

Mixing angle and phase correlations from A_5 with generalized CP and their prospects for discovery

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The observed leptonic mixing pattern could be explained by the presence of a discrete flavor symmetry broken into residual subgroups at low energies. In this scenario, a residual generalized charge parity (CP) symmetry allows the parameters of the Pontecorvo-Maki-Nakagawa-Sakata matrix, including Majorana phases, to be predicted in terms of a small set of input parameters. In this article, we study the mixing parameter correlations arising from the symmetry group A_5 including generalized CP subsequently broken into all of its possible residual symmetries. Focusing on those patterns which satisfy present experimental bounds, we then provide a detailed analysis of the measurable signatures accessible to the planned reactor, superbeam and neutrinoless double-beta decay experiments. We also discuss the role which could be played by high-precision measurements from longer term projects such as the Neutrino Factory. This work provides a concrete example of how the synergies of the upcoming experimental program allow flavor symmetric models to be thoroughly investigated. Indeed, thanks to the rich tapestry of observable correlations, we find that each step of the experimental program can make important contributions to the assessment of such flavor-symmetric patterns, and ultimately all patterns that we have identified can be excluded, or strong evidence found for their continued relevance.

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I. INTRODUCTION

The existence of three families of fermions in the standard model (SM), identical in all properties apart from their masses, is as yet unexplained by any physical principle or mechanism. Moreover, the discovery that both quarks and leptons permit complementary but distinct descriptions in terms of the flavor states which diagonalize the weak interactions and the states which diagonalize their mass terms has shown that the connection between families betrays a precise structure which is an essential component in our description of the physical world. Explaining the origins of this flavor structure has been a recurring theme in proposed extensions of the SM. One such program is the application of discrete flavor symmetries, predominately in the lepton sector, where the flavor quantum numbers are associated with a new symmetry and particles are assigned to its irreducible representations. This can provide a way to unify the three families into a single mathematical object. However, as the lepton masses are known to be distinct, any non-Abelian symmetry can only be exact above the scale of mass generation. Nevertheless, its existence at high energy shapes the theory, and the residual symmetries which survive the breaking procedure at low energies can play an important role in the structure of flavor observables.

The paradigm of a non-Abelian flavor symmetry breaking into residual symmetries has been used by many authors to make predictions about the six mixing angles and phases which constitute the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and parametrize neutrino mixing: θ_{12} , θ_{13} , θ_{23} , the Dirac phase δ and two Majorana phases α_{21} and α_{31} . For a recent review of such models see e.g. Ref. [1]. Although many of the earliest models were designed to predict a very small value of θ_{13} , a prediction now firmly ruled out [2–4], a number of models remain consistent with the current data, many of which are based on groups taken from the $\Delta(6n^2)$ family: $\Delta(96)$ [5,6], $\Delta(150)$ [7], $\Delta(600)$ [8] and $\Delta(1536)$ [9]. This connection was strengthened in Ref. [10] which showed that, based on only a few generic model building assumptions, if the full PMNS matrix is to be specified by the symmetry alone (so-called “direct” models [1]), the only possible predictions which would agree with current data are those arising (minimally) from $\Delta(6n^2)$. In general, however, the existence of residual symmetries amongst the leptonic mass terms may not fully specify the mixing pattern. In these “semidirect” models [1] the symmetries reduce the degrees of freedom necessary to describe the mixing parameters by defining a correlation between previously independent parameters (sometimes known as mixing sum rules [11–14]). Often these correlations between mixing angles and phases can be derived from quite generic analyses of the residual symmetries present in a system without needing to specify a full UV-complete

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theory [15–21]. As such, focusing on these relations can be an effective way to compare a wide class of models to data [17–24].

To date, the fine structure of the PMNS matrix has been inaccessible to experiment, preventing the study of subtle parameter correlations. Current measurements of the mixing angles have 3σ uncertainties of around 6.9% on θ_{12} , 7.3% on θ_{13} and 16.7% on θ_{23} [25].¹ All three of the CP phases are unconstrained at this significance level; although, some low-significance hints for a maximally CP -violating value of the Dirac phase $\delta \approx 3\pi/2$ have been observed [25,26]. Therefore the upcoming experimental work will focus on two key topics: the precision determination of the mixing angles and the first measurements of the CP phase δ . This will open the door for studies of the correlations between mixing angles, and between mixing angles and phases, that are predicted by models of flavor symmetries.

In the most popular formulation of models with discrete flavor symmetries, the constraints on the mass matrices used to derive the PMNS matrix cannot remove a number of complex phase degrees of freedom. This results in an inability to predict the Majorana phases and, in general, lessens the predictivity of the model. However, by imposing a *generalized CP symmetry* (GCP), phase information may be accessible and dictated by the flavor structure itself. This can lead to very predictive scenarios, where all six mixing parameters are related to a small number of input parameters [27]. GCPs were first explored in the context of discrete and continuous groups in Ref. [28]; however, they have recently been revived due to the question of consistency between a CP symmetry and discrete flavor group [27,29,30]. This has led to interesting work studying the predictions of models with imposed flavor and GCP symmetries for a number of groups such as A_4 [29,31], S_4 [27,32–35], $\Delta(48)$ [36,37] and $\Delta(96)$ [38] along with more comprehensive analyses of the families $\Delta(3n^2)$ and $\Delta(6n^2)$ [39–41]. (See also Refs. [42,43] for further applications of GCP symmetries.)

In this article, we present a detailed analysis of a single group: the alternating group on five elements, A_5 . This was first introduced to leptonic flavor physics in Ref. [44] via the study of golden ratio mixing [45,46]: a possible pattern of the PMNS matrix with $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and a value of θ_{12} related to the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$, $\tan \theta_{12} = 1/\varphi$. This pattern has been shown to be a prediction of a number of different models based on A_5 [44,46–50]; however, it is not the only fully specified mixing pattern associated with direct models based on this group. If a \mathbb{Z}_3 subgroup is preserved among the charged leptons and a $\mathbb{Z}_2 \times \mathbb{Z}_2$ is preserved among the neutrinos, a pattern with $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and $\cos \theta_{12} = \varphi/\sqrt{3}$ can be found [21,51,52].

¹For alternative global analyses of oscillation data, see Ref. [26].

Further patterns are also possible when a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry is preserved in the charged leptons while a different $\mathbb{Z}_2 \times \mathbb{Z}_2$ remains in the neutrino sector, which predicts a large value of θ_{13} , $\theta_{13} \approx 17.9^\circ$ [52].

Needless to say, the patterns above are in severe tension with the current global data by dint of their θ_{13} predictions alone; however, the possibility remains that symmetries such as these do not completely survive at low energies and that a semidirect approach may remain viable. In this work, we consider the group A_5 with a GCP symmetry, deriving the most general GCP transformation which could be implemented for this group. We assume that the flavor group with GCP is broken into a set of residual symmetries at low energies insufficient to fix all of the oscillation parameters. We compute all possible predictions for the induced correlations amongst the mixing parameters. These are compared to the current data, and we identify those which are compatible with the current bounds. The viable patterns that we identify are highly predictive, expressing all six parameters of the PMNS matrix in terms of a single unphysical angle. We take particular care in assessing the phenomenology of the correlations between mixing angles and phases for these viable models: discussing their accessibility to reactor, long-baseline and neutrinoless double-beta decay experiments, and highlighting particularly interesting signatures to be tested. Although we have restricted our attention to the group A_5 , this can be seen as an illustrative choice and we stress the rich but moreover *readily testable* phenomenology which exists in the residual symmetry framework, much of which arises from the predictions taken as a whole instead of resting on single generic types of parameter correlation.

The work presented in this paper is divided into two main parts. In Secs. II–III, we discuss the assumptions behind our framework, and explain the technical steps in our derivation of the correlations. In Sec. IV, we focus on the predictions themselves, presenting some simplified formulas for the correlations between observable quantities and identifying their most interesting phenomenological signatures. We also study a number of ways that the correlations can be tested by upcoming reactor, superbeam and neutrinoless double-beta decay experiments, as well as possible longer-term experiments such as neutrino factories. We make our concluding remarks in Sec. V.

II. RESIDUAL FLAVOR AND GENERALIZED CP SYMMETRIES

We assume the presence of a discrete flavor symmetry, G , at high energies. To unify the three flavors, we assume that the fields are assigned to a three-dimensional irreducible representation of this group, and the general multiplets of leptons Ψ transform as

$$\Psi_\alpha \rightarrow \rho(g)_{\alpha\beta} \Psi_\beta,$$

where $\rho: G \rightarrow \text{GL}(3, \mathbb{C})$ represents a unitary representation of G .² Ensuring the existence of the three-dimensional irreducible representation ρ restricts us to non-Abelian groups.

As neutrinos are known to oscillate, they cannot have degenerate masses and therefore the non-Abelian flavor group, G , cannot be a symmetry of our low-energy effective Lagrangian. Therefore, we assume that the full flavor symmetry must be broken at low energies into two Abelian residual symmetry groups, G_e and G_ν , which are unbroken in the charged-lepton and neutrino sectors, respectively. We denote the leptonic mass terms in the low-energy effective theory by

$$-\mathcal{L}_{\lambda\nu} = \overline{(e_L)_\alpha}(m_\lambda)_{\alpha\beta}(e_R)_\beta + \overline{(e_R)_\alpha}(m_\lambda^\dagger)_{\alpha\beta}(e_L)_\beta + \frac{1}{2}\overline{(\nu_L^c)_\alpha}(m_\nu)_{\alpha\beta}(\nu_L)_\beta + \frac{1}{2}\overline{(\nu_L)_\alpha}(m_\nu^\dagger)_{\alpha\beta}(\nu_L^c)_\beta, \quad (1)$$

where the greek indices are flavor indices and the mass matrices are 3×3 and complex valued. Due to the anticommutation of the fermionic fields, one can show that the Majorana mass matrix m_ν is restricted to being complex symmetric.

We assume that there exist residual symmetries acting on the left-hand charged and neutral leptons. If we denote a general element of these subgroups by $g_e \in G_e$ and $g_\nu \in G_\nu$, the fields transform according to the following rules:

$$(e_L)_\alpha \rightarrow \rho(g_e)_{\alpha\beta}(e_L)_\beta, \quad \text{and} \quad (\nu_L)_\alpha \rightarrow \rho(g_\nu)_{\alpha\beta}(\nu_L)_\beta.$$

Combining these relations with the Lagrangian in Eq. (1) leads us to matrix relations which the mass terms must satisfy if the residual symmetries are to be preserved at low energies,

$$m_\lambda m_\lambda^\dagger = \rho(g_e)^\dagger (m_\lambda m_\lambda^\dagger) \rho(g_e), \quad (2)$$

$$m_\nu = \rho(g_\nu)^\text{T} m_\nu \rho(g_\nu). \quad (3)$$

These relations constrain the forms of the mass matrices and, as we show, knowledge of their existence can be used to derive a form of the PMNS matrix, U_{PMNS} .

Working from a bottom-up perspective, we deduce the phenomenological consequences of a given choice of residual symmetries G_e and G_ν . The choice of residual flavor groups is constrained in two ways. First, G_e and G_ν must be subgroups of the unbroken group G . Secondly, the possible residual flavor symmetries must be subgroups of the largest symmetry allowed by the mass terms in Eq. (1).

²Throughout this paper we assume that all representations are unitary and therefore all group elements are represented by unitary matrices.

These maximal symmetries are best identified in the basis where both mass terms are diagonal. In this basis, the most general symmetry of the charged lepton mass matrices is $U(1)^3$, which has discrete subgroups of the form $G_e = \mathbb{Z}_m$ for any m or a direct product of such groups. For the neutrino residual symmetry, the argument changes due to the assumed Majorana nature of the mass term. In this case the largest possible symmetry is smaller, $\mathbb{Z}_2 \times \mathbb{Z}_2$, which leaves us with only two choices, $G_\nu = \mathbb{Z}_2$ or $G_\nu = \mathbb{Z}_2 \times \mathbb{Z}_2$.

In addition to a flavor symmetry G , we also assume the presence of a GCP symmetry. A CP symmetry is understood as a combination of charge conjugation and a parity transformation; however, a generalized CP symmetry is one which also acts on the flavor indices while making this transposition. In Ref. [29], it was shown that ensuring the consistency of a discrete flavor symmetry and a CP symmetry often requires the introduction of a nontrivial generalized CP symmetry.

We define our generalized CP symmetry to act on a set of fields Ψ_α as

$$\Psi_\alpha \rightarrow X_{\alpha\beta} \Psi_\beta^c,$$

where $X_{\alpha\beta}$ is assumed to be a unitary matrix so as to preserve the kinetic terms in the Lagrangian and Ψ^c denotes the conventional CP conjugate appropriate for the Lorentz representation of the field Ψ . If a discrete flavor symmetry is present, G , then a generalized CP symmetry must satisfy a consistency equation [27,29]

$$X\rho(g)^*X^* = \rho(g'), \quad (4)$$

where g and g' are elements of G . For a faithful representation ρ , this relation can be seen as establishing a mapping from g to g' which preserves the structure of the group and therefore defines a group automorphism. In Ref. [30], it was pointed out that a physical GCP transformation must be restricted to a single irreducible representation, and is related to a class-inverting automorphism of G , meaning that g' is mapped to an element in the conjugacy class of g^{-1} . We restrict our consideration to involutory GCP transformations, requiring that the application of the transformation twice is equivalent to the identity, and therefore the X matrix satisfies an additional constraint

$$XX^* = 1. \quad (5)$$

As with the flavor symmetry G , for our GCP symmetry to leave the Lagrangian invariant the mass matrices must satisfy further constraints. The GCP symmetry exchanges the Hermitian conjugate terms in the Lagrangian of Eq. (1), which remains invariant if the mass matrices obey the relations

$$X^T m_\nu X = m_\nu^*, \quad (6)$$

$$X^\dagger (m_\lambda m_\lambda^\dagger) X = (m_\lambda m_\lambda^\dagger)^*. \quad (7)$$

If Eqs. (6)–(7) are unbroken relations at low energies, it can be shown that all CP -violating effects of the PMNS matrix vanish [27,29]. However, if only one of these relations is preserved, the consistency of the flavor and CP symmetries leads to novel constraints on the PMNS matrix. In this paper, we assume that the GCP symmetry is broken in the charged-lepton sector but is preserved in the neutrino sector. For this to be consistent, the X matrix must map the elements of the neutrino residual symmetry to themselves,

$$X \rho(g_\nu)^* X^* = \rho(g_\nu). \quad (8)$$

In summary, we assume a discrete flavor symmetry G and a GCP symmetry implemented by X at high-energy scales which is assumed to break into a subgroup G_e acting on the charged-lepton mass terms and another subgroup G_ν which along with the GCP symmetry acts on the neutrino mass terms. This leads to a system of constraints which the mass matrices must satisfy: Eqs. (2)–(3), Eq. (6) and Eq. (8). In the next section we show how knowledge of these constraints alone can be used to predict the PMNS matrix, including its Majorana phases.

A. Constructing the PMNS matrix using symmetry constraints

Constraints on the mass matrices, as derived above, lead to restrictions on their allowed form, and subsequently to the matrices required to diagonalize them. We focus first on the charged leptons. A constraint on the charged-lepton mass matrix of the form in Eq. (2) can be rephrased as a statement of commutation

$$[\rho(g_e), (m_\lambda m_\lambda^\dagger)] = 0.$$

As the matrix $\rho(g_e)$ is unitary and the matrix $H = m_\lambda m_\lambda^\dagger$ is Hermitian, there exists a basis such that they are simultaneously diagonalized,

$$\begin{aligned} \exists U_e \text{ s.t. } U_e^\dagger U_e &= 1, & \rho(g_e)_d &= U_e^\dagger \rho(g_e) U_e, \\ H_d &= U_e^\dagger H U_e, \end{aligned}$$

where $\rho(g_e)_d$ and H_d denote diagonal forms of the matrices $\rho(g_e)$ and H . As the charged leptons have distinct masses, H is full rank. This implies that U_e is unique (up to rephasing and reordering of columns). If $\rho(g_e)$ is also known to be full rank, U_e can also be found by diagonalizing this operator. In this way, by insisting on the relation in Eq. (2), we can compute U_e solely from the group element $\rho(g_e)$, and the symmetry alone specifies the mixing

matrix. However, a complication arises if $\rho(g_e)$ is not full rank. In this case, $\rho(g_e)$ does not have a unique diagonalizing matrix, as in any basis in which it takes diagonal form, further $SU(2)$ transformations can be performed freely in its degenerate eigenspace. Without knowledge of the mass matrix, our knowledge of $\rho(g_e)$ will only allow the identification of the family of diagonalizing matrices of $\rho(g_e)$, and U_e must take a more general form

$$U_e = U_0 R_e(\phi, \gamma) \Phi,$$

where U_0 is any matrix which diagonalizes $\rho(g_e)$, $R_e(\phi, \gamma)$ is a complex rotation in the degenerate subspace of $\rho(g_e)$ by an angle ϕ with a phase γ and Φ is a diagonal matrix of phases.

In the neutrino sector, we have three constraints to consider on the mass terms: one from the flavor symmetry, one from the GCP symmetry and one ensuring their consistency. Under a change of flavor basis, the matrix X is mapped to

$$X \rightarrow U^\dagger X U^*.$$

As X is unitary and symmetric, Takagi factorization allows us to express it as $X = \Omega \Omega^T$ for some unitary matrix Ω which implies that we can choose a basis where X becomes trivial [27]. In fact, this basis is not unique and the remaining freedom can be used to further diagonalize $\rho(g_\nu)$. In this basis the constraint in Eq. (6) implies that the mass matrix is real valued

$$(\Omega^T m_\nu \Omega)_{\alpha\beta} \in \mathbb{R}.$$

As $\rho(g_\nu)$ is diagonal and commutes with this matrix, we know that the mass matrix in this basis must be diagonal up to a basis change in the degenerate subspace of $\rho(g_\nu)$. As it is purely real, the most general additional basis transformation required to bring it into diagonal form is a rotation in two dimensions

$$U_\nu = \Omega R_\nu(\theta),$$

where θ is the angle describing the real rotation. There remains the possibility that the diagonal mass matrix is not positive definite, in which case a diagonal rephasing must occur. Without further knowledge of the mass matrix this cannot be predicted, and in consequence, the Majorana phases can be predicted only up to $\pm\pi$ or multiples thereof.³

We see that the GCP symmetry in the neutrino sector has specified a special basis in which the residual flavor-symmetry elements are diagonal and the mass matrix is

³We always work with the Particle Data Group parametrization of the PMNS matrix [52], in which the Majorana phases are defined by the diagonal matrix $\text{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}})$ which take physical values on the intervals $\alpha_{ij} \in [0, 2\pi)$.

real. In this way, GCP symmetries help to fix some of the rephasing degrees of freedom associated with diagonalizing matrices and allow for the prediction of Majorana phases. Combining the results for the charged-lepton sector and the neutrino sector, we find the full PMNS matrix is given by

$$U_{\text{PMNS}} = \Phi R_e(\phi, \gamma) U_0^\dagger \Omega R_\nu(\theta), \quad (9)$$

where R_e and R_ν denote two unspecified rotations ($R_e = 1$ if $\text{ord}(g_e) > 2$). We make two further simplifications: Φ is removed by rephasing the charged leptons, and we note that the angles θ and ϕ need only be defined over the interval $\theta, \phi \in [0, \pi)$, as shifts by π can be absorbed by unphysical redefinitions of the complex phases.

III. MIXING PATTERNS FROM A_5

The preceding section showed how the assumed residual flavor and GCP symmetries can lead to expressions for the PMNS matrix. In this section, we derive the possible mixing matrices which arise by this method for the group A_5 . First we discuss the structure of A_5 and the possible subgroups eligible to be taken as residual symmetries. Then we derive the form of the most general GCP transformation. In the subsequent subsections, we consider all viable combinations of CP and residual subgroups and present those patterns which are consistent with the current global data [25].

A. Subgroups and GCP symmetries

The group A_5 can be defined as the group of even permutations on five elements. It has the abstract presentation

$$\langle S, T | S^2 = T^5 = (ST)^3 \rangle,$$

where S and T are the *generators* of the group and all group elements can be expressed by a word made from these distinguished elements. The structure of this group and its representation theory have been discussed in the physics literature before (see e.g. Refs. [44] and [48]) and we abstain from deriving the explicit representations, deferring the reader to these references instead. However, we briefly recap those features of the group and its representations most pertinent to our subsequent analysis.

We assume that the lepton doublets are assigned to a three-dimensional representation. A_5 has two distinct three-dimensional irreducible representations, and in the following we always work with the representation 3 of Ref. [48]. We have checked that the final results do not change if we choose the alternative three-dimensional representation instead. For our chosen representation, the generators S and T can be expressed by

$$S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } T = \frac{1}{2} \begin{pmatrix} 1 & -\varphi & -\varphi_g \\ \varphi & -\varphi_g & -1 \\ -\varphi_g & 1 & \varphi \end{pmatrix},$$

where $\varphi_g = \frac{1-\sqrt{5}}{2}$ represents the Galois conjugate⁴ of φ . We note that this is a real representation and in our chosen basis all group elements are real; it also forms a subgroup of $SU(3)$.

The nonidentity elements of A_5 are either of order 2, 3 or 5 and can be partitioned into four conjugacy classes: one of order 2 elements (15 members), one of order 3 elements (20 elements) and two of order 5 elements (12 members each). The center of A_5 is trivial, and the identity alone forms one additional conjugacy class.

As described in Sec. II, to compute the PMNS matrix from residual symmetries, we must first find U_e , the matrix which diagonalizes the generator of the residual symmetry of the charged-lepton mass term. To do so, we must identify the eligible residual symmetry groups for G_e . This symmetry must be an Abelian subgroup of A_5 , of which there are four kinds (up to isomorphism). Three of these subgroups are cyclic groups: \mathbb{Z}_2 , \mathbb{Z}_3 and \mathbb{Z}_5 . These are the groups generated by a single element g , and its members are those powers of g less than or equal to its order

$$\langle g \rangle = \{g^n | \text{s.t. } n \leq \text{ord}(g)\}.$$

As any element can be taken as the generator of a cyclic group, there are 15 distinct subgroups of \mathbb{Z}_2 in A_5 , ten of \mathbb{Z}_3 and six of \mathbb{Z}_5 . The diagonalizing basis for these groups is simply the basis which diagonalizes the generator $\rho(g)$.

In addition to the cyclic subgroups, there are also noncyclic Abelian subgroups in A_5 . These are isomorphic to the Klein four group $\mathbb{Z}_2 \times \mathbb{Z}_2$ which is generated by distinguished pairs of order-2 elements,

$$\langle g_1, g_2 | g_1^2 = g_2^2 = (g_1 g_2)^2 \rangle.$$

In fact, the 15 order-2 elements in A_5 can be divided into five triplets which (with the identity) define distinct four-element groups. For these noncyclic groups, the diagonalizing basis is defined as that which diagonalizes the two generators simultaneously.

Therefore, the different choices for the residual symmetry of the charged-lepton mass term can be divided into four categories depending on the preserved subgroup: $G_e \in \{\mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_5, \mathbb{Z}_2 \times \mathbb{Z}_2\}$. For the residual symmetry of the neutrino Majorana mass term, we are restricted to taking subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_2$, leaving us with two options: a single \mathbb{Z}_2 or the full Klein group $\mathbb{Z}_2 \times \mathbb{Z}_2$.

⁴In this case Galois conjugation exchanges the two solutions of the minimal polynomial over the rationals $x^2 - x - 1 = 0$.

B. Deriving X

The matrix X which implements the generalized CP symmetry, as discussed in Sec. II, must satisfy $XX^* = 1$ and be related to a class-inverting automorphism of the group,

$$\forall g \in A_5, \exists h_g \in A_5 \quad \text{s.t.} \\ (X^* \rho(g) X)^* = \rho(h_g^{-1}) \rho(g^{-1}) \rho(h_g),$$

where ρ is our chosen irreducible representation, generated by the matrices S and T . We derive the most general form of X for the group A_5 by exploiting our knowledge of the automorphism structure of the group. The automorphism group of A_5 is S_5 (see e.g. [53]), and we identify two important subgroups: *inner* and *outer* automorphisms. The inner automorphism group, $\text{Inn}(A_5)$, comprises those automorphisms which can be represented by conjugation by a group element,

$$\phi_h \in \text{Inn}(A_5) \Leftrightarrow \forall g \in A_5, \quad \phi_h(g) = h^{-1}gh.$$

This group can be found by considering the map from element ($h \in A_5$) to inner automorphism ($\phi_h(g) = h^{-1}gh$), and applying the first isomorphism theorem,

$$\text{Inn}(A_5) \cong A_5 / \mathcal{Z}(A_5) \cong A_5,$$

where the final step uses the fact that A_5 has trivial center, $\mathcal{Z}(A_5) = 1$. Therefore, the inner automorphisms of A_5 are given by A_5 itself. The outer automorphism group is defined as the quotient of the full automorphism group by the inner automorphism group. For A_5 it follows from our discussion above that this is the unique group of two elements \mathbb{Z}_2 . Our derivation of X is greatly simplified by A_5 being an *ambivalent* group, where each element is conjugate to its inverse. For such groups, the class-inverting automorphisms are also class preserving. All inner automorphisms of a group are class preserving, but the two properties are not equivalent as there do exist class-preserving outer automorphisms for some groups [54,55]. However, for the case of A_5 we have a single nontrivial outer automorphism to check, and this automorphism maps elements of order 5 from one conjugacy class to the other. Therefore, in the present case, we conclude that the class-preserving automorphisms are precisely the inner automorphisms.

We can therefore simplify our defining constraint on X ,

$$\exists h \in A_5, \forall g \in A_5 \quad (X^* \rho(g) X)^* = \rho(h^{-1}) \rho(g) \rho(h),$$

where the element h is the same for all elements g . As we are working with a real representation, we can always change basis so that all group elements are given by real matrices, and we use this fact with Eq. (5) to make further simplifications

$$\forall g \in A_5 \quad X \rho(g) X^* = \rho(h^{-1}) \rho(g) \rho(h),$$

which is equivalent to a commutation relation,

$$[\rho(h)X, \rho(g)] = 0.$$

We can then invoke Schur's lemma to infer that as $\rho(h)X$ commutes with all the elements of an irreducible representation, it must be a scalar matrix: $\rho(h)X = \lambda 1$, for some complex constant λ . Requiring that $XX^* = 1$ constrains $\rho(h^2) = 1/|\lambda|^2$. However, by closure the element on the left must be a member of A_5 and, as our representation is unitary, we conclude that λ is just a complex phase, $\lambda = e^{i\theta}$ for $\theta \in \mathbb{R}$. Therefore, h must be an order-2 element, and the most general form of X which implements an involutory class-inverting automorphism for A_5 is given by

$$X = e^{i\theta} \rho(h) \quad \text{s.t.} \quad \text{ord}(h) = 2.$$

In our basis, the consistency relation in Eq. (8) implies that the X matrix must commute with the generator S of the residual \mathbb{Z}_2 symmetry in the neutrino sector. Therefore not all choices of h can be consistently implemented, and there will be only three nontrivial X matrices (up to global phases) for any given S . These are the three elements of the Klein four group associated with S . If we work in the basis where this group is diagonal, we find that

$$X_1 = e^{i\theta} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

and X_2 and X_3 can be defined as permutations of this matrix, where the row of the positive entry is denoted by the subscript. It is necessary for us to find the basis in which X is trivial. The necessary change of basis is given for X_i by Ω_i , where

$$\Omega_1 = e^{i\theta/2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{pmatrix},$$

and similar definitions hold for $i = 2$ and $i = 3$. In this basis, the generator of the residual symmetry in the neutrino sector is diagonal and the GCP action is trivial. However, due to the degenerate subspace in the \mathbb{Z}_2 generator, it remains possible that the neutrino mass matrix is only block diagonal and requires an orthogonal transformation to fully diagonalize it. This rotation must be in the plane of the degenerate subspace for the matrix S . Given these two elements, the most general form of the matrix which maps between neutrino flavor and mass bases is given by

$$U_\nu = \Omega R(\theta),$$

where $R(\theta)$ is an orthogonal matrix effecting a rotation in either the 12-, 13- or 23-plane. We also note at this point that the overall phase included in our definition of Ω can be seen to have no physical effect, and will be set to zero in what follows.

In the following subsections, we consider all possible residual symmetry groups in the charged-lepton sector. In the first three sections we consider the charged-lepton residual symmetry to be given by each of the 10 \mathbb{Z}_3 subgroups, 6 \mathbb{Z}_5 subgroups and 5 $\mathbb{Z}_2 \times \mathbb{Z}_2$ subgroups. For each of these subgroups, the basis is found which diagonalizes its elements, U_e . Due to the order of these subgroups, this diagonalizing matrix is predicted exactly with no remaining degrees of freedom [e.g. $R_e = 1$ in Eq. (9)]. The PMNS matrix is then constructed combining U_e with one of the forms of U_ν found above by consideration of the residual CP symmetry,

$$U_{\text{PMNS}} = U_e^\dagger \Omega R(\theta).$$

Finally in Sec. III F, we consider less restrictive symmetries when U_e is not fully specified by symmetry alone ($R_e \neq 1$).

For each configuration considered in this section, the arbitrariness in eigenvector ordering and phasing is accounted for by considering all permutations of rows and columns of the PMNS matrix. From these permuted matrices, we compute the mixing angles and phases, and these are compared to global data. We report all patterns of mixing angles found by this process which are consistent with the current 3σ regions as reported in Ref. [25].

C. Predictions from $G_e = \mathbb{Z}_3$ and $G_\nu = \mathbb{Z}_2$

When the residual symmetry in the charged leptons is taken as \mathbb{Z}_3 , the diagonalizing matrix of the residual symmetry generator T is uniquely specified (up to diagonal rephasings and permutations). We have considered the 10 \mathbb{Z}_3 subgroups of A_5 which could act as the residual symmetry of the charged leptons. Although many different group elements lead to viable mixing patterns, all viable solutions can be described by a single matrix after a suitable permutation and redefinition of the unphysical parameters. This leads to a single viable set of correlations between the angles. The angles can be derived from the PMNS matrix,

$$U_{\text{PMNS}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta), \quad (10)$$

where $R_{13}(\theta)$ denotes a rotation in the 13-plane by an angle θ . This leads to the following expressions for the mixing angles:

$$\sin^2 \theta_{12} = \frac{1}{3-2\sin^2 \theta}, \quad \sin^2 \theta_{13} = \frac{2}{3} \sin^2 \theta,$$

$$\sin^2 \theta_{23} = \frac{1}{2}.$$

This pattern is continuously connected to the tribimaximal mixing pattern [56] which is recovered at $\theta = 0$, and is an explicit example of a trimaximal pattern [57] where $|U_{\alpha 2}| = 1/\sqrt{3} \forall \alpha \in \{e, \mu, \tau\}$. We have plotted these mixing angle predictions for the full range of the unobservable parameter θ in Fig. 1 in which the colored regions show the current 3σ global intervals from Ref. [25]. Although meeting our criterion of having mixing angles within the current 3σ limits, we point out that this pattern's prediction for θ_{12} is currently excluded at greater than 2σ .

The Dirac phase for this pattern depends discretely on the value of θ . It can be shown that

$$\delta = \begin{cases} \frac{3\pi}{2} & \theta \in (0, \frac{\pi}{2}), \\ \frac{\pi}{2} & \theta \in (\frac{\pi}{2}, \pi), \end{cases}$$

while the Majorana phases can be shown to take CP -conserving values $\{\alpha_{21}, \alpha_{31}\} \subseteq \{0, \pi\}$ for all values of θ . From Fig. 1, we see that there are two intervals of the unphysical parameter which lead to mixing angles which satisfy the current global 3σ bounds. Due to the symmetry of the expressions, these two solutions offer identical predictions for values of θ_{12} , θ_{13} and θ_{23} ; however, one of these values lies in a region with $\delta = \frac{\pi}{2}$ while the other predicts $\delta = \frac{3\pi}{2}$. Therefore, there are two sets of predictions from the order-3 elements, differing only in their prediction for the Dirac CP phase.

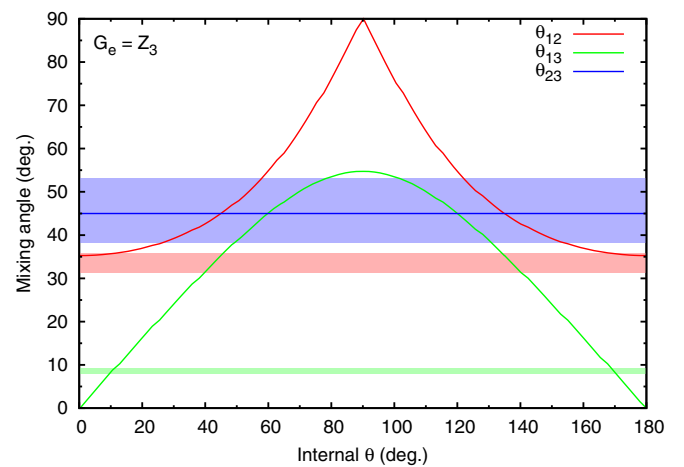


FIG. 1 (color online). Mixing angles for \mathbb{Z}_3 as a function of the internal parameter θ . This pattern predicts $|\sin \delta| = 1$ and $\sin \alpha_{21} = \sin \alpha_{31} = 0$. The shaded regions show the 3σ allowed region for the corresponding mixing angle according to current global data [25].

D. Predictions from $G_e = \mathbb{Z}_5$ and $G_\nu = \mathbb{Z}_2$

If the residual charged-lepton symmetry is assumed to be $G_e = \mathbb{Z}_5$, there are six subgroups which we need to consider which could lead to a viable set of mixing parameters. We find two distinct sets of correlations which are viable for some range of the unphysical parameter, leading to three distinct sets of mixing angle predictions.

The first set of predictions can be derived from the following matrix,

$$U_{\text{PMNS}} = \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & -\frac{i}{\sqrt{2+\varphi}} & 0 \\ -\frac{i}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{4+2\varphi}} & -\frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta), \quad (11)$$

which leads to mixing angles expressed by

$$\sin^2\theta_{12} = \frac{1}{1 + \varphi^2 \cos^2\theta}, \quad \sin^2\theta_{13} = \frac{\sin^2\theta}{1 + \varphi_g^2},$$

$$\sin^2\theta_{23} = \frac{1}{2}.$$

The mixing angle predictions from this pattern are shown on the left of Fig. 2 as a function of the unphysical parameter θ . In this case, the Dirac phase is maximally CP violating with $\cos\delta = 0$. However, as with the order-3 elements, the sign of $\sin\delta$ depends on the parameter θ ,

$$\delta = \begin{cases} \frac{3\pi}{2} & \theta \in (0, \frac{\pi}{2}), \\ \frac{\pi}{2} & \theta \in (\frac{\pi}{2}, \pi), \end{cases}$$

and the Majorana phases are again given by CP -conserving values, $\{\alpha_{21}, \alpha_{31}\} \subseteq \{0, \pi\}$, although the precise values cannot be determined in this framework.

The second viable pattern arising from \mathbb{Z}_5 is shown on the right panel of Fig. 2. This can be derived from a matrix similar to Eq. (11) but distinct in the relative phasing between the columns,

$$U_{\text{PMNS}} = \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ -\frac{i}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & -\frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta). \quad (12)$$

This relative phase difference, which arises from the choice of alignment between the matrix implementing the GCP symmetry X and the generator S of the residual \mathbb{Z}_2 symmetry of the neutrino mass term, crucially affects the mixing angle θ_{23} and leads to mixing angles which can be expressed by

$$\sin^2\theta_{12} = \frac{1}{1 + \varphi^2 \cos^2\theta}, \quad \sin^2\theta_{13} = \frac{\sin^2\theta}{1 + \varphi_g^2},$$

$$\sin^2\theta_{23} = \frac{1}{2} \frac{(\sin\theta + \sqrt{1 + \varphi^2 \cos^2\theta})^2}{1 + \varphi^2 \cos^2\theta}.$$

All CP phases take CP -conserving values for this pattern of mixing parameters. The precise value of the Dirac phase again depends on θ ,

$$\delta = \begin{cases} 0 & \theta \in (0, \frac{\pi}{2}), \\ \pi & \theta \in (\frac{\pi}{2}, \pi). \end{cases}$$

The appearance of CP conservation can be explained as, although it was not imposed explicitly, the generalized CP symmetry remains accidentally unbroken in the charged-lepton sector. This second pattern leads to two distinct allowed intervals in θ once we restrict the mixing angles to lie in the current 3σ intervals. Due to the symmetry of the

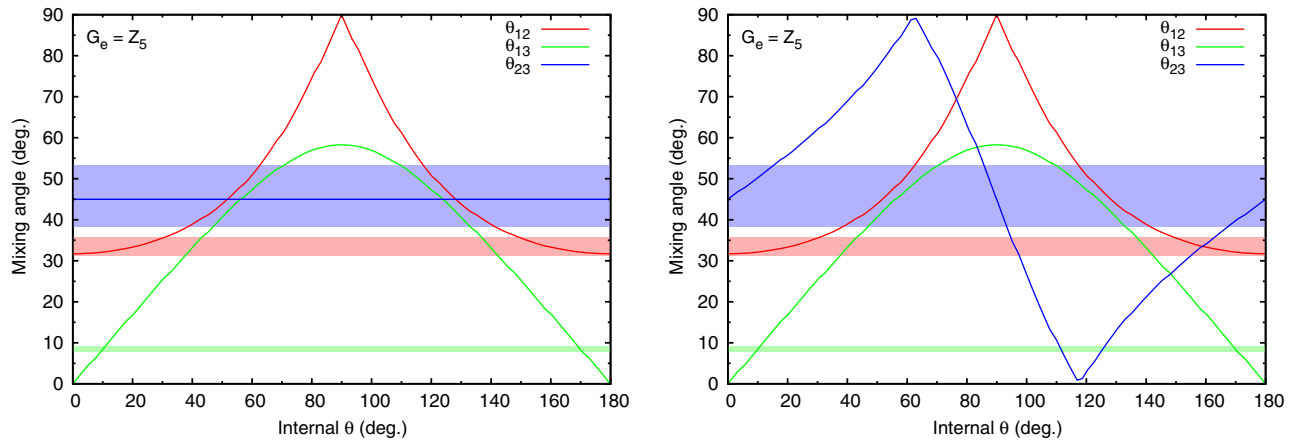


FIG. 2 (color online). The two patterns of mixing angles for \mathbb{Z}_5 as a function of the internal parameter θ . Both patterns predict $\sin\alpha_{21} = \sin\alpha_{31} = 0$. The right pattern predicts $|\sin\delta| = 0$ while the left pattern predicts $|\sin\delta| = 1$. The shaded regions show the 3σ allowed region for the corresponding mixing angle according to current global data [25].

curves, these two viable sets of mixing angles are distinguished only by their predictions for θ_{23} .

Both of the above mixing patterns are continuous extensions of the well-known GR mixing pattern (also known as GR1 or GRA) [45,46], which is found at $\theta = 0$. If we expand in the small parameter $r \equiv \sqrt{2} \sin \theta_{13}$ [11], we find that both patterns arising from \mathbb{Z}_5 lead to the prediction for $s \equiv \sqrt{3} \sin \theta_{12} - 1$ [11] given by

$$s = \sqrt{\frac{3}{2+\varphi}} - 1 + \sqrt{\frac{3}{2+\varphi}} \frac{r^2}{4} + \mathcal{O}(r^4). \quad (13)$$

The two patterns are distinguished by their predictions for θ_{23} ; however, both can be seen as subcases of a more general model predicting the θ_{12} correlation in Eq. (13) and further obeying a linearized atmospheric sum rule first derived in Ref. [17],

$$a = \frac{1-\varphi}{\sqrt{2}} r \cos \delta + \mathcal{O}(r^2, a^2).$$

From this relation, it is clear how the maximal angle of the first pattern $a = 0$ is associated with the vanishing of $\cos \delta$, while the contrasting nontrivial predictions of θ_{23} in the second pattern are due to the CP -conserving value of δ , $\sin \delta = 0$. However, the correlation of maximal atmospheric mixing and CP -conserving values of δ is not an artefact of linearization and holds exactly, as is shown in the right panel of Fig. 2. We consider these correlated maximal predictions as a measurable signature in Sec. IVB.

E. Predictions from $G_e = \mathbb{Z}_2 \times \mathbb{Z}_2$ and $G_\nu = \mathbb{Z}_2$

The only noncyclic Abelian subgroup in A_5 is the Klein four group $\mathbb{Z}_2 \times \mathbb{Z}_2$. If we take this as the residual symmetry in the charged-lepton sector, we find two patterns of mixing angles which differ only by their predictions for θ_{23} . Consistent predictions exist for all of the 3σ range of θ_{13} , and the predictions for the other mixing angles can be seen in Fig. 3.

The first pattern can be derived from the following mixing matrix:

$$U_{\text{PMNS}} = \frac{1}{2} \begin{pmatrix} \varphi & \varphi_g & -1 \\ \varphi_g & 1 & -\varphi \\ -1 & -\varphi & \varphi_g \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix}. \quad (14)$$

From this matrix, we find the mixing angles can be expressed by

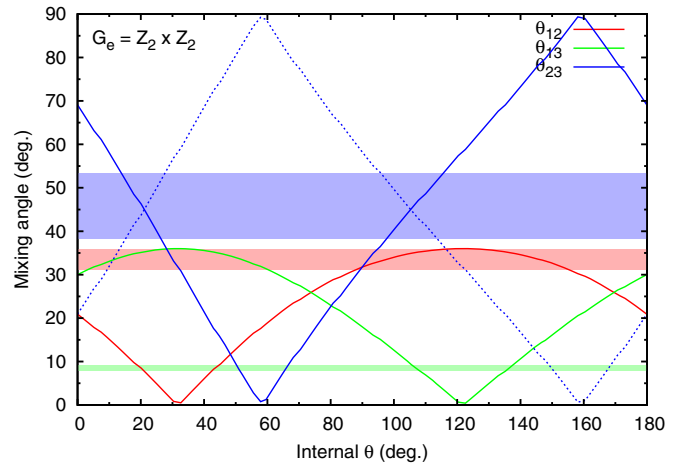


FIG. 3 (color online). Allowed mixing angles for $G_e = \mathbb{Z}_2 \times \mathbb{Z}_2$ as a function of the unphysical parameter θ . There are two possible sets of predictions of the mixing angles which have the same θ_{12} and θ_{13} predictions but distinct θ_{23} predictions (solid and dotted lines) related by the mapping $\theta_{23} \rightarrow \frac{\pi}{2} - \theta_{23}$. All complex phases are CP conserving for these patterns: $\sin \delta = \sin \alpha_{21} = \sin \alpha_{31} = 0$. The shaded regions show the 3σ allowed region for the corresponding mixing angle according to current global data [25].

$$\begin{aligned} \sin^2 \theta_{12} &= \frac{1 + \varphi_g [\cos^2 \theta + \sin(2\theta)]}{3 - \varphi_g [\sin^2 \theta - \sin(2\theta)]}, \\ \sin^2 \theta_{13} &= \frac{1 + \varphi_g [\sin^2 \theta - \sin(2\theta)]}{4}, \\ \sin^2 \theta_{23} &= \frac{1 + \varphi [\cos^2 \theta - \sin(2\theta)]}{3 - \varphi_g [\sin^2 \theta - \sin(2\theta)]}. \end{aligned}$$

For this pattern the complex phases are given by CP -conserving values: $\sin \delta = 0$ and $\{\alpha_{21}, \alpha_{31}\} \subseteq \{0, \pi\}$. The true value of δ can be shown to depend on θ ,

$$\delta = \begin{cases} 0 & 31.7^\circ < \theta < 58.3^\circ \text{ or } 121.7^\circ < \theta < 159.1^\circ, \\ \pi & \text{else.} \end{cases}$$

This dependence on θ looks complex, but the boundaries of the $\delta = 0$ regions can be seen in Fig. 3 to be those values of θ for which one mixing angle is either 0° or 90° , and closed form expressions can be derived for these values from the mixing angle formulas above. For the matrix shown here, this means that only the $\delta = \pi$ solution agrees with the global data. However, when considering all permutations the alternative CP -conserving solution can also be found.

The prediction for θ_{23} can be expressed as an atmospheric sum rule to first order in r ,

$$a = \sqrt{\frac{2}{1+\varphi^2}} - 1 + \frac{\varphi}{1+\varphi_g^2} r + \mathcal{O}(r^2).$$

This relation, being derived from a noncyclic symmetry in the charged-lepton sector, has to the best of our knowledge

not been presented before in general analyses of atmospheric sum rules [17].

A permutation in the θ_{23} plane acting from the left of the PMNS matrix effects a mapping of $\theta_{23} \rightarrow \frac{\pi}{2} - \theta_{23}$. For this reason, both of the expressions defined above have a complementary pattern with an inverted θ_{23} . These alternative patterns are shown by dashed lines in Fig. 3.

F. Predictions with two and three degrees of freedom

So far we have analyzed the cases when $G_e \in \{\mathbb{Z}_3, \mathbb{Z}_5, \mathbb{Z}_2 \times \mathbb{Z}_2\}$ and $G_\nu = \mathbb{Z}_2$. The patterns resulting from these groups have a single degree of freedom (d.o.f.), the unphysical angle θ , controlling their mixing parameter predictions. There are, however, more general scenarios where the three angles and three phases of the PMNS matrix are specified by two or three input parameters.

The cases with two degrees of freedom arise from the choice $G_e = \mathbb{Z}_2$ while the neutrino symmetry is enlarged to the full Klein group, $G_\nu = \mathbb{Z}_2 \times \mathbb{Z}_2$. The symmetry of the charged-lepton mass terms is insufficient to uniquely specify the diagonalizing matrix of the mass matrix, and requires in general a further two-dimensional complex rotation,

$$R_e(\theta, \gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta e^{i\gamma} \\ 0 & -\sin \theta e^{-i\gamma} & \cos \theta \end{pmatrix}.$$

However, the neutrino symmetry has been enlarged, and so U_ν is uniquely specified by the symmetry generators. We have scanned over all such combinations and found that no viable patterns arise from this scenario.

There is one more combination of residual symmetries possible in our construction: $G_e = \mathbb{Z}_2$ and $G_\nu = \mathbb{Z}_2$. This is an extension of the previous case, where the charged-lepton symmetry introduces two parameters but now the neutrino residual symmetry also requires a single real parameter to diagonalize the most general mass matrix. In principle, there is no reason to discount these patterns. They are consistent with the idea that the full flavor group has broken into residual subgroups implying correlations on the flavor observables. However, the increased number of parameters reduces the predictivity of the theory (three inputs, six outputs). We have scanned over such groups and verified that there are eligible patterns which match all the global data. Some of these patterns feature nonconstant Majorana and Dirac phase predictions, and many are not simply related to the patterns that we have found in more restrictive schemes. However, due to the larger parameter space and reduced predictivity, we do not attempt to present any results of this type.

IV. PHENOMENOLOGICAL PROSPECTS

In the preceding sections we have derived all patterns of mixing angles and phases which are possible with an A_5

symmetry with generalized CP broken into residual symmetries. They depend upon a single real angle, θ , and can all be brought into agreement with current global data [25] for a suitable restriction of its range. Eliminating the unphysical parameter θ leads to a set of correlated predictions between observables which are testable by oscillation experiments and searches for neutrinoless double-beta decay. In this section, we discuss the prospects for present and future experiments to constrain these patterns and derive simple versions of the predicted parameter correlations which may be useful experimentally. We stress that the correlations identified in this paper will be tested at almost every stage in the experimental program of the next few decades. Near term results from T2K [58] and NO ν A [59] can be expected on the maximality of θ_{23} and δ , and in the medium term, new reactor and long-baseline experiments such as JUNO [60], RENO-50 [61], DUNE,⁵ T2HK [62] and possibly ESS ν B [63] should bring us increased precision on θ_{12} , θ_{23} and δ . Finally, for the most stringent tests of the models in question, the option remains to construct a more ambitious facility such as the Neutrino Factory (NF) [64]. In a complementary direction, neutrinoless double-beta decay experiments will further sensitivity to this decay, providing evidence on the Majorana nature of neutrinos and, in combination with precise measurements of the neutrino masses, e.g. from cosmological studies, could potentially constrain the value of one of the Majorana phases. Many of these observations are largely independent and we can expect significant evidence either in favor of, or ruling out, the patterns identified in this paper.

A. Precision measurements of θ_{12}

The viable sets of mixing parameters which we have found above predict correlations in θ_{12} and θ_{13} , and therefore very precise measurements of these angles have the potential to discriminate between flavor-symmetric patterns, or to rule them out entirely [23].

The upcoming medium-baseline reactor (MR) neutrino oscillation experiments, such as JUNO [60] and RENO-50 [61], expect to make very precise, subpercent measurements of the oscillation parameter θ_{12} . The precision on θ_{13} , currently dominated by measurements from Daya Bay [2] and RENO [3], is not expected to be significantly improved by the next generation of reactor facilities. Therefore, the first significant test of the predictions of this paper will come from increased precision on θ_{12} independently of θ_{13} . We have identified three distinct predictions for θ_{12} ; if we fix θ_{13} to its current best fit [25] these are

$$\theta_{12} = 35.71^\circ, \quad \theta_{12} = 32.11^\circ, \quad \theta_{12} = 35.14^\circ,$$

for preserved charged-lepton subgroups \mathbb{Z}_3 , \mathbb{Z}_5 and $\mathbb{Z}_2 \times \mathbb{Z}_2$, respectively. Given that the expected precision

⁵The new name for the LBNF/ELBNF project.

of the MR experiments is at the level of 0.1° or around 0.3% for θ_{12} , a strong discriminatory power exists between the values of the mixing angles predicted by these correlations. The difference between the predicted values of all models under consideration is always greater than 0.26° over the current 3σ interval for θ_{13} , and in many cases significantly greater. Therefore we can expect these experiments to identify with considerable confidence if any of the charged-lepton residual symmetries are consistent with observation.

In the framework discussed in this article, each model predicts a continuous correlation between the values of θ_{13} and θ_{12} . If one of the predictions above appears to agree with data, it would be desirable to test the correlation between parameters itself. These correlations can be conveniently expressed as expansions in the dimensionless parameter $r \equiv \sqrt{2} \sin \theta_{13}$ [11]. The current global best fits give $\theta_{13} \approx 8.50^\circ$ [25] which translates to $r \approx 0.2$; the second-order corrections are therefore suppressed by a factor of $1/25$. Expressed in this way, the predictions for $\sin \theta_{12}$ associated with the charged-lepton subgroups \mathbb{Z}_3 , \mathbb{Z}_5 and $\mathbb{Z}_2 \times \mathbb{Z}_2$ (respectively) can be expanded in the following relations,

$$\begin{aligned} \sin \theta_{12} &= \frac{1}{\sqrt{3}} \left(1 + \frac{r^2}{4} \right) + \mathcal{O}(r^4), \\ \sin \theta_{12} &= \frac{1}{\sqrt{1 + \varphi^2}} \left(1 + \frac{r^2}{4} \right) + \mathcal{O}(r^4), \\ \sin \theta_{12} &= \frac{\sqrt{2 + \varphi_g}}{2} - \frac{2 - \varphi_g}{\sqrt{2 + \varphi_g}} \frac{r^2}{8} + \mathcal{O}(r^4). \end{aligned}$$

Expressing these in terms of the angles themselves, we find

$$\theta_{12} = 35.27^\circ + 10.13^\circ r^2 + \mathcal{O}(r^4), \quad (15)$$

$$\theta_{12} = 31.72^\circ + 8.85^\circ r^2 + \mathcal{O}(r^4), \quad (16)$$

$$\theta_{12} = 36.00^\circ - 19.72^\circ r^2 + \mathcal{O}(r^4). \quad (17)$$

The approximations in Eqs. (15)–(17) have been plotted against the unapproximated expressions for θ_{12} in Fig. 4. We see that these relations depend only slightly on θ_{13} , which first appears at the order $\mathcal{O}(r^2)$, leading to subdegree-level corrections.

The formulas above show that the predictions for θ_{12} only vary by 0.07° , 0.06° and 0.13° (for \mathbb{Z}_3 , \mathbb{Z}_5 and $\mathbb{Z}_2 \times \mathbb{Z}_2$, respectively) over the current 3σ region for θ_{13} . This is of the order of the target precision of the MR experiments, and it is therefore unlikely that the θ_{12} – θ_{13} correlations themselves will be tested at a significant level even if precision on θ_{13} were to be greatly improved. There are no currently planned facilities which could further improve the precision on θ_{12} .

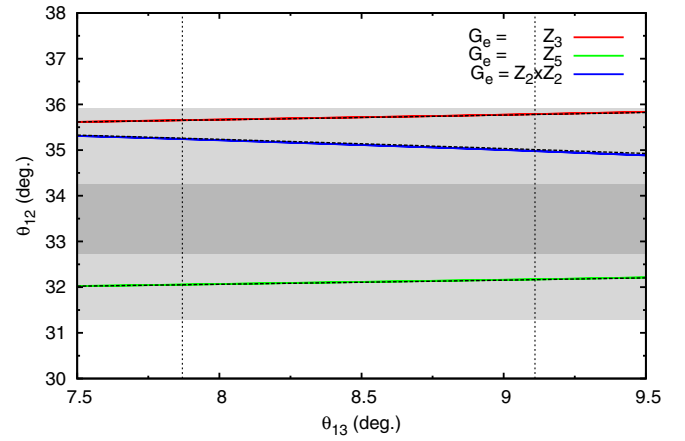


FIG. 4 (color online). Predictions for θ_{12} as a function of θ_{13} . All of the patterns of mixing parameters associated with a given charged-lepton residual symmetry have the same prediction (solid lines). The dashed line close to each prediction shows the linearized predictions in Eqs. (15)–(17). The grey regions show the 1 and 3σ allowed regions for θ_{12} and the vertical lines show the 3σ range for θ_{13} from current global data [25].

B. Maximal-maximal predictions for θ_{23} and δ

The current and upcoming generation of long-baseline experiments will be able to place important constraints on the parameters θ_{23} and δ . Measuring θ_{23} and δ independently will provide valuable information on the viability of flavor-symmetric models; however, in the patterns that we have identified the maximality of θ_{23} is significantly correlated with the value of δ . In four of these patterns (excluding different Majorana phase assignments), two from $G_e = \mathbb{Z}_3$ and two from $G_e = \mathbb{Z}_5$ predict a maximal value of θ_{23} and a maximal amount of CP violation,

$$\theta_{23} = \frac{\pi}{4} \quad \text{and} \quad |\sin \delta| = 1.$$

Therefore, the joint determination of these parameters around these maximal values would be a particularly interesting measurement from the point of view of GCP model building.

Testing the maximality of these parameters is an attainable goal for current and future oscillation experiments. After its full period of data taking, T2K expects to be able to exclude maximal θ_{23} at the 90% C.L. for $|\sin^2(2\theta_{23}) - 0.5| > 0.05$ – 0.07 largely independently of the value of δ [58]. Measuring δ itself is significantly harder; however, the maximal CP -violating values considered here are the most accessible. T2K can expect to be able to exclude $0 \lesssim \delta \lesssim \pi$ ($\pi \lesssim \delta \lesssim 2\pi$) at the 90% C.L. for a true value of $\delta = 3\pi/2$ ($\delta = \pi/2$) [58]. This would allow T2K to distinguish between $\delta = \pi/2$ and $\delta = 3\pi/2$ if one of them is true at least the 90% C.L. NO ν A can also be expected to contribute to this measurement [59] with a similar power for excluding $\delta = \pi/2$ and $\delta = 3\pi/2$.

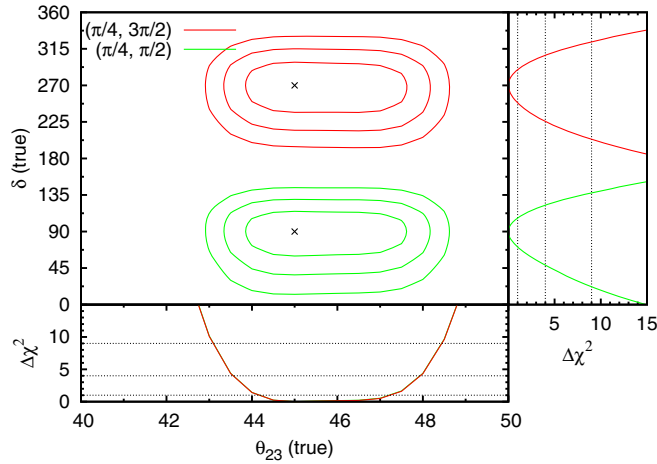


FIG. 5 (color online). Red (green) lines show the exclusion regions at 1, 2 and 3σ for $\theta_{23} = \pi/4$ and $\delta = 3\pi/2$ ($\delta = \pi/2$) expected at LBNF with a 34 kton LAr detector after 5 + 5 years running. In this region outside the curves, the two sets of predictions can be excluded at the given confidence. The side panels show the appropriate marginalized $\Delta\chi^2$ and the 1, 2 and 3σ confidence levels (one d.o.f).

Although these exclusions are expected to be individually statistically weak, they would constitute valuable information on the validity of the models studied in this paper and would provide strong encouragement for future work by the next generation of oscillation experiments.

In the medium term, new long-baseline experiments are expected with significantly improved sensitivities, in particular to the phase δ , allowing the maximal-maximal predictions to be further tested. To estimate the potential for excluding these models with future facilities, we have run a simulation of LBNE/DUNE using the GLOBES package [65]. Our simulation is based on the detector responses files and fluxes made available by the LBNE collaboration in Ref. [66]. We point out that thanks to its more ambitious design, it seems likely that the DUNE project can significantly improve the sensitivity computed here.⁶ However, without access to updated experimental information, making a quantitative assessment of the extent of this improvement is challenging. We assume a 700 kW beam operating at 120 GeV, a detector based on liquid argon-TPC technology with a mass of 34 kton, and overall systematic errors of 5% for both the signal and background normalizations.⁷ The results of these simulations are shown in Fig. 5, where we present the regions of true parameter space for which the combinations of $(\theta_{23}, \delta) = (\pi/4, \pi/2)$ and $(\pi/4, 3\pi/2)$ can be excluded after five years neutrino

⁶For the latest information on the DUNE design see Ref. [67].

⁷Over the lifetime of our simulation, the experiment expects around 795 (174) $\nu_\mu \rightarrow \nu_e$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$) charged-current events for $\delta = 0$. This is roughly equivalent to the rates for the LBNE-LE configuration described in Ref. [68] with a total exposure of 180 kton yr.

and five years antineutrino running. We find that these patterns can be excluded at 3σ if the true value of θ_{23} satisfies $\theta_{23} \lesssim 43.0^\circ$ or $\theta_{23} \gtrsim 48.3^\circ$, or if the true value of δ is outside the intervals $90^{+48^\circ}_{-69^\circ}$ or $270^{+53^\circ}_{-67^\circ}$. Here we see the importance of observing both θ_{23} and δ for excluding our models. A measurement of θ_{23} alone will not be able to distinguish between the models which predict θ_{23} -maximality and the model from $\mathbb{Z}_2 \times \mathbb{Z}_2$ which predicts values of θ_{23} which differ from maximality by only around 2° . However, these models have maximally distinct predictions for δ , and as we have shown, the measurement of δ alone would be able to separate these cases at 3σ .

C. Dirac CP -conserving patterns and precision measurements of θ_{23}

Those patterns which instead make nonmaximal predictions of θ_{23} also predict CP -conserving values of δ , such that $|\cos \delta| = 1$, and we can expect constraints to be placed on these models by the attempts to discover leptonic CP violation—a standard search for the next-generation of CP -sensitive oscillation experiments [62,63,68–71]. It has been shown that LBNO running with a beam derived from the SPS accelerator at CERN could rule out leptonic CP conservation at 3σ for around 45% (65%) of the parameter space for a detector mass of 20 kton (70 kton). This could be increased to 70% (80%) with an upgraded beam power [70]. LBNE has predicted a similar sensitivity to CP violation [68], with the ultimate reach also depending crucially on the planned series of upgrades to detector mass and beam power. With a 10 kton detector and six years of data using a 1.2 MW beam, the measurement could be made for 33% of the parameter space at 3σ . This rises to 40% of the parameter space at 5σ once the detector mass has been increased to 34 kton and six years more data has been collected. Finally, a beam power upgrade to 2.3 MW could increase this to 60% of the parameter space at 5σ [68]. The T2HK and ESS ν B proposals also show strong sensitivity to CP violation, both using a megaton-scale water Čerenkov detector and MW power beams. T2HK has shown that it can expect a discovery of CP violation over 76% (58%) of the parameter space at 3σ (5σ) [72]. A similar reach is possible with ESS ν B, which expects a 3σ (5σ) discovery of CP violation after ten years of data taking over 74% (50%) of the parameter space [63].

If the current experimental program fails to discover CP violation in the leptonic sector and $\sin \delta$ is discovered to be small, there are four distinct patterns from our model which would remain in agreement with the data. These models can be tested by the increased precision on measurements of θ_{23} expected from next-generation long-baseline facilities. In all cases of this kind, we find predictions coming in pairs. The model associated with a \mathbb{Z}_5 residual symmetry predicts

$$\theta_{23} = 45^\circ \pm 25.04^\circ r + \mathcal{O}(r^2), \quad (18)$$

while the predictions for a residual $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry are given by

$$\begin{aligned}\theta_{23} &= 31.72^\circ + 55.76^\circ r + \mathcal{O}(r^2), \\ \theta_{23} &= 58.28^\circ - 55.76^\circ r + \mathcal{O}(r^2).\end{aligned}\quad (19)$$

The pairs of sum rules given by Eq. (18) or (19) are related by the octant degeneracy, $\theta_{23} \rightarrow \frac{\pi}{2} - \theta_{23}$, as can be seen clearly in Fig. 6 where the approximations above are shown against the full predictions. We note that in contrast to those for θ_{12} , these relations depend on r at linear order. Therefore, they are far more sensitive to the precise correlation between parameters, and the measurement of the correlation itself becomes more accessible.

The first discriminating factor between these solutions will come from improved precision on θ_{23} and the resolution of the octant degeneracy. In these models we predict nonmaximal mixing, and a successful determination of the octant would provide early evidence in their favor. This would be most challenging for the model based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ for which θ_{23} differs from 45° by between 2.5° and 0.8° over the current 3σ range of θ_{13} . The model based on \mathbb{Z}_5 instead predicts greater deviations from maximal atmospheric mixing, ranging between 4.8° and 5.6° over the same interval. Studies of the potential for the current generation of oscillation experiments, of which T2K and NO ν A play the most important role, suggest that the octant can be established at 3σ (2σ) for deviations from maximality greater than around 6° (4°) [73,74]. This precludes the current generation from separating between the two predictions of $\mathbb{Z}_2 \times \mathbb{Z}_2$, but would allow for 2σ evidence for those predictions coming from our model based on \mathbb{Z}_5 . This discovery potential will be improved by the next

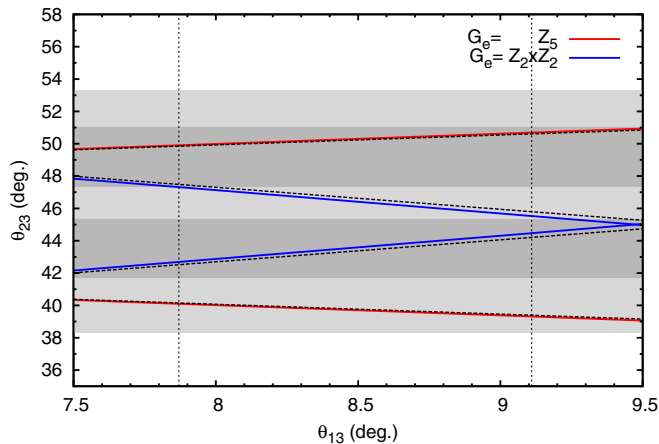


FIG. 6 (color online). θ_{23} as a function of θ_{13} for the patterns which predict CP conservation (solid lines). The dashed line close to each solid line shows the linearized expression in Eqs. (18)–(19). The grey regions show the 1 and 3σ allowed regions for θ_{23} while the vertical lines show the 3σ range for θ_{13} from current global data [25].

generation of oscillation experiments. In Ref. [68] it is shown that with an exposure of 60 kton years, LBNE could determine the octant at 3σ if the true value of θ_{23} deviates from maximality by more than 4° – 5° . However, the best bounds could come from T2HK by studying atmospheric neutrino data. A 3σ determination of the octant is expected to be possible after ten years of data taking for true values $|\sin^2\theta_{23} - 0.5| > 0.04$ – 0.06 corresponding to deviations between 2° and 3° [62]. Although exclusion of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ pattern would be unlikely, the two predictions from \mathbb{Z}_5 would be distinguishable.

To go beyond the octant measurement, higher precision will be necessary to separate between the \mathbb{Z}_5 and $\mathbb{Z}_2 \times \mathbb{Z}_2$ predictions, or indeed to test their specific correlations with θ_{13} . The difference between these two predictions varies between 2.4° and 4.8° over the current allowed regions. Therefore degree-level precision will be required to distinguish between them, even in the presence of greatly improved knowledge of θ_{13} . In Ref. [68] it is shown that the minimal 10 kton LBNE configuration running for six years would have a precision of around 1° at 1σ for true values of θ_{23} around 51° , which increases as we approach θ_{23} maximality to a 1σ width of around 2.5° . Similarly, T2HK shows that around the point expected to give the worst sensitivity to θ_{23} , the 90% C.L. width is around 2° – 3° [62]. These results suggest that a significant discrimination between these models would be challenging with these setups; however, evidence in favor of these models would be possible at low significance and if observed in conjunction with an absence of observable CP violation, this would present a concrete hypothesis for future work.

D. Long-term prospects

We have seen in the previous sections that although the next generation of superbeam and reactor experiments will be able to test the consistency of the patterns that we have identified, much of their discriminatory power relies on excluding maximal angles and phases. Testing the continuous correlations predicted in our models, for example between θ_{12} and θ_{13} or between θ_{23} and θ_{13} , would require higher precision.

The only proposed experiment capable of pushing the precision frontier beyond the results of the next-generation superbeams is the NF [75], which produces a beam with low systematic uncertainties from the decay of stored muons [64]. A NF would be able to improve our knowledge of the mixing parameters in a number of ways, but for the present purposes it serves two main roles. First, such a facility would greatly increase the precision on δ , with an ultimate 1σ precision estimated at around 5° [76,77]. This could allow many of our models to be excluded independently of their other parameter correlations. Secondly, a NF would improve the precision in the determination of θ_{13} and θ_{23} [76,78], allowing it to better test the θ_{23} – θ_{13} correlations discussed in the previous

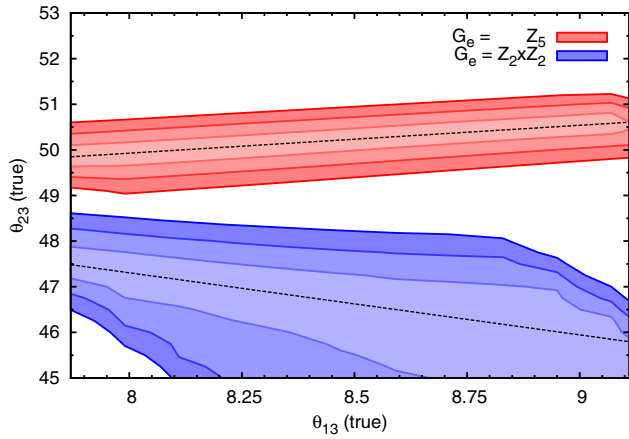


FIG. 7 (color online). The regions of true parameter space for which the upper-octant relations in Eqs. (18)–(19) can be excluded at 1, 2 and 3 σ by a 2000 km, 10 GeV LENF using a magnetized iron neutrino detector (MIND). The two regions start to overlap at low values of θ_{13} at the 5 σ confidence level.

subsection. To quantify this possibility, we have performed a simulation of a representative low-energy Neutrino Factory [79,80] to compute the regions of true parameter space which would allow two candidate models to be (individually and collectively) excluded. Our simulations assume a 2000 km baseline and a stored-muon energy of 10 GeV which is close to the optimal configuration for CP violation discovery [78,81–83]. For our detector, we take a MIND [84,85], assumed to be in a toroidal magnetic field allowing for muon charge identification, with a fiducial mass of 100 kton. The detector response is described by a set of migration matrices provided by Ref. [86]. Backgrounds to the appearance channel signal come from both charge- and flavor-misidentified events, as well as the secondary decay products arising from τ^\pm decays in the detector. An overall 1% (10%) uncertainty is taken on the signal (background). Our results are shown in Fig. 7, where the blue (red) shaded regions show the area of true parameter space for which the $Z_2 \times Z_2$ (Z_5) correlation would not be able to be excluded. We see that the two colored regions do not overlap, and therefore all points in this parameter space allow for the exclusion of at least one of the correlations at 3 σ or higher significance.

E. Neutrinoless double-beta decay

Over the next decade, the new generation of neutrinoless double-beta ($0\nu\beta\beta$) decay experiments will significantly increase the sensitivity to this rare process. For the first time these experiments will probe the region of parameter space associated with the inverse hierarchical spectrum. These experiments aim to establish that neutrinos are Majorana in nature, but can also provide valuable information on the neutrino mass spectrum and, through the global analysis of mixing parameters and a precise measurement of the

neutrino mass scale, evidence for the value of one of the Majorana phases.

The $0\nu\beta\beta$ decay rate is proportional to the effective Majorana mass $|m_{ee}|$ (see e.g. Ref. [52,87]), which is given by

$$|m_{ee}| = \left| \sum_{k=1}^3 U_{ek}^2 m_k \right|, \\ = |m_1 \cos^2 \theta_{12} \cos^2 \theta_{13} + m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} e^{i\alpha_{21}} \\ + m_3 \sin^2 \theta_{13} e^{i(\alpha_{31} - 2\delta)}|, \quad (20)$$

where α_{21} and α_{31} are Majorana phases and δ is the Dirac phase. The predicted values of $|m_{ee}|$ depend crucially on the neutrino masses. The latter can be ordered in two ways: normal ordering (NO; $m_1 < m_2 < m_3$) or inverted ordering (IO; $m_3 < m_1 < m_2$). As the parameters Δm_{21}^2 and $|\Delta m_{31}^2|$ are known from oscillation physics, there is a single degree of freedom remaining amongst the masses. This is typically taken to be the lightest neutrino mass, m_1 (m_3) for NO (IO), which we denote in both cases by m_l [87]. The parameter space available to $|m_{ee}|$ can be further divided into three particularly interesting regions based on the true value of m_l . The first is for *quasidegenerate* masses (QD) where $m_l \gtrsim 0.1$ eV, in which the splitting between masses is a small correction to approximately degenerate values. For smaller values of m_l there are two parameter regions: one for *normal hierarchical* masses (NH; $m_1 < m_2 \ll m_3$) and the other for *inverted hierarchical* masses (IH; $m_3 \ll m_1 < m_2$). In order to better understand the predictions of our models, we have first computed the predicted values of $|m_{ee}|$ in the generic case, assuming only that the mixing parameters lie in their current 3 σ allowed ranges [25]. These predictions for NO (IO) are shown as the blue (red) region in Fig. 8. For quasidegenerate and IH spectra, there exist lower bounds on $|m_{ee}|$ [88]: for IH $|m_{ee}| \gtrsim 0.015$ eV, and for QD $|m_{ee}| \gtrsim 0.03$ – 0.04 eV. For NO there is no nonzero lower bound as $|m_{ee}|$ can vanish due to a cancellation between terms in Eq. (20).

There are many experiments that are searching for $0\nu\beta\beta$ decay or are in various stages of planning and construction. Recent experiments that have set limits on the effective Majorana mass are CUORICINO, GERDA, EXO-200, KamLAND-Zen and NEMO-3. Both CUORICINO [90] and GERDA [91] have measured a lower $|m_{ee}|$ limit of 200 meV. EXO-200 [92] and KamLAND-Zen [93] have probed even further with a lower $|m_{ee}|$ limit of 69 and 140 meV respectively. This set of experiments has successfully explored the QD region of parameter space; however the IO region still remains unexplored. The future goal of upcoming experiments will be to fully probe the IO region and measure $|m_{ee}|$ down to 10–20 meV. The experiments involved in these future searches, such as SuperNEMO [94], CUORE [95], AMoRE [96], COBRA [97], nEXO [98], MOON [99], SNO+ [100], XMASS [101], BEXT

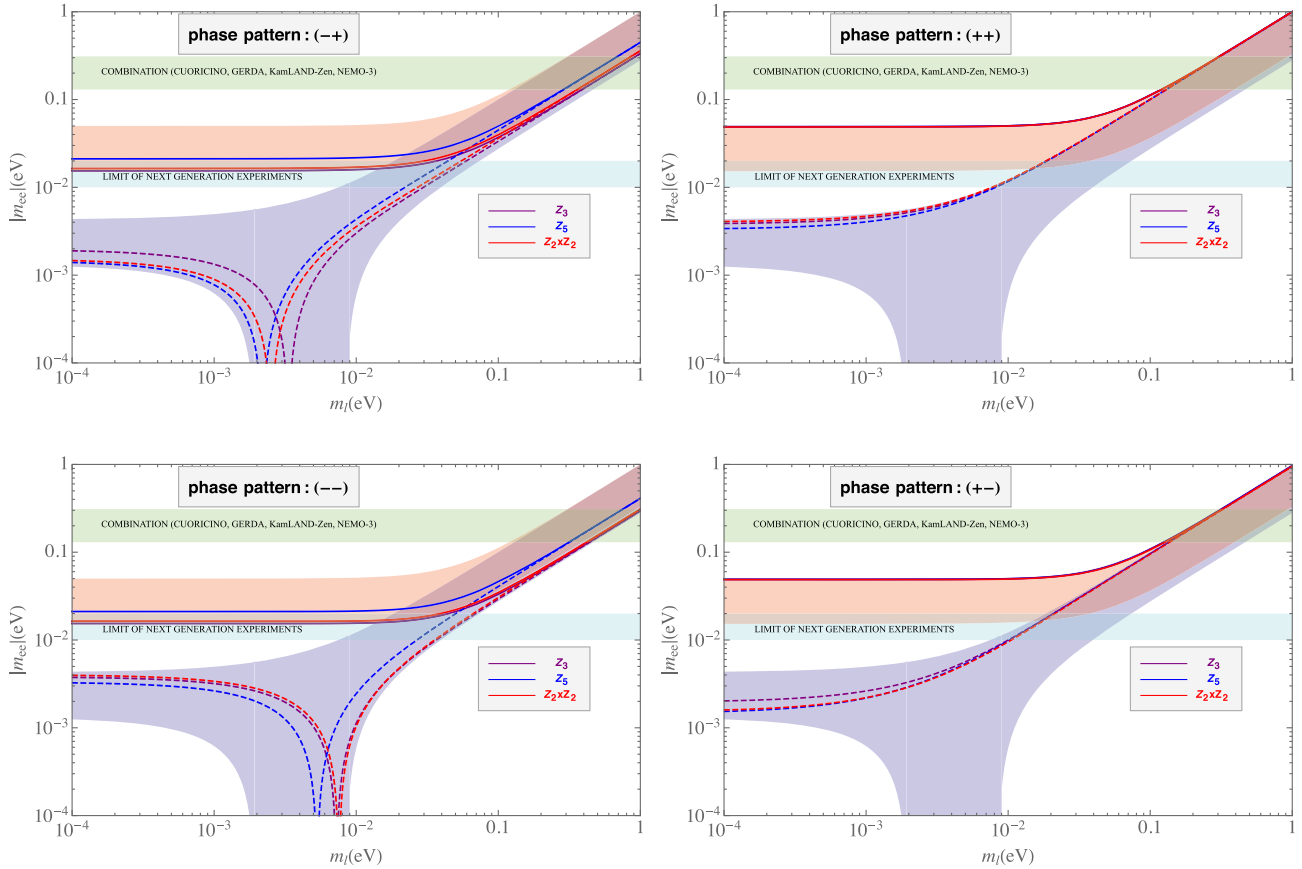


FIG. 8 (color online). $|m_{ee}|$ versus the lightest neutrino mass for the IO (NO) for the solid (dashed) lines. The predictions in a given panel all have the same phase assignment, shown in the top left of the plot. The red (blue) shaded region shows the most general predictions for $|m_{ee}|$ with IO (NO) obtained by varying the oscillation parameters over their current 3σ global ranges [25]. The green band indicates the combined limit using data from CUORICINO, EXO-200, GERDA, KamLAND-Zen and NEMO-3 [89] and the light blue band displays the projected limit of future experiments.

[102] and KamLAND2-Zen [93] amongst others, will be of the tonne or multitonne scale (see e.g. Ref. [103,104] for an in-depth discussion).

The predicted values of $|m_{ee}|$ for the case of A_5 with GCP can be calculated from the leptonic mixing matrices of Eqs. (10)–(11) and Eq. (14) for both IO and NO. The complex phases only influence $|m_{ee}|$ through the combinations $e^{i\alpha_{21}}$ and $e^{i(\alpha_{31}-2\delta)}$, and we denote the phases of our predictions by an ordered pair of \pm signs e.g. $(+-)$ when $\alpha_{21} = 0$ and $\alpha_{21} - 2\delta = \pi$. As $|m_{ee}|$ does not depend on θ_{23} , patterns which only differ by this angle will be degenerate and each preserved charged-lepton subgroup leads to a single prediction for each mass ordering and phase assignment. Figure 8 shows the predicted values from the mixing patterns in this paper for each charged-lepton residual symmetry G_e . In these plots, we have neglected a small width to each line which comes from varying θ_{13} and the neutrino mass-squared splittings over their allowed ranges, instead fixing these at their best-fit values from Ref. [25].

We focus first on the IO spectra. The phase assignments $(++)$ and $(+-)$, shown on the top row of Fig. 8, predict

large values of $|m_{ee}|$, close to the upper boundary of the IO region obtained using the 3σ global data. These predictions are very similar for all models. This can be understood as the term in $|m_{ee}|$ proportional to m_3 only has a subdominant effect: it is not only multiplied by the small number $\sin^2 \theta_{13}$, but is further suppressed for IH by the small value of m_3 itself. If we neglect this term, the resulting approximation at leading order is independent of θ_{12} up to corrections of the order $\mathcal{O}(\Delta m_{21}^2/\Delta m_{31}^2)$. It is feasible that experiments such as CUORE, KamLAND2-Zen, BEXT and nEXO will be able to explore this topmost region of the IH parameter space and test these predictions. Furthermore, distinguishing between them will be beyond their scope due to the small predicted differences and substantial experimental and theoretical uncertainties on $|m_{ee}|$. For the phase assignments $(-+)$ and $(--)$, shown on the bottom row of Fig. 8, we see values of $|m_{ee}|$ that are further suppressed and which exhibit more model dependence. Once again, the suppression of the m_3 term explains the similarity between the two phase assignments. The lower values compared to the $(++)$ case arise from the relative phase difference between the m_1 and m_2 terms: at leading order

$|m_{ee}| = \sqrt{|\Delta m_{31}^2|} \cos^2 \theta_{13} \cos^2(2\theta_{12})$ [88,105]. This effect is evident if we compare \mathbb{Z}_5 and \mathbb{Z}_3 cases: the larger θ_{12} value of \mathbb{Z}_3 accounts for the more pronounced cancellation and therefore lower $|m_{ee}|$ than that of \mathbb{Z}_5 . These predictions are beyond the reach of many of the facilities discussed so far; although KamLAND2-Zen aims to set limits near the predictions for \mathbb{Z}_5 and, if capable of testing the full IO region, should be accessible to the next generation of experiments. Although lying in a region of parameter space that is harder to explore, the greater model dependence for these phase assignments would make it easier to distinguish between models than with the $(++)$ and $(+-)$ cases. There exists a separation of around 5 meV between the predictions for \mathbb{Z}_5 and the other subgroups; however, it is unlikely such a resolution on $|m_{ee}|$ would be attainable in the foreseeable future. For NO, we see quite different behavior. In the quasidegenerate region, the mass-squared splittings are negligible and the predictions for the IO and NO cases effectively coincide. However, in the limit of vanishing m_1 the situation is very different. In this limit it is the relative phase between the m_2 and m_3 terms which dominates the magnitude of $|m_{ee}|$, which leads to larger predictions for the phase assignments $(++)$ and $(--)$, while suppressing the predictions of $(+-)$ and $(-+)$. Although exploring the NH region experimentally is beyond the scope of any planned experiment, if $0\nu\beta\beta$ decays are not observed and oscillation physics establishes that the neutrino masses are NO, it would be of paramount importance to try and test $|m_{ee}|$ values in the NH region. Due to the rich interplay between relative phases, these models make quite different predictions across this parameter space. In fact, all mixing angle patterns discussed in this paper could accommodate a value of $|m_{ee}|$ near the top of the current NH region allowed by global data. Although such an observation would add further support to any prediction of this paper which was

still consistent with experimental data, to further discriminate between these models it would be necessary to provide complementary information on the absolute mass scale.

V. CONCLUSIONS

Assessing the viability of flavor-symmetric models of the leptonic sector is an accessible target for precision measurements from present and future neutrino oscillation experiments. In this article, we have presented a detailed analysis of a particular theoretical scenario: the flavor symmetry A_5 with a generalized CP symmetry breaking into residual subgroups at low energies. We have identified the most general form of the generalized CP transformation, and studied the full group for consistent residual symmetries. Our analysis results in six distinct sets of mixing angle predictions each with an additional eight possible combinations of phases which are shown in Table I. These depend at most on a single real parameter, and predict testable correlations between certain parameters. In addition, the Majorana phases for all of our predictions are CP conserving. These patterns can be classified by the residual symmetry in the charged-lepton mass terms: \mathbb{Z}_3 , \mathbb{Z}_5 and $\mathbb{Z}_2 \times \mathbb{Z}_2$. A symmetry of \mathbb{Z}_3 predicts maximal θ_{23} , maximal CP violation from δ and a value of θ_{12} that lies close to the upper boundary of the 3σ global fit data. There are two distinct patterns which arise from a preserved \mathbb{Z}_5 residual symmetry. These share a common θ_{12} prediction which lies close to the lower boundary of the 3σ global fit data; however, one prediction has maximal θ_{23} and a maximally CP -violating value of δ while the other has nonmaximal θ_{23} and CP -conserving values of δ . The patterns arising from a preserved subgroup $\mathbb{Z}_2 \times \mathbb{Z}_2$ also share a common θ_{12} which lies above the current 1σ region. In this case both θ_{23} predictions are nonmaximal and the value of δ is CP conserving.

We then discussed the phenomenology of our predictions, focusing on the role which current and future reactor, superbeam and neutrinoless double-beta decay experiments can play. The predictions for θ_{12} should be testable at high significance by the next generation of reactor neutrino experiments, such as JUNO and RENO-50. These experiments can be expected to distinguish between the different models; however, testing the precise correlations between θ_{12} and θ_{13} will most probably remain beyond the reach of any foreseen experiment. A particularly interesting feature of the patterns found in this paper is the correlated maximality of θ_{23} and δ , and also nonmaximal θ_{23} and CP -conserving values of δ . Testing these correlations is a feasible goal for current and future superbeam experiments. T2K and NO ν A can be expected to collect early evidence if such a pattern obtains, and we have shown that DUNE will be able to identify such a pattern over a significant part of the parameter space. For the CP -conserving patterns, the deviations from $\theta_{23} = \pi/4$ are expected to be measurable at 3σ by the next generation of superbeams for the preserved

TABLE I. Numerical predictions for the correlations found in this paper. The dimensionless parameter $r \equiv \sqrt{2} \sin \theta_{13}$ is constrained by global data to lie in the interval $0.19 \lesssim r \lesssim 0.22$ at 3σ . The predictions for θ_{12} and θ_{23} shown here neglect terms of order $\mathcal{O}(r^4)$ and $\mathcal{O}(r^2)$, respectively. Following the method of this paper, the Majorana phases can only be predicted modulo π and the values in the fourth column hold for all phases.

G_e	θ_{12}	θ_{23}	$\sin \alpha_{ji}$	δ
\mathbb{Z}_3	$35.27^\circ + 10.13^\circ r^2$	45°	0	90°
			0	270°
\mathbb{Z}_5	$31.72^\circ + 8.85^\circ r^2$	$45^\circ \pm 25.04^\circ r$	0	0°
			0	180°
		0	90°	
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$36.00^\circ - 34.78^\circ r^2$	$31.72^\circ + 55.76^\circ r$	0	270°
			0	0°
		0	180°	
		$58.28^\circ - 55.76^\circ r$	0	0°
			0	180°

subgroup \mathbb{Z}_5 , but not for $\mathbb{Z}_2 \times \mathbb{Z}_2$. Ultimately separating between these models at 3σ significance across the whole parameter space could be done using a Neutrino Factory after ten years of data taking. An attractive feature of the theoretical scenario in this work is its ability to predict Majorana phases, and, therefore, observables for neutrinoless double-beta decay experiments. We have seen that in the case of inverted mass ordering, two of the possible Majorana phase combinations predict the discovery of neutrinoless double-beta decay at upcoming experiments. In the longer term, the exploration of the full parameter space for inverted hierarchical mass spectra could allow all of our patterns with this mass spectrum to be confirmed independently of oscillation physics.

In conclusion, we find that the combination of the flavor symmetry A_5 with a generalized CP symmetry allows for a number of viable predictions to be made for the mixing angles and phases. These predictions specify parameter correlations which present good targets for each stage of the next decade of the experimental program.

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Note added.—Recently, preprints of two similar works were made available [106,107] which also study the mixing patterns arising from the residual symmetries of A_5 with GCP. The patterns derived in the first part of our paper, previously presented at NuPhys 2014 [108], are in agreement with those found in Refs. [106,107]. Although, case II in Ref. [106] is omitted in our analysis as its predictions fall outside the 3σ global intervals used in this work. Our phenomenological work significantly extends the analysis in these papers.

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