PHYSICAL REVIEW D **92,** 094012 (2015)

Seesaw mechanism at electron-electron colliders

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(Received 9 September 2015; published 6 November 2015)

We consider the Standard Model with right-handed neutrinos to explain the masses of active neutrinos by the seesaw mechanism. Since active neutrinos as well as heavy neutral leptons are Majorana fermions in this case, the lepton number violating process can be induced. We discuss the inverse neutrinoless double beta decay $e^-e^- \to W^-W^-$ in the framework of the seesaw mechanism and its detectability at future colliders. It is shown that the cross section can be 17 fb for $\sqrt{s} = 3$ TeV even with the stringent constraint from the neutrinoless double beta decays if three (or more) right-handed neutrinos exist. In such a case, the future e^-e^- colliders can test lepton number violation mediated by a right-handed neutrino lighter than about 10 TeV.

DOI: 10.1103/PhysRevD.92.094012 PACS numbers: 13.66.Lm, 11.30.Fs, 14.60.St

I. INTRODUCTION

The origin of neutrino masses is one of the most important questions in particle physics at present. Various oscillation experiments have provided the mass squared differences and mixing angles of active neutrinos very precisely [1]; however, the mass ordering, violation of CP symmetry, and fundamental property of massive neutrinos (i.e., Majorana or Dirac particles) are still unknown. The simplest way to explain the neutrino masses is to add right-handed neutrinos to the Standard Model (SM). The smallness of active neutrino masses can be explained by the large Majorana masses of right-handed neutrinos thanks to the seesaw mechanism [2–8].

In the seesaw mechanism, the mass eigenstates of neutrinos are three active neutrinos which have tiny masses observed in oscillation experiments, and heavy neutral leptons (HNLs) which are almost identical to the right-handed states. The HNL masses are determined by Majorana masses and independent from the electroweak Higgs mechanism. They can take arbitrary values as long as they are sufficiently heavy to realize the seesaw mechanism. This is the reason why such HNLs can cause interesting phenomena in various aspects of particle physics and cosmology.

Right-handed neutrinos can explain the baryon asymmetry of the Universe. The well-known scenario is the canonical leptogenesis [9] in which they need to be heavier than $\mathcal{O}(10^9)$ GeV [10] (or $\mathcal{O}(10^6)$ GeV when the nonthermal production is realized [11]). The resonant leptogenesis with quasidegenerate right-handed neutrinos [12] can be effective with much smaller masses. Moreover, if we use the flavor oscillation of right-handed neutrinos, the required mass can be as small as $\mathcal{O}(1)$ MeV [13–15].

In addition, a right-handed neutrino can play an important role in astrophysics. It can be the dark matter candidate with $\mathcal{O}(1)$ keV mass [16]. This particle can also explain other phenomena, such as the pulsar kick [17] (for a review, see Ref. [18]). Further, right-handed neutrinos with $\mathcal{O}(0.1)$ GeV may be important for the supernova explosion [19].

Right-handed neutrinos are required by grand unified theories based on SO(10) [20,21] or larger symmetry groups. These particles can also accomplish the bottom-tau Yukawa unification [22] which is necessary for realistic grand unified theories. It has been discussed that they are favored to be lighter than $\mathcal{O}(10^7)$ GeV due to the argument of the electroweak naturalness [23].

In this way, right-handed neutrinos in the seesaw mechanism are well-motivated particles beyond the SM. The direct test of HNLs is possible if they are light and their mixings are large. So far various experiments have been performed to search for HNLs (for example, see reviews [24–27]).

The noble consequence of the seesaw mechanism is that active neutrinos and HNLs are Majorana particles, and hence the lepton number violating processes can appear. A well-known example is the neutrinoless double beta ($0\nu\beta\beta$) decay $(Z, A) \to (Z + 2, A) + 2e^-$, which violates the lepton number by two units [28].

In this paper, we consider another example of such processes,

$$e^-e^- \to W^-W^-, \tag{1}$$

which is called the "inverse neutrinoless double beta decay" [29] (see Fig. 1), which cannot also happen in the SM. The e^-e^- collision at high energy is one attractive option of the future lepton colliders such as the

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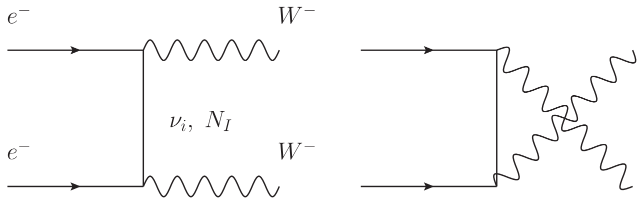

FIG. 1. Feynman diagrams for $e^-e^- \to W^-W^-$.

International Linear Collider (ILC [30]) and the Compact Linear Collider (CLIC [31]). Various aspects of the process (1) have been studied so far [29,32–40].

The advantages of the process (1) over the $0\nu\beta\beta$ decay for the test of the lepton number violation are (i) the signal event is clean and (ii) the prediction is free from the uncertainty in the nuclear matrix element. Furthermore, we should mention that these processes are complementary tests for the lepton number violation. As we will show, HNLs can give a destructive contribution to the $0\nu\beta\beta$ decay so that no signal could be observed. Even in this case, the inverse $0\nu\beta\beta$ decay could be observed since the energy scales of these processes are different.

In this paper, we shall revisit the inverse $0\nu\beta\beta$ decay, paying special attention to the following points: (i) We study the process in the concrete model, i.e., the SM extended by $\mathcal{N}$ right-handed neutrinos with the seesaw mechanism, and try to derive the theoretical prediction being specific to the model. (ii) We take fully into account the interference effects between active neutrinos and HNLs for both the $0\nu\beta\beta$ and the inverse $0\nu\beta\beta$ processes. (iii) We estimate the upper bounds on the cross section depending on the number of right-handed neutrinos. We use a corrected cross section (for the difference from previous studies, see the Appendix). In addition, we take into account the possibilities of the fine-tuning in parameters of the model, which have not been studied thoroughly.

We find that the maximal cross section can be 0.47 fb (17 fb) for the center-of-mass energy $\sqrt{s} = 500$ GeV (3 TeV), avoiding the stringent constraint from the $0\nu\beta\beta$ decay if there are three right-handed neutrinos. This is a contrast to the previous results. Therefore, the process (1) can be a good target of the future lepton colliders.

The rest of this paper is organized as follows. In Sec. II, we define the model parameters and list existing constraints on them. In Sec III, we derive upper bounds on the cross sections of the process (1) in the cases with one, two, and three right-handed neutrinos in turn. Finally, Sec. IV is devoted to conclusions. We add an Appendix to present the definitions of variables in the cross section of $e^-e^- \to W^-W^-$.

II. STANDARD MODEL WITH RIGHT-HANDED NEUTRINOS

Let us first explain the framework of the present analysis. We consider the SM extended by right-handed neutrinos ν_R with Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \bar{\nu}_{RI} i\partial_\mu \gamma^\mu \nu_{RI} - F_{\alpha I}\bar{L}_\alpha \Phi \nu_{RI} - \frac{M_I}{2}\bar{\nu}_{RI}\nu^c_{RI} + \text{H.c.}. \tag{2}$$

Here, we shall assume $|M_{D\alpha I}| \equiv |F_{\alpha I}|\langle\Phi\rangle \ll M_I (\alpha = e,\mu,\tau; I = 1,\ldots,\mathcal{N})$ in order to realize the seesaw mechanism. In this case, the mass eigenstates of neutrinos are three active neutrinos ν_i $(i = 1,2,3)$ with masses m_i and $\mathcal{N}$ HNLs N_I with masses $\simeq M_I$. The mass ordering of HNLs can be chosen as $M_1 \le M_2 \le \cdots \le M_{\mathcal{N}}$ without loss of generality. The mixing of neutrinos in the charged current interactions is then written as

$$\nu_{L\alpha} = \sum_{i=1}^{3} U_{\alpha i}\nu_i + \sum_{I=1}^{\mathcal{N}} \Theta_{\alpha I} N_I^c, \tag{3}$$

where U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix of active neutrinos, while $\Theta_{\alpha I}$ represent the mixings of HNLs obtained by the diagonalization of the $(3+\mathcal{N})\times(3+\mathcal{N})$ neutrino mass matrix including radiative corrections.

In this model, the masses and mixings of active neutrinos and HNLs must satisfy the following relations:

$$0 = \sum_{i}^{3} U_{\alpha i}U_{\beta i} m_i + \sum_{I=1}^{\mathcal{N}} \Theta_{\alpha I}\Theta_{\beta I} M_I. \tag{4}$$

These relations hold if the Majorana mass terms for the left-handed neutrinos are absent at the tree level in the canonical seesaw mechanism discussed here. We should note that such Majorana masses are induced by radiative corrections [41,42] and would alter the relation (4) desperately. We assume that such corrections are sufficiently suppressed by artificial fine-tuning.

For the inverse $0\nu\beta\beta$ decay (1), only the component with $\alpha = \beta = e$,

$$0 = m^\nu_{\rm eff} + \sum_I \Theta^2_{eI} M_I, \tag{5}$$

is relevant. We call this equation “the seesaw relation” from now on. We define $m^\nu_{\rm eff}$ by

$$m^\nu_{\rm eff} \equiv \sum_{i=1}^{3} U^2_{ei} m_i. \tag{6}$$

In the considered model, both active neutrinos and HNLs are Majorana fermions, and then they induce $0\nu\beta\beta$ decay. The half-life of the decay is expressed as

$$T^{-1}_{1/2} = \mathcal{A}\frac{m_p^2}{\langle p^2\rangle^2}|m_{\rm eff}|^2. \tag{7}$$

Here and hereafter, we use the notation and the results for the $0\nu\beta\beta$ decay given in Ref. [43]. The effective neutrino mass in the considered model is given by

$$m_{\rm eff} = m_{\rm eff}^{\nu} + \sum_{I=1}^{\mathcal{N}} \Theta_{eI}^2 M_I f_I. \quad (8)$$

Here, the first term represents the contribution from the active neutrinos, which is given by Eq. (6), and the second term denotes the contributions from HNLs, in which the suppression of the nuclear matrix element for $M_I \gg \sqrt{\langle p^2 \rangle} \simeq 200$ MeV is taken into account by the function f_I [43],

$$f_I \simeq \frac{\langle p^2 \rangle}{\langle p^2 \rangle + M_I^2}. \quad (9)$$

As for the $0\nu\beta\beta$ decay of ^{136}Xe, the lower bound on the half-life is $T_{1/2} > 3.4 \times 10^{25}$ yr at 90% C.L. [44], which can be translated into the upper bound on the effective neutrino mass as

$$|m_{\rm eff}| < m_{\rm eff}^{\rm UB} = (0.185\text{–}0.276)\ {\rm eV}, \quad (10)$$

where we have taken into account the uncertainties estimated in Ref. [43]. On the other hand, the half-life bound of ^{76}Ge is $T_{1/2} > 3.0 \times 10^{25}$ yr at 90% C.L. [45]. In this case, the bound on $|m_{\rm eff}|$ is

$$|m_{\rm eff}| < m_{\rm eff}^{\rm UB} = (0.213\text{–}0.308)\ {\rm eV}. \quad (11)$$

We then apply the bound (10) throughout this analysis in order to make the conservative analysis.

Furthermore, the mixings of HNLs are constrained by various experiments [24–27]. In the following analysis, we consider the case when the mass of the lightest HNL is $M_1 \gtrsim 3$ GeV in order to avoid the stringent constraints from the search experiments by using the decays of π, K, and D mesons. Even in this case, there is an upper limit on $|\Theta_{eI}|^2$ in the region $M_I < m_Z$ from the search for HNLs at the Large Electron-Positron collider (LEP) experiment by the decay of the Z boson [46]. In the mass range $M_I \simeq$ 6–50 GeV, the bound is given as $|\Theta_{eI}|^2 < 2.1 \times 10^{-5}$ at 95% C.L. There is also an upper bound on the mixing angle which comes from the electroweak precision tests. The recent bound at 90% C.L. [47] is

$$|\Theta_e|^2 \equiv \sum_I |\Theta_{eI}|^2 < |\Theta_e|^2_{\rm EW} = 2.1 \times 10^{-3}. \quad (12)$$

These are the upper bounds on the mixings which are important for the analysis below.[1]

[1]Our analysis can also be applied to processes $\mu^-\mu^- \to W^-W^-$ and $e^-\mu^- \to W^-W^-$ as in Ref. [39]. The latter process is, however, strongly constrained by the $\mu \to e\gamma$ experiment [47,48], $|\sum_I \Theta_{eI}^* \Theta_{\mu I}| < 10^{-5}$ (90% C.L.).

III. INVERSE NEUTRINOLESS DOUBLE BETA DECAY $e^-e^- \to W^-W^-$

Now, we are in the position to discuss the inverse neutrinoless double beta decay (1). The cross section of the process in the model with $\mathcal{N}$ right-handed neutrinos is given by

$$\frac{d\sigma_{\mathcal{N}}}{d\cos\theta} = \frac{G_F^2 \beta_W}{32\pi} [|A_t|^2 B_t + |A_u|^2 B_u + (A_t A_u^* + {\rm H.c.}) B_{tu}]. \quad (13)$$

Here, the definitions of variables in this equation are presented in the Appendix. Notice that only A_t and A_u depend on the parameters of HNLs, namely, M_I and Θ_{eI}.

In the following analysis, we consider the detectability of $e^-e^- \to W^-W^-$ by taking the center-of-mass energy as $\sqrt{s} = 0.5$, 1, and 3 TeV. We assume the integrated luminosity of 100 fb^{-1}, which can be achieved by $\mathcal{O}(10^7)$ s run of the ILC or CLIC [49] (the luminosities of the e^-e^- colliders are expected to be the same order as e^+e^- colliders [31]).

Before discussing the realistic cases, we shall consider the two extreme cases. The first one is the limit when all HNLs are sufficiently heavy (i.e., $M_I \gg \sqrt{s}$) and only three active neutrinos effectively take part in the process. In this case, the cross section is given by [34][2]

$$\sigma_0 = \frac{3G_F^2 |m_{\rm eff}^{\nu}|^2}{4\pi} = 0.96 \times 10^{-18}\ {\rm fb} \left(\frac{|m_{\rm eff}^{\nu}|}{0.28\ {\rm eV}} \right)^2. \quad (14)$$

The cross section is determined solely by the effective neutrino mass $m_{\rm eff}^{\nu}$ of active neutrinos in the $0\nu\beta\beta$ decay. It is seen that σ_0 is too small to be accessible by future colliders.

The other limit is that all HNLs are sufficiently light (i.e., $M_I \ll m_W \ll \sqrt{s}$). In this case, it is found from the seesaw relation (5) that

$$A_t = \frac{1}{t} \left(m_{\rm eff}^{\nu} + \sum_{I=1}^{\mathcal{N}} \Theta_{eI}^2 M_I \right) = 0, \quad (15)$$

and then $A_u = 0$, which results in the vanishing cross section. In other words, the seesaw relation (5) ensures the unitarity of the process (1) at high energies $\sqrt{s} \gg M_I$ [34].

From now on, we will study the cross section of $e^-e^- \to W^-W^-$ in the framework of the seesaw model with $\mathcal{N} = 1$, 2, and 3 right-handed neutrino(s). Especially, we shall discuss the impacts of various constraints on the masses and mixings of HNLs discussed in the previous section.

[2]The prefactor of σ_0 in Eq. (14) is three times larger than the estimations in Refs. [34,39] (see the discussion in the Appendix).

A. Case with one right-handed neutrino

Let us first consider the SM with only one right-handed neutrino. In this case, there is one HNL N_1 with mass M_1 and mixing $\Theta_{\alpha 1}$ in addition to active neutrinos. This model is incompatible with the oscillation data since there appear two massless states among active neutrinos. We shall, however, discuss it in order to make our point clear.

One might think that N_1 with a relatively large mixing allowed by Eq. (12) may give rise to a significant contribution to $e^- e^- \to W^- W^-$. This is, however, not true in the considered case. The seesaw relation (5) tells that the mixing cannot be large but $\Theta_{e1}^2 = -m_{\rm eff}^\nu / M_1$, and then A_t is suppressed as

$$A_t = m_{\rm eff}^\nu \left(\frac{1}{t} - \frac{1}{t - M_1^2} \right). \qquad (16)$$

This shows that the cross section becomes independent on the mixing of N_1 but is determined from the effective mass $m_{\rm eff}^\nu$ and M_1. In Fig. 2, we show the cross section $e^- e^- \to W^- W^-$ with $|m_{\rm eff}^\nu| = 0.276$ eV.

It is seen that the cross section approaches to σ_0 (14) for $M_1 \gg \sqrt{s}$ due to the decoupling of N_1 as mentioned before. On the other hand, when $M_1 \ll m_W$, the cross section behaves as

$$\sigma_1 = \frac{1}{6\pi} \frac{G_F^2 |m_{\rm eff}^\nu|^2 M_1^4 s^2}{m_W^8}, \qquad (17)$$

and it vanishes for $M_1 \to 0$ as expected.

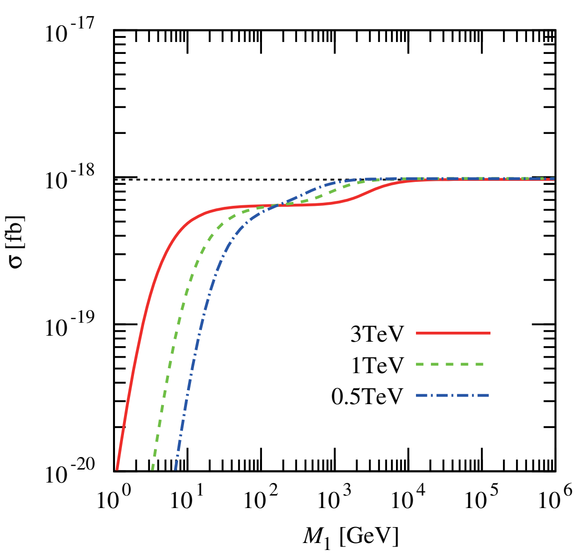

FIG. 2 (color online). Cross section of $e^- e^- \to W^- W^-$ in the model with one right-handed neutrino ($\mathcal{N} = 1$) for $\sqrt{s} = 3$ TeV (the red solid line), 1 TeV (the green dashed line), and 0.5 TeV (the blue dotted-dashed line). The horizontal line (the black dotted line) represents σ_0 in (14). Here we take $|m_{\rm eff}^\nu| = 0.276$ eV.

In the model with $\mathcal{N} = 1$, therefore, the cross section is too small to be observed at future colliders. This is the direct consequence of the seesaw mechanism; namely, the mixing Θ_{e1} is determined by $m_{\rm eff}^\nu$ from the seesaw relation (5).

B. Case with two right-handed neutrinos

Next, we discuss the case with two right-handed neutrinos, where there appear two HNLs N_1 and N_2. Notice that the lightest active neutrino in this case is massless (i.e., $m_1 = 0$ for the normal hierarchy and $m_3 = 0$ for the inverted hierarchy).

In this case, by using the seesaw relation (5) and $|\Theta_e|^2$ (12), the mixing angles of N_1 and N_2 are given by[3]

$$\Theta_{e1}^2 = + \frac{M_2}{M_1 + M_2} |\Theta_e|^2 - \frac{m_{\rm eff}^\nu}{M_1 + M_2}, \qquad (18)$$

$$\Theta_{e2}^2 = - \frac{M_1}{M_1 + M_2} |\Theta_e|^2 - \frac{m_{\rm eff}^\nu}{M_1 + M_2}. \qquad (19)$$

We find that, when $M_1 \ll M_2$ and $|m_{\rm eff}^\nu|/M_1 \ll |\Theta_e|^2$, the large mixing of N_1, $\Theta_{e1}^2 \simeq |\Theta_e|_{\rm EW}^2$, can be realized. This is because of the cancellation between the contributions of N_1 and N_2 in the seesaw relation (5) [33,34]. Some mechanism is desirable to stabilize such a fine-tuning, for example, by a discrete flavor symmetry. This issue is, however, beyond the scope of our analysis (similar situations have been discussed in Refs. [50,51]).

The function A_t is now

$$A_t = - \frac{(M_2 - M_1) M_1 M_2}{(t - M_2^2)(t - M_1^2)} |\Theta_e|^2 - \frac{m_{\rm eff}^\nu [(M_2^2 - M_1 M_2 + M_1^2) t - M_1^2 M_2^2]}{t (t - M_1^2)(t - M_2^2)}. \qquad (20)$$

It is then seen that, by neglecting the second term suppressed by $m_{\rm eff}^\nu$, the hierarchical mass pattern $M_1 \ll M_2$ enhances the matrix element of the process. In this limit, A_t is dominated by the contribution of N_1 as

$$A_t \simeq \frac{M_1 |\Theta_e|^2}{t - M_1^2}, \qquad (21)$$

which leads to the cross section

$$\sigma_2 \simeq \frac{G_F^2 s |\Theta_e|^4}{8\pi} H(r_1), \qquad (22)$$

where $r_1 = M_1^2/s$ and $H(r_1)$ is $H \simeq 6 r_1$ for $M_1 \ll m_W$, $H \simeq 2 r_1$ for $m_W \ll M_1 \ll \sqrt{s}$, and $H \simeq 1/(2 r_1)$ for

[3] Θ_{e1}^2 can be taken to be real and positive without loss of generality. Here, we consider the case when Θ_{e2}^2 is real and negative. The importance of the relative phases between the mixings has been discussed in Refs. [36,37].

$M_1 \gg \sqrt{s}$. By taking $m_W/\sqrt{s} \to 0$, it is approximately given by

$$H(r) = r(2+3r)\left[\frac{1}{1+r} - \frac{2r}{1+2r}\ln\left(1+\frac{1}{r}\right)\right]. \tag{23}$$

It is then found that $H(r_1)$ takes its maximal value

$$H(r_1)|_{\mathrm{MAX}} = 0.206 \tag{24}$$

at $r_1 = 0.542$ (i.e., $M_1 = 0.736\sqrt{s}$). If we applied the upper bound on $|\Theta_e|^2$ given in Eq. (12), the maximal cross section $\sigma_2 = 17$ fb could be obtained for $\sqrt{s} = 3$ TeV. Unfortunately, such a large cross section cannot be realized for the case with $\mathcal{N} = 2$ because we have to take into account the severe constraint from the $0\nu\beta\beta$ decay.

The effective neutrino mass in this case is estimated as

$$m_{\mathrm{eff}} = (f_1 - f_2)\frac{M_1 M_2}{M_1 + M_2}|\Theta_e|^2 + \left(1 - \frac{f_1 M_1 + f_2 M_2}{M_1 + M_2}\right)m^\nu_{\mathrm{eff}}. \tag{25}$$

The upper bound on $|\Theta_e|^2$ is then given by Eq. (12) for $M_1 > M_*$, while the more stringent bound from the $0\nu\beta\beta$ decay,[4]

$$|\Theta_e|^2 < \frac{(m^{\mathrm{UB}}_{\mathrm{eff}} + |m^\nu_{\mathrm{eff}}|)M_1 M_2}{\langle p^2\rangle(M_2 - M_1)}, \tag{26}$$

is imposed for $M_1 < M_*$ where M_* is determined from the ratio M_1/M_2 as

$$M_* = \left(1 - \frac{M_1}{M_2}\right)\frac{|\Theta_e|^2_{\mathrm{EW}}\langle p^2\rangle}{m^{\mathrm{UB}}_{\mathrm{eff}} + |m^\nu_{\mathrm{eff}}|}. \tag{27}$$

Here, we have assumed that $M_1^2, M_2^2 \gg \langle p^2\rangle$.

By using the upper bound on the mixing angle listed above, we can estimate the maximal value of the cross section of $e^-e^- \to W^-W^-$. In Fig. 3, we show how this maximal value depends on the mass ratio when $\sqrt{s} = 3$ TeV. Here, we choose $M_2/M_1 = 1.01$, 1.1, and 100. It can be seen that the largest cross section is obtained by taking $M_2/M_1 \to \infty$. (Note that the difference in the cross section between $M_2/M_1 = \infty$ and 100 is very small.)

We then show in Fig. 4 the upper bound on the cross section by taking $\sqrt{s} = 0.5$, 1, and 3 TeV. Notice that, when $M_1 < M_* = \mathcal{O}(10^5)$ GeV, the maximal cross section is given by

$$\sigma_2 = \frac{G_F^2(m^{\mathrm{UB}}_{\mathrm{eff}} + |m^\nu_{\mathrm{eff}}|)^2}{8\pi}\frac{s^2}{\langle p^2\rangle^2}r_1 H(r_1). \tag{28}$$

The function $r_1 H(r_1)$ becomes constant for $r_1 \gg 1$. For $M_1 > M_*$, the cross section falls as r_1^{-1}. The maximal cross section is then given by

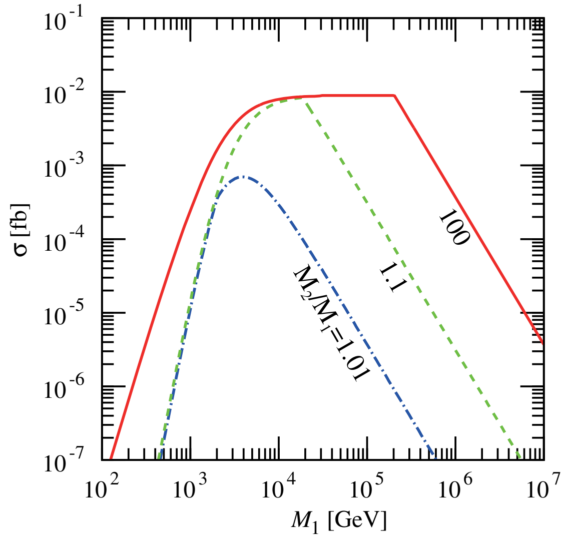

FIG. 3 (color online). Cross section of $e^-e^- \to W^-W^-$ in the model with two right-handed neutrinos ($\mathcal{N} = 2$) for $\sqrt{s} = 3$ TeV. We take $M_2 = 1.1M_1$ (the blue dotted-dashed line), $M_2 = 2M_2$ (the green dashed line), and $M_2 = 100M_2$ (the red solid line). Here, we consider the inverted hierarchy case with $m^{\mathrm{UB}}_{\mathrm{eff}} = 0.276$ eV and $|m^\nu_{\mathrm{eff}}| = 4.79 \times 10^{-3}$ eV.

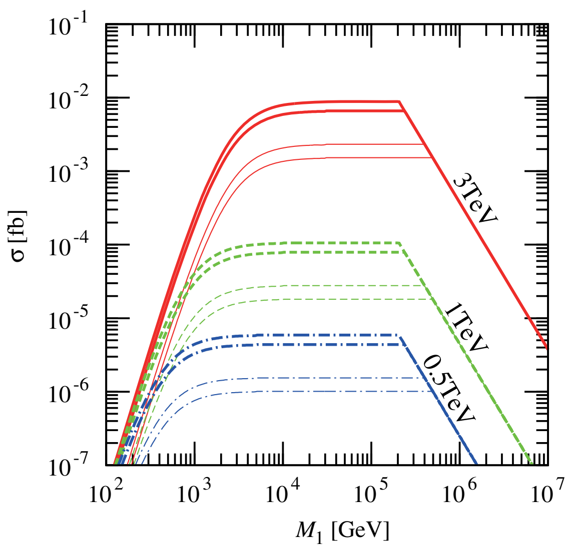

FIG. 4 (color online). Cross section of $e^-e^- \to W^-W^-$ in the model with two right-handed neutrinos with $M_1 \ll M_2$ for $\sqrt{s} = 3$ TeV (the red solid lines), 1 TeV (the green dashed lines), and 0.5 TeV (the blue dotted-dashed lines). In each set of lines, the upper one is for the inverted hierarchy case with $|m^\nu_{\mathrm{eff}}| = 4.79 \times 10^{-3}$ eV, and the lower one is for the normal hierarchy case with $|m^\nu_{\mathrm{eff}}| = 3.66 \times 10^{-3}$ eV. We use $m^{\mathrm{UB}}_{\mathrm{eff}} = 0.276$ and 0.185 eV for the thick and thin lines.

[4]When the masses of N_1 and N_2 are sufficiently degenerate, we can avoid the constraint from the $0\nu\beta\beta$ decay. In this case, however, A_t in Eq. (20) approaches to (16), which leads to the suppression of the cross section (see also Fig. 3).

$$\sigma_2 < 8.9\times 10^{-3}\ \mathrm{fb}\left(\frac{m_{\mathrm{eff}}^{\mathrm{UB}}+|m_{\mathrm{eff}}^{\nu}|}{(0.276+0.048)\ \mathrm{eV}}\right)^2 \times\left(\frac{0.178\ \mathrm{GeV}}{\sqrt{\langle p^2\rangle}}\right)^4\left(\frac{\sqrt{s}}{3\ \mathrm{TeV}}\right)^4. \tag{29}$$

The maximal value changes depending on the mass hierarchy of active neutrinos. The cross section in the inverted hierarchy can be larger than that in the normal hierarchy since $|m_{\mathrm{eff}}^{\nu}|$ can be larger: $|m_{\mathrm{eff}}^{\nu}| < 3.66\times 10^{-3}$ eV for the normal hierarchy and $|m_{\mathrm{eff}}^{\nu}| < 4.79\times 10^{-2}$ eV for the inverted hierarchy. Here, we have estimated these values by using the central values of the mass squared differences and the mixing angles of active neutrinos in Ref. [1] and by varying the possible values of the *CP* violating phases in the PMNS matrix. Notice that, as shown in Fig. 4, there exists an uncertainty in the upper bound on the cross section from the nuclear matrix element shown in Eq. (10).

The upper bound on the cross section scales as s^2, and hence the collisions of electrons with higher energies are desired. In fact, the possible way to observe the inverse $0\nu\beta\beta$ decay is to realize $\sqrt{s}=3$ TeV at CLIC with the integrated luminosity larger than $\mathcal{O}(10^2)$ fb. In such a case, the number of the signal event can be larger than unity.

C. Case with three right-handed neutrinos

Finally, let us discuss the case with three right-handed neutrinos ($\mathcal{N}=3$). We will show that this case allows the larger cross section than the previous cases. Our basic idea is the following: Let us consider three HNLs with hierarchical masses $M_1 \ll M_2 \ll M_3$. Then, the large cross section of $e^-e^- \to W^-W^-$ can be induced by N_2 with mass $M_2 \sim \sqrt{s}$ and mixing $|\Theta_{e2}|^2 \simeq |\Theta_e|^2_{\mathrm{EW}}$ (12). The seesaw relation can be realized even with the large mixing $|\Theta_{e2}|^2$ because of the cancellation between N_2 and N_3, which is similar to the case with $\mathcal{N}=2$. In addition, the constraint from the $0\nu\beta\beta$ decay can be avoided by the cancellation in m_{eff} between N_2 and N_1. This is the reason why $\mathcal{N}\geq 3$ is required for having the large cross section of the inverse $0\nu\beta\beta$ decay in the framework of the seesaw mechanism.

Now, we discuss this point in detail. By using the seesaw relation (5) and the effective mass (8), we may express the mixing angles of N_1 and N_3 in terms of that of N_2:

$$\Theta_{e1}^2 = \frac{-1}{(f_1-f_3)M_1}[(f_2-f_3)M_2\Theta_{e2}^2 - m_{\mathrm{eff}} + (1-f_3)m_{\mathrm{eff}}^{\nu}], \tag{30}$$

$$\Theta_{e3}^2 = \frac{+1}{(f_1-f_3)M_1}[(f_1-f_2)M_2\Theta_{e2}^2 + m_{\mathrm{eff}} - (1-f_1)m_{\mathrm{eff}}^{\nu}]. \tag{31}$$

It is then found that A_t is

$$A_t = -\frac{M_2(M_2^2-M_1^2)(M_3^2-M_2^2)(t+\langle p^2\rangle)\Theta_{e2}^2}{(M_2^2+\langle p^2\rangle)(t-M_1^2)(t-M_2^2)(t-M_3^2)} - \frac{m_{\mathrm{eff}}(M_1^2+\langle p^2\rangle)(M_3^2+\langle p^2\rangle)}{\langle p^2\rangle(t-M_1^2)(t-M_3^2)} + \frac{m_{\mathrm{eff}}^{\nu}M_1^2M_3^2(t+\langle p^2\rangle)}{\langle p^2\rangle t(t-M_1^2)(t-M_3^2)}. \tag{32}$$

By neglecting the second and third terms suppressed by m_{eff} and m_{eff}^{ν}, the matrix element can be maximal when the masses of three HNLs are hierarchical, namely, $M_1 \ll M_2 \ll M_3$. In this case, the mixing angles become

$$\Theta_{e1}^2 \simeq -\frac{M_1}{M_2}\Theta_{e2}^2, \tag{33}$$

$$\Theta_{e3}^2 \simeq -\frac{M_2}{M_3}\Theta_{e2}^2, \tag{34}$$

and hence $|\Theta_{e1}|^2$, $|\Theta_{e3}|^2 \ll |\Theta_{e2}|^2$, so $|\Theta_{e2}|^2 \simeq |\Theta_e|^2$, which results in

$$A_t \simeq \frac{M_2|\Theta_e|^2}{t-M_2^2}. \tag{35}$$

In this limit, the cross section is given by

$$\sigma_3 \simeq \frac{G_F^2 s|\Theta_e|^4}{8\pi}H(r_2), \tag{36}$$

where $r_2 = M_2^2/s$. The result is similar to Eq. (22) for $\mathcal{N}=2$.

Thus, the upper bound on the cross section can be obtained by the maximal value of the mixing angle, which is shown in Fig. 5. Here, we take $M_1 = 3$ GeV to avoid the stringent constraint on $|\Theta_{e1}|^2$ from the direct search experiments. Notice that the bound on $|\Theta_e|^2$ comes from the search at LEP [46] for $M_2 \lesssim m_Z$ and comes from the electroweak precision test (12) for $M_2 \gtrsim m_Z$. Since the former bound is stringent, the cross section is too small to be observed for that mass range. The maximal cross section is obtained when $M_2 \simeq 0.736\sqrt{s}$ as

$$\sigma_3 < 17\ \mathrm{fb}\left(\frac{\sqrt{s}}{3\ \mathrm{TeV}}\right)^2\left(\frac{|\Theta_e|^2}{|\Theta_e|^2_{\mathrm{EW}}}\right)^2. \tag{37}$$

Compared with Eq. (29), the maximal value of the cross section in the $\mathcal{N}=3$ case can be much larger than the previous cases.

In Fig. 6, we show the sensitivity limit of $|\Theta_{e2}|^2$ together with the current upper limits (for the perturbativity bound $|\Theta_{e2}|^2 < 4\pi\langle\Phi\rangle^2/(M_2M_3)$, see Ref. [52]). Interestingly, the inverse $0\nu\beta\beta$ decay process can probe HNL (N_2 in this

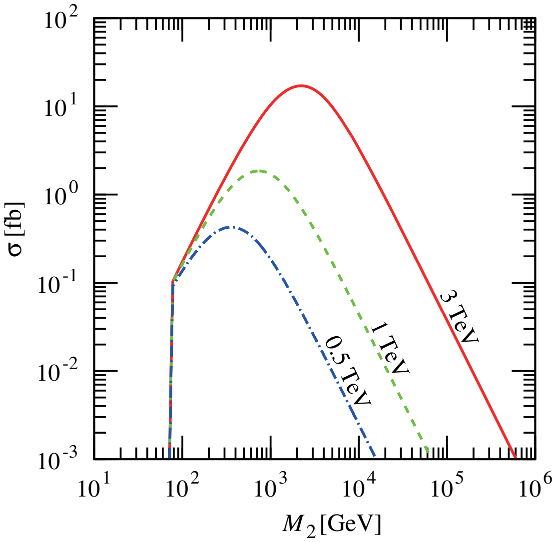

FIG. 5 (color online). Cross section of $e^-e^- \to W^-W^-$ in the model with three right-handed neutrinos ($\mathcal{N} = 3$) with $M_1 \ll M_2 \ll M_3$ for $\sqrt{s} = 3$ TeV (the red solid line), 1 TeV (the green dashed line), and 0.5 TeV (the blue dotted-dashed line).

case) of which the mass is much heavier than the center-of-mass energy ($M_2 \gg \sqrt{s}$) because it is induced as the virtual effect of HNLs. This is one of the most important advantages of $e^-e^- \to W^-W^-$ in searching HNLs [34,38].

It should be noted that the process (1) is induced by the collision of two electrons with negative helicity. Thus, this reaction will be turned off simply by using one of the electron beams with positive helicity [32]. This can offer

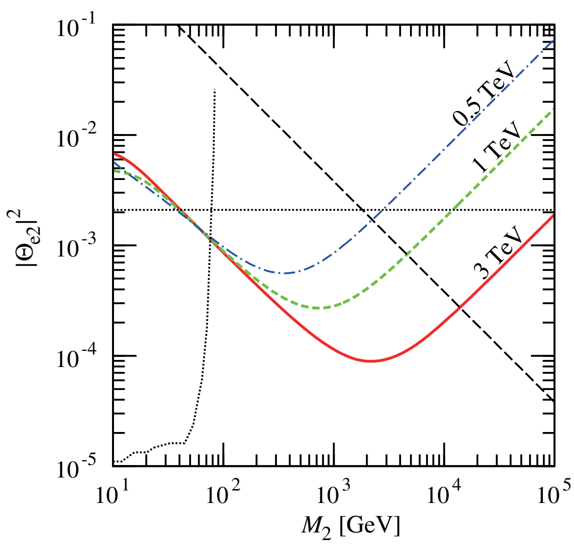

FIG. 6 (color online). Sensitivity limit of $|\Theta_{e2}|^2$ from $e^-e^- \to W^-W^-$ in the model with three right-handed neutrinos ($\mathcal{N} = 3$) for $\sqrt{s} = 3$ TeV (the red solid line), 1 TeV (the green dashed line), and 0.5 TeV (the blue dotted-dashed line). The black dotted lines show the upper bounds from Refs. [46] and [47]. The black long-dashed line shows the upper bound from the perturbativity of Yukawa couplings when $M_3 = 10^5$ GeV.

the important test for this process by using the polarized beam which is specific to the linear colliders.

We should note that the large cross section shown in Eq. (37) is possible when the HNL N_1 with $M_1 \ll M_2$ and $\Theta_{e1}^2 \simeq -(M_1/M_2)\Theta_{e2}^2$ exists in order to avoid the stringent constraint from the $0\nu\beta\beta$ decay. If this is the case, such a light HNL N_1 is also a good target for the future experiments. The searches by using charmed meson decay [53,54], $e^+e^- \to \nu N_1$ [40,55], and $Z \to \nu N_1$ [56] may provide the complementary test for the model discussed in this analysis.

Here, we have derived the sensitivity limit of the mixing by just counting the numbers of the signal event assuming no background event. The possible background processes have been studied in Refs. [32,38,40], which may reduce the sensitivity. However, the usage of the polarized beam can make the cross section four times larger, and the longer duration of the experiment can increase the number of events. These issues must be studied in detail to have the precise sensitivity limit; however, they are beyond the scope of this analysis.

So far, we have discussed the case with the hierarchical mass pattern $M_1 \ll M_2 \ll M_3$ with $M_2 \sim \sqrt{s}$. When $M_2 \gg \sqrt{s}$, N_2 decouples from the process, and the cross section becomes small. Even in this case, when $M_1 \sim \sqrt{s} \ll M_2 \ll M_3$, the situation becomes the same as the case with $\mathcal{N} = 2$. Then, the maximal cross section is given by Eq. (29).

Note that Eq. (36) is independent from $m_{\rm eff}$. This means that the inverse $0\nu\beta\beta$ decay can happen even if the $0\nu\beta\beta$ decay could not be observed. This is because the energy scales of the former process ($\sqrt{s} \sim 1$ TeV) and the latter one ($\sqrt{\langle p^2 \rangle} \sim 0.1$ GeV) are much different. Therefore, the $0\nu\beta\beta$ decay and the inverse one are regarded as complementary tests for the violation of lepton number in nature.

Finally, we comment on the case with $\mathcal{N} > 3$. The maximal cross section is the same as that with $\mathcal{N} = 3$. This is because the severe constraints from the seesaw relation and the $0\nu\beta\beta$ decay can be avoided as described above, but the inevitable upper bound (12) on $|\Theta_e|^2$, which is just the sum of $|\Theta_{eI}|^2$, gives the cross section as shown in Eq. (37).

IV. CONCLUSIONS

We have discussed the inverse $0\nu\beta\beta$ decay process $e^-e^- \to W^-W^-$ in the SM with $\mathcal{N}$ right-handed neutrinos. We have estimated for the first time the maximal cross sections of this process depending on $\mathcal{N}$ by taking into account the seesaw relation on the mixings of active neutrinos and HNLs as well as the severe constraints from direct search experiments, electroweak precision measurements, and $0\nu\beta\beta$ decay. The maximal cross sections are realized in limited regions of the parameter space, and the cross sections are orders of magnitude smaller if there is no fine-tuning.

For the $\mathcal{N}=1$ case, the seesaw relation restricts the cross section to be smaller than $\mathcal{O}(10^{-18})$ fb, so it cannot be seen at future experiments. For the $\mathcal{N}=2$ case, the cross section can be $\sigma_2 \sim 10^{-2}$ fb for $\sqrt{s}=3$ TeV. It may then be observable at CLIC, but it is impossible at ILC. For the $\mathcal{N}=3$ case, the larger cross section as $\sigma_3 = 0.47$ (17) fb for $\sqrt{s}=0.5$ (3) TeV can be obtained. In this case, both ILC and CLIC can search the inverse $0\nu\beta\beta$ decay, which would reveal the fate of the lepton number and the origin of the neutrino masses.

ACKNOWLEDGMENTS

T. A. was supported by JSPS KAKENHI Grants No. 25400249, No. 26105508, and No. 15H01031.

APPENDIX: DEFINITION OF VARIABLES

In this Appendix, we define the variables used in the cross section of $e^-e^- \to W^-W^-$ in Eq. (13). Here, we work in the center-of-mass frame, and θ is a scattering angle of one of the outgoing W^-s. Mandelstam variables are defined as usual,

$$t = -\frac{s}{2}(1-\beta_W\cos\theta) + m_W^2, \tag{A1}$$

$$u = -\frac{s}{2}(1+\beta_W\cos\theta) + m_W^2, \tag{A2}$$

where $\beta_W \equiv \sqrt{1-4r_W}$ with $r_W \equiv m_W^2/s$. The variables A_t and A_u are given by

$$A_t = \sum_{i=1}^{3}\frac{U_{ei}^2 m_i}{t-m_i^2} + \sum_{I=1}^{\mathcal{N}}\frac{\Theta_{eI}^2 M_I}{t-M_I^2}, \tag{A3}$$

$$A_u = A_t|_{t\to u}. \tag{A4}$$

We have found a difference in the definition of A_t from Refs. [36,37]. Here, we have corrected the errors in the denominators of A_t as $2t \to t$.

The range of $|t|$ is given by $(1-2r_W+\beta_W)s/2 \geq |t| \geq (1-2r_W-\beta_W)s/2$ and then $|t| \geq m_W^2[r_W + \mathcal{O}(r_W^2)]$. Therefore, $m_i^2/|t| \ll 1$ for the realistic range of $\sqrt{s}$. In this case, A_t is

$$A_t = \frac{m_{\rm eff}^\nu}{t} + \sum_{I=1}^{\mathcal{N}}\frac{\Theta_{eI}^2 M_I}{t-M_I^2}. \tag{A5}$$

On the other hand, the variables B_t, B_u, and B_{tu} are given by

$$B_t = (1-4r_W)t^2 - 4r_W(1-2r_W)st + 4(1-r_W)r_W^2 s^2, \tag{A6}$$

$$B_u = B_t|_{t\to u}, \tag{A7}$$

$$B_{tu} = (1-4r_W)tu + 4r_W^3 s^2. \tag{A8}$$

A typo in Ref. [39] is corrected in Eq. (A6).

Since we are considering $\sqrt{s} \geq 500$ GeV, we can approximate $m_W^2 \ll s$. We cannot, however, approximate $m_W^2 \ll |t|$, since $|t| \simeq m_W^4/s < m_W^2$ at $\theta \simeq 0$. Indeed,

$$\int_{-1}^{1} d\cos\theta \frac{m_W^4}{t^2} = 2 + \mathcal{O}(r_W), \tag{A9}$$

so we have to take the limit $m_W^2/s \to 0$ after integration when we consider the contribution of light neutrinos. This is the reason that the cross section in Eq. (14) is three times larger than that in previous studies [34,39].

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