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Hyperon polarization from the twist-3 distribution in unpolarized proton-proton collision

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We investigate the transverse polarization of a hyperon produced in the high-energy unpolarized proton-proton collision, $pp \to \Lambda^{\uparrow} X$, based on the collinear twist-3 factorization formalism. We focus on the contribution from the twist-3 distribution in one of the unpolarized proton and the transversity fragmentation function for the final hyperon. Utilizing the "master formula" for the soft-gluon-pole cross section, we clarify the reason for why it receives only the derivative term of the twist-3 distribution. We also present the first computation of the soft-fermion-pole cross section and found that it vanishes. This means that the derivative of the soft-gluon-pole function is the only source for the twist-3 cross section from the unpolarized twist-3 distribution, which provides a useful basis for a phenomenological analysis.

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I. INTRODUCTION

Understanding the origin of the transverse polarization of a hyperon (hereafter denoted as Λ) produced from the unpolarized proton-proton (proton-nucleus) collision, $pp \to \Lambda^{\uparrow} X$ [1–3], has been a big challenge since its first discovery in the 1970s, along with the left-right asymmetry observed in the pion production, $p^{\uparrow} p \to \pi X$ [4–11]. This is because the conventional framework of perturbative QCD led to a negligible effect for these asymmetries [12]. Since only one hadron in the reaction is transversely polarized, they are collectively called transverse single-spin asymmetries (SSA). In the collinear factorization of perturbative QCD, the SSA appears as a twist-3 observable [13], and thus it was necessary to explore the formalism beyond twist-2 for a proper description of SSAs. Extensive study has been performed along this line and the collinear twist-3 formalism has been well established [14–21]. While there have been many works on the SSA in $p^{\uparrow} p \to \pi X$ [19,22–34], there are only a few works on the hyperon polarization in pp collision [35,36]. We will address this issue based on the collinear twist-3 formalism.

Twist-3 effects are, in general, represented in terms of multiparton (quark-gluon and purely gluonic) correlations in the distribution and fragmentation functions. In particular, the real twist-3 distributions give rise to the cross section through the pole part of an internal propagator in the hard scattering part, which supplies the phase necessary to cause naively T-odd SSAs. For SSAs in the inclusive hadron production in the hadron-hadron collision, those poles are classified as the soft-gluon pole (SGP) and the soft-fermion pole (SFP). For the SGP contribution, a convenient "master formula" was invented [17,26,37,38], which reduces the corresponding partonic hard scattering part to a 2-parton $\to$ 2-parton scattering cross section, and simplifies the actual calculation greatly. Owing to this master formula, the structure of the SGP cross section has become transparent. For example, for the case of $p^{\uparrow} p \to \{\pi, \gamma\} X$, the formula clarified [17,26] why the *total* partonic hard part for the SGP contribution from the twist-3 quark-gluon correlation function in the polarized nucleon becomes the same as the twist-2 unpolarized partonic cross section for $pp \to \pi X$ except for the kinematic and color factors, and why the SGP function $G_F(x,x)$ appears in the combination of $dG_F(x,x)/dx - G_F(x,x)$ in the cross section, which was found in [25] by a direct calculation. The SFP contribution for the above processes has also been derived [27,39], and its potential importance was also explored for $p^{\uparrow} p \to \pi X$ [28,29]. In addition to the quark-gluon correlation in the polarized nucleon, this process receives other twist-3 contributions and extensive works have been performed for the multigluon correlation in the polarized nucleon [33,40], the twist-3 distribution in the unpolarized nucleon [23,24] and the twist-3 fragmentation function for π [19,34].

Under this circumstance, we will discuss, in this paper, the contribution from the twist-3 quark-gluon correlation function $E_F(x_1, x_2)$ in the unpolarized nucleon to the hyperon polarization in $pp \to \Lambda^{\uparrow} X$. This twist-3 distribution is chiral-odd and is combined with the chiral-odd transversity fragmentation function for $\Lambda^{\uparrow}$. As another source for this observable, the twist-3 fragmentation function for the final $\Lambda^{\uparrow}$ also contributes, on which we shall work in a future study.[1] The purpose of this paper is twofold: We first present the rederivation of the SGP cross section in terms of the master formula. The SGP cross section, in general, consists of the derivative and

[1]In [41], the complete twist-3 cross section including the twist-3 fragmentation contribution for the leptoproduction of the polarized Λ, $ep \to \Lambda^{\uparrow} X$, has been derived.

nonderivative terms. The former was first calculated in [35] and the latter was shown to vanish in [36], which is different from the case of $p^\uparrow p \to \{\pi, \gamma\}X$. We will clarify why this happens in the light of the master formula. Next we present the first calculation of the SFP contribution to complete the cross section from the twist-3 distribution. As in $p^\uparrow p \to \pi X$ the effect of SFP has a potential importance to the cross section. However, this contribution turns out to vanish after summing all the diagrams.

The remainder of the paper is organized as follows: in Sec. II, we introduce the nonperturbative functions relevant to the present study. In Sec. III, we present the rederivation of the SGP cross section using the master formula for the twist-3 distribution contribution to $pp \to \Lambda^\uparrow X$. In Sec. IV, we present the first calculation for the SFP contribution. Section V is devoted to a brief summary of the present work.

II. TWIST-3 DISTRIBUTION AND THE TRANSVERSITY FRAGMENTATION FUNCTION

We consider the inclusive production of a transversely polarized spin-1/2 hyperon (represented by Λ) from the unpolarized proton-proton collision,

$$p(p) + p(p') \to \Lambda^\uparrow(P_h, S_\perp) + X, \tag{1}$$

where p, p' and P_h represent the momenta of each hadron and $S_\perp$ is the polarization vector for Λ. In the framework of the collinear factorization, this hyperon polarization can be generated by the two types of mechanisms, i.e., the effect of the twist-3 distribution function in one of the unpolarized protons and that of the twist-3 fragmentation function for the polarized Λ. In this paper, we focus on the former contribution.

We first introduce the nonperturbative functions relevant to our study. The twist-2 unpolarized quark and gluon densities, $f_1^q(x)$ and $f_1^g(x)$, are defined as

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p|\bar{\psi}_j(0)\psi_i(\lambda n)|p\rangle = \frac{1}{2}(\not{p})_{ij} f_1^q(x) + \cdots, \tag{2}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p|F^{\alpha n}(0)F^{\beta n}(\lambda n)|p\rangle = -\frac{1}{2} x f_1^g(x) g_\perp^{\alpha\beta}(p) + \cdots, \tag{3}$$

where ψ_i is the quark field with spinor index i and the projection tensor is defined as $g_\perp^{\alpha\beta}(p) \equiv g^{\alpha\beta} - p^\alpha n^\beta - p^\beta n^\alpha$ with the lightlike vector satisfying $p \cdot n = 1$. $F^{\alpha n} \equiv F^{\alpha\gamma} n_\gamma$ represents the gluon's field strength tensor. Here and below we suppress the gauge link operators in the distribution and fragmentation functions for simplicity.

The twist-3 distribution function $E_F(x_1, x_2)$ in the unpolarized nucleon is defined as [23,35]

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle p|\bar{\psi}_j(0) g F^{\alpha n}(\mu n)\psi_i(\lambda n)|p\rangle = \frac{M_N}{4} \epsilon^{\alpha\beta n p}(\gamma_5 \gamma_\beta \not{p})_{ij} E_F(x_1, x_2) + \cdots, \tag{4}$$

where we introduced the nucleon mass M_N to define the function $E_F(x_1, x_2)$ as dimensionless. From Hermiticity and PT-invariance, $E_F(x_1, x_2)$ is real and symmetric as $E_F(x_1, x_2) = E_F(x_2, x_1)$. Another twist-3 distribution function $E_D(x_1, x_2)$ obtained by replacing $gF^{\alpha w}(\mu n)$ in (4) by the covariant derivative $D^\alpha(\mu n) = \partial^\alpha - igA^\alpha(\mu n)$ is related to $E_F(x_1, x_2)$ as

$$E_D(x_1, x_2) = \mathcal{P}\left(\frac{1}{x_1 - x_2}\right) E_F(x_1, x_2), \tag{5}$$

where $\mathcal{P}$ indicates the principal value. This relation shows that $E_F(x_1, x_2)$ is the only independent twist-3 quark-gluon correlation function in the unpolarized nucleon.

The twist-2 fragmentation function $H_1(z)$ for the polarized Λ is defined as [35]

$$\frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{i\frac{\lambda}{z}} \langle 0|\psi_i(\lambda w)|\Lambda(P_h, S_\perp)X\rangle\langle\Lambda(P_h, S_\perp)X|\bar{\psi}_j(0)|0\rangle = (\gamma_5 \not{S}_\perp \not{p}_c)_{ij} H_1(z) \cdots, \tag{6}$$

where w^μ is a lightlike vector satisfying $P_h \cdot w = 1$, and $p_c \equiv P_h/z$ is the momentum of the quark fragmenting into $\Lambda^\uparrow$. The combination of the two chiral-odd functions $E_F(x_1, x_2)$ and $H_1(z)$ can generate the transverse polarization of the hyperon.

III. SGP CONTRIBUTION TO $p + p \to \Lambda^\uparrow + X$

According to the general formalism of the twist-3 calculation for SSAs [16], the SGP component of a twist-3 distribution function in the nucleon contributes as derivative and nonderivative terms. For the case of the SGP contribution of $p^\uparrow p \to \{\pi, \gamma\}X$, it was shown that the derivative and the nonderivative terms have the common partonic hard cross sections and thus the total SGP function contributes to the cross section in the particular combination. The origin of this simplification was clearly understood in terms of the master formula which shows the *total* SGP partonic hard cross section is connected to the twist-2 unpolarized cross section for $pp \to \{\pi, \gamma\}X$ with different color factors [17,26]. For the case of $pp \to \Lambda^\uparrow X$, the

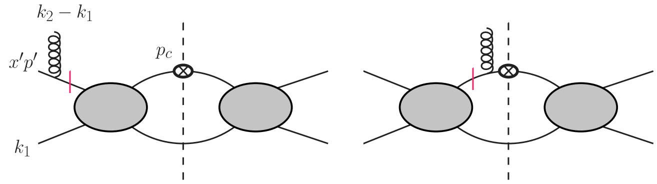

FIG. 1 (color online). Diagrammatic representation of the hard part for the SGP contribution. The left (right) diagram corresponds to ISI (FSI) and gives $S^I_{\lambda\beta}(k_1,k_2)$ $[S^F_{\lambda\beta}(k_1,k_2)]$. The SGP is given as a pole part of the bared propagator. The circled cross indicates the fragmentation insertion for $\Lambda^\uparrow$. Each blob represents the $2\to 2$ scattering amplitude.

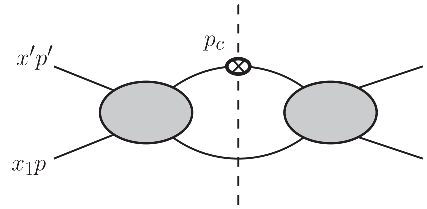

FIG. 2. Diagrammatic representation for $\tilde{S}^{I,F}_{\beta\gamma}(x_1p, x'p', p_c)$.

derivative term was first calculated in [35] and the nonderivative terms were also calculated in [36] and were shown to vanish, indicating that the SGP contribution is the sole contribution. Here we calculate the SGP cross section for $pp\to\Lambda^\uparrow X$ in the light of the master formula and clarify the origin for the vanishing nonderivative terms. We emphasize that the master formula itself is valid for this process as well, while its outcome differs from the case of $p^\uparrow p\to\{\pi,\gamma\}X$, and this difference does not imply the "violation" of the master formula as claimed in [36].

Applying the formalism developed in [16], one can obtain the SGP contribution to $pp\to\Lambda^\uparrow X$ from the following formula:

$$E_{P_h}\frac{d\Delta\sigma}{d^3P_h}=\frac{iM_N}{64\pi^2 s}\int\frac{dx'}{x'}f_1(x')\int\frac{dz}{z^2}H_1(z)\int\frac{dx_1}{x_1}\int dx_2 E_F(x_1,x_2)\epsilon^{\alpha\beta np}\frac{\partial}{\partial k_2^\alpha}(S^I_{\lambda\beta}(k_1,k_2)p^\lambda+S^F_{\lambda\beta}(k_1,k_2)p^\lambda)\Big|_{k_i=x_ip},\tag{7}$$

where $s=(p+p')^2$ is the square of the center of mass energy, and the partonic hard parts $S^{I,F}_{\lambda\beta}(k_1,k_2)p^\lambda$ are obtained from diagrams shown in Fig. 1 by taking the spinor and color traces with the appropriate projections for the distribution and fragmentation functions in (2), (3), (4) and (6).[2] $S^I_{\lambda\beta}$ and $S^F_{\lambda\beta}$ correspond to the initial-state interaction (ISI) and the final-state interaction (FSI), respectively. In $S^{I,F}_{\lambda\beta}(k_1,k_2)$, the Lorentz index λ corresponds to that for the coherent gluon line with the momentum k_2-k_1 in Fig. 1, and β is for the projection of $E_F(x_1,x_2)$ in (4). The SGP contribution was obtained by a direct computation without referring to the master formula in [35,36]. Here we take an alternative approach based on the master formula which reduces the SGP cross section to a $2\to 2$ partonic cross section. Following the procedure described in [17,26,37,38,42], one obtains the hard part as

$$\epsilon^{\alpha\beta np}\frac{\partial S^I_{\lambda\beta}(k_1,k_2)p^\lambda}{\partial k_2^\alpha}\Big|^{\text{SGP}}_{k_i=x_ip}=\left[\frac{-1}{x_2-x_1+i\epsilon}\right]^{\text{pole}}\epsilon^{\alpha\beta np}S_\perp^\gamma\frac{d}{d(x'p'^\alpha)}\tilde{S}^I_{\beta\gamma}(x_1p,x'p',p_c),\tag{8}$$

$$\epsilon^{\alpha\beta np}\frac{\partial S^F_{\lambda\beta}(k_1,k_2)p^\lambda}{\partial k_2^\alpha}\Big|^{\text{SGP}}_{k_i=x_ip}=\left[\frac{1}{x_1-x_2+i\epsilon}\right]^{\text{pole}}\epsilon^{\alpha\beta np}\times\left[S_\perp^\gamma\frac{d}{dp_c^\alpha}+\frac{1}{p\cdot p_c}\{(p\cdot S_\perp)g^\gamma_\alpha-S_{\perp\alpha}p^\gamma\}\right]\tilde{S}^F_{\beta\gamma}(x_1p,x'p',p_c),\tag{9}$$

where $\epsilon^{\alpha\beta np}S_\perp^\gamma\tilde{S}^I_{\beta\gamma}(x_1p,x'p',p_c)$ represents the partonic cross section for the $2\to 2$ scattering process, $q(x_1p)+b(x'p')\to q(p_c)+q(x_1p+x'p'-p_c)$ ($b=q$ or g), as shown in Fig. 2. It is obtained by the spinor projection $\epsilon^{\alpha\beta np}\gamma_5\gamma_\beta x_1\not{p}$ for the initial parton with momentum x_1p, the unpolarized projection for the parton b with momentum $x'p'$, and the projection $\gamma_5\not{S}_\perp\not{p}_c$ for the final parton fragmenting into the polarized Λ, but it has the same color factor as the ISI diagrams in Fig. 1.[3] Except for the color factor, the twist-2 partonic cross section for the

[2]Since we included an extra x_1 into $S^{I,F}_{\lambda\beta}$ to project out $E_F(x_1,x_2)$, the integration measure became dx_1/x_1.

[3]We have factored out the spin vector $S_\perp^\gamma$ to define $\tilde{S}^I_{\beta\gamma}(x_1p,x'p',p_c)$ for later convenience.

spin-transfer reaction $p^\uparrow(p,S_{N\perp})+p(p')\to\Lambda^\uparrow(P_h,S_\perp)+X$ [43] can be written as $S^\beta_{N\perp}S^\gamma_\perp\tilde{S}^I_{\beta\gamma}(x_1p,x'p',p_c)$. $\tilde{S}^F_{\beta\gamma}(x_1p,x'p',p_c)$ in (9) is defined similarly but with the color factors for the FSI diagrams in Fig. 1, and thus $\tilde{S}^I_{\beta\gamma}$ and $\tilde{S}^F_{\beta\gamma}$ differ only in the color factors. In taking the derivative $\frac{d}{dx'p'^\alpha}$ in (8), the on-shell form $p'^\mu=\left(p'^+=\frac{\vec{p}'^{\,2}_\perp}{2p'^-},p'^-,\vec{p}'_\perp\right)$ needs to be used. Likewise the derivative $\frac{d}{dp_c^\alpha}$ needs to be taken with $p_c^\mu=(\frac{\vec{p}_{c\perp}^{\,2}}{2p_c^-},p_c^-,\vec{p}_{c\perp})$ in (9). We note the appearance of the extra terms in (9) for FSI compared with (8) for ISI, which is different from the case of the $G_F(x,x)$ contribution to $p^\uparrow p\to\pi X$.

To calculate the derivatives $\frac{d\tilde{S}^I_{\beta\gamma}}{dx'p'^\alpha}$ and $\frac{d\tilde{S}^F_{\beta\gamma}}{dp_c^\alpha}$ in (8) and (9), we note that $\tilde{S}^{\rm Y}_{\beta\gamma}(xp,x'p',p_c)$ (Y $=I$, F) can be expanded as

$$\tilde{S}^{\rm Y}_{\beta\gamma}(xp,x'p',p_c)=a_1^{\rm Y}g_{\beta\gamma}+a_2^{\rm Y}xp_\beta xp_\gamma+a_3^{\rm Y}x'p'_\beta x'p'_\gamma+a_4^{\rm Y}p_{c\beta}p_{c\gamma}+a_5^{\rm Y}x'p'_\beta xp_\gamma$$
$$+a_6^{\rm Y}xp_\beta x'p'_\gamma+a_7^{\rm Y}p_{c\beta}xp_\gamma+a_8^{\rm Y}xp_\beta p_{c\gamma}+a_9^{\rm Y}p_{c\beta}x'p'_\gamma+a_{10}^{\rm Y}x'p'_\beta p_{c\gamma}, \tag{10}$$

where each coefficient $a_i^{\rm Y}=a_i^{\rm Y}(\hat{s},\hat{t},\hat{u})$ $(i=1,\ldots,10)$ is a scalar function of the Mandelstam variables, $\hat{s}=(xp+x'p')^2$, $\hat{t}=(xp-p_c)^2$ and $\hat{u}=(x'p'-p_c)^2$. We further introduce the partonic cross sections $\hat{\sigma}_i^{\rm Y}(\hat{s},\hat{t},\hat{u})$ (Y $=I$, F) by the relation

$$a_i^{\rm Y}(\hat{s},\hat{t},\hat{u})=\hat{\sigma}_i^{\rm Y}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}), \tag{11}$$

for later use. Here we work in a frame in which p and p' are collinear. The derivative $d/dx'p'^\alpha$ and d/dp_c^α on $a_i^{\rm Y}$ can be performed through $\hat{u}$ to obtain[4]

$$\frac{d}{d(x'p'^\alpha)}a_i^I(\hat{s},\hat{t},\hat{u})=-2p_{c\alpha}\frac{\partial}{\partial\hat{u}}a_i^I(\hat{s},\hat{t},\hat{u}), \tag{12}$$

$$\frac{d}{dp_c^\alpha}a_i^F(\hat{s},\hat{t},\hat{u})=-2\left(\frac{\hat{s}}{\hat{t}}\right)p_{c\alpha}\frac{\partial}{\partial\hat{u}}a_i^F(\hat{s},\hat{t},\hat{u}). \tag{13}$$

Using (8) and (10) in (7), one can express the cross section from the ISI in terms of a_i^I $(i=1,\ldots,10)$. It is easy to see that only $a_{1,9}^I$ survive and one obtains

$$\begin{aligned}E_{P_h}\frac{d\Delta\sigma^{\rm ISI}}{d^3P_h}&=-\frac{M_N}{64\pi s}\int\frac{dx'}{x'}f_1(x')\int\frac{dz}{z^2}H_1(z)\int\frac{dx}{x}E_F(x,x)\epsilon^{\alpha\beta np}S^\gamma_\perp\frac{d}{d(x'p'^\alpha)}\tilde{S}^I_{\beta\gamma}(xp,x'p',p_c)\\&=-\frac{M_N}{32\pi s}\int\frac{dx'}{x'}f_1(x')\int\frac{dz}{z^2}H_1(z)\int\frac{dx}{x}E_F(x,x)\epsilon^{p_cpnS_\perp}\left(\frac{\partial}{\partial\hat{u}}a_1^I+\frac{1}{2}a_9^I\right)\\&=-\frac{M_N}{32\pi s}\int\frac{dx'}{x'}f_1(x')\int\frac{dz}{z^2}H_1(z)\int\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u})\epsilon^{p_cpnS_\perp}\\&\quad\times\left[x\frac{dE_F(x,x)}{dx}\frac{1}{\hat{u}}\hat{\sigma}_1^I+E_F(x,x)\left(-\frac{1}{\hat{u}}\hat{\sigma}_1^I+\frac{1}{2}\hat{\sigma}_9^I\right)\right],\end{aligned} \tag{14}$$

where we used the relation $(\hat{s}\frac{\partial}{\partial\hat{s}}+\hat{t}\frac{\partial}{\partial\hat{t}}+\hat{u}\frac{\partial}{\partial\hat{u}})\hat{\sigma}_1^{\rm Y}(\hat{s},\hat{t},\hat{u})=0$ which follows from the scale invariance property $\hat{\sigma}_1^{\rm Y}(\lambda\hat{s},\lambda\hat{t},\lambda\hat{u})=\hat{\sigma}_1^{\rm Y}(\hat{s},\hat{t},\hat{u})$.

Repeating the same steps for the FSI contribution, one obtains the corresponding cross section as

$$\begin{aligned}E_{P_h}\frac{d\Delta\sigma^{\rm FSI}}{d^3P_h}&=\frac{M_N}{32\pi s}\int\frac{dx'}{x'}f_1(x')\int\frac{dz}{z^2}H_1(z)\int\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u})\epsilon^{p_cpnS_\perp}\\&\quad\times\left[x\frac{dE_F(x,x)}{dx}\left(\frac{\hat{s}}{\hat{t}\,\hat{u}}\right)\hat{\sigma}_1^F+E_F(x,x)\left(\frac{\hat{s}}{\hat{t}}\right)\left(-\frac{1}{\hat{u}}\hat{\sigma}_1^F+\frac{1}{2}\hat{\sigma}_9^F\right)\right],\end{aligned} \tag{15}$$

[4]In taking the derivative with respect to $p'^\alpha_\perp$, we first keep $p'_\perp\neq0$ to take the derivative and then put $p'_\perp=0$.

which is formally the same form as the ISI contribution (14) except for the kinematic factor $\hat{s}/\hat{t}$. From (14) and (15) one sees that the SGP function does not appear in the combination of $x\frac{d}{dx}E_F(x,x) - E_F(x,x)$ if $\hat{\sigma}_9^{I,F} \neq 0$. In fact we found, by the direct calculation, the relation $\hat{\sigma}_9^{I,F} = 2\hat{\sigma}_1^{I,F}/\hat{u}$ in all the channels. This means that only the derivative of the SGP function contributes to $pp \to \Lambda^\uparrow X$, which was found by [36]. This is in contrast to the case of $p^\uparrow p \to \{\pi, \gamma\}X$ [26], where the SGP function appears in the combination of $x\frac{d}{dx}G_F(x,x) - G_F(x,x)$. This way the final form of the SGP contribution to $pp \to \Lambda^\uparrow X$ is obtained as follows [35,36]:

$$E_{P_h}\frac{d\Delta\sigma^{\rm SGP}}{d^3P_h} = \frac{\pi M_N \alpha_s^2}{s}\epsilon^{P_h p n S_\perp}\sum_{a,b,c}\int\frac{dx'}{x'}f_1^b(x')\int\frac{dz}{z^3}H_1^c(z)\int dx\frac{dE_F^a(x,x)}{dx}\sigma_{ab\to c}\delta(\hat{s}+\hat{t}+\hat{u}), \tag{16}$$

where the partonic cross section in each channel is given by

$$\sigma_{qq'\to q} = \frac{1}{N^2}\frac{2\hat{s}}{\hat{t}^2} - \frac{1}{N^2}\frac{\hat{s}^2}{\hat{t}^3}, \qquad \sigma_{qq\to q} = \sigma_{qq'\to q} - \left(\frac{1}{N}+\frac{1}{N^3}\right)\frac{\hat{s}}{\hat{t}\,\hat{u}} + \frac{1}{N^3}\frac{\hat{s}^2}{\hat{t}^2\hat{u}},$$
$$\sigma_{q\bar{q}'\to q} = \left(\frac{N^2-2}{N^2}\right)\frac{\hat{s}}{\hat{t}^2} - \frac{1}{N^2}\frac{\hat{s}^2}{\hat{t}^3}, \qquad \sigma_{q\bar{q}\to q} = \sigma_{q\bar{q}'\to q\bar{q}'} + \frac{1}{N^3}\frac{1}{t} + \frac{1}{N^3}\frac{\hat{s}}{\hat{t}^2},$$
$$\sigma_{q\bar{q}\to\bar{q}} = -\frac{1}{N^3}\frac{1}{\hat{u}} + \left(\frac{1}{N}+\frac{1}{N^3}\right)\frac{\hat{s}}{\hat{t}\,\hat{u}},$$
$$\sigma_{qg\to q} = -\frac{N^2}{N^2-1}\frac{\hat{u}}{t^2} + \frac{1}{N^2-1}\frac{1}{\hat{u}} - \frac{1}{N^2(N^2-1)}\frac{\hat{s}}{\hat{t}\,\hat{u}} - \frac{1}{(N^2-1)}\frac{2\hat{s}^2}{\hat{t}^3}, \tag{17}$$

where $N = 3$ is the number of colors for a quark. Summarizing this section, we have shown that the partonic hard cross section for the SGP contribution from the twist-3 distribution to $pp \to \Lambda^\uparrow X$ can also be reduced to a certain $2 \to 2$ scattering cross section (master formula), and the relation can be conveniently used to derive the cross section. We have also shown that the particular relation for the above $2 \to 2$ scattering cross section leads to the result that only the derivative term with $dE_F(x,x)/dx$ appears in the SGP cross section for $pp \to \Lambda^\uparrow X$, which is in contrast the case of $p^\uparrow p \to \{\pi,\gamma\}X$ where the SGP function appears through the combination $x\frac{d}{dx}G_F(x,x) - G_F(x,x)$.

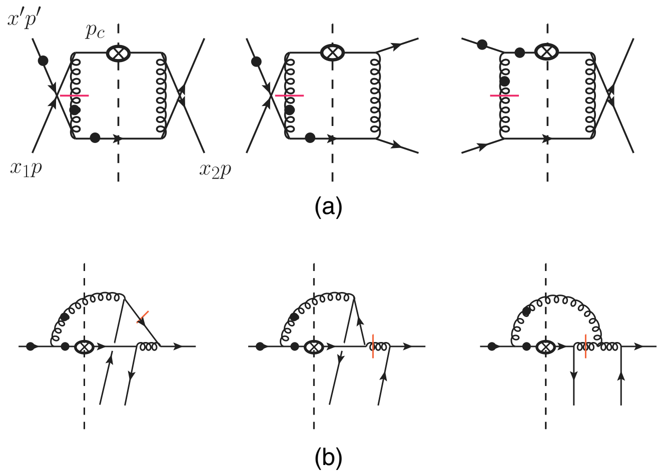

FIG. 3 (color online). Lowest order diagrams for the hard part of the SFP contribution in the $qq' \to qq'$ and $qq \to qq$ channels. The twist-3 distribution contributes from the lower side of each diagram. For each diagram, three diagrams corresponding to a different attachment of the coherent gluon line to one of the dots need to be considered. The barred propagator gives rise to SFP. Mirror diagrams should also be included.

IV. SFP CONTRIBUTION TO $p + p \to \Lambda^\uparrow + X$

In this section we present the first derivation for the SFP contribution from the twist-3 unpolarized quark distribution. According to the formalism of [16], it receives only the nonderivative contribution and is given by

$$E_{P_h}\frac{d\Delta\sigma}{d^3P_h} = \frac{iM_N}{64\pi^2 s}\int\frac{dx'}{x'}f_1(x')\int\frac{dz}{z^2}H_1(z) \times\int dx_1\int dx_2 E_F(x_1,x_2)\epsilon^{\alpha\beta n p}\mathcal{P}\left(\frac{1}{x_1-x_2}\right) \times S_{\alpha\beta}^{\rm SFP}(x_1 p, x_2 p), \tag{18}$$

where $S_{\alpha\beta}^{\rm SFP}(x_1 p, x_2 p)$ represents the hard part which is obtained by calculating the diagrams shown in Figs. 3–5. The meaning of the Lorentz indices is the same as $S_{\lambda\beta}^{\rm I,F}(k_1,k_2)$ in (7). By the direct calculation of the diagrams, it turned out that the SFP contribution completely vanishes in all channels after summing over all the diagrams.[5] As a result, only the derivative term of SGP contribution survives in the final cross section formula.

[5]The SFP contribution from E_F to $p^\uparrow p \to \gamma X$ was also shown to vanish in [42].

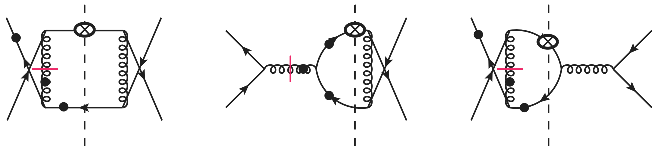

FIG. 4 (color online). The same as Fig. 3, but for the $q\bar{q} \to q\bar{q}$ channel. Diagrams for the $\bar{q}q \to q\bar{q}$ channel are obtained by reversing the arrows of the quark lines and shifting the fragmentation insertion to the other quark line crossing the final-state cut.

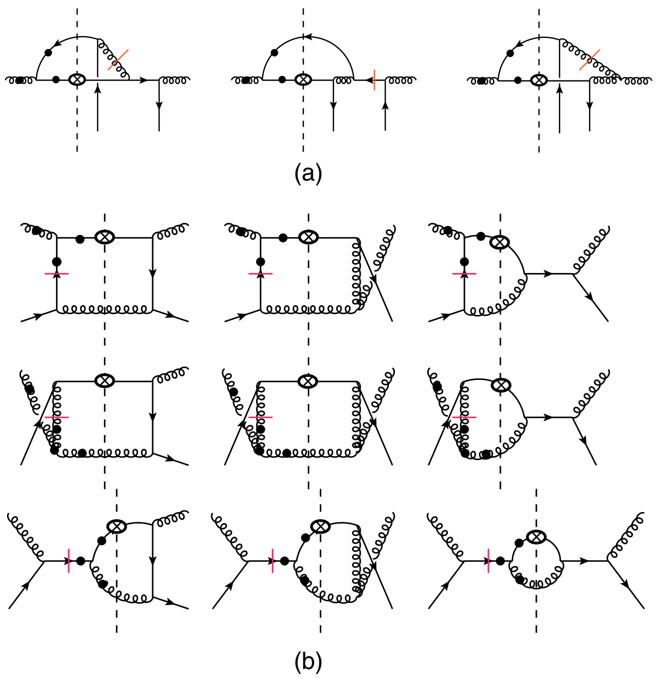

FIG. 5 (color online). The same as Fig. 3, but for the $qg \to qg$ channel.

V. SUMMARY

In this paper we have studied the transverse polarization of a hyperon produced in the unpolarized proton-proton collision, $pp \to \Lambda^{\uparrow} X$. This is an example of the single spin asymmetry (SSA) in high-energy inclusive reactions and has been a long-standing issue since its discovery in the 1970s. In perturbative QCD, this phenomenon occurs as a twist-3 effect in the framework of the collinear factorization. We have focused on the effect of the twist-3 quark-gluon correlation function in one of the initial protons combined with the transversity fragmentation function for the final hyperon. In principle, the cross section appears as the SGP and SFP contributions, and we presented the full derivation for those contributions. The SGP cross section was derived some time ago [35,36], which showed that only the derivative of the SGP function $dE_F(x,x)/dx$ contributes, unlike the case for the SSA in $p^{\uparrow} p \to \{\pi, \gamma\} X$. We presented a rederivation of the SGP cross section in the light of the master formula [17,26] and have clarified the origin of the difference between the two processes. We also calculated the SFP cross section for the first time and showed that it vanishes in all channels after summing over all diagrams. We conclude from this study that the twist-3 effect in the unpolarized proton is embodied solely through the derivative of the SGP function, $dE_F(x,x)/dx$, in $pp \to \Lambda^{\uparrow} X$, which provides a useful basis for future phenomenological study on the hyperon polarization. For the complete understanding of the origin of the hyperon polarization, however, we need to include the twist-3 fragmentation function for the final $\Lambda^{\uparrow}$. We will address this issue in a future publication.

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