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Quark and gluon distribution functions in a viscous quark-gluon plasma medium and dilepton production via $q\bar{q}$ annihilation

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Viscous modifications to the thermal distributions of quark-antiquarks and gluons have been studied in a quasiparticle description of the quark-gluon-plasma medium created in relativistic heavy-ion collision experiments. The model is described in terms of quasipartons that encode the hot QCD medium effects in their respective effective fugacities. Both shear and bulk viscosities have been taken in to account in the analysis, and the modifications to thermal distributions have been obtained by modifying the energy-momentum tensor in view of the nontrivial dispersion relations for the gluons and quarks. The interactions encoded in the equation of state induce significant modifications to the thermal distributions. As an implication, the dilepton production rate in the $q\bar{q}$ annihilation process has been investigated. The equation of state is found to have a significant impact on the dilepton production rate along with the viscosities.

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I. INTRODUCTION

There are strong indications from relativistic heavy-ion collider (RHIC) experiments at Brookhaven National Laboratory (BNL) concerning the creation of strongly coupled quark-gluon-plasma (QGP) [1] that possess near-perfect fluidity. These observations on the QGP are mainly corroborated by two of the most striking findings of the RHIC, viz., the large elliptic flow shown by QGP, and the large jet quenching [1] at the RHIC. The former (elliptic flow) led to the near-perfect fluid picture and latter (jet quenching) indicated toward the strongly coupled picture of the QGP. Preliminary results from heavy-ion collisions at the LHC [2,3] reconfirm a similar picture of the QGP. There are interesting possibilities for observing the other higher-order flow parameters (dipolar and triangular, etc.) at the LHC, that are crucial for the quantitative understanding of collectivity and the viscous coefficients of the QGP [4,5].

The strongly coupled picture of the QGP is seen to be consistent with the lattice simulations of the QCD equation of state (EoS) [6–8]. The EoS is an important quantity that plays crucial role in deciding the bulk and transport properties of the QGP. Therefore, it needs to be implemented in an appropriate way as a model for the equilibrium state of the QGP while investigating its properties within the framework of semiclassical transport theory. Furthermore, the form of local thermal distribution functions that describe the hydrodynamic expansion of the QGP liquid must contain the effect from the realistic EoS. This sets the motivation for the present investigations. For the temperatures higher than the QCD transition temperature, T_c, this issue can be addressed by adopting the quasiparticle approaches [9–12]. The way to couple the nontrivial dispersion (single particle energy) for the effective degrees of freedom of the QGP in those quasiparticle approaches to the transport theory is the modification in the definition of the $T^{\mu\nu}$ [13,14].

On the other hand, the transport coefficient of the QGP (shear viscosity η and bulk viscosity ζ) is essential to understand and characterizes its liquid state and the hydrodynamic evolution in heavy-ion collisions. A tiny value of $\eta/\mathcal{S}$ (here $\mathcal{S}$, denotes the entropy density) can be associated with the near-perfect fluid picture and the strongly coupled nature of the QGP provided that the $\zeta/\mathcal{S}$ is relatively smaller. Theoretical investigations suggest that this is true for the temperatures not very close to T_c where bulk viscosity is large [14–16]. Several phenomenological and theoretical investigations do suggest that the QGP indeed possesses a very tiny value of the $\eta/\mathcal{S}$ [17–19].

Moreover, in certain situations, the temperature behavior of the ζ and/or η leads to cavitation, and it may cause the hydrodynamic evolution of the QGP to stop before the freeze-out is actually reached [20–23]. Both the bulk and shear viscosities play a vital role in deciding the observed properties of final state hadrons in the RHIC [24]. Furthermore, these transport coefficients have a significant impact on the important phenomena such as heavy quark transport [25] and photon and dilepton production in heavy-ion collisions [21,26–31]. All these investigations call for an appropriate modeling of viscous modified thermal distribution functions of quarks and gluons in the QGP medium. Importantly, such modifications naturally encode hot QCD medium effects through the QGP EoS (described in terms of the quasiparticle approaches at high temperature).

The present analysis is devoted to obtaining the viscous modified thermal distributions for quarks and gluons in the QGP medium, within the framework of transport

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theory, coupling it with a recently proposed effective fugacity quasiparticle model [32]. As an implication of these distribution functions, the dilepton production rate via the $q\bar{q}$ annihilation process is analyzed, and significant modifications are obtained, as compared to those obtained by considering a viscous quark-gluon medium with nonmodified particle distribution functions. The ideal QCD/QGP EoS refers to the system of ultrarelativistic noninteracting gas of quarks-antiquarks and gluons (the Stefan–Boltzmann limit of hot QCD).

The paper is organized as follows. Section II deals with a recently proposed quasiparticle description of hot QCD in terms of effective quasiparton distribution functions along with how it modifies the kinetic theory definition of the energy-momentum tensor. Furthermore, the modifications to the thermal distributions of the quasiparticles (quasi-gluons and quasiquarks) in the presence of dissipation that is induced by shear and bulk viscosity of the QGP are obtained by coupling the kinetic theory with the hydrodynamic description of the QGP. In Sec. III, the dilepton production rate is investigated by employing these viscous modified thermal distribution functions, and interesting observations are discussed. Section IV articulates the conclusions and future directions.

II. VISCOUS MODIFICATION TO QUARK AND GLUON THERMAL DISTRIBUTION FUNCTIONS

The determination of transport properties of any fluid is subject to the matter of moving away from equilibrium followed by adopting either the transport theory approach or equivalently the field theory approach utilizing the well-known Green–Kubo formulas [33]. Once these transport coefficients such as shear and bulk viscosities are known, it is pertinent to ask what kind of modifications are induced to the momentum distributions of the fluid degrees of freedom.

Now, to obtain the modified distribution function of quarks and gluons which describe the viscous QGP, first we need an appropriate modeling of the equilibrium state of the QGP in terms of its degrees of freedom. To that end, we employ a recently proposed quasiparticle description of the QGP [32] as a model for its equilibrium state. This is followed by the linear perturbation induced in terms of shear and bulk viscous effects adopting the quadratic *ansatz*[13] (quadratic in terms of momentum dependence). The viscous modifications to the momentum distributions of quasi-partons (quarks and gluons) are obtained by equating the kinetic theory expression for the energy-momentum tensor, $T^{\mu\nu}$ to its hydrodynamic decomposition. Let us first briefly review the quasiparticle model followed by the $T^{\mu\nu}$ obtained from this model.

A. Quasiparticle description of hot QCD

Let us now discuss the quasiparticle understanding of hot QCD medium effects employed in the present analysis, recently proposed by Chandra and Ravishankar [32]. This description has been developed in the context of the recent $(2+1)$-lattice QCD equation of state (lQCD EoS) [7] at physical quark masses. There are more recent lattice results with the improved actions and more refined lattices [8], for which we need to revisit the model with a specific set of lattice data specially to define the effective gluonic degrees of freedom. This is beyond the scope of the present analysis. Henceforth, we will stick with the one set of lattice data utilized in the model [32].

The model initiates with an *ansatz* that the lQCD EoS can be interpreted in terms of noninteracting quasipartons having effective fugacities, z_g, z_q, which encode all the interaction effects, where z_g denotes the effective gluon fugacity and z_q denotes the effective quark fugacity, respectively [32]. In this approach, the hot QCD medium is divided into two sectors, viz., the effective gluonic sector and the matter sector (light quark sector and strange quark sector). The former refers to the contribution of gluonic action to the pressure which also involves contributions from the internal fermion lines. On the other hand, the latter involve interactions among quarks and antiquarks as well as their interactions with gluons. The *ansatz* can be translated to the form of the equilibrium distribution functions, $f_{\rm eq} \equiv \{f^g_{\rm eq}, f^q_{\rm eq}, f^s_{\rm eq}\}$ (this notation will be useful later while writing the transport equation in quark and gluonic sectors in compact notations), as follows,

$$f^{g,q}_{\rm eq} = \frac{z_{g,q}\exp(-\beta E_p)}{(1\mp z_{g,q}\exp(-\beta E_p))},$$
$$f^{s}_{\rm eq} = \frac{z_{q}\exp(-\beta\sqrt{p^2+m^2})}{(1+z_{q}\exp(-\beta\sqrt{p^2+m^2}))}, \qquad (1)$$

where $E_p = |\vec{p}| \equiv p$ is for gluons and light quarks and $\sqrt{p^2+m^2}$ for strange quarks (m denotes the mass of the strange quark) and $\beta = 1/T$ in the natural units. The *minus* sign is for gluons, and the *plus* sign is for quark-antiquarks. The quarks and antiquarks possess the same distribution functions since we are working at the zero baryon chemical potential. The determination of $f_{\rm eq}$ is achieved by fixing the temperature dependence of the effective fugacities z_g and z_q from the QGP EoS which in our case is the lQCD EoS (for details, we refer the reader to Ref. [32]). Effective fugacity in our quasiparticle model should not be confused with the presence of any chemical potential. It does not indicate the presence of any conserved quantity in the medium. Its physical significance is described below.

It is worth emphasizing that the effective fugacity is not merely a temperature dependent parameter which encodes the hot QCD medium effects. It is very interesting and

physically significant and can be understood in terms of the effective number density of quasiparticles in hot QCD medium and equivalently in terms of an effective virial expansion [32]. Interestingly, its physical significance reflects in the modified dispersion relation both in the gluonic and matter sectors by looking at the thermodynamic relation of energy density $\epsilon = -\partial_\beta \ln(Z)$. One thus finds that the effective fugacities modify the single quasiparton energy as follows:

$$\begin{aligned} \omega_g &= p + T^2\partial_T \ln(z_g) \\ \omega_q &= p + T^2\partial_T \ln(z_q) \\ \omega_s &= \sqrt{p^2 + m^2} + T^2\partial_T \ln(z_q). \end{aligned} \tag{2}$$

This leads to the new energy dispersions for gluons (ω_g), light quarks/antiquarks (ω_q), and strange quark-antiquarks (ω_s). These dispersion relations can be explicated as follows. The second term in the right-hand side of Eq. (2) is like the gap in the energy spectrum due to the presence of quasiparticle excitations. This makes the model more in the spirit of the Landau's theory of Fermi liquids. Detailed discussions regarding the interpretation and physical significance of z_g and z_q are discussed at a length in Refs. [19,32]. Note that the quasiparticle model is reliable for the temperatures that are higher than T_c, and hence in this situation, the effects induced by the strange quark mass can be neglected (the strange quark mass is around 100 MeV in the EoS, and the model is reliable beyond $1.5T_c$). In this case, we can describe the hot QCD EoS as a system with the effective gluons ($f^g_{\rm eq}$) and the effective quark-antiquarks ($f^q_{\rm eq}$) as the degrees of freedom. The effective fugacity model has further been employed to study the anisotropic hot QCD matter and quarkonia dissociation [34] and to study the heavy-quark drag/diffusion coefficients in the QGP medium [25], leading to the significant impact of the realistic QGP equation of sate on both these important phenomena.

Note that there are other quasiparticle descriptions of hot QCD medium effects, viz., the effective mass models [10], effective mass models with gluon condensate [12], quasiparticle models with Polyakov loop [11], along with our effective fugacity model. Our model is fundamentally distinct from these models, and the differences are discussed at a length in Ref. [32]. The major difference between our model and the effective mass models is in their philosophies themselves. The effective fugacities in our model are not the effective masses. However, these can be interpreted as effective mass in a limiting case $p \ll T^2\partial_T \ln(z_{g/q})$ [32]. Another substantive difference between the two approaches can be seen in terms of group velocity, v_{gr}, which is not the same in two approaches. In the effective mass approaches, the group velocity of quasiparticles depends upon the thermal mass parameter $v_g r = p/\sqrt{(p2 + m(T)^2}$. In contrast, in our model, the description does not touch the v_{gr}. One can alternatively interpret the effective fugacities in terms of effective mass, $m_{\rm eff} \equiv g'T$ (g': effective coupling), as $z_{g,q} \equiv \exp(-m_{\rm eff}|_{g,q}/T)$. The effective coupling, g', comes out to be less than unity for $T \geq 1.3T_c$ (in both the gluonic and quark sectors) [32].

B. Modification to the thermal distributions

Shear and bulk viscosities are essential to understand the space-time evolution of the QGP during its hydrodynamic expansion. Physically, shear viscosity accounts for the entropy production during the anisotropic expansion of the system maintaining its volume constant; on the other hand, bulk viscosity accounts for the entropy production while the volume of the system changes at a constant rate (isotropic expansion). Since these transport coefficients are related to the nonequilibrium properties of the fluid, this requires going beyond the equilibrium modeling of the fluid within the linear response theory.

The general linear response (Chapman–Enskog) formalism assumes a small perturbation of the thermal equilibrium distribution (considering the small perturbation around the equilibrium distributions of the quark-antiquarks and gluons) as

$$f(\vec{p}, \vec{r}) = f_0(p) + \delta f, \tag{3}$$

where

$$f_0(p) = \frac{z_{g,q}\exp(-\beta u^\mu p_\mu)}{(1 \mp z_{g,q}\exp(-\beta u^\mu p_\mu))} \tag{4}$$

denotes the local thermal equilibrium distribution function in Eq. (5) in the absence of viscous effects. The quantity, δf, is the linear perturbation which encodes the viscous effects as described below. Here, g stands for quasigluons and q for quasiquarks (we have also neglected the mass of the strange quark which is justified at high temperature), u^μ is the 4-velocity of the fluid, and $\beta = 1/T$. The isotropic distribution, $f_0(p)$, is reduced to $f_{\rm eq}$ in the local rest frame of the fluid (LRF).

Now, using $T\partial f_0/\partial(u^\mu p_\mu) = -f_0(1 \pm f_0)$, the linear perturbation δf can be expressed as [18]

$$\delta f \equiv f(\vec{p}) - f_0(p) = f_0(p)(1 \pm f_0(p))f_1(\vec{p}). \tag{5}$$

Here, plus is for gluons, and minus is for the quark-antiquarks. The perturbation $f_1 \equiv \{f_{1g}, f_{1q}\}$ (combined notation for quarks and gluons) can be thought of as a change in the argument of f_0 as ($\beta u^\mu p_\mu \to \beta u^\mu p_\mu - f_1(\vec{p}, \vec{r})$) [18] and can be thought of as a local fugacity factor leading to following form of the near-equilibrium distributions:

$$f_g(\vec{p}) = \frac{z_g \exp(-\beta u^\mu p_\mu + f_{1g})}{1 - z_g \exp(-\beta u^\mu p_\mu + f_{1g})}$$
$$f_q(\vec{p}) = \frac{z_q \exp(-\beta u^\mu p_\mu + f_{1q})}{1 + z_q \exp(-\beta u^\mu p_\mu + f_{1q})}. \quad (6)$$

Note that Eq. (5) is obtained by expanding Eq. (6) and keeping only the linear term in the perturbation, f_1. Next, we discuss the energy-momentum tensor for the QGP fluid obtained from these distribution functions that is essential for determining the form of f_1 in terms of shear and the bulk viscosities.

1. Energy-momentum tensor

To obtain viscous modifications to the quark-antiquarks and gluon distribution functions, we need to couple the fluid dynamic description of the QGP to the kinetic theory description. At the point of freeze-out in heavy-ion collisions, the fluid dynamic description of QGP should smoothly change to the particle description (hardonization). This is understood in terms of the matching of the energy-momentum tensor, $T^{\mu\nu}$, in these two descriptions. The viscous hydrodynamic, $T^{\mu\nu}$ in the QGP phase must be equated to its kinetic theory definition [in terms of $f(\vec{p})$]. To achieve the continuity of of the $T^{\mu\nu}$, in our case, the kinetic theory definition of $T^{\mu\nu}$ needs to be revised in a way that it must capture the hot QCD medium effects in terms of nontrivial dispersion relations and the effective fugacities. We further have to satisfy the the Landau–Lifshitz (LL) conditions [35] discussed below.

The energy density and the pressure can be obtained in terms of quasigluons and quasiquarks in our quasiparticle model [32] as

$$\epsilon = \int \frac{d^3\vec{p}}{8\pi^3}(\nu_g \omega_g f_g^{\rm eq} + \nu_q \omega_q f_q^{\rm eq})$$
$$\mathcal{P} = -\frac{1}{\beta}\nu_g \int \frac{d^3\vec{p}}{8\pi^3}\ln(1 - z_g \exp(-\beta p)) + \frac{1}{\beta}\nu_q \int \frac{d^3\vec{p}}{8\pi^3}\ln(1 + z_q \exp(-\beta p)). \quad (7)$$

We use the notation $\nu_g = 2(N_c^2 - 1)$ for gluonic degrees of freedom, $\nu_q = 2 \times 2 \times N_c \times 3$ ($N_c = 3$ in the present case).

In kinetic theory $T^{\mu\nu}$ is obtained from the single particle momentum distributions as

$$T^{\mu\nu} = \sum_{g,q} \int \frac{d^3\vec{p}}{8\pi^3} \frac{p^\mu p^\nu}{\omega} f(\vec{p}). \quad (8)$$

It is emphasized in Ref. [14] that the above expression of $T^{\mu\nu}$ cannot simply be utilized in the present case, since it does not capture the nontrivial dispersions of quasiparticles. In other words, the thermodynamic consistency condition is not satisfied with this expression of $T^{\mu\nu}$ yielding incorrect expressions for the energy density and the pressure.

This issue has recently been addressed in Ref. [14] by arguing for a modified form of the $T^{\mu\nu}$, in a similar spirit as it is done in the effective mass quasiparticle models [13]:

$$T^{\mu\nu} = \sum_{g,q}\left\{\int \frac{d^3\vec{p}}{(2\pi)^3\omega} p^\mu p^\nu f(\vec{p}) + \int \frac{d^3\vec{p}}{(2\pi)^3 p\omega}(\omega - p) p^\mu p^\nu f_0(p) + \int \frac{d^3\vec{p}}{(2\pi)^3}(\omega - p) u^\mu u^\nu f_0(p)\right\}. \quad (9)$$

One can clearly realize the presence of the factors, $T^2 \frac{d\ln(z_g)}{dT}$ and $T^2 \frac{d\ln(z_g)}{dT}$, in the expression for $T^{\mu\nu}$ in Eq. (9) that are the part of the modified dispersions. The second term in the right-hand side of Eq. (9) ensures the correct expression for the pressure, and the third term ensures the correct expression for the energy density, and hence the definition of $T^{\mu\nu}$ incorporates the thermodynamic consistency condition correctly. In view of the reliability of the quasiparticle descriptions of hot QCD for temperature beyond the QCD transition temperature, we may ignore the strange quark mass effects. In this case, the QGP can be described by massless quasigluons and massless quasiquarks having nontrivial dispersion relations. Therefore, in Eq. (9), $\omega \equiv (\omega_g, \omega_q)$, and summation is over the gluons and quarks. On the other hand, the effective mass quasiparticle models include the terms containing the temperature derivative of the effective mass in the modified $T^{\mu\nu}$ [13]. It is to be noted that the effective mass models are fundamentally different from the effective fugacity model [14] employed here. The differences can be understood in terms of the modified dispersions in the two cases. The major difference can be realized in terms of nonchanging particle velocities in the effective fugacity model, in contrast to the effective mass models. Moreover, the effective fugacity model can be understood in terms of charge renormalization in the hot QCD; on the other hand, the effective mass quasiparticle models are motivated by mass renormalization in the hot QCD medium.

To realize the LL conditions ($u_\mu T^{\mu\nu} u_\nu = e$ and $u_\mu \delta T^{\mu\nu} = 0$), we can resolve the $T^{\mu\nu}$ as

$$T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu}. \quad (10)$$

From Eq. (9),

$$T_0^{\mu\nu} = \sum_{g,q}\left\{\int \frac{d^3\vec{p}}{(2\pi)^3\omega} p^\mu p^\nu f_0(p) + \int \frac{d^3\vec{p}}{(2\pi)^3 p\omega}(\omega - p) p^\mu p^\nu f_0(p) + \int \frac{d^3\vec{p}}{(2\pi)^3}(\omega - p) u^\mu u^\nu f_0(p)\right\},$$
$$\delta T^{\mu\nu} = \sum_{g,q} \int \frac{d^3\vec{p}}{(2\pi)^3\omega} p^\mu p^\nu \delta f(\vec{p}). \quad (11)$$

$T_0^{\mu\nu}$ which gets the modifications from the EoS leads to the right expressions for the energy density and pressure

following the LL condition. The form of $\delta f(\vec{p}) \equiv f_0(p)(1 \pm f_0(p))f_1(p)$ here is based on the quadratic *ansatz* [see Eq. (13)] which also follows the LL condition (since $u_\mu \pi^{\mu\nu} = 0$, $u^\mu \Delta_{\mu\nu} = 0$, $\Delta^{\mu\nu} := u^\mu u^\nu - g^{\mu\nu}$).

On the other hand, the fluid dynamic definition of $T^{\mu\nu}$ in the presence of shear and bulk viscous effects is given as

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}, \tag{12}$$

where Π and $\pi^{\mu\nu}$ are the shear and bulk parts of the viscous stress tensor.

The form of the perturbations f_1 to the thermal distributions of gluons and quarks can be obtained in terms of the Π and $\pi^{\mu\nu}$ by relating the two definitions (kinetic theory and fluid dynamic) of the $T^{\mu\nu}$. The two definitions can be matched through the following quadratic *ansatz* for $f_1(\vec{p})$ [13],

$$f_1(\vec{p}) = \frac{1}{(\epsilon + P)T^2}\left(\frac{p^\mu p^\nu}{2} C_1 \pi_{\mu\nu} + \frac{C_2}{5} p^\mu p^\nu \Delta_{\mu\nu}\Pi\right), \tag{13}$$

where the coefficients C_1 and C_2 are obtained by the matching of the two definitions of $T^{\mu\nu}$ in the LRF. This follows from the fact that shear and bulk viscosities are Lorentz-invariant quantities and can conveniently be obtained in the LRF of the fluid. The factor $\epsilon + P \equiv \mathcal{S}T$ is introduced for convenience, since for the QGP in the RHIC we consider viscosities scaled with entropy density ($\mathcal{S}$). While matching hydrodynamic and kinetic theory descriptions, one should ensure the Landau–Lifshitz matching conditions. The modified form of the $T^{\mu\nu}$ in Eq. (12) ensures LL-matching conditions in the temperature range where our quasiparticle model is valid [14].

Next, utilizing the notations in Eq. (5) and matching the right-hand sides of Eq. (9) and Eq. (13) in the LRF, we obtain

$$\begin{aligned}
\Pi\delta^{ij} + \pi^{ij} &= \frac{\nu_g}{\mathcal{S}T^3}\int \frac{d^3\vec{p}}{8\pi^3\omega_g} p^i p^j p^l p^m f_g(1 + f_g) \\
&\quad \times \left(C_1 \pi_{lm} + \frac{C_2}{5}\Pi\right) \\
\Pi\delta^{ij} + \pi^{ij} &= \frac{\nu_q}{\mathcal{S}T^3}\int \frac{d^3\vec{p}}{8\pi^3\omega_q} p^i p^j p^l p^m f_q(1 - f_q) \\
&\quad \times \left(C_1 \pi_{lm} + \frac{C_2}{5}\Pi\delta_{lm}\right).
\end{aligned} \tag{14}$$

Here, l and m are contracted and summed over. Note that the shear and bulk viscous parts of the stress in the gluonic and quark sectors are distinguished by their respective transport coefficients (their total value is obtained by adding up appropriately the gluonic and quark contributions). The fluid velocity fields for quark-antiquark and gluonic degrees of freedom are assumed to be the same in the QGP fluid. The integral over the momentum in the above equations can be expressed as in Ref. [13], $I_{g,q}(\delta^{ij}\delta^{lm} + \delta^{il}\delta^{jm} + \delta^{im}\delta^{jl})$ (the subscripts g and q are used to distinguish the gluonic and the matter sector), where

$$\begin{aligned}
I_g &= \frac{1}{15\mathcal{S}T^3}\nu_g \int \frac{d^3\vec{p}}{8\pi^3\omega_g} p^4 f_g(1 + f_g) \\
I_q &= \frac{1}{15\mathcal{S}T^3}\nu_q \int \frac{d^3\vec{p}}{8\pi^3\omega_q} p^4 f_q(1 - f_q).
\end{aligned} \tag{15}$$

Now, from Eq. (14) in the gluonic sector,

$$C_1 = C_2 = \frac{1}{I_g}, \tag{16}$$

and in the matter sector,

$$C_1 = C_2 = \frac{1}{I_q}. \tag{17}$$

Now, the viscous modified thermal distributions of gluons and quarks in the QGP in terms of $I \equiv (I_{g,q})$ reads,

$$f(\vec{p}) = f_{\rm eq} + \frac{f_{\rm eq}(1 \pm f_{\rm eq})}{\mathcal{S}T^3}\left(\frac{p^\mu p^\nu}{2I}\pi_{\mu\nu} + \frac{p^\mu p^\nu \Delta_{\mu\nu}\Pi}{5I}\right). \tag{18}$$

As mentioned earlier, $f_{\rm eq} \equiv (f_g, f_q)$.

Let us discuss the validity of the above expression of the viscous modified thermal distributions. The validity criterion is simply $(f - f_{\rm eq}) \ll f_{\rm eq}$ (near-equilibrium condition). In other words, for the validity of our formalism, the viscous corrections ($\pi^{\mu\nu}$ and Π) must induce small corrections to the equilibrium distribution of the gluons and quarks. This translates to the condition

$$\frac{p^\mu p^\nu \pi_{\mu\nu}}{2} + \frac{p^\mu p^\nu \delta_{\mu\nu}\Pi}{5} \ll \mathcal{S}T^3(1 \pm f_{\rm eq})I. \tag{19}$$

Next, we consider a case where the integral displayed in Eq. (15) can be solved analytically. In the limit $T^2\partial_T(z_{g,q})/p \ll 1$ (high temperature limit), we can obtain analytic expressions for I_g and I_q as

$$\begin{aligned}
I_g &= \frac{4\nu_g T^3}{\pi^2\mathcal{S}}\mathrm{PolyLog}[5, z_g] \\
I_q &= -\frac{4\nu_q T^3}{\pi^2\mathcal{S}}\mathrm{PolyLog}[5, -z_q].
\end{aligned} \tag{20}$$

The $\mathrm{PolyLog}[n, x]$ function appearing in Eq. (20) is having the series representation $\mathrm{PolyLog}[n, x] = \sum_{k=1}^{\infty}\frac{x^k}{k^n}$ (the convergence of the series is subject to the condition that $|x| \leq 1$). The Stefan–Boltzmann (SB) limit (the employment of ideal QGP EoS) is obtained only asymptotically (by putting $z_{g,q} \equiv 1$) in right-hand side of Eq. (20).

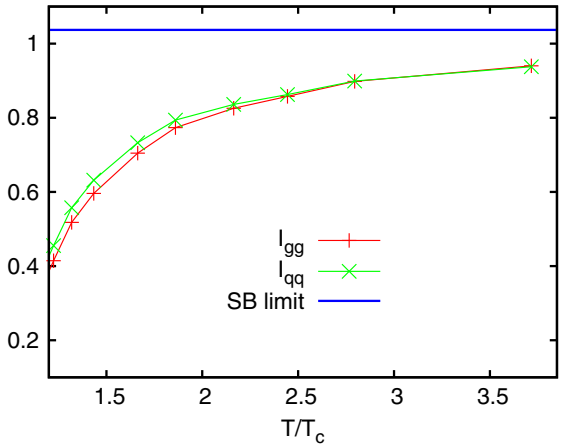

FIG. 1 (color online). Behavior of I_{gg} and I_{qq} as functions of T/T_c. The temperature dependences of the z_g and z_q are taken from Ref. [32]. Clearly, the modifications induced by the EoS are quite significant even at higher temperatures as compared to the ideal QGP EoS.

It can easily be seen that I_g and I_q are of the order of unity in the case of the ideal EoS. This is also realized in Ref. [13]. To see the difference in these two cases, we plot the quantities I_{gg} and I_{qq}, defined as

$$I_{gg} \equiv \frac{I_g \pi^2 \mathcal{S}}{4\nu_g T^3} = \mathrm{PolyLog}[5, z_g]$$
$$I_{qq} \equiv \frac{15 I_q \pi^2 \mathcal{S}}{64\nu_g T^3} = -\frac{16}{15}\mathrm{PolyLog}[5, -z_q], \qquad (21)$$

for the ideal QGP EoS, and lQCD EoS (the temperature dependence of z_g and z_q are taken from Ref. [32]) in Fig. 1. Here, we use the identities $\mathrm{PolyLog}[5, 1] = \zeta(5)$ and $-\mathrm{PolyLog}[5, -1] = \frac{15}{16}\zeta(5)$ to obtain I_g and I_q in the case of the ideal EoS.

Clearly I_{gg} and I_{qq} will approach their SB limit that is $\zeta(5)$ asymptotically. The interaction effects are significant even at $3.5T_c$. Therefore, one cannot simply ignore these effects while obtaining the viscous modified forms of the thermal distributions of gluons and quarks in the QGP medium. This crucial observation has been realized in the case of the effective mass quasiparticle model in Ref. [13]. Next, we shall investigate the significance of such viscous modified thermal distributions of gluons and quarks in the context of dilepton production.

III. EFFECTS OF THE EOS AND VISCOSITIES ON DILEPTON PRODUCTION VIA $q\bar{q}$ ANNIHILATION

The dilepton production in the QGP medium has dominant contributions from the $q\bar{q}$ annihilation process via the mechanism $q\bar{q} \to \gamma^* \to l^+ l$. The kinetic theory expression for the dilepton production rate for a given dilepton mass and momentum is given by [36]

$$\frac{dN}{d^4x d^4p} = \int\int \frac{d^3\vec{p}_1}{(2\pi)^3}\frac{d^3\vec{p}_2}{(2\pi)^3} f(E_1, T) f(E_2, T) \times \frac{M^2 g^2 \sigma(M^2)}{2E_1 E_2}\delta^4(P - p_1 - p_2), \qquad (22)$$

where the 4-momenta $p_{1,2} = (E_{1,2}, \vec{p}_{1,2})$ are of the quark and antiquark, respectively, with $E_{1,2} = \sqrt{p_{1,2}^2 + m^2} \approx |\vec{p}_{1,2}|$, if one neglects the quark masses. The quantity $M^2 = (E1 + E2)^2 - (\vec{p}_1 + \vec{p}_2)^2$ is the invariant mass of the intermediate virtual photon. Here, g is the degeneracy factor, and $\sigma(M^2)$ is the thermal dilepton production cross section. Here, $P = p_0 = E1 + E2, \vec{p} = \vec{p}_1 + \vec{p}_2$ is the 4- momentum of the dileptons.

The quantity $f(E, T)$ is the quark (antiquark) distribution function in thermal equilibrium, $f(E, T) = \frac{1}{1 + z_q^{-1}\exp(-E/T)}$ (this form is in view of the effective quasiparticle model based on realistic QGP EoS). In the case of the ideal QGP EoS, the factor z_q will be replaced by unity, as is done in most of the works on dilepton production in the QGP medium in the literature. We shall see that the EoS effects are quite significant, even after employing the high temperature limit of the quark (anti-quark) distribution functions. Recall from the previous section that the realistic EoS strongly influences the viscous modified portion of the thermal distributions of gluons and quarks (antiquarks). In the present analysis, we are interested in the invariant masses that are larger compared to the temperature, T. In this limit, we can take the high temperature limit of quark (antiquark) equilibrium thermal distribution functions (replacing Fermi–Dirac distribution with classical Maxwell–Boltzmann distribution in the case of the ideal QGP EoS) as

$$f(E, T) \to z_q \exp\left(-\frac{E}{T}\right), \qquad (23)$$

where $E = |\vec{p}| \equiv p$. The form will remain the same for the quarks and antiquarks since the baryon chemical potential is zero here. It is straightforward to observe from Eq. (22) that the effects coming from the EoS are of the order z_q^2 (this quantity is quite significant even at $2T_c$). In other words, the dilepton production rate is modulated by a factor z_q^2. Let us now proceed to explore the impact of the EoS and the viscous modifications to the dilepton production rate.

Next, we employ the result obtained in Eq. (18) for the viscous modified quark (antiquark) distribution function $f(\vec{p})$ and take its high temperature limit and analyze the shear and bulk viscous contributions one by one. In this limit, the viscous modified quark (antiquark) distribution functions become

$$f(\vec{p}) = z_q \exp\left(-\frac{p}{T}\right)\left[1 + \frac{(1 - z_q \exp(-\frac{p}{T}))}{\mathcal{S}T^3} \times \left(\frac{p^\mu p^\nu}{2I}\pi_{\mu\nu} + \frac{p^\mu p^\nu \Delta_{\mu\nu}\Pi}{5I}\right)\right]$$
$$f(\vec{p}) \approx z_q \exp\left(-\frac{p}{T}\right)\left[1 + \frac{1}{\mathcal{S}T^3}\left(\frac{p^\mu p^\nu}{2I}\pi_{\mu\nu} + \frac{p^\mu p^\nu \Delta_{\mu\nu}\Pi}{5I}\right)\right]. \tag{24}$$

Note that the first term in the above equation accounts for the equilibrium part of the quark (antiquark) thermal distribution, the second encodes the shear viscous effects, and the third one encodes the bulk viscous effects.

The impact of the viscosity corrections to the dilepton production rate are obtained by invoking Eq. (24) to Eq. (22). Next, we rewrite the dileption rate in the component form as follows,

$$\frac{dN}{d^4x d^4p} = \frac{dN^{(0)}}{d^4x d^4p} + \frac{dN^{(\eta)}}{d^4x d^4p} + \frac{dN^{(\zeta)}}{d^4x d^4p}. \tag{25}$$

The notations η and ζ are introduced since $\pi^{\mu\nu}$, Π involve them as the first-order transport coefficient in their definitions. The three terms in the right hand side of Eq. (25) have been computed for the ideal QGP EoS in Ref. [28]. It is straightforward to employ the approach to compute them it the present case (differences are there in the definition of the distribution functions). The first term is given by the following integral:

$$\frac{dN^{(0)}}{d^4x d^4p} = \int\int \frac{d^3\vec{p}_1}{(2\pi)^3}\frac{d^3\vec{p}_2}{(2\pi)^3} z_q^2 \exp\left(-\frac{E_1 + E_2}{T}\right) \times \frac{M^2 g^2 \sigma(M^2)}{2E_1E_2}\delta^4(p - p_1 - p_2). \tag{26}$$

This integral is well known in the literature [36] in the case of $z_q = 1$. Since z_q is independent of the momentum of the particles, so the integral can be evaluated in the same way as [36]

$$\frac{dN^{(0)}}{d^4x d^4p} = \frac{z_q^2}{2}\frac{M^2 g^2 \sigma(M^2)}{2\pi^5}\exp\left(-\frac{p_0}{T}\right). \tag{27}$$

The modification to the rate due to the shear viscosity at first order $\pi^{\alpha\beta} \equiv 2\eta\sigma^{\alpha\beta}$, where $\sigma^{\alpha\beta}$ is the Navier–Stokes tensor, $\sigma_{\alpha\beta} = \frac{1}{2}(\nabla_\alpha u_\beta - \nabla_\beta u_\alpha) - \frac{1}{3}\delta_{\alpha\beta}\Theta$, $\nabla_\alpha = \Delta^\mu_\alpha d_\mu$, $\Theta = d^\mu u_\mu$ can be obtained from the following equation:

$$\frac{dN^{(\eta)}}{d^4x d^4p} = \int\int \frac{d^3\vec{p}_1}{(2\pi)^3}\frac{d^3\vec{p}_2}{(2\pi)^3} z_q^2 \exp\left(-\frac{E_1 + E_2}{T}\right) \times \frac{M^2 g^2 \sigma(M^2)}{2E_1E_2}\left[\frac{\eta}{2I_q\mathcal{S}T^3}(p_1^\alpha p_1^\beta + p_2^\alpha p_2^\beta)\sigma_{\alpha\beta}\right] \times \delta^4(p - p_1 - p_2). \tag{28}$$

Following the analysis of Ref. [27], we obtain the following expression for the shear viscous correction of the rate:

$$\frac{dN^{(\eta)}}{d^4x d^4p} = \frac{-z_q^2}{4\nu_q T^3 \mathrm{PolyLog}[5, -z_q]/\pi^2\mathcal{S}} \times \frac{1}{2}\frac{M^2 g^2 \sigma(M^2)}{(2\pi)^5}\exp\left(-\frac{p_0}{T}\right) \times \frac{2}{3}\left[\frac{\eta}{2\mathcal{S}T^3}p^\alpha p^\beta \sigma_{\alpha\beta}\right]. \tag{29}$$

Now, the third term which is the correction to the rate due to the bulk viscosity (at first order, $\Pi \equiv -\zeta\Theta$, where Θ is the expansion rate of the fluid) can be evaluated from the following expression:

$$\frac{dN^{(\zeta)}}{d^4x d^4p} = \int\int \frac{d^3\vec{p}_1}{(2\pi)^3}\frac{d^3\vec{p}_2}{(2\pi)^3} z_q^2 \exp\left(-\frac{E_1 + E_2}{T}\right) \times \frac{M^2 g^2 \sigma(M^2)}{2E_1E_2}\left[\frac{2\zeta}{10 I_q\mathcal{S}T^3}(p_1^\alpha p_1^\beta + p_2^\alpha p_2^\beta)\Delta_{\alpha\beta}\Theta\right] \times \delta^4(p - p_1 - p_2). \tag{30}$$

This integral can be evaluated using the analysis of Ref. [37] as

$$\frac{dN^{(\zeta)}}{d^4x d^4p} = \frac{-z_q^2}{4\nu_q T^3 \mathrm{PolyLog}[5, -z_q]/\pi^2\mathcal{S}} \times \frac{1}{2}\frac{M^2 g^2 \sigma(M^2)}{(2\pi)^5}\exp\left(-\frac{p_0}{T}\right) \times \left[\left(\frac{2}{3}\frac{2\zeta}{10\mathcal{S}T^3}p^\alpha p^\beta \Delta_{\alpha\beta}\Theta\right) - \frac{2}{5}\frac{\zeta}{4\mathcal{S}T^3}M^2\Theta\right]. \tag{31}$$

The full expression for the rate displayed in Eq. (26) can be obtained by combining Eqs. (27)–(31). These expressions reduce to those obtained in Ref. [28] (the expressions obtained by employing the ideal QCD EoS) by substituting $z_q = 1$ (in this case, $I_q \approx 1$ as already described in Ref. [13]).

If we ignore the viscous corrections, it is obvious that the EoS induced modifications appear as a factor, z_q^2. On the other hand, the shear and bulk viscous corrections to the dilepton production rate get a factor of $R_q = \frac{-z_q^2\pi^2}{\nu_q \mathrm{PolyLog}[5,-z_q]}$ [of which the SB limit is $16/15\zeta(5)$], as a modification

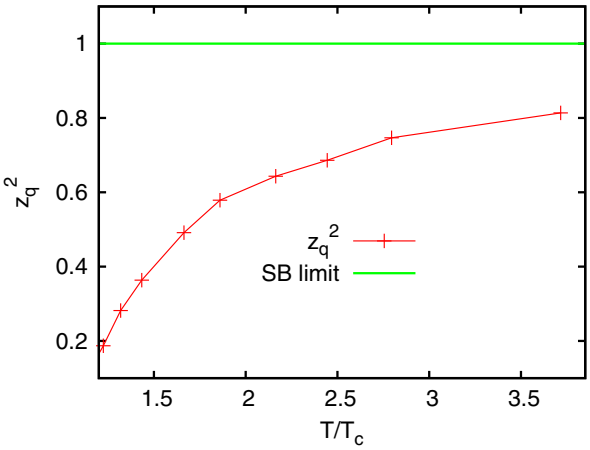

FIG. 2 (color online). The behavior of z_q^2 as a function of T/T_c is shown along with its SB limit ($z_q \to 1$). The temperature dependence of the effective quark fugacity, z_q, is taken from Ref. [32].

from the EoS. We have plotted both of these factors, employing the quasiparticle model for $(2+1)$-flavor QCD [32] in Figs. 2 and 3. On looking at the temperature behavior of both these factors, we can safely say that all three terms in the dilepton rate in Eqs. (27), (29), and (31) get significant modifications from the QGP EoS. From Figs. 2 and 3, both z_q^2 and R_q approach their respective SB limits only asymptotically.

Let us discuss the interesting observations that can be made out, based on the results of the dilepton production rate obtained in the viscous environment and the realistic EoS. Since the quantities η and ζ are the phenomenological

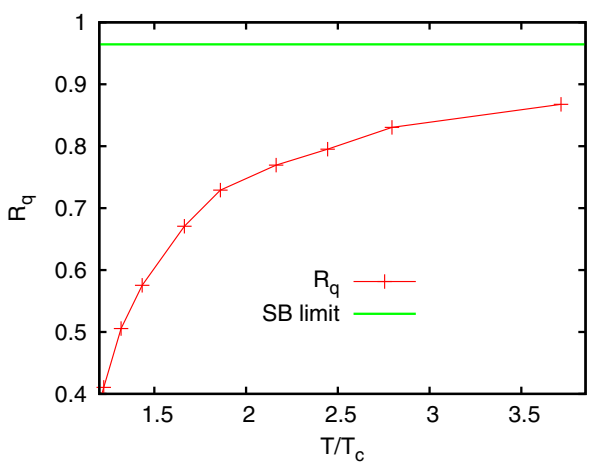

FIG. 3 (color online). The behavior of R_q (the modification factor to the viscous contribution to the dilepton rate induced by the realistic EoS) as function of T/T_c is shown along with its SB limit [$R_q \to 1/\zeta(5)$]. The temperature dependence of the effective quark fugacity, z_q, is taken from Ref. [32]. Here, we assume that η and ζ are the phenomenological parameters for the QGP and assumed to be same for realistic and ideal QGP EoSs.

numbers, they can safely be assumed to be the same in the case of the ideal and the realistic equations of state. Therefore, the role of the viscous corrections in both the cases will be qualitatively similar. However, the quantitative differences are mainly induced by the EoS. The realization that the dilepton production is sensitive to the EoS has also been seen in the recent work of Deng *et al.* [38].

Finally, the EoS induces significant modifications to the viscous modified thermal distribution functions. These modifications play a significant role in the dilepton production rate in the RHIC. The rate is suppressed significantly as compared to that obtained by employing the ideal EoS (the modifications are of the order of z_q^2 in the absence of the viscosities). Let us now proceed to the quantitative understanding of these effects from the realistic EoS and the viscosities.

We choose the value of $\eta/S = \frac{1}{4\pi}$ (the number from the AdS/CFT [Kovtun-Son-Starinets (KSS) bound] [39]). On the other hand, the bulk viscosity of QGP can taken according to the studies from strongly interacting gauge theories [40] as

$$\frac{\zeta}{S} = 2\frac{\eta}{S}\left(-c_s^2 + \frac{1}{3}\right). \tag{32}$$

Here, c_s^2 is the speed of sound squared. The temperature dependence of ζ/S is dictated by η/S and the speed of sound in the QGP phase.

IV. DILEPTON SPECTRA FROM HEAVY-ION COLLISIONS

We now study the effect of the modified gluon and quark, antiquark distribution functions, and viscosity on thermal dilepton spectra produced from the QGP in the heavy-ion collision experiments. The evolution of the fireball is modeled using relativistic hydrodynamics. In this qualitative study, we use one-dimensional boost-invariant scaling flow to analyze the system [41]. We choose the parametrization $t = \tau \cosh \eta_s$ and $z = \tau \sinh \eta_s$ for the coordinates, with the proper time $\tau = \sqrt{t^2 - z^2}$ and space-time rapidity $\eta_s = \frac{1}{2}\ln[\frac{t+z}{t-z}]$. With the fluid 4-velocity expressed as $u^\mu = (\cosh \eta_s, 0, 0, \sinh \eta_s)$, we can write down the equation governing the longitudinal expansion of the plasma as

$$\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P}{\tau} = 0, \tag{33}$$

where we have neglected the effect of viscosity on the expansion as the significant contribution to particle production comes from the viscous modified rates [28]. To close the system, we use the lQCD EoS [7]. With critical temperature $T_C = 180$ MeV, we take the initial conditions relevant to the RHIC energies, $\tau_0 = 0.5$ fm/c and $T_0 = 310$ MeV, in our calculation. By numerically solving the energy equation,

Eq. (33), we obtain the temperature profile $T(\tau)$. We note that for $\tau = \tau_f = 6.1$ fm/c the system reaches T_c.

Once we obtain the temperature profile, the particle spectra can be calculated by integrating the viscous modified particle production rates over the space-time history of the collisions,

$$\frac{dN}{p_T dp_T dM dy} = (4\pi M)\pi R_A^2 \int_{\tau_0}^{\tau_f} d\tau\tau \int_{-y_n}^{y_n} d\eta_s \left(\frac{1}{2}\frac{dN}{d^4x d^4p}\right), \tag{34}$$

where within the Bjorken model the volume element is given by $d^4x = d^2x_T d\eta_s \tau d\tau = \pi R_A^2 d\eta_s \tau d\tau$, with $R_A = 1.2A^{1/3}$ representing the radius of the nucleus used for the collision (for Au, $A = 197$). Here τ_0 and τ_f are the initial and final values of proper time that we are interested in (i.e. the duration of the QGP phase in the present analysis). The production rates calculated in Sec. III need to be modified while considering a longitudinally expanding system. This is done by replacing $\exp(-\frac{E}{T})$ of Eq. (23) with $\exp(-\frac{u.p}{T})$ in rate expressions of Eq. (25). With the 4-momentum of the dilepton parametrized as $p^\alpha = (m_T \cosh y, p_T \cos\phi_p, p_T \sin\phi_p, m_T \sinh y)$, where $m_T^2 = p_T^2 + M^2$, we get $u.p = m_T \cosh(y - \eta_s)$. The other factors appearing in the rate Eqs. (29) and (31) are now given as

$$p^\alpha p^\beta \sigma_{\alpha\beta} = \frac{2}{3\tau} p_T^2 - \frac{4}{3\tau} m_T^2 \sinh^2(y - \eta_s), \tag{35}$$

$$p^\alpha p^\beta \Delta_{\alpha\beta}\Theta = -\frac{p_T^2}{\tau} - \frac{m_T^2}{\tau}\sinh^2(y - \eta_s). \tag{36}$$

We use the temperature dependent ζ/S and constant value of $\eta/S = 1/4\pi$ as prescribed in the previous section for our calculations. All results are presented for the $y = 0$ case only.

We plot the transverse momentum spectra of thermal dileptons produced from the viscous QGP in Fig. 4 for invariant masses $M = 1$, 2 GeV. To understand the impact of the EoS effects to the dilepton production through the modified distribution functions, we compare the results in a case where such effects are ignored ($z_q = 1$, $I_q = 1$). Note that in both these situations we choose the lQCD EoS for the purpose of hydrodynamical evolution of the temperature. Solid lines represent the rate while considering the modified distribution functions, and the dotted lines represent the rate while ignoring such effects.

We observe significant modifications to the spectra while using the lQCD EoS through the modified distribution functions for the quark-antiquarks. From the curves, it is clear that effect of these terms is to suppress the particle spectra. Significant suppression is observed for all the values of transverse momentum; e.g. at $p_T = 0.5$ GeV, suppression is about 89% for $M = 2$ GeV, which reduces to about 79% for $p_T = 1$ GeV and about 47% for $p_T = 2$ GeV.

We also observe that the suppression of low p_T particles is strong, indicating that the effect of the modifications is more dominant in the later stages of collision, when the system is near T_c. The high p_T particles, produced predominantly during the early stages of the evolution of the system, are also affected by these modifications, albeit less compared to the low p_T region. This behavior can be understood by observing the behavior of the EoS induced modifications to the dilepton rate in Fig. 3. Since these terms are getting multiplied with the dilepton rates employing the ideal EoS, they suppress the spectra at low p_T.

Next, we look into the behavior of various dissipative terms in the dilepton production. In Fig. 5, we plot the dilepton yield as a function of the transverse momentum of the pair for the invariant mass $M = 0.525$ GeV. In this plot, we show the effect of various viscosity terms in the total rate.

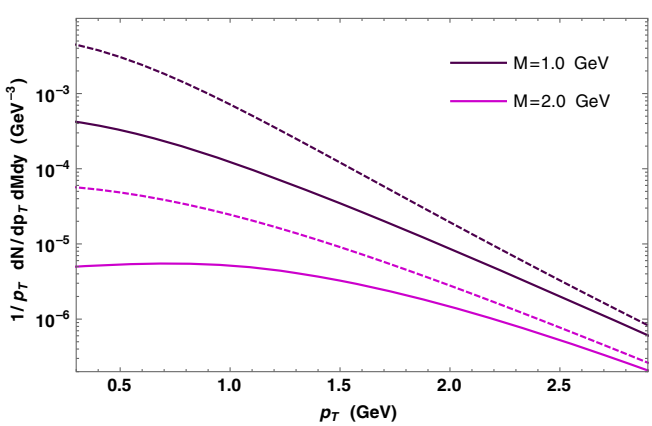

FIG. 4 (color online). Thermal dilepton yield from viscous quark-gluon plasma at RHIC energies for different dilepton-invariant masses using the modified distribution functions. Dotted curves represent the rates calculated ignoring the EoS modifications on the thermal distribution functions.

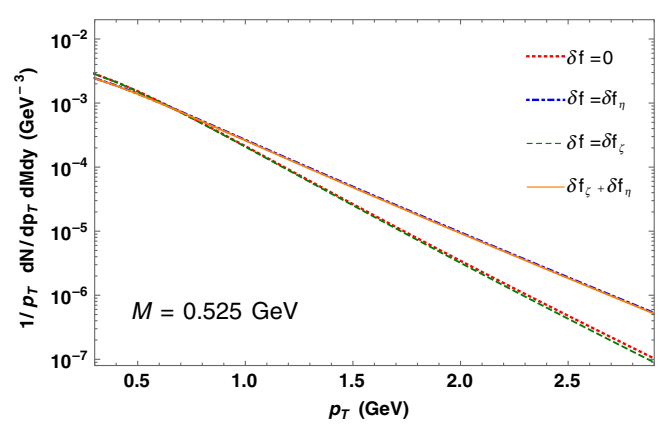

FIG. 5 (color online). Transverse momentum spectra for the thermal dileptons (with invariant mass $M = 0.525$ GeV) from the viscous quark-gluon plasma produced in RHIC energies. The effect of viscosities is highlighted.

First, we plot the spectra without considering the viscous effects in the calculations ($\delta f = 0$ case). As emphasized earlier, the presence of z_q^2 terms in the expression, as we saw from Eq. (27), will lead to the overall suppression of the spectra. One can see that inclusion of the effect of bulk viscosity ($\delta f = \delta f_\zeta$) has only a marginal effect. As observed before in [28], at high p_T, the effect of bulk viscosity is to suppresses the spectra; for example, at $p_T = 2$ GeV suppression is about 7%. Next, we consider only the effect of shear viscosity (the $\delta f = \delta f_\eta$ case) in the spectra. Shear viscosity significantly enhances the particle production, and its effect becomes stronger as p_T increases. This can be understood from the presence of the first (positive) term on the right-hand side of Eq. (35). At $p_T = 2$ GeV the enhancement of spectra due to shear viscosity is around 177%. Since the shear viscosity coefficient $\eta/S > \zeta/S$ in the entire temperature regime we are interested in, its effect is expected to be more dominant. As before [26,28], we observe that even the lowest value of the shear viscosity $\sim 1/4\pi$ has a significant effect on the spectra. Consequently, when we consider the fully viscous case ($\delta f = \delta f_\eta + \delta f_\zeta$), the spectra get highly enhanced due to the shear viscosity, albeit marginal suppression of the spectra due to the bulk viscosity. For instance, at $p_T = 2$ GeV the total enhancement of the spectra is 170%.

It is worth emphasizing that the main source of thermal dileptons in the QGP medium is the quark-antiquark annihilation processes considered here. There are other higher-order processes that can also contribute to the thermal dilepton production [42,43]. Such higher-order processes are not considered in the present analysis. It may further be noted that the thermal dileptons from the annihilation process are dominant in the regime of intermediate invariant mass $1 < M < 3$ GeV and transverse momentum of the pair, p_T, in that range [44,45]. We intend to extend our present studies by incorporating higher-order contributions in view of the quasiparticle description in the near future.

V. CONCLUSIONS

In conclusion, the form of viscous modified thermal distribution functions for quasiquarks and quasigluons is obtained in the QGP medium by systematically employing the realistic EoS for the QGP (the lQCD EoS). As an implication, the impact of them is demonstrated on the dilepton production via $q\bar{q}$ annihilation in the RHIC. The EoS also induces significant modifications to the viscous modified thermal distributions of the gluons and quark-antiquarks that constitute the QGP. The effects are equally significant in deciding the dilepton production rate in the viscous QGP medium. In particular, even in the high temperature regime, where the hot QCD medium effects are weaker, the realistic EoS and viscosities play a crucial role.

Finally, coupling the present analysis to the relativistic second-order viscous hydrodynamic evolution of the QGP and the impact of the temperature dependence of the shear and bulk viscosities on the dilepton production rate will be matters of future investigation.

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