

# Light wave propagation through a dilaton-Maxwell domain wall

J. R. Morris<sup>1,\*</sup> and A. Schulze-Halberg<sup>2,†</sup>

<sup>1</sup>*Department of Physics, Indiana University Northwest, 3400 Broadway, Gary, Indiana 46408, USA*

<sup>2</sup>*Department of Mathematics and Department of Physics, Indiana University Northwest, 3400 Broadway, Gary, Indiana 46408, USA*

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We consider the propagation of electromagnetic waves through a dilaton-Maxwell domain wall of the type introduced by Gibbons and Wells [G. W. Gibbons and C. G. Wells, *Classical and Quantum Gravity* **11**, 2499 (1994)]. It is found that if such a wall exists within our observable Universe, it would be absurdly thick, or else have a magnetic field in its core which is much stronger than observed intergalactic fields. We conclude that it is highly improbable that any such wall is physically realized.

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## I. INTRODUCTION

Starting from a five-dimensional Kaluza-Klein theory, which is toroidally compactified to yield an effective four-dimensional dilaton-Maxwell theory, we find exact background solutions describing a dilatonic domain wall which entraps magnetic flux, which has previously been described by Gibbons and Wells [1]. This type of domain wall is interesting, not only because it traps magnetic flux, but also because it is nontopological in origin, i.e., the solution is not stabilized by a nontrivial topology of the vacuum manifold. (However, this stability issue was examined in [2], where it was determined that the Gibbons-Wells wall is indeed stable against small fluctuations in the scalar and magnetic fields.) Particles, including both fermions and bosons, can scatter from a topological domain wall in various ways (see [3–5] for example). In particular, the scattering of scalar bosons from such walls has been examined in [6]. In addition, a coupling of a scalar dilaton field (with a simple quartic potential) to matter and electromagnetic fields has been studied in [7], where it was proposed that the existence of a dilaton domain wall might give rise to spatial variations in the fine structure constant  $\alpha$ . We do not concern ourselves with specifics of such a type of scenario involving a Gibbons-Wells wall, but simply point out that interactions of dilatonic walls with matter and electromagnetic fields may indeed be of physical importance.

The main focus here is on the propagation of electromagnetic waves in the dilaton-Maxwell domain wall. Exact solutions for the wave equation are found, and it is determined that there is a critical frequency above which there are transmitted traveling waves which are damped in amplitude as the distance  $|x|$  from the core of the wall tends to infinity. We argue, however, that the wall is transparent to essentially all electromagnetic waves if the effective

dielectric function is to have very small spatial variation. We speculate on some observable consequences of the existence of such a domain wall. The existence of such a solitonic structure would support the possibility of the existence of extra compactified space dimensions. However, what we infer is that the wall's magnetic field is either too large in comparison to an intergalactic field strength, or else the thickness of the wall is absurdly large. We conclude that it seems improbable that a Gibbons-Wells wall is physically realized in our observable Universe.

## II. THE DILATON-MAXWELL DOMAIN WALL

### A. Equations of motion

We start with a 5d action, using a 5d metric described by  $d\tilde{s}_5^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu - b^2 (x^\mu) dy^2$  with signature  $(+, -, -, -, -)$  which is dimensionally reduced by toroidal compactification and rewritten in a 4d Einstein conformal frame, subsequently taking the form (see, e.g., Ref. [8] for details)

$$S = \int d^4x \sqrt{g} \left\{ \frac{1}{2\kappa^2} R[g_{\mu\nu}] + \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi + e^{2\tilde{\kappa}\varphi} \left( \mathcal{L} - \frac{\Lambda}{\kappa^2} \right) \right\} \quad (1)$$

where  $\kappa^2 = 8\pi G$ ,  $g = |\det g_{\mu\nu}|$  with  $g_{\mu\nu}$  being the Einstein metric, and  $\mathcal{L}$  is the matter Lagrangian written in terms of the 4d Jordan frame metric  $\tilde{g}^{\mu\nu}$ . Here  $2\tilde{\kappa} = -\sqrt{2/3}\kappa$  is a constant, we have assumed  $\partial_5 = \partial/\partial y = 0$  (no Kaluza-Klein modes), and we use natural units with  $\hbar = c = k_B = 1$ . The extra dimensional scale factor is denoted by  $b = e^{-2\tilde{\kappa}\varphi}$  and the relation between the 4d Jordan frame metric  $\tilde{g}_{\mu\nu}$  and the 4d Einstein frame metric  $g_{\mu\nu}$  is given by

$$g_{\mu\nu} = b\tilde{g}_{\mu\nu} = e^{-2\tilde{\kappa}\varphi} \tilde{g}_{\mu\nu}, \quad \tilde{g}^{\mu\nu} = e^{-2\tilde{\kappa}\varphi} g^{\mu\nu}. \quad (2)$$

The Lagrangian forming  $\mathcal{L}$  is taken to be

$$\mathcal{L} = -\frac{1}{4} \tilde{F}^{\mu\nu} \tilde{F}_{\mu\nu} = -\frac{1}{4} e^{-4\tilde{\kappa}\varphi} F^{\mu\nu} F_{\mu\nu} = -\frac{1}{2} e^{-4\tilde{\kappa}\varphi} (\mathbf{B}^2 - \mathbf{E}^2) \quad (3)$$

\*jmorris@iun.edu

†axgeschu@iun.edu, xbat Axel@gmail.com

where  $\tilde{F}^{\mu\nu} = \tilde{g}^{\mu\alpha}\tilde{g}^{\nu\beta}F_{\alpha\beta}$ , with  $\tilde{F}_{\mu\nu} = F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ . Also, following [1], we focus on the flat space version where the Ricci scalar  $R[g_{\mu\nu}] = 0$  and the metric is Minkowski,  $g_{\mu\nu} = \eta_{\mu\nu}$ , and we set the cosmological constant to zero,  $\Lambda = 0$ .

Now, the equations of motion that follow from (1), along with the Bianchi identity, are given by

$$\square\varphi + \frac{1}{2}\tilde{\kappa}e^{-2\tilde{\kappa}\varphi}F^{\mu\nu}F_{\mu\nu} = 0 \quad (4)$$

$$\nabla_\mu(e^{-2\tilde{\kappa}\varphi}F^{\mu\nu}) = 0, \quad \nabla_\mu(*F^{\mu\nu}) = 0 \quad (5)$$

where  $\square = \partial_t^2 - \nabla^2$  and the electromagnetic dual tensor is  $*F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ . The set of equations in (5) is just the set of Maxwell equations

$$\begin{aligned} \nabla \cdot \mathbf{D} &= 0, & \nabla \times \mathbf{H} - \dot{\mathbf{D}} &= 0, \\ \nabla \cdot \mathbf{B} &= 0, & \nabla \times \mathbf{E} + \dot{\mathbf{B}} &= 0 \end{aligned} \quad (6)$$

with  $\mathbf{D} = \epsilon\mathbf{E}$  and  $\mathbf{B} = \mu\mathbf{H}$ , where the effective dielectric and permeability functions are  $\epsilon = \mu^{-1}$  with

$$\mu = \epsilon^{-1} = e^{2\tilde{\kappa}\varphi} \quad (7)$$

and the index of refraction is  $\sqrt{\epsilon\mu} = 1$ . We can rewrite (4) now in terms of  $\mathbf{D}$  and  $\mathbf{H}$ :

$$\nabla^2\varphi - \partial_t^2\varphi = -\tilde{\kappa}e^{2\tilde{\kappa}\varphi}(\mathbf{H}^2 - \mathbf{D}^2). \quad (8)$$

### B. The background ansatz

An exact, static solution set can be found for the above equations of motion. This solution set then serves as a background for the scattering of electromagnetic (EM) waves. The background solution is that of a dilatonic domain wall entrapping magnetic flux, originally discovered by Gibbons and Wells [1]. For this static solution, we set  $\mathbf{D} = 0$ ,  $\mathbf{E} = 0$ ,  $\mathbf{H} = (0, 0, H)$ , where  $H$  is a constant, and  $\mathbf{B} = (0, 0, B) = \mu\mathbf{H}$ . We find that the Maxwell equations (6) are then satisfied, and the equation of motion for the dilaton field  $\varphi$  of (8) then reduces to

$$(\partial_x^2 + \partial_y^2)\varphi = -\tilde{\kappa}H^2e^{2\tilde{\kappa}\varphi} \quad (9)$$

where we assume that  $\partial_z\varphi = 0$ . This equation is recognized as the 2D Euclidean Liouville equation [9] whose solution is given by [1,2,9,10]

$$\mu(\zeta) = e^{2\tilde{\kappa}\varphi(\zeta)} = \frac{4}{\tilde{\kappa}^2H^2} \frac{|f'(\zeta)|^2}{(1 + |f(\zeta)|^2)^2} \quad (10)$$

where  $\zeta = x + iy$  and  $f(\zeta)$  is a holomorphic function of  $\zeta$  and  $f'(\zeta) = df(\zeta)/d\zeta$ . Let us choose  $f(\zeta)$  to take the form  $f(\zeta) = e^{M\zeta}$ . Then (10) produces a static domain wall solution [1,2,11]

$$\begin{aligned} \mu(x) &= e^{2\tilde{\kappa}\varphi(x)} = \left(\frac{M}{\tilde{\kappa}H}\right)^2 \frac{1}{\cosh^2(Mx)} = \left(\frac{M}{\tilde{\kappa}H}\right)^2 \text{sech}^2(\bar{x}), \\ \bar{x} &\equiv Mx. \end{aligned} \quad (11)$$

The constant  $M$  has a canonical dimension of mass, so that the coordinate  $\bar{x} = Mx$  is dimensionless, as is the factor  $(M/\tilde{\kappa}H)$ . This domain wall solution depends only upon  $x$ , and not upon  $y$ , and the width of the wall is represented by  $a = M^{-1}$ . The magnetic field is  $B(x) = \mu(x)H \propto \text{sech}^2\bar{x}$ , which maximizes in the wall's core and falls to zero asymptotically. The magnetic flux per unit length of the domain wall is [1,2,11]

$$\frac{\Phi_{\text{mag}}}{L_y} = \frac{1}{L_y} \int_{-\infty}^{\infty} \int_0^{L_y} B(x) dx dy = \frac{2M}{\tilde{\kappa}^2H}. \quad (12)$$

## III. ELECTROMAGNETIC WAVE PROPAGATION

### A. Wave equations and exact solutions

We now examine the scattering of electromagnetic (EM) waves from the wall background ansatz of (11), except now we denote the static magnetic  $B$  and  $H$  fields of the wall by  $B_0$  and  $H_0$ , and denote those of electromagnetic waves by  $B$  and  $H$ . The basic formalism for EM scattering from a dilatonic wall (with normal incidence) with arbitrary  $\epsilon(x)$  and  $\gamma(x) = \ln \epsilon(x)$  is described in Sec. IVa of [12]. We use results presented there to describe EM wave fields with nonvanishing components  $E_y(x, t)$  and  $B_z(x, t)$  propagating in the  $\pm x$  direction. The electromagnetic field equations can be reduced to [12]

$$\partial_x^2 B_z - \partial_t^2 B_z + (\partial_x^2 \gamma) B_z + (\partial_x \gamma) \partial_x B_z = 0 \quad (13)$$

and we assume fields of the form

$$E_y(x, t) = E(x, \omega)e^{-i\omega t}, \quad B_z(x, t) = B(x, \omega)e^{-i\omega t}. \quad (14)$$

From the field equations we then have

$$E = -\frac{i}{\omega} [\partial_x B + (\partial_x \gamma) B]. \quad (15)$$

We again define the dimensionless coordinate  $\bar{x} = Mx$ , and using  $\partial_x \gamma = 2M \tanh \bar{x}$  and  $\partial_x^2 \gamma = 2M^2 \text{sech}^2 \bar{x}$ , (13) can be written as

$$\partial_{\bar{x}}^2 B + 2(\tanh \bar{x}) \partial_{\bar{x}} B + \left(\frac{\omega^2}{M^2} + 2\text{sech}^2 \bar{x}\right) B = 0. \quad (16)$$

Now we change coordinates according to

$$\bar{x}(u) = \text{arctanh}(u), \quad B[\bar{x}(u)] = \sqrt{u^2 - 1} \hat{B}(u). \quad (17)$$

This renders our Eq. (16) in the form

$$(1 - u^2)^2 \hat{B}''(u) - 2u \hat{B}'(u) + \left[ \frac{M^2(2u^2 - 1) - \omega^2}{M^2(u^2 - 1)} \right] \hat{B}(u) = 0, \quad (18)$$

where here the prime denotes differentiation with respect to the argument  $u$ . The general solution to this equation is given by

$$\hat{B}(u) = c_1 P_1^\mu(u) + c_2 Q_1^\mu(u), \quad \mu = \sqrt{1 - \frac{\omega^2}{M^2}}. \quad (19)$$

After reverting the change of coordinate (17), the latter solution takes the form

$$B = \operatorname{sech}(\bar{x}) \left\{ c_1 \frac{(\xi - \mu)(1 + \xi)^{\mu/2}}{\Gamma(2 - \mu)(1 - \xi)^{\mu/2}} + c_2 \frac{\pi(1 - \xi^2)^{-\mu/2} [(1 + \xi)^\mu (\xi - \mu) \cos(\pi\mu) - (1 - \xi)^\mu (\xi + \mu)]}{2 \sin(\pi\mu) \Gamma(2 - \mu)} \right\}. \quad (22)$$

The behavior of the solution (22) can be described as follows. First we observe that the factor  $\operatorname{sech}(\bar{x})$  is responsible for damping, as  $|\bar{x}|$  tends toward infinity. The term inside the curly brackets is in general complex valued. Its qualitative behavior depends principally on the quantity  $\mu$ , which depends upon  $\omega/M$ . We can distinguish two cases, assuming without restriction that  $c_1$  and  $c_2$  are real valued.

- (1)  $\omega \leq M$ , i.e., *real root*,  $(1 - \omega^2/M^2) \geq 0$ : The solution (22) is real, bounded everywhere, has two zeros (one positive, one negative) and goes to zero as  $|x|$  tends to infinity. These properties are independent of  $c_1$  and  $c_2$ . These solutions are nonoscillatory outside of the domain wall.
- (2)  $\omega > M$ , i.e., *imaginary root*,  $1 - \omega^2/M^2 < 0$ : The solution (22) is complex. Both real and imaginary parts are bounded everywhere, have an infinite number of zeros and go to zero as  $|\bar{x}|$  tends to infinity. The set of zeros is unbounded from both below and above. These properties are independent of  $c_1$  and  $c_2$ .

### B. An estimate for the domain wall mass parameter $M$ and width $a = M^{-1}$

Since dilatonic-matter effects are expected to be nearly negligible and nearly undetectable at this time, we focus on the case where the effective dielectricity in vacuum  $\epsilon(\bar{x})$  does not wander far from unity. We define  $\epsilon_0 = 1$  for the case of no dilaton coupling to EM fields, i.e., ordinary electrodynamics, and consider the case where

$$B = \operatorname{sech}(\bar{x}) [c_1(\omega) P_1^\mu(\xi) + c_2(\omega) Q_1^\mu(\xi)] \quad (20)$$

where  $P$  and  $Q$  are Legendre functions;  $c_1(\omega)$  and  $c_2(\omega)$  are  $\bar{x}$  independent parameters, which, in general, can depend upon the frequency  $\omega$ ; and

$$\mu = \sqrt{1 - \frac{\omega^2}{M^2}}, \quad \xi = \tanh \bar{x}, \quad (21)$$

where the index  $\mu$  of (21) is not to be confused with the permeability function defined earlier. Due to the fact that the lower index on both  $P$  and  $Q$  is an integer, the solution (20) degenerates to an elementary function, which is given by

$\Delta\epsilon(\bar{x}) = \epsilon(\bar{x}) - \epsilon_0 \ll 1$ . Furthermore, we consider the possibility where there is just one dilaton-Maxwell wall within the observable Universe, and we roughly estimate the width of the wall to be on the order of the Hubble length,  $a \sim |x_C| \sim l_H \sim 10^{10}$  light-years  $\sim 10^{26}$  m. The distance  $|x_C|$  serves as a long distance cutoff, as the wall's surface energy density (tension)  $\Sigma(x)$  diverges with distance away from the wall according to [2]  $\Sigma(|x|) = (M/\tilde{\kappa}^2)|x|$ , so that

$$\Sigma(|x_C|) = \frac{M}{\tilde{\kappa}^2} |x_C|. \quad (23)$$

Our expression for the effective dielectric constant is

$$\epsilon(\bar{x}) = e^{-2\tilde{\kappa}\varphi} = \left( \frac{\tilde{\kappa}H_0}{M} \right)^2 \cosh^2(\bar{x}) \quad (24)$$

where  $-2\tilde{\kappa} = \sqrt{2/3}\kappa$  and  $\bar{x} = Mx = x/a$ . We therefore examine the limit where  $\kappa|\varphi| \ll 1$  and  $\epsilon(\bar{x}) \ll 1 + \epsilon_0 = 2$ . We therefore want to consider  $|\bar{x}_C|$  to be not far from order unity so that the  $\cosh^2(\bar{x})$  term has little variance, even over large distances. (We expect standard classical EM theory and QED to hold to a high degree of precision everywhere within the observable Universe, with dilatonic effects being extremely small.) We choose to set  $|\frac{\tilde{\kappa}H_0}{M}| = 1$ , which fixes  $H_0$  in terms of  $M$ , so that

$$\epsilon(\bar{x}) = \cosh^2(\bar{x}) = 1 + \bar{x}^2 + O(\bar{x}^4) \approx 1 + \bar{x}^2. \quad (25)$$

We then have that  $\Delta\epsilon(\bar{x}) = \epsilon(\bar{x}) - 1 = \bar{x}^2 = (Mx)^2 \ll 1$ . Setting  $|\bar{x}| = |\bar{x}_C| = M|x_C| \lesssim 1$  we have  $M \lesssim 1/|x_C|$  and

therefore a wall thickness  $a = M^{-1} \gtrsim |x_C| \sim l_H \sim 10^{26}$  m, where  $l_H$  is the Hubble length. So for  $a \sim |x_C|$  on the order of the Hubble length, we have a very thick wall, extending through the observable Universe. EM radiation of essentially all wavelengths is much smaller than the wall thickness, i.e.,  $\lambda \ll a = M^{-1} \sim l_H$  for essentially all radiation with wavelength smaller than the Hubble length, and therefore we have  $\omega/M > 1$  for essentially all radiation. Therefore, all radiation travels through a very thick wall ( $\lambda \ll a = M^{-1}$ ) where  $\epsilon(x)$  varies very slowly.

#### IV. WAVE PROPAGATION THROUGH A THICK WALL

##### A. Wave equations and approximate solutions

The propagation of EM waves through a thick wall with arbitrary, but slowly varying,  $\epsilon(x)$  has been described in [12]. Instead of repeating the calculational details presented there, we simply recap some of the highlights. We define the function  $\gamma(x) = \ln \epsilon(x)$  and consider  $\partial_x \epsilon$  and  $\partial_x \gamma$  to be sufficiently small. Let us now examine the scattering of electromagnetic (EM) waves from the wall background ansatz of the dilaton-Maxwell wall, except now we denote the static magnetic  $B$  and  $H$  fields of the wall by  $B_0$  and  $H_0$ , and denote those of electromagnetic waves by  $B$  and  $H$ . The basic formalism for EM scattering from a dilatonic wall (with normal incidence) with arbitrary  $\epsilon(x)$  and  $\gamma(x) = \ln \epsilon(x)$  is described in Sec. IVa of [12], and the reader is referred there for calculational details. We use results presented there to describe EM wave fields with non-vanishing components  $E_y(x, t)$  and  $B_z(x, t)$  propagating in the  $\pm x$  direction. For notational simplicity, we simply write  $E = E_y$  and  $B = B_z$ . The electromagnetic field equations can be reduced to [12]

$$B'' - \ddot{B} + \gamma'' B + \gamma' B' = 0, \quad E = -\frac{i}{\omega} [B' + \gamma' B] \quad (26)$$

where we assume the fields to have the time dependence  $e^{-i\omega t}$  and a prime stands for differentiation with respect to  $x$ .

We now take the magnetic field  $B(x, t)$  to be of the form

$$B(x, t) = A e^{i\phi(x)} e^{-i\omega t} \quad (27)$$

where the amplitude  $A$  is a real constant and  $\phi(x)$  is a phase function, which may be complex valued, in general. The wave equation for  $B$  then gives an equation for the function  $\phi$ ,

$$i\phi'' - \phi'^2 + \omega^2 + \gamma'' + i\phi'\gamma' = 0. \quad (28)$$

For the expectedly small dilatonic effect on the wave equations for  $E$  and  $B$ , we note that for the case  $\epsilon = \text{const}$  and  $\gamma' = 0$  we have the usual solution  $\phi' = \pm\omega$  and  $\phi = \pm\omega x$ . The  $+$ ( $-$ ) solution describes waves traveling

in the  $+x$ ( $-x$ ) direction. Approximations can be made for the case of slowly varying  $\epsilon(x)$  which lead to the approximate solutions for the EM fields [12]

$$B_{\pm}(x, t) = A \left( \frac{\epsilon}{\epsilon_0} \right)^{-1/2} e^{\pm i\omega x} e^{-i\omega t} \quad (29)$$

and

$$E_{\pm}(x, t) = \left( \pm 1 - i \frac{\gamma'}{2\omega} \right) A \left( \frac{\epsilon}{\epsilon_0} \right)^{-1/2} e^{\pm i\omega x} e^{-i\omega t} \quad (30)$$

where  $\epsilon_0$  is just a constant which can be set to unity. Note that the effective amplitude of the magnetic field is  $A \left( \frac{\epsilon}{\epsilon_0} \right)^{-1/2}$ . Since we assume that  $\Delta\epsilon \ll 1$ , the effective amplitude varies only mildly over any distance of interest.

##### B. Reflection and transmission coefficients

The Poynting vector is given by  $\mathbf{S} = \text{Re}(\mathbf{E} \times \mathbf{H}^*) = \epsilon \text{Re}(\mathbf{E} \times \mathbf{B}^*)$ . This can be applied to each of the  $\pm$  propagating waves, which after some algebra, yields [12]

$$\begin{aligned} (S_x)_{\pm} &= \epsilon \text{Re}(\mathbf{E}_{\pm} \times \mathbf{B}_{\pm}^*) = \pm \epsilon |\mathbf{B}_{\pm}|^2 \\ &= \pm \epsilon \left[ A^2 \frac{\epsilon_0}{\epsilon} \right] = \pm \epsilon_0 A^2. \end{aligned} \quad (31)$$

This shows that  $(S_x)_{\pm}$  is  $x$  independent, which by Poynting's theorem, indicates that no energy or momentum is lost by either the  $+$  or  $-$  traveling waves, which in turn implies that the transmission and reflection coefficients are given by  $T = 1$  and  $R = 0$ , respectively, for waves with  $\omega \gg |\gamma'| = M |\tanh(\bar{x})|$ , i.e.,  $\omega/M \gg |\tanh(\bar{x})|$ . We have argued that, within the scenario considered here,  $\omega/M > 1$  for all EM waves of interest, and in fact, due to the extreme smallness of  $M$ , we have  $\omega/M \gg 1$  for all frequencies of interest. The wall is therefore transparent to all EM radiation.

#### V. OBSERVATIONAL CONSEQUENCES

Since  $T = 1$  and  $R = 0$ , it would therefore seem that there would be few observational consequences. However, we recall that the setting  $|\frac{\bar{\kappa} H_0}{M}| = 1$  will fix the value of the wall-entrapped  $H_0$  field in terms of the mass parameter  $M$ . We therefore have

$$|H_0| = \frac{M}{|\bar{\kappa}|} = \frac{2M}{\sqrt{2/3}\kappa} = 2\sqrt{\frac{3}{2}} \frac{M}{\kappa} = 2\sqrt{\frac{3}{2}} \frac{M}{\sqrt{8\pi G}} \sim MM_P \quad (32)$$

where  $M_P = 1/\kappa = 1/\sqrt{8\pi G} \sim 10^{18}$  GeV is the reduced Planck mass. Our estimate of  $M^{-1} \sim 10^{26}$  m gives  $M \sim 10^{-26} \text{ m}^{-1} \sim 10^{-42}$  GeV, so that

$$|H_0| \sim MM_P \sim 10^{-24} \text{ GeV}^2 \sim 10^{-4} \text{ G} \quad (33)$$

where we have used the conversion that 1 Tesla (T) is  $10^4$  Gauss (G), approximately given by  $1 \text{ T} = 10,000 \text{ G} \sim 200 \text{ eV}^2$ , or  $1 \text{ GeV}^2 \sim 10^{16} \text{ T} \sim 10^{20} \text{ G}$ . This value of  $|H_0|$  in the center of the wall is much larger than an intergalactic magnetic field strength of  $\sim 10^{-15} \text{ G}$  [13]. Therefore, setting  $a \sim l_H$  results in a magnetic field in space that is larger than that observed in intergalactic regions.

If, on the other hand, we set  $|H_0| \sim 10^{-15} \text{ G} \sim 10^{-35} \text{ GeV}^2$  and use  $M \sim |H_0|/M_P$ , we can determine  $a = M^{-1}$ . Doing this, we have

$$M \sim \frac{|H_0|}{M_P} \sim 10^{-53} \text{ GeV},$$

$$a = M^{-1} \sim 10^{53} \text{ GeV}^{-1} \sim 10^{37} \text{ m}. \quad (34)$$

Since a characteristic Hubble length is  $l_H \sim 10^{26} \text{ m}$ , this estimate gives  $a \sim 10^{11} l_H$ , i.e.,  $10^{11}$  Hubble lengths. In this case the wall is ridiculously thick, spanning many observable universes.

## VI. SUMMARY AND CONCLUSIONS

In summary, it is found that for a single dilaton-Maxwell wall, the setting of  $|\frac{\tilde{\kappa}H_0}{M}| = 1$  along with  $a \sim l_H$  results in a magnetic field that is too strong, while setting  $|H_0|$  to the value of an intergalactic magnetic field results in a domain wall spanning many observable universes, with  $a \sim 10^{11} l_H$ . On the other hand, if  $|\frac{\tilde{\kappa}H_0}{M}| \neq 1$ , we have a value of  $\epsilon(x)$  that wanders too far from unity in at least some regions of space, even if  $\cosh^2(\bar{x}) \sim 1$ , which does not seem to be supported by observation. If there were many thinner walls where  $\epsilon(x)$  could differ from unity for  $|x|/a > 1$ , one would expect some observed periodic spatial variation in the fine structure constant  $\alpha$ , which does not seem reasonable. (However, other topological dilaton domain wall models that do not require a large distance cutoff may possibly allow for a mild variation of  $\alpha$  [7].) We conclude that a dilaton-Maxwell domain wall is not likely to be physically realized within our observable Universe.

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