0.90	0.89	1.60	1.09	0.08	0.15
	Physical values	1.68	2.33	0.93	0.91	1.32	1.28	0.15	0.25
With decuplet, covariant	Chiral limit	−2.14(38)	−1.43(33)	−2.72(33)	0.89	0.67(9)	−1.69(28)	0.07	−3.51(38)
	Physical values	−1.64(33)	−1.03(33)	−2.30(33)	0.90	0.47(8)	−1.25(25)	0.13	−3.02(33)

intermediate spin-3/2 states. The novel results of the present work concern both the SU(3) extension of fully covariant ChPT to the order p^3 and the inclusion of the spin-3/2 decuplet up to order $p^{7/2}$. Empirical results exist only for the nucleon case, where in both versions the inclusion of explicit decuplet states is crucial for finding an agreement between phenomenology and BχPT. In particular, it is the tree-level diagram with virtual decuplet baryons that gives the dominant extra contribution. This also carries over to the SU(3) case, where all contributions from K and η loops turn out to be negligible. However, in SU(3) the two baryons Σ^- and Ξ^- are prominent as their photon transitions to the corresponding decuplet states are forbidden in SU(3) symmetry. As a result, the decuplet tree-level contributions are not present and their polarizabilities in pure BχPT remain nearly unchanged, meaning that also most of the uncertainties connected to the decuplet inclusion drop out. Since experimental polarizability measurements for these baryons are improbable, comparisons to results from lattice QCD would be very interesting, as polarizabilities to chiral order p^3 are leading-order predictions of BχPT. The γ_0 results for the hyperons, especially the ones for Σ^- and Ξ^-, can therefore serve as a benchmark for other calculations in this sector. At this point it seems a necessity to extend the present calculation to the cases of the electric and magnetic polarizabilities α_E and β_M of the nucleon and the baryon octet, especially the Σ^- and Ξ^-, where probably a similar situation as above occurs with respect to the inclusion of decuplet states.

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APPENDIX A: BASIC NOTATIONS OF CHPT

The matrices of pseudoscalar mesons ϕ, photons Q, and baryons B are given by

$$\phi = \sum_{a=1}^{8} \lambda_a \phi^a = \sqrt{2}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \quad \text{(A1)}$$

$$Q = \frac{1}{2}\left(\lambda_3 + \frac{\lambda_8}{\sqrt{3}}\right) = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}, \quad \text{(A2)}$$

and

$$B = \frac{1}{\sqrt{2}}\sum_{a=1}^{8} \lambda_a B^a = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \quad \text{(A3)}$$

The decuplet states Δ^{ijk} are specified as

$$\Delta^{111} = \Delta^{++}, \qquad \Delta^{112} = \frac{1}{\sqrt{3}}\Delta^+, \qquad \Delta^{122} = \frac{1}{\sqrt{3}}\Delta^0,$$
$$\Delta^{222} = \Delta^-, \qquad \Delta^{113} = \frac{1}{\sqrt{3}}\Sigma^{*+}, \qquad \Delta^{123} = \frac{1}{\sqrt{6}}\Sigma^{*0},$$
$$\Delta^{223} = \frac{1}{\sqrt{3}}\Sigma^{*-}, \qquad \Delta^{133} = \frac{1}{\sqrt{3}}\Xi^{*0},$$
$$\Delta^{233} = \frac{1}{\sqrt{3}}\Xi^{*-}, \qquad \Delta^{333} = \Omega^-. \quad \text{(A4)}$$

The Dirac tensors $\gamma^{\mu\nu}$ are defined as

$$\gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu] \quad \text{and} \quad \gamma^{\mu\nu\lambda} = \frac{1}{4}\{[\gamma^\mu, \gamma^\nu], \gamma^\lambda\}. \quad \text{(A5)}$$

APPENDIX B: LOOP INTEGRALS AND DIMENSIONAL REGULARIZATION

The integrals for Figs. 1 and 2 are taken in d dimensions and later dimensionally regularized to the normal four-dimensional Minkowski space. To calculate the crossed diagrams, the simple substitutions

$$\omega \leftrightarrow -\omega \quad \text{and} \quad \epsilon^* \leftrightarrow \epsilon \quad \text{(B1)}$$

have to be performed. To obtain the final numerical results for the forward spin polarizability, the integrands of the structure constants of the $\epsilon^*\epsilon$ terms are expanded up to the order ω^3. The coefficients of the third order of the expansion are then used to evaluate the integrals.

The following loop integrals are of interest in this work:

$$\int \frac{\mathrm{d}^d z}{(2\pi)^d} \frac{1}{(z^2 - \Delta)^n} = \frac{(-1)^n \mathrm{i}}{(4\pi)^{d/2}} \frac{\Gamma(n - \frac{d}{2})}{\Gamma(n)\Delta^{n-\frac{d}{2}}}, \quad \text{(B2)}$$

$$\int \frac{\mathrm{d}^d z}{(2\pi)^d} \frac{z^\mu z^\nu}{(z^2 - \Delta)^n} = \frac{(-1)^{n-1} \mathrm{i}}{(4\pi)^{d/2}} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)\Delta^{n-\frac{d}{2}-1}} \frac{g^{\mu\nu}}{2}, \quad \text{(B3)}$$

$$\int \frac{\mathrm{d}^d z}{(2\pi)^d} \frac{z^\mu z^\nu z^\rho z^\sigma}{(z^2 - \Delta)^n} = \frac{(-1)^n \mathrm{i}}{(4\pi)^{d/2}} \frac{\Gamma(n - \frac{d}{2} - 2)}{\Gamma(n) \Delta^{n - \frac{d}{2} - 2}} \frac{g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}}{4}. \tag{B4}$$

As a result, one has to dimensionally regularize the integrals, obtaining the expressions

$$\lambda_1(\Delta) = \frac{\Gamma(1 - \frac{d}{2})}{(4\pi)^{d/2} \Delta^{1 - \frac{d}{2}}} = -\frac{\Delta}{16\pi^2} \left(\frac{2}{\epsilon} - \log\left(\frac{\Delta}{\mu}\right) + \log(4\pi) - \gamma_E + 1 + \mathcal{O}(\epsilon) \right),$$
$$\lambda_2(\Delta) = \frac{\Gamma(2 - \frac{d}{2})}{(4\pi)^{d/2} \Delta^{2 - \frac{d}{2}}} = \frac{1}{16\pi^2} \left(\frac{2}{\epsilon} - \log\left(\frac{\Delta}{\mu}\right) + \log(4\pi) - \gamma_E + \mathcal{O}(\epsilon) \right),$$
$$\lambda_3(\Delta) = \frac{\Gamma(3 - \frac{d}{2})}{(4\pi)^{d/2} \Delta^{3 - \frac{d}{2}}} = \frac{1}{16\pi^2 \Delta},$$
$$\lambda_4(\Delta) = \frac{\Gamma(4 - \frac{d}{2})}{(4\pi)^{d/2} \Delta^{4 - \frac{d}{2}}} = \frac{1}{16\pi^2 \Delta^2},$$

where $\epsilon = 4 - d$ and μ is the scale parameter set to the proton mass in this work. For regularization, the minimal subtraction would have to be performed in the extended on-mass shell scheme, where terms proportional to

$$\frac{2}{\epsilon} + \log(4\pi) - \gamma_E + 1 \tag{B5}$$

are subtracted. It is interesting to note that in this work no diagram had to be renormalized, as at order $p^{7/2}$ no divergent or power-counting breaking terms contribute to γ_0.

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