Superfield effective potential for the two-form field

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We develop a theory describing the superfield extension of the two-form field coupled to the usual chiral and real scalar superfield and find the one-loop Kählerian effective potential in this theory.

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I. INTRODUCTION

The supersymmetric field theory models are constructed on the base of specific supermultiplets represented by corresponding superfields [1,2]. The most used supermultiplets are the chiral one, widely used for the description of the scalar matter, and the vector one, naturally describing supersymmetric extensions of gauge theories. However, the set of possible supermultiplets is much larger. The most important examples are presented in Ref. [2].

One of the important although less studied multiplets is the tensor one described by the spinor chiral superfield. Originally, it was introduced in Ref. [3], where it was shown to describe a gauge theory. Furthermore, it was demonstrated in Ref. [4] that this superfield allows one to construct the supersymmetric extension of the BF gauge theory in four-dimensional space-time, allowing thus for the superfield description of the models involving the antisymmetric tensor field, which is essentially important within the string theory context [5] as well as within the quantum gravity context [6]. While in Ref. [4] the free action for this theory was constructed, it is natural to make the next step, that is, to couple this theory to matter, which is as usual represented by a chiral scalar superfield, and to study the low-energy effective action in the resulting theory.

In our previous work [7], the coupling of the spinor chiral gauge superfield to chiral matter has been considered, and the leading one-loop contribution to the effective potential has been calculated. However, the action considered in Ref. [7] does not involve the terms responsible for the BF action. Therefore, we propose another theory which, from one side, is similar in some aspects to the model discussed in Ref. [7] and, from another side, involves the BF terms, allowing one thus to treat the BF theory in a manner analogous to Ref. [7].

Within our studies, we consider the composite theory whose action involves, first, the usual superfield Maxwell term describing the dynamics of the real scalar gauge superfield, second, the action for the spinor chiral superfield involving the gauge-invariant BF term, and, third, the coupling of these gauge fields to chiral matter. For this theory, we calculate the low-energy effective action described by the Kählerian effective potential.

The structure of the paper reads as follows. In Sec. II, we formulate the model involving two gauge fields and matter. In Sec. III, we perform the one-loop calculations. In the summary, we discuss the results.

II. THE MODEL

We start with the Abelian gauge theory describing two gauge fields, the real scalar one V and the chiral spinor one ψ_α:

$$S_k = \frac{1}{2}\int d^6 z W^\alpha W_\alpha - \frac{1}{2}\int d^8 z G^2, \tag{1}$$

where

$$W_\alpha = i\bar{D}^2 D_\alpha V, \qquad G = -\frac{1}{2}(D^\alpha \psi_\alpha + \bar{D}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}}). \tag{2}$$

The theory (1) is gauge invariant, while the corresponding gauge transformations look like

$$\delta V = i(\bar{\Lambda} - \Lambda), \qquad \delta\psi_\alpha = i\bar{D}^2 D_\alpha L,$$
$$\delta\bar{\psi}_{\dot{\alpha}} = -iD^2\bar{D}_{\dot{\alpha}} L, \tag{3}$$

where, as in Ref. [2], the parameters Λ and $\bar{\Lambda}$ are chiral and antichiral, respectively, and $L = \bar{L}$ is a real one.

We can introduce mass terms for the theory (1). They are given by [3]

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$$S_m = \frac{im}{2}\left[\int d^6 z\psi^\alpha W_\alpha - \int d^6\bar{z}\bar{\psi}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}\right] + \frac{m_\psi^2}{4}\left[\int d^6 z\psi^\alpha\psi_\alpha + \int d^6\bar{z}\bar{\psi}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}}\right] + \frac{m_V^2}{2}\int d^8 z V^2. \quad (4)$$

Actually, Eq. (4), considered at $m_V = 0$, describes the superfield BF model [4]. In that paper, the dimensional reduction of that model has been carried out, and a mass generation mechanism for the Kalb-Ramond field was performed without the loss of gauge and supersymmetry invariance.

Therefore, let us now consider the theory whose action is given by $S = S_k + S_m$. We note that the term $\int d^8 z G^2$ in its action is necessary, since if this term were absent, one could simply eliminate the ψ_α and $\bar{\psi}_{\dot{\alpha}}$ through their equations of motion, thus reducing the theory to a simple supersymmetric QED.

Now, we can obtain the equations of motion for the model given by a sum of (1) and (4). For the superfields V, ψ_α, and $\bar{\psi}_{\dot{\alpha}}$, respectively, they look like

$$\frac{\delta(S_k + S_m)}{\delta V} = iD_\alpha W^\alpha - mG + m_V^2 V = 0, \quad (5)$$

$$2\frac{\delta(S_k + S_m)}{\delta\psi_\alpha} = \bar{D}^2 D^\alpha G - imW^\alpha - m_\psi^2\psi^\alpha = 0, \quad (6)$$

$$2\frac{\delta(S_k + S_m)}{\delta\bar{\psi}_{\dot{\alpha}}} = D^2\bar{D}^{\dot{\alpha}} G + im\bar{W}^{\dot{\alpha}} - m_\psi^2\bar{\psi}^{\dot{\alpha}} = 0. \quad (7)$$

It follows from (5)–(7) that, first, the superfield strengths W_α and G satisfy the field equations [7]

$$(\Box - m^2)W^\alpha = 0, \quad (\Box - m^2)G = 0, \quad \text{for } m_V = m_\psi = 0; \quad (8)$$

second, the gauge superfields V, ψ_α, and $\bar{\psi}_{\dot{\alpha}}$ satisfy the field equations [1]

$$(\Box - m_V^2)V = 0, \qquad (\Box - m_\psi^2)\psi^\alpha = 0, \qquad (\Box - m_\psi^2)\bar{\psi}^{\dot{\alpha}} = 0, \quad \text{for } m = 0. \quad (9)$$

We conclude that both the superfield strengths and the gauge superfields satisfy Klein-Gordon equations. Notice that (4) is not invariant under the gauge transformations (3) unless $m_V = m_\psi = 0$. As a consequence of this fact, Eqs. (8) are invariant under the gauge transformations, but Eqs. (9) are not.

In order to overcome the lack of gauge symmetry of the theory $S_k + S_m$, for $m_V \neq 0$ and $m_\psi \neq 0$, let us generalize (4) by introducing the Stückelberg superfields Ω, $\bar{\Omega}$, and N in the following way:

$$S'_m = \frac{im}{2}\left[\int d^6 z\psi^\alpha W_\alpha - \int d^6\bar{z}\bar{\psi}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}\right] + \frac{m_\psi^2}{4}\left\{\int d^6 z\left[\psi^\alpha - \frac{i}{m_\psi}\bar{D}^2 D^\alpha N\right]\left[\psi_\alpha - \frac{i}{m_\psi}\bar{D}^2 D_\alpha N\right] + \int d^6\bar{z}\left[\bar{\psi}^{\dot{\alpha}} + \frac{i}{m_\psi}D^2\bar{D}^{\dot{\alpha}}N\right]\left[\bar{\psi}_{\dot{\alpha}} + \frac{i}{m_\psi}D^2\bar{D}_{\dot{\alpha}}N\right]\right\} + \frac{m_V^2}{2}\int d^8 z\left[V + \frac{i}{m_V}(\Omega - \bar{\Omega})\right]^2, \quad (10)$$

where these new superfields transform as

$$\delta\Omega = m_V\Lambda, \qquad \delta\bar{\Omega} = m_V\bar{\Lambda}, \qquad \delta N = m_\psi L. \quad (11)$$

By construction, the action (10) (and $S_k + S'_m$) is invariant under the gauge transformations (3) and (11).

Note that there are mixed Stückelberg and gauge superfield terms in (10). This makes the one-loop calculations more cumbersome. However, if one fixes the gauge through adding the gauge-fixing term of the form

$$S_{\rm GF} = -\frac{1}{\alpha}\int d^8 z\left(\bar{D}^2 V - i\alpha m_V\frac{\bar{D}^2}{\Box}\bar{\Omega}\right)\left(D^2 V + i\alpha m_V\frac{D^2}{\Box}\Omega\right) - \frac{1}{8\beta}\int d^8 z(D^\alpha\psi_\alpha - \bar{D}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}} + 2i\beta m_\psi N)^2, \quad (12)$$

where α and β are the gauge-fixing parameters, the mixed terms are eliminated. Of course, since the gauge symmetry in this theory is Abelian, the ghosts completely decouple.

Up to now, we have considered only the free theory. Now, let us introduce its coupling to the matter represented as usual by chiral and antichiral scalar fields [1]. It is known that, under the usual gauge transformation, the chiral and antichiral matter superfields transform as [2]

$$\Phi' = e^{2ig\Lambda}\Phi, \qquad \bar{\Phi}' = \bar{\Phi}e^{-2ig\bar{\Lambda}}. \quad (13)$$

Then, we introduce the following gauge-invariant action involving coupling of matter and gauge fields [8] studied also in Ref. [9] within the cosmic strings context:

$$S_M = \int d^8 z\bar{\Phi}e^{2gV}\Phi e^{4hG}. \quad (14)$$

The coupling constants g and h have mass dimensions 0 and -1, respectively (the last fact implies the nonrenormalizability of the theory; however, renormalizable and gauge-invariant couplings of superfields ψ_α and $\bar{\psi}_{\dot{\alpha}}$ simply do not exist).

Finally, the complete supersymmetric massive gauge theory we study here is described by the sum of (1), (10), (12), and (14), that is,

$$S = -\frac{1}{2}\int d^8z V\left(-D^\alpha\bar{D}^2D_\alpha + \frac{1}{\alpha}\{D^2,\bar{D}^2\}\right)V - \frac{1}{8}\int d^8z\left\{\left(1+\frac{1}{\beta}\right)[\psi_\alpha D^\alpha D^\beta\psi_\beta + \bar{\psi}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\bar{D}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}}] + 2\left(1-\frac{1}{\beta}\right)\psi_\alpha D^\alpha\bar{D}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}}\right\} + \frac{m}{2}\int d^8z V(D^\alpha\psi_\alpha + \bar{D}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}}) + \frac{m_V^2}{2}\int d^8z V^2 + \frac{1}{2}\int d^8z\left[(D^\alpha\psi_\alpha)\frac{m_\psi^2}{2\Box}(D^\beta\psi_\beta) + (\bar{D}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}})\frac{m_\psi^2}{2\Box}(\bar{D}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}})\right] + \int d^8z\bar{\Phi}e^{2gV}\Phi e^{-2h(D^\alpha\psi_\alpha+\bar{D}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}})} + (\ldots), \tag{15}$$

where the dependence on the gauge superfields is given explicitly. Here the dots are for the contributions involving the Stückelberg superfields which completely decouple, giving only a trivial contribution to the effective action. Finally, notice that there is a nonlocality which was introduced in order to rewrite the mass term as an integral over the whole superspace.

Now, let us calculate the effective action for our theory. It is known [1] that, in the matter sector, the low-energy effective action in theories involving chiral and antichiral matter fields is characterized by the Kählerian effective potential (KEP) depending only on the background matter fields but not on their derivatives. Within this paper, we concentrate namely on calculating the KEP $K(\Phi,\bar{\Phi})$ in our theory.

The standard method of calculating the effective action is based on the methodology of the loop expansion [10,11]. To do this, we make a shift $\Phi\to\Phi+\phi$ in the superfield Φ (together with the analogous shift for $\bar{\Phi}$), where now Φ is a background (super)field and ϕ is a quantum one. Since our aim in this paper will consist in consideration of the KEP, we assume that the gauge superfields V, ψ_α, and $\bar{\psi}_{\dot{\alpha}}$ are purely quantum ones. In the one-loop approximation, one should keep only the quadratic terms in the quantum superfields. Therefore, (15) implies the following quadratic action of quantum superfields:

$$S_2[\bar{\Phi},\Phi;\bar{\phi},\phi,\psi_\alpha,\bar{\psi}_{\dot{\alpha}},V] = S_q + S_{\rm int}, \tag{16}$$

$$S_q = \frac{1}{2}\int d^8z\left[-V\Box\left(\Pi_{1/2}+\frac{1}{\alpha}\Pi_0\right)V - \frac{1}{2}\psi_\alpha D^\alpha\bar{D}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}} + 2\bar{\phi}\phi\right], \tag{17}$$

$$S_{\rm int} = \frac{1}{2}\int d^8z\left\{(m-8gh\bar{\Phi}\Phi)V(D^\alpha\psi_\alpha+\bar{D}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}}) + 2(2g)\bar{\Phi}V\phi + 2(2g)\Phi\bar{\phi}V + (m_V^2+(2g)^2\bar{\Phi}\Phi)V^2 - 4h\bar{\Phi}(D^\alpha\psi_\alpha+\bar{D}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}})\phi - 4h\Phi\bar{\phi}(D^\alpha\psi_\alpha+\bar{D}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}}) + (D^\alpha\psi_\alpha)\left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+\frac{m_\psi^2}{2\Box}+(2h)^2\bar{\Phi}\Phi\right]D^\beta\psi_\beta + (\bar{D}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}})\left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+\frac{m_\psi^2}{2\Box}+(2h)^2\bar{\Phi}\Phi\right]\bar{D}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}} + 2\left[\frac{1}{4\beta}+(2h)^2\bar{\Phi}\Phi\right](D^\alpha\psi_\alpha)\bar{D}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}}\right\}, \tag{18}$$

where the terms involving derivatives of the background superfields were omitted, being irrelevant for us. Here, we use the projection operators $\Pi_{1/2}\equiv-\Box^{-1}D^\alpha\bar{D}^2D_\alpha$ and $\Pi_0\equiv\Box^{-1}\{D^2,\bar{D}^2\}$.

The one-loop approximation does not depend on the manner of splitting the Lagrangian into free and interacting parts, since, at this order, one should deal only with the quadratic action of quantum superfields [12]. Usually, the propagators are defined from the background-independent terms, and the vertices are defined from the ones involving couplings of quantum superfields with the background ones. However, as a matter of convenience, here we will extract the propagators from S_q and treat the remaining terms as interaction vertices. Therefore, we obtain from S_q the propagators

$$\langle V(1)V(2)\rangle = -\frac{1}{p^2}(\Pi_{1/2}+\alpha\Pi_0)_1\delta_{12},$$
$$\langle \psi_\alpha(1)\bar{\psi}_{\dot\alpha}(2)\rangle = \frac{4p_{\alpha\dot\alpha}}{p^4}\delta_{12},$$
$$\langle \phi(1)\bar{\phi}(2)\rangle = \frac{1}{p^2}\delta_{12}. \tag{19}$$

It is convenient to transfer the covariant derivatives from the vertices to the propagators of the two-form superfield. This will allow us to define new scalar propagators written in terms of projection operators. To do it, let us employ some tricks used in Ref. [7]: one can observe from (18) that there is a factor $D^\alpha\bar{D}^2$ in a vertex associated to one end of the propagator $\langle\psi_\alpha(1)\bar{\psi}_{\dot\alpha}(2)\rangle$, and there is a factor $\bar{D}^{\dot\alpha}D^2$ in the other vertex at the other end of the same propagator. Thus, we absorb these covariant derivatives into a redefinition of the propagator $\langle\psi_\alpha(1)\bar{\psi}_{\dot\alpha}(2)\rangle$ (instead of associating them to vertices) and define a new scalar field $\psi = D^\alpha\psi_\alpha$ whose propagator is

$$\langle\psi(1)\bar{\psi}(2)\rangle \equiv D_1^\alpha\bar{D}_1^2\bar{D}_2^{\dot\alpha}D_2^2\langle\psi_\alpha(1)\bar{\psi}_{\dot\alpha}(2)\rangle = 4(\Pi_{1/2})_1\delta_{12}, \tag{20}$$

where we took into account that $\bar{D}_2^{\dot\alpha}D_2^2\delta_{12} = -D_1^2\bar{D}_1^{\dot\alpha}\delta_{12}$. We proceed in the same way with vertices and propagators involving ϕ and $\bar{\phi}$.

Finally, redefining the propagators and vertices in this manner, we get

$$\langle V(1)V(2)\rangle = -\frac{1}{p^2}(\Pi_{1/2}+\alpha\Pi_0)_1\delta_{12}, \tag{21}$$

$$\langle\psi(1)\bar{\psi}(2)\rangle = \langle\bar{\psi}(1)\psi(2)\rangle = 4(\Pi_{1/2})_1\delta_{12}, \tag{22}$$

$$\langle\phi(1)\bar{\phi}(2)\rangle = -(\Pi_-)_1\delta_{12}, \qquad \langle\bar{\phi}(1)\phi(2)\rangle = -(\Pi_+)_1\delta_{12}, \tag{23}$$

where $\Pi_- \equiv \Box^{-1}\bar{D}^2D^2$ and $\Pi_+ \equiv \Box^{-1}D^2\bar{D}^2$ are projection operators. The new interaction part of the action looks like

$$\tilde{S}_{\rm int} = \frac{1}{2}\int d^8z\Big\{2MV(\psi+\bar{\psi}) + 2(2g)\bar{\Phi}V\phi + 2(2g)\Phi\bar{\phi}V + (m_V^2+(2g)^2\bar{\Phi}\Phi)V^2 + \psi\left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+M_\psi\right]\psi + \bar{\psi}\left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+M_\psi\right]\bar{\psi} + 2\left[\frac{1}{4\beta}+(2h)^2\bar{\Phi}\Phi\right]\psi\bar{\psi}\Big\}, \tag{24}$$

where $M \equiv \frac{1}{2}(m - 8gh\bar{\Phi}\Phi)$ and $M_\psi \equiv \frac{m_\psi^2}{2\Box} + (2h)^2\bar{\Phi}\Phi$ (notice that M_ψ, although it characterizes the mass term, has zero mass dimension). We note that we redefined the theory in terms of scalar superfields only, which allows one to simplify the calculations drastically.

In the next section, namely the new propagators (21)–(23) and the new vertices (24), written only in terms of scalar superfields, will be used.

III. ONE-LOOP CALCULATIONS

So, let us proceed with calculating the KEP. The usual methods of its calculation are performed by means of perturbative series in powers of $\hbar$, the so-called loop expansion [1,10], namely,

$$K(\Phi,\bar{\Phi}) = K^{(0)}(\Phi,\bar{\Phi}) + \hbar K^{(1)}(\Phi,\bar{\Phi}) + \hbar^2K^{(2)}(\Phi,\bar{\Phi}) + \cdots. \tag{25}$$

The tree approximation can be read from the classical action (14) by replacing g and h by zero, yielding

$$K^{(0)}(\Phi,\bar{\Phi}) = \Phi\bar{\Phi}. \tag{26}$$

In order to calculate the one-loop contribution $K^{(1)}(\Phi,\bar{\Phi})$, we will use the methodology of summation over supergraphs originally elaborated in Ref. [13] and applied in many other examples including Ref. [7].

We proceed in three steps. First, we draw all the one-loop supergraphs allowed by (24). Second, we discard supergraphs involving covariant derivatives of Φ and $\bar{\Phi}$ and calculate the contributions of each supergraph, with the external momenta equal to zero, to the effective action. Finally, we sum all contributions and calculate the integral over the momenta. The result will be just the KEP.

Because of the known properties of the supersymmetric projectors, that is, $\Pi_{1/2}\Pi_- = \Pi_-\Pi_{1/2} = \Pi_{1/2}\Pi_+ = \Pi_+\Pi_{1/2} = 0$, and the fact that there is no any spinor covariant derivative in the vertices (24), it follows from (22) and (23) that the mixed contributions containing both $\langle\psi(1)\bar{\psi}(2)\rangle$ and $\langle\phi(1)\bar{\phi}(2)\rangle$ propagators cannot arise. Therefore, the set of the one-loop supergraphs contributing to the effective action in the theory under consideration are of four types.

In our graphical notation, solid lines denote $\langle\phi(1)\bar{\phi}(2)\rangle$ propagators, the dashed ones denote $\langle\psi(1)\bar{\psi}(2)\rangle$ propagators, the wavy ones denote $\langle V(1)V(2)\rangle$ propagators, and the double ones denote Φ or $\bar{\Phi}$ background superfields.

Let us start the calculations of the one-loop supergraphs involving only the gauge superfield propagators $\langle V(1)V(2)\rangle$ in the internal lines connecting the vertices $(m_V^2+(2g)^2\bar{\Phi}\Phi)V^2$. Such supergraphs are the simplest

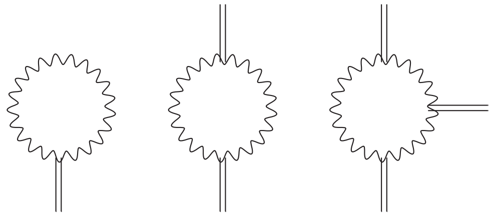

FIG. 1. One-loop supergraphs composed by propagators $\langle V(1)V(2)\rangle$.

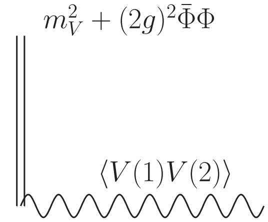

FIG. 2. A typical vertex in one-loop supergraphs involving $(m_V^2 + (2g)^2\bar{\Phi}\Phi)V^2$.

and exhibit structures given in Fig. 1. Of course, if we had taken the particular case $\alpha = 1$, such one-loop corrections would be zero, because the corrections would not contain any $D^2\bar{D}^2$ acting on the Grassmann delta function.

We can compute all the contributions by noting that each supergraph above is formed by n links, like those shown in Fig. 2.

Hence, the contribution of this link is simply given by

$$Q_{12} = (m_V^2 + (2g)^2\bar{\Phi}\Phi)_1\left[-\frac{1}{p^2}(\Pi_{1/2} + \alpha\Pi_0)\right]_1\delta_{12}. \quad (27)$$

Therefore, the contribution of a loop formed by n such links is given by

$$\begin{aligned}(I_a)_n &= \int d^4x\frac{1}{2n}\int d^4\theta_1 d^4\theta_3\ldots d^4\theta_{2n-1}\\ &\quad\times\int\frac{d^4p}{(2\pi)^4}Q_{13}Q_{35}\ldots Q_{2n-3,2n-1}Q_{2n-1,1}\\ &= \int d^8z\int\frac{d^4p}{(2\pi)^4}\frac{1}{2n}\left(-\frac{m_V^2 + (2g)^2\bar{\Phi}\Phi}{p^2}\right)^n\\ &\quad\times(\Pi_{1/2} + \alpha^n\Pi_0)\delta_{\theta\theta'}|_{\theta=\theta'},\end{aligned} \quad (28)$$

where we integrated by parts the expression $(I_a)_n$ and used the usual properties of the projection operators.

The contribution for the effective action is given by the sum of all supergraphs $(I_a)_n$:

$$\Gamma_a^{(1)} = \sum_{n=1}^{\infty}(I_a)_n = \int d^8z\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\left\{-\ln\left[1 + \frac{m_V^2 + (2g)^2\bar{\Phi}\Phi}{p^2}\right] + \ln\left[1 + \alpha\frac{m_V^2 + (2g)^2\bar{\Phi}\Phi}{p^2}\right]\right\}. \quad (29)$$

Notice that this contribution vanishes at $\alpha = 1$ (Feynman gauge), as it should.

Let us proceed the calculation of the second type of one-loop supergraphs, which involve the $\langle\phi(1)\bar{\phi}(2)\rangle$ and $\langle V(1)V(2)\rangle$ propagators in the internal lines connecting the vertices $(2g)\Phi\bar{\phi}V$ and $(2g)\bar{\Phi}V\phi$. Such supergraphs exhibit the structure shown in Fig. 3. Certainly, if we had taken the particular case $\alpha = 0$, such one-loop corrections would not contribute to the effective action, because $\langle V(1)V(2)\rangle \sim \Pi_{1/2}$ and $\Pi_{1/2}\Pi_- = \Pi_{1/2}\Pi_+ = 0$.

To sum over arbitrary numbers of insertions of vertices $(m_V^2 + (2g)^2\bar{\Phi}\Phi)V^2$ into the gauge propagators, it is convenient to define a "dressed" propagator where the summation over all vertices $(m_V^2 + (2g)^2\bar{\Phi}\Phi)V^2$ is performed (see Fig. 4), which, as a result, is equal to

$$\begin{aligned}\langle V(1)V(2)\rangle_D &= \langle V(1)V(2)\rangle + \int d^4\theta_3\langle V(1)V(3)\rangle[m_V^2 + (2g)^2\bar{\Phi}\Phi]_3\langle V(3)V(2)\rangle\\ &\quad + \int d^4\theta_3 d^4\theta_4\langle V(1)V(3)\rangle[m_V^2 + (2g)^2\bar{\Phi}\Phi]_3\langle V(3)V(4)\rangle[m_V^2 + (2g)^2\bar{\Phi}\Phi]_4\langle V(4)V(2)\rangle + \cdots.\end{aligned} \quad (30)$$

Finally, we arrive at

$$\langle V(1)V(2)\rangle_D = -\left[\frac{\Pi_{1/2}}{p^2 + m_V^2 + (2g)^2\bar{\Phi}\Phi} + \frac{\alpha\Pi_0}{p^2 + \alpha(m_V^2 + (2g)^2\bar{\Phi}\Phi)}\right]_1\delta_{12}. \quad (31)$$

Then, we notice that each supergraph above (see Fig. 3) is formed by n links depicted in Fig. 5, each of which yields the contribution

$$\begin{aligned}R_{13} &= \int d^4\theta_2[(2g)\Phi]_1\left\{-\left[\frac{\Pi_{1/2}}{p^2 + m_V^2 + (2g)^2\bar{\Phi}\Phi} + \frac{\alpha\Pi_0}{p^2 + \alpha(m_V^2 + (2g)^2\bar{\Phi}\Phi)}\right]_1\delta_{12}\right\}[(2g)\bar{\Phi}]_2[-(\Pi_-)_2\delta_{23}]\\ &= \left[\frac{\alpha(2g)^2\bar{\Phi}\Phi}{p^2 + \alpha(m_V^2 + (2g)^2\bar{\Phi}\Phi)}\Pi_-\right]_1\delta_{13}.\end{aligned} \quad (32)$$

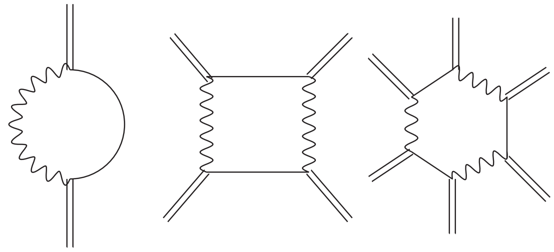

FIG. 3. One-loop supergraphs composed by propagators $\langle\phi(1)\bar{\phi}(2)\rangle$ and $\langle V(1)V(2)\rangle$.

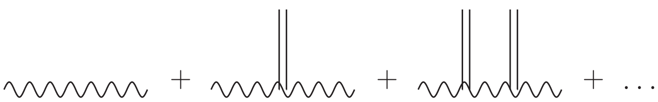

FIG. 4. Dressed propagator.

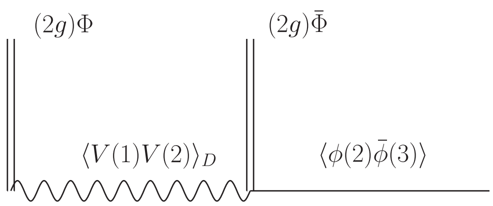

FIG. 5. A typical link in one-loop supergraphs in a mixed sector.

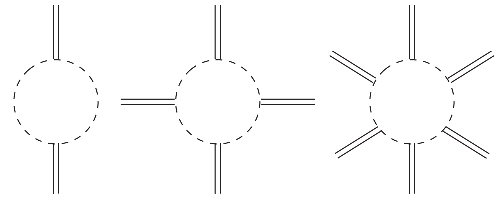

FIG. 6. One-loop supergraphs composed by propagators $\langle\psi(1)\bar{\psi}(2)\rangle$.

FIG. 7. Dressed propagator $\langle\psi(1)\bar{\psi}(2)\rangle_D$. The vertices are $[\frac{1}{4\beta}+(2h)^2(\Phi\bar{\Phi})]\psi\bar{\psi}$.

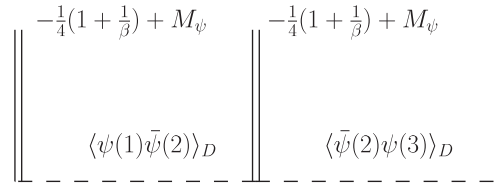

FIG. 8. A typical vertex in one-loop supergraphs involving $[-\frac{1}{4}(1+\frac{1}{\beta})+M_\psi]\psi^2$ and $[-\frac{1}{4}(1+\frac{1}{\beta})+M_\psi]\bar{\psi}^2$.

Therefore, the contribution of a supergraph formed by n such links is given by

$$\begin{aligned}(I_b)_n &= \int d^4x\frac{1}{n}\int d^4\theta_1 d^4\theta_3\ldots d^4\theta_{2n-1}\int\frac{d^4p}{(2\pi)^4}R_{13}R_{35}\ldots R_{2n-3,2n-1}R_{2n-1,1}\\ &= \int d^8z\frac{1}{n}\int\frac{d^4p}{(2\pi)^4}\left(\frac{\alpha(2g)^2\bar{\Phi}\Phi}{p^2+\alpha(m_V^2+(2g)^2\bar{\Phi}\Phi)}\right)^n\Pi_-\delta_{\theta\theta'}|_{\theta=\theta'}.\end{aligned} \tag{33}$$

By using $\Pi_-\delta_{\theta\theta'}|_{\theta=\theta'}=-1/p^2$, we get the effective action

$$\Gamma_b^{(1)}=\sum_{n=1}^{\infty}(I_b)_n=\int d^8z\frac{1}{p^2}\ln\left[\frac{p^2+\alpha m_V^2}{p^2+\alpha(m_V^2+(2g)^2\bar{\Phi}\Phi)}\right]. \tag{34}$$

We notice that at $\alpha=0$ (Landau gauge) this expression vanishes.

By summing (29) and (34), we get

$$\Gamma_a^{(1)}+\Gamma_b^{(1)}=\int d^8z\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\left\{-\ln\left[1+\frac{m_V^2+(2g)^2\bar{\Phi}\Phi}{p^2}\right]+\ln\left[1+\alpha\frac{m_V^2}{p^2}\right]\right\}. \tag{35}$$

Notice that (35) is explicitly gauge independent for the massless case $m_V=0$. However, even in the massive case, the α dependence is trivial, since the last logarithm does not depend on the background superfields and hence can be disregarded.

Now, let us sum over the vertices $[-\frac{1}{4}(1+\frac{1}{\beta})+M_\psi]\psi^2$ and $[-\frac{1}{4}(1+\frac{1}{\beta})+M_\psi]\bar{\psi}^2$. The corresponding supergraphs exhibit their structures in Fig. 6 with only an even number of vertices. Since we can insert an arbitrary number of vertices $[\frac{1}{4\beta}+(2h)^2(\Phi\bar{\Phi})]\psi\bar{\psi}$ into the propagators $\langle\psi(1)\bar{\psi}(2)\rangle$, we must introduce the dressed propagator $\langle\psi(1)\bar{\psi}(2)\rangle_D$ (see Fig. 7). Therefore, this dressed propagator is equal to

$$\langle\psi(1)\bar\psi(2)\rangle_D = \langle\psi(1)\bar\psi(2)\rangle + \int d^4\theta_3 \langle\psi(1)\bar\psi(3)\rangle[(2h)^2\bar\Phi\Phi]_3\langle\psi(3)\bar\psi(2)\rangle + \int d^4\theta_3 d^4\theta_4 \langle\psi(1)\bar\psi(3)\rangle[(2h)^2\bar\Phi\Phi]_3\langle\psi(3)\bar\psi(4)\rangle[(2h)^2\bar\Phi\Phi]_4\langle\psi(4)\bar\psi(2)\rangle + \cdots. \tag{36}$$

By using (22) and proceeding as above, we arrive at

$$\langle\psi(1)\bar\psi(2)\rangle_D = \left(\frac{4\beta\Pi_{1/2}}{\beta-1-4\beta(2h)^2\bar\Phi\Phi}\right)_1 \delta_{12}. \tag{37}$$

Afterwards, we can compute all the contributions by noting that each one-loop supergraph above is formed by n vertices like those given by Fig. 8.

Hence, the contribution of this vertex is given by

$$L_{13} = \int d^4\theta_2 \left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+M_\psi\right]_1 \left[\left(\frac{4\beta\Pi_{1/2}}{\beta-1-4\beta(2h)^2\bar\Phi\Phi}\right)_1\delta_{12}\right]\left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+M_\psi\right]_2\left[\left(\frac{4\beta\Pi_{1/2}}{\beta-1-4\beta(2h)^2\bar\Phi\Phi}\right)_2\delta_{23}\right] = \left(\frac{\beta+1-4\beta M_\psi}{\beta-1-4\beta(2h)^2\bar\Phi\Phi}\Pi_{1/2}\right)^2_1\delta_{13}. \tag{38}$$

It follows from the result above that the contribution of a supergraph formed by n vertices is given by

$$(I_c)_n = \int d^4x \frac{1}{2n}\int d^4\theta_1 d^4\theta_3 \ldots d^4\theta_{2n-1}\int \frac{d^4p}{(2\pi)^4} L_{13}L_{35}\ldots L_{2n-3,2n-1}L_{2n-1,1} = \int d^8z \frac{1}{2n}\int \frac{d^4p}{(2\pi)^4}\left(\frac{\beta+1-4\beta M_\psi}{\beta-1-4\beta(2h)^2\bar\Phi\Phi}\Pi_{1/2}\right)^{2n}\Pi_{1/2}\delta_{\theta\theta'}|_{\theta=\theta'}. \tag{39}$$

On one hand, for $\beta=0$, we get ($\Pi_{1/2}\delta_{\theta\theta'}|_{\theta=\theta'} = 2/p^2$)

$$(I_c)_n = \int d^8z \frac{1}{n}\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}. \tag{40}$$

This integral over the momenta vanishes within the dimensional regularization scheme. Therefore, we get

$$\Gamma_c^{(1)} = 0, \quad \text{for } \beta = 0. \tag{41}$$

On the other hand, for $\beta\neq 0$, we obtain the effective action

$$\Gamma_c^{(1)} = \sum_{n=1}^{\infty}(I_c)_n = -\int d^8z\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2} \times \ln\left\{1-\left[\frac{\beta+1-4\beta(-\frac{m_\psi^2}{2p^2}+(2h)^2\bar\Phi\Phi)}{\beta-1-4\beta(2h)^2\bar\Phi\Phi}\right]^2\right\}, \tag{42}$$

for $\beta\neq 0$. Moreover, we used $M_\psi \equiv -\frac{m_\psi^2}{2p^2}+(2h)^2\bar\Phi\Phi$. In particular, if $m_\psi=0$, Eq. (42) also vanishes within the dimensional regularization scheme.

Finally, let us evaluate the last type of one-loop supergraphs, which involve the propagators $\langle\psi(1)\bar\psi(2)\rangle$ and $\langle V(1)V(2)\rangle$ in the internal lines connecting the vertices $MV\psi$ and $MV\bar\psi$ (see Fig. 9). As before, we can insert an arbitrary number of vertices $[\frac{1}{4\beta}+(2h)^2(\Phi\bar\Phi)]\psi\bar\psi$ into the propagators $\langle\psi(1)\bar\psi(2)\rangle$. Moreover, we can also insert an arbitrary number of pairs of the vertices $[-\frac{1}{4}(1+\frac{1}{\beta})+M_\psi]\psi^2$ and $[-\frac{1}{4}(1+\frac{1}{\beta})+M_\psi]\bar\psi^2$ into $\langle\psi(1)\bar\psi(2)\rangle$. Since $\langle\psi(1)\bar\psi(2)\rangle$ has already been dressed by $[\frac{1}{4\beta}+(2h)^2(\Phi\bar\Phi)]\psi\bar\psi$ in (36) and (37), it follows that the desired dressed propagator $\langle\psi(1)\bar\psi(2)\rangle_{2D}$ can be obtained through the summation over all pairs of the vertices $[-\frac{1}{4}(1+\frac{1}{\beta})+M_\psi]\psi^2$ and $[-\frac{1}{4}(1+\frac{1}{\beta})+M_\psi]\bar\psi^2$ into $\langle\psi(1)\bar\psi(2)\rangle_D$ (see Fig. 10). Therefore, we get

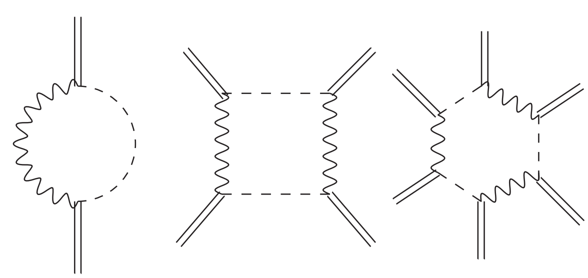

FIG. 9. One-loop supergraphs composed by propagators $\langle\psi(1)\bar\psi(2)\rangle$ and $\langle V(1)V(2)\rangle$.

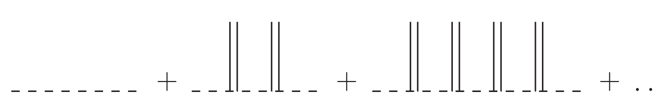

FIG. 10. Dressed propagator $\langle\psi(1)\bar\psi(2)\rangle_{2D}$.

$$\langle\psi(1)\bar{\psi}(2)\rangle_{2D} = \langle\psi(1)\bar{\psi}(2)\rangle_D + \int d^4\theta_3 d^4\theta_4 \langle\psi(1)\bar{\psi}(3)\rangle_D \left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+M_\psi\right]_3 \langle\bar{\psi}(3)\psi(4)\rangle_D \times \left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+M_\psi\right]_4 \langle\psi(4)\bar{\psi}(2)\rangle_D + \int d^4\theta_3 d^4\theta_4 d^4\theta_5 d^4\theta_6 \langle\psi(1)\bar{\psi}(3)\rangle_D \times \left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+M_\psi\right]_3 \langle\bar{\psi}(3)\psi(4)\rangle_D \left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+M_\psi\right]_4 \langle\psi(4)\bar{\psi}(5)\rangle_D \times \left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+M_\psi\right]_5 \langle\bar{\psi}(5)\psi(6)\rangle_D \left[-\frac{1}{4}\left(1+\frac{1}{\beta}\right)+M_\psi\right]_6 \langle\psi(6)\bar{\psi}(2)\rangle_D + \cdots. \tag{43}$$

After some algebraic work, we find

$$\langle\psi(1)\bar{\psi}(2)\rangle_{2D} = (f(\bar{\Phi}\Phi)\Pi_{1/2})_1\delta_{12}, \tag{44}$$

where

$$f(\bar{\Phi}\Phi) \equiv \frac{[1-\beta(1-4(2h)^2\bar{\Phi}\Phi)]p^4}{(1-4(2h)^2\bar{\Phi}\Phi)p^4 + [1+\beta(1-4(2h)^2\bar{\Phi}\Phi)]m_\psi^2 p^2 + \beta m_\psi^4}. \tag{45}$$

As before, we can compute all the contributions by noting that each supergraph above (Fig. 9) is formed by n links depicted in Fig. 11, each of which yields the contribution

$$N_{13} = \int d^4\theta_2 (M)_1 \left\{-\left[\frac{\Pi_{1/2}}{p^2+m_V^2+(2g)^2\bar{\Phi}\Phi} + \frac{\alpha\Pi_0}{p^2+\alpha(m_V^2+(2g)^2\bar{\Phi}\Phi)}\right]_1 \delta_{12}\right\}(M)_2 \times [(f\Pi_{1/2})_2\delta_{23}] = \left(\frac{-fM^2\Pi_{1/2}}{p^2+m_V^2+(2g)^2\bar{\Phi}\Phi}\right)_1 \delta_{13}. \tag{46}$$

Hence, the contribution of a supergraph formed by n such links is given by

$$(I_d)_n = \int d^4x \frac{1}{2n}\int d^4\theta_1 d^4\theta_3 \ldots d^4\theta_{2n-1}\int \frac{d^4p}{(2\pi)^4} N_{13}N_{35}\ldots N_{2n-3,2n-1}N_{2n-1,1} = \int d^8z \frac{1}{2n}\int \frac{d^4p}{(2\pi)^4}\left(\frac{-fM^2}{p^2+m_V^2+(2g)^2\bar{\Phi}\Phi}\right)^n \Pi_{1/2}\delta_{\theta\theta'}|_{\theta=\theta'}. \tag{47}$$

Again, by using $\Pi_{1/2}\delta_{\theta\theta'}|_{\theta=\theta'} = 2/p^2$, we get the effective action

$$\Gamma_d^{(1)} = \sum_{n=1}^{\infty}(I_d)_n = -\int d^8z \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\ln\left[1+\frac{fM^2}{p^2+m_V^2+(2g)^2\bar{\Phi}\Phi}\right]. \tag{48}$$

For $\beta = 0$, we get the total one-loop KEP (up to terms independent on the background superfields) by summing (35), (41), and (48):

$$K_{\beta=0}^{(1)}(\bar{\Phi}\Phi) = -\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\ln\left\{p^2+m_V^2+(2g)^2\bar{\Phi}\Phi + \frac{M^2p^2}{(1-4(2h)^2\bar{\Phi}\Phi)p^2+m_\psi^2}\right\}, \tag{49}$$

where we substituted the explicit form of f for $\beta = 0$.

For $\beta \neq 0$, we obtain the total one-loop KEP by summing (35), (42), and (48):

$$K_{\beta\neq 0}^{(1)}(\bar{\Phi}\Phi) = -\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\left\{\ln\left\{1-\left[\frac{\beta+1-4\beta(-\frac{m_\psi^2}{2p^2}+(2h)^2\bar{\Phi}\Phi)}{\beta-1-4\beta(2h)^2\bar{\Phi}\Phi}\right]^2\right\} + \ln\left\{p^2+m_V^2+(2g)^2\bar{\Phi}\Phi + \frac{[1-\beta(1-4(2h)^2\bar{\Phi}\Phi)]M^2p^4}{(1-4(2h)^2\bar{\Phi}\Phi)p^4+[1+\beta(1-4(2h)^2\bar{\Phi}\Phi)]m_\psi^2p^2+\beta m_\psi^4}\right\}\right\}, \tag{50}$$

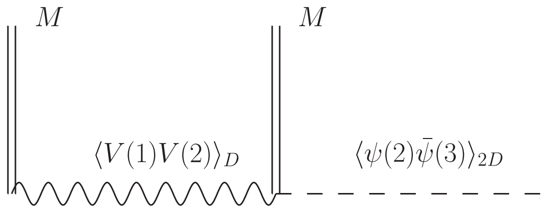

FIG. 11. A typical vertex in one-loop supergraphs involving $MV\psi$ and $MV\bar{\psi}$.

where we substituted the explicit form of f (45). Notice that (50) depends on the gauge parameter β, but it does not depend on the gauge parameter α (one should recall that β corresponds to the gauge fixing for the ψ_α field, and α—for the real V gauge field, and the gauge independence, that is, α independence of the one-loop KEP in the super-QED involving only chiral matter and the V field, is a well-known fact [13]).

Unfortunately we did not succeed in performing the momentum integrals (50) analytically and finding an explicit expression for the β-dependent term in a most generic case. Therefore, in order to proceed with the calculation and solve explicitly the integral above, at least in certain cases, we will consider two characteristic examples where the final result is expressed in closed form and in terms of elementary functions.

As our first example, let us take $m_\psi = 0$ in (49) and (50). It follows that

$$K^{(1)}_{m_\psi=0}(\bar{\Phi}\Phi) = -\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\left\{\ln\left\{1-\left[\frac{\beta+1-4\beta(2h)^2\bar{\Phi}\Phi}{\beta-1-4\beta(2h)^2\bar{\Phi}\Phi}\right]^2\right\} + \ln\left\{p^2+m_V^2+(2g)^2\bar{\Phi}\Phi+\frac{[1-\beta(1-4(2h)^2\bar{\Phi}\Phi)]M^2}{1-4(2h)^2\bar{\Phi}\Phi}\right\}\right\}. \quad (51)$$

The first integral in this expression vanishes within the dimensional reduction scheme. The second one is well known and, in the limit $\omega \to 2$, gives

$$K^{(1)}_{m_\psi=0} = K^{(1)}_{m_\psi=0,\mathrm{div}}(\bar{\Phi}\Phi) + K^{(1)}_{m_\psi=0,\mathrm{fin}}(\bar{\Phi}\Phi), \quad (52)$$

where

$$K^{(1)}_{m_\psi=0,\mathrm{div}}(\bar{\Phi}\Phi) = -\frac{1}{16\pi^2(2-\omega)}\left[m_V^2+(2g)^2\bar{\Phi}\Phi+\frac{[1-\beta(1-4(2h)^2\bar{\Phi}\Phi)]M^2}{1-4(2h)^2\bar{\Phi}\Phi}\right], \quad (53)$$

$$K^{(1)}_{m_\psi=0,\mathrm{fin}}(\bar{\Phi}\Phi) = \frac{1}{16\pi^2}\left[m_V^2+(2g)^2\bar{\Phi}\Phi+\frac{[1-\beta(1-4(2h)^2\bar{\Phi}\Phi)]M^2}{1-4(2h)^2\bar{\Phi}\Phi}\right] \times \ln\frac{1}{\mu^2}\left[m_V^2+(2g)^2\bar{\Phi}\Phi+\frac{[1-\beta(1-4(2h)^2\bar{\Phi}\Phi)]M^2}{1-4(2h)^2\bar{\Phi}\Phi}\right], \quad (54)$$

$M = \frac{1}{2}(m-8gh\bar{\Phi}\Phi)$, and μ is an arbitrary scale required on dimensional grounds. If we take the particular case of $\beta=-1$ and $m_V=0$, we recover the result of Ref. [7].

As our second example, let us consider $\beta=0$. Hence, we need only to calculate (49). The procedure to calculate it is quite analogous to the one reported in Ref. [14]. Therefore, we get

$$K^{(1)}_{\beta=0} = K^{(1)}_{\beta=0,\mathrm{div}}(\bar{\Phi}\Phi) + K^{(1)}_{\beta=0,\mathrm{fin}}(\bar{\Phi}\Phi), \quad (55)$$

where

$$K^{(1)}_{\beta=0,\mathrm{div}}(\bar{\Phi}\Phi) = -\frac{1}{16\pi^2(2-\omega)}\left[m_V^2+(2g)^2\bar{\Phi}\Phi+\frac{M^2}{1-4(2h)^2\bar{\Phi}\Phi}\right], \quad (56)$$

$$K^{(1)}_{\beta=0,\mathrm{fin}}(\bar{\Phi}\Phi) = \frac{1}{16\pi^2}\left[\Omega_+\ln\left(\frac{\Omega_+}{\mu^2}\right)+\Omega_-\ln\left(\frac{\Omega_-}{\mu^2}\right)-\Omega_3\ln\left(\frac{\Omega_3}{\mu^2}\right)\right]. \quad (57)$$

Moreover, we introduced a shorthand notation:

$$\Omega_{\pm} = \frac{1}{2}\left\{m_V^2 + (2g)^2\bar{\Phi}\Phi + \frac{m_\psi^2 + M^2}{1-4(2h)^2\bar{\Phi}\Phi} \pm \sqrt{\left[m_V^2 + (2g)^2\bar{\Phi}\Phi + \frac{m_\psi^2 + M^2}{1-4(2h)^2\bar{\Phi}\Phi}\right]^2 - 4m_\psi^2(m_V^2 + (2g)^2\bar{\Phi}\Phi)}\right\},$$
$$\Omega_3 = \frac{m_\psi^2}{1-4(2h)^2\bar{\Phi}\Phi}. \quad (58)$$

Notice that one-loop results (52) and (55) are both divergent. Moreover, we notice that the divergent parts (53) and (56) are nonpolynomial, and, to eliminate the divergences, it would be necessary to introduce an infinite number of counterterms and an infinite number of unknown parameters in order to cancel the ultraviolet divergences appearing in the quantum corrections, so that the theory would not have any predictive power. However, it reflects the fact we already mentioned above, that theory under consideration is nonrenormalizable and must be interpreted as an effective field theory for the low-energy domain [15]. It is also clear that, in the case $h = 0$, we notice that the divergent terms (53) and (56) are proportional to $\bar{\Phi}\Phi$. Therefore, we can implement a one-loop counterterm such as the one used in the supersymmetric quantum electrodynamics [13] to eliminate the divergences. However, in this case, the coupling between chiral matter and chiral spinor gauge fields is switched off; therefore, the spinor gauge field completely decouples, and the theory reduces to the usual super-QED.

IV. SUMMARY

We considered the Abelian superfield gauge theory involving two gauge fields: the real scalar one and the spinor one. The essentially new feature of this theory consists in the fact that it essentially involves the BF term, thus opening the way for constructing more sophisticated supersymmetric models involving the antisymmetric tensor fields.

The theory we consider represents itself as an alternative model involving two gauge fields, different from the one considered earlier in Ref. [7]. For this theory, we calculated the one-loop Kählerian effective potential, which turns out to be divergent, since the only possible gauge-invariant coupling of matter to the spinor gauge field turns out to be nonrenormalizable. However, treating this theory as an effective model for a description of the low-energy limit of the string theory (one can recall that the antisymmetric tensor field naturally emerges within the string context, playing there an important role; see, e.g., Ref. [16]), we can implement a natural cutoff of the order of the characteristic string mass.

Further application of this study could consist in development of supersymmetric extensions of more sophisticated theories involving the BF theory as an ingredient.

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