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Optical properties of black holes in the presence of a plasma: The shadow

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We have studied photon motion around axially symmetric rotating Kerr black holes in the presence of a plasma with radial power-law density. It is shown that in the presence of a plasma, the observed shape and size of the shadow changes depending on the (i) plasma parameters, (ii) black hole spin, and (iii) inclination angle between the observer plane and the axis of rotation of the black hole. In order to extract the pure effect of the plasma influence on the black hole image, the particular case of the Schwarzschild black hole has also been investigated and it has been shown that the photon sphere around the spherically symmetric black hole is left unchanged under the plasma influence; however, the Schwarzschild black hole shadow size in the plasma is reduced due to the refraction of the electromagnetic radiation in the plasma environment of the black hole. The study of the energy emission from the black hole in plasma environment shows that in the presence of the plasma the maximal energy emission rate from the black hole decreases.

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I. INTRODUCTION

The study of astrophysical processes in the plasma medium surrounding a black hole becomes very interesting and important due to the evidence for the presence of black holes at the centers of the galaxies [1–3]. For example, the gravitational lensing in inhomogeneous and homogeneous plasma around black holes has been recently studied in [4–8] as an extension of vacuum studies (see, e.g., [9,10]).

From the literature it is known that the black hole shadow can be revealed by the gravitational lensing effect, see, e.g., [2,11–13]. If the black hole is placed between a bright source and a far observer, the dark zone is created in the source image by a photon falling inside the black hole which is commonly called the shadow of the black hole. Recently, this effect has been investigated by many authors for the different black holes (see, e.g., [14–17]). The silhouette shape of an extremely rotating black hole has been investigated by Bardeen [18]. Our previous studies on the shadow of the black hole are related to the non-Kerr [13], Hořava-Lifshitz [17], Kerr-Taub-NUT [19], and Myers-Perry [20] black holes. A new coordinate-independent formalism for characterization of a black-hole shadow has been recently developed in [21].

The shape of the black hole is determined through a boundary of the shadow which can be studied by application of the null geodesic equations. The presence of a plasma in the vicinity of black holes changes the equations of motion of photons which may lead to the modification of the black hole shadow by the influence of a plasma. In this paper our main goal is to consider the silhouette of the shadow of an axially symmetric black hole using the equations of motion for photons in a plasma with radial power-law density. We would like to underline that very recently, influence of a non-magnetized cold plasma with the radially dependent density to black hole shadow has been studied in [7] using the different alternate approach. In addition, [22] has studied the photon motion around the black hole surrounded by a plasma.

The paper is arranged as follows. In Sec. II, we consider the equations of motion of photons around an axially symmetric black hole in the presence of a plasma. In Sec. III we study the shadow of the axial-symmetric black hole in the presence of a plasma. As a particular case in subsections III A and III B, we study the shadow and the energy emission from the spherically symmetric black hole. Finally, in Sec. IV we briefly summarize our results.

Throughout the paper, we use a system of geometric units in which $G = 1 = c$. Greek indices run from 0 to 3.

II. PHOTON MOTION AROUND THE BLACK HOLE IN THE PRESENCE OF A PLASMA

The rotating black hole is described by the spacetime metric, which in the standard Boyer-Lindquist coordinates can be written in the form

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta, \tag{1}$$

with [23]

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$$g_{00} = -\left(1 - \frac{2Mr}{\Sigma}\right),$$
$$g_{11} = \frac{\Sigma}{\Delta},$$
$$g_{22} = \Sigma,$$
$$g_{33} = \left[(r^2 + a^2) + \frac{2a^2 Mr\sin^2\theta}{\Sigma}\right]\sin^2\theta,$$
$$g_{03} = -\frac{2Mar\sin^2\theta}{\Sigma},$$
$$\Delta = r^2 + a^2 - 2Mr, \qquad \Sigma = r^2 + a^2\cos^2\theta, \tag{2}$$

where as usual M and a are the total mass and the spin parameter of the black hole.

In this paper we will consider a plasma surrounding the central axially symmetric black hole. The refraction index of the plasma will be $n = n(x^i, \omega)$, where the photon frequency measured by the observer with velocity u^α is ω. In this case the effective energy of the photon has the form $\hbar\omega = -p_\alpha u^\alpha$. The refraction index of the plasma as a function of the photon four-momentum has been obtained in [24] and has the following form,

$$n^2 = 1 + \frac{p_\alpha p^\alpha}{(p_\beta u^\beta)^2}, \tag{3}$$

and for the vacuum case one has the relation $n = 1$. The Hamiltonian for the photon around an arbitrary black hole surrounded by a plasma has the following form:

$$H(x^\alpha, p_\alpha) = \frac{1}{2}[g^{\alpha\beta} p_\alpha p_\beta + (n^2 - 1)(p_\beta u^\beta)^2] = 0. \tag{4}$$

Following the derivation of a gravitational redshift discussed in [25], we will assume that the spacetime stationarity allows the existence of a timelike Killing vector ξ^α obeying the Killing equations,

$$\xi_{\alpha;\beta} + \xi_{\beta;\alpha} = 0. \tag{5}$$

Then one can introduce two frequencies of electromagnetic waves using the null wave-vector k^α; the first one is the frequency measured by an observer with four-velocity u^α and defined as

$$\omega \equiv -k^\alpha u_\alpha, \tag{6}$$

while the second one is the frequency associated with the timelike Killing vector ξ^α and defined as

$$\omega_\xi \equiv -k^\alpha \xi_\alpha. \tag{7}$$

The frequency (6) depends on the observer chosen and is, therefore, a function of position, while the frequency (7) is a conserved quantity that remains unchanged along the trajectory followed by the electromagnetic wave. One can apply this property to measure how the frequency changes with the radial position and is redshifted in the spacetime. Assume the Killing vector to have components

$$\xi^\alpha \equiv (1, 0, 0, 0); \qquad \xi_\alpha \equiv g_{00}(-1, 0, 0, 0), \tag{8}$$

so that $\omega_\xi = k_0 = \text{const}$. The frequency of an electromagnetic wave emitted at radial position r and measured by an observer with four-velocity $u^\alpha\{1/\sqrt{-g_{00}}, 0, 0, 0\}$ parallel to ξ^α (i.e., a static observer) will be governed by the following equation:

$$\sqrt{-g_{00}}\,\omega(r) = \omega_\xi = \text{const}. \tag{9}$$

One may introduce a specific form for the plasma frequency for analytic processing, assuming that the refractive index has the general form

$$n^2 = 1 - \frac{\omega_e^2}{\omega^2}, \tag{10}$$

where ω_e is usually called plasma frequency. Now we use the Hamilton-Jacobi equation which defines the equation of motion of the photons for a given spacetime geometry [4,22,24],

$$\frac{\partial S}{\partial \sigma} = -\frac{1}{2}\left[g^{\alpha\beta} p_\alpha p_\beta - (n^2 - 1)\left(p_0\sqrt{-g^{00}}\right)^2\right], \tag{11}$$

where $p_\alpha = \partial S/\partial x^\alpha$. Using a method of separation of variables, the Jacobi action S can be written as [13,23]

$$S = \frac{1}{2}m^2\sigma - \mathcal{E}t + \mathcal{L}\phi + S_r(r) + S_\theta(\theta), \tag{12}$$

where $\mathcal{L}$, $\mathcal{E}$ are conservative quantities as angular momentum and energy of the test particles.

For trajectories of the photons, we have the following set of the equations:

$$\Sigma\frac{dt}{d\sigma} = a(\mathcal{L} - n^2\mathcal{E}a\sin^2\theta) + \frac{r^2 + a^2}{\Delta}[(r^2 + a^2)n^2\mathcal{E} - a\mathcal{L}], \tag{13}$$

$$\Sigma\frac{d\phi}{d\sigma} = \left(\frac{\mathcal{L}}{\sin^2\theta} - a\mathcal{E}\right) + \frac{a}{\Delta}[(r^2 + a^2)\mathcal{E} - a\mathcal{L}], \tag{14}$$

$$\Sigma\frac{dr}{d\sigma} = \sqrt{\mathcal{R}}, \tag{15}$$

$$\Sigma\frac{d\theta}{d\sigma} = \sqrt{\Theta}, \tag{16}$$

can be derived from the Hamilton-Jacobi equation, where the functions $\mathcal{R}(r)$ and $\Theta(\theta)$ are introduced as

$$\mathcal{R} = [(r^2+a^2)\mathcal{E} - a\mathcal{L}]^2 + (r^2+a^2)^2(n^2-1)\mathcal{E}^2 - \Delta[\mathcal{K} + (\mathcal{L} - a\mathcal{E})^2], \quad (17)$$

$$\Theta = \mathcal{K} + \cos^2\theta\left(a^2\mathcal{E}^2 - \frac{\mathcal{L}^2}{\sin^2\theta}\right) - (n^2-1)a^2\mathcal{E}^2\sin^2\theta, \quad (18)$$

and the Carter constant as $\mathcal{K}$.

For calculation examples one needs the analytical expression of the plasma frequency ω_e which for the electron plasma has the following form,

$$\omega_e^2 = \frac{4\pi e^2 N(r)}{m_e}, \quad (19)$$

where e and m_e are the electron charge and mass, respectively, and $N(r)$ is the plasma number density. Following the work by [22], here we consider a radial power-law density,

$$N(r) = \frac{N_0}{r^h}, \quad (20)$$

where $h \geq 0$, such that

$$\omega_e^2 = \frac{k}{r^h}. \quad (21)$$

As an example, here we get the value for power h as 1 [22]. For this value, we plot the radial dependence of the effective potential V_{eff} of the radial motion of the photons defined as

$$\left(\frac{dr}{d\sigma}\right)^2 + V_{\mathrm{eff}} = 1. \quad (22)$$

The radial dependence of the effective potential for different values of the plasma refraction n and black hole spin a has been presented in Fig. 1. In Fig. 1 the left plot corresponds to the case when refraction parameter of the plasma is $n^2 = 0.2, 0.44, 0.89$ (dotted, dashed, and solid lines, respectively) at the position $r = 3M$; the middle plot corresponds to the case when the refraction parameter is $n^2 = 0.19, 0.42, 0.88$ corresponding to dotted, dashed, and solid lines, respectively, at the position $r = 3M$; the right plot represents the radial dependence of the effective potential when the refraction parameter is $n^2 = 0.14, 0.39, 0.88$, corresponding to dotted, dashed, and solid lines, respectively, at the position $r = 3M$.

III. THE SHADOW OF A BLACK HOLE IN THE PRESENCE OF A PLASMA

In this section we consider the shadow cast by a black hole surrounded by a plasma. If the black hole surrounded by a plasma is originated between the light source and the observer, then the latter can observe the black spot on the bright background. The observer at infinity can only observe the light beam scattered away, and due to capturing of the photons by the black hole the shaded area on the sky would appear. This spot corresponds to the shadow of the black hole, and its boundary can be defined using the equation of motion of photons given by expressions (13)–(16) around the black hole surrounded by the plasma.

In order to describe the apparent shape of the black hole surrounded by the plasma, we need to consider the closed orbits around it. Since the equations of motion depend on conserved quantities $\mathcal{E}$, $\mathcal{L}$ and the Carter constant $\mathcal{K}$, it is convenient to parametrize them using the normalized parameters $\zeta = \mathcal{L}/E$ and $\eta = \mathcal{K}/E^2$. The silhouette of the black hole shadow in the presence of the plasma can be found using the conditions

$$\mathcal{R}(r) = 0 = \partial\mathcal{R}(r)/\partial r.$$

Using these equations one can easily find the expressions for the parameters ζ and η in the form

$$\zeta = \frac{\mathcal{B}}{\mathcal{A}} + \sqrt{\frac{\mathcal{B}^2}{\mathcal{A}^2} - \frac{\mathcal{C}}{\mathcal{A}}}, \quad (23)$$

$$\eta = \frac{(r^2+a^2-a\zeta)^2 + (r^2+a^2)^2(n^2-1)}{\Delta} - (\zeta - a)^2, \quad (24)$$

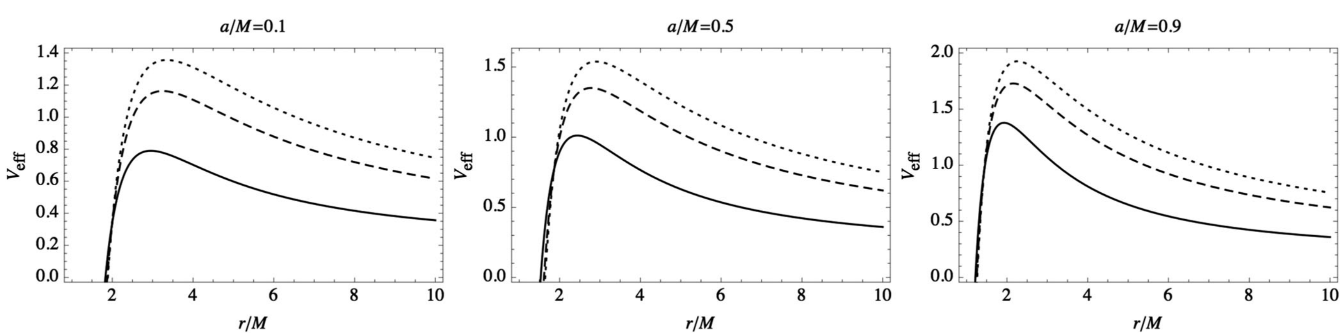

FIG. 1. The radial dependence of the effective potential of radial motion of photons for the different values of rotation parameter a and refraction index n of the plasma. Here the quantity V_{eff} is normalized by the energy of the photon $\mathcal{E}$.

where we have used the following notations,

$$\mathcal{A} = \frac{a^2}{\Delta}, \tag{25}$$

$$\mathcal{B} = \frac{a^2 - r^2}{M - r}\frac{Ma}{\Delta}, \tag{26}$$

$$\mathcal{C} = n^2\frac{(r^2 + a^2)^2}{\Delta} + \frac{2r(r^2 + a^2)n^2 + (r^2 + a^2)^2 nn'}{M - r}, \tag{27}$$

and prime denotes the differentiation with respect to radial coordinate r.

The boundary of the black hole's shadow can be fully determined through the expessions (23)–(24). However, the shadow will be observed at the "observer's sky," which can be referenced by the celestial coordinates related to the real astronomical measurements. The celestial coordinates are defined as

$$\alpha = \lim_{r_0\to\infty}\left(-r_0^2 \sin\theta_0 \frac{d\phi}{dr}\right), \tag{28}$$

$$\beta = \lim_{r_0\to\infty} r_0^2 \frac{d\theta}{dr}. \tag{29}$$

Using the equations of motion (13)–(16) one can easily find the relations for the celestial coordinates in the form

$$\alpha = -\frac{\zeta}{n\sin\theta}, \tag{30}$$

$$\beta = \frac{\sqrt{\eta + a^2 - n^2a^2\sin^2\theta - \zeta^2\cot^2\theta}}{n}, \tag{31}$$

for the case when the black hole is surrounded by a plasma.

In Fig. 2 the shadow of the rotating black hole for the different values of black hole rotation parameter a, inclination angle θ_0 between the observer and the axis of the rotation is represented. In this figure we choose the plasma frequency in the form $\omega_e/\omega_\xi = k/r$. From Fig. 2 one can observe the change of the size and shape of the rotating black hole surrounded by the plasma. The physical reason for this is due to the gravitational redshift of photons in the gravitational field of the black hole. The frequency change due to the gravitational redshift affects the plasma refraction index.

A. Shadow of a nonrotating black hole

Now in order to extract pure plasma effects, we will concentrate on the special case when the black hole is nonrotating, and the size of the black hole shadow can be observed (see, e.g., [7]). In the case of the static black hole, the shape of the black hole is a circle, and the radius of the shadow will be changed by the plasma effects. Using the expressions (30) and (31), one can easily find the radius of the shadow of a static black hole surrounded by a plasma in the form

$$R_{sh} = \frac{1}{n(r-M)}[2r^3(r-M)n^2 + r^4nn'(r-M) - 2r^2M^2 + 2Mr^2\{nr^2(n+rn') - (4n+3rn') \times nMr + M^2(1+3n^2+2rnn')\}^{1/2}]^{1/2}, \tag{32}$$

where r is the position of the last unstable circular orbit of photons defined by $dr/d\sigma = 0$ and $\partial V_{\rm eff}/\partial r = 0$. In the absence of the plasma one has the standard value of the photon sphere radius as $r = 3M$, and the shadow radius as $R_{sh} = 3\sqrt{3}\,M$ [26,27]. In the presence of the plasma, we will have a different value for the photon sphere radius and, consequently, a different shadow radius for the boundary of the black hole shadow. In Fig. 3 the dependence of the radius of shadow of the static black hole from the plasma parameters has been presented which shows that the radius of the shadow of black hole surrounded by inhomogeneous plasma decreases. It is similar to the results of the paper [7].

B. Emission energy of black holes in plasma

For completeness we evaluate the rate of energy emission from the black hole in a plasma using the expression for the Hawking radiation at the frequency Ω as [28,29]

$$\frac{d^2E(\Omega)}{d\Omega dt} = \frac{2\pi^2\sigma_{\rm lim}}{\exp\Omega/T - 1}\Omega^3, \tag{33}$$

where $T = \kappa/2\pi$ is the Hawking temperature and κ is the surface gravity. Here, for simplicity, we consider the special case when the black hole is nonrotating and the background spacetime is spherically symmetric.

At the horizon the temperature T of the black hole is

$$T = \frac{1}{4\pi r_+}. \tag{34}$$

The limiting constant $\sigma_{\rm lim}$,

$$\sigma_{\rm lim} \approx \pi R_{sh}^2,$$

defines the value of the absorption cross section vibration for a spherically symmetric black hole and R_{sh} is given by expression (32).

Consequently, one can get

$$\frac{d^2E(\Omega)}{d\Omega dt} = \frac{2\pi^3 R_{sh}^2}{e^{\Omega/T} - 1}\Omega^3,$$

so that the energy of radiation of a black hole in plasma depends on the size of its shadow.

The dependence of the energy emission rate on the frequency for the different values of plasma parameters ω_e

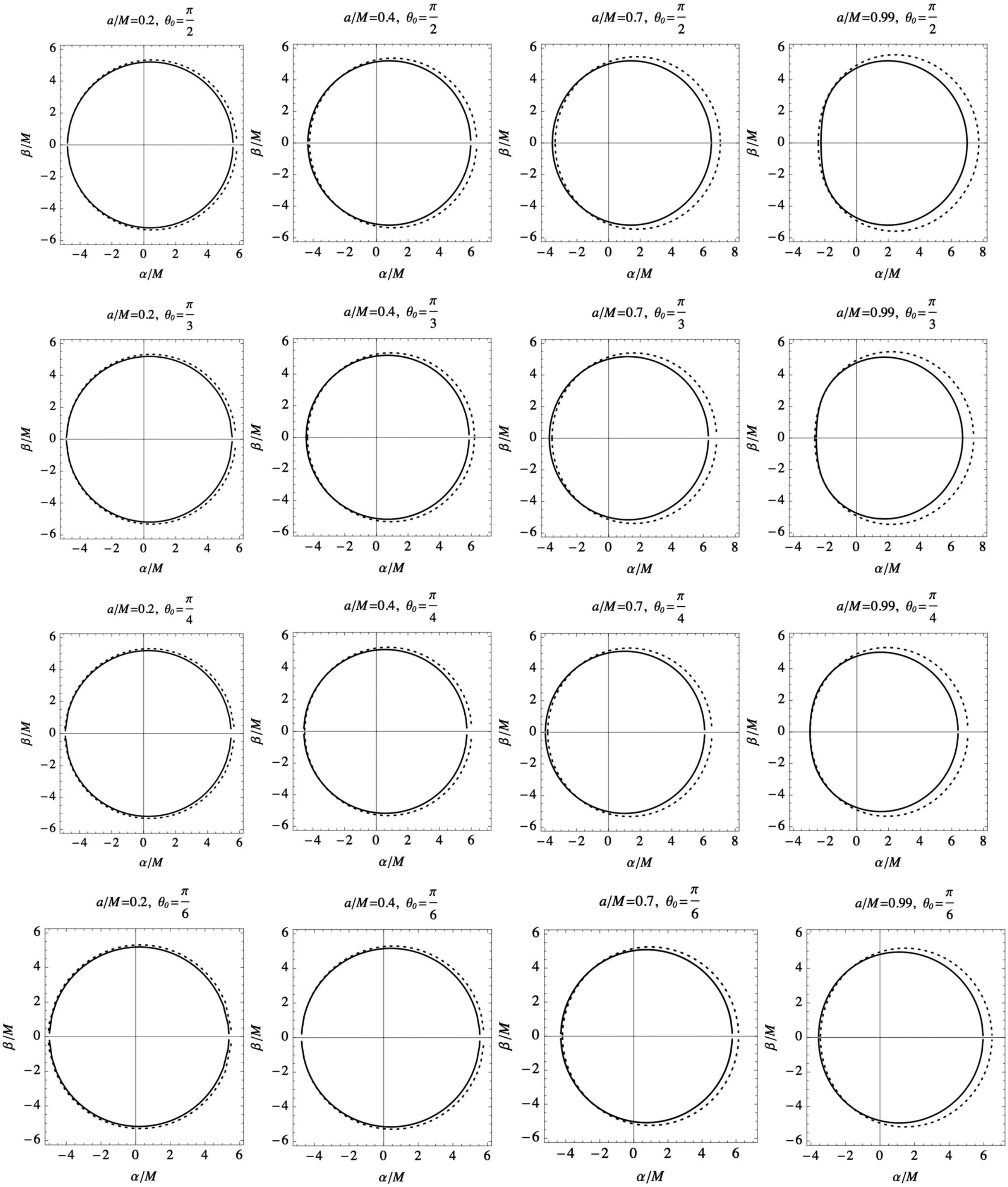

FIG. 2. The shadow of the black hole surrounded by a plasma for the different values of the rotation parameter a, inclination angle θ_0 between observer and the axis of the rotation, and the refraction index n. The solid lines in the plots correspond to the vacuum case, while for dashed lines we choose the plasma frequency $\omega_e/\omega_\xi = k/r$ and $(k/M)^2 = 0.5$.

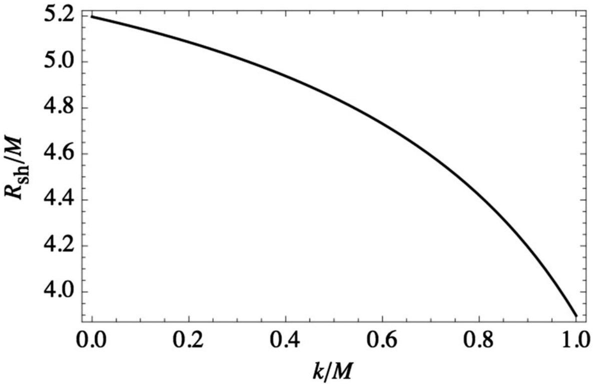

FIG. 3. The dependence of the Schwarzschild black hole shadow on the plasma frequency parameter. Here we take the plasma frequency as $\omega_e/\omega_\xi = k/r$.

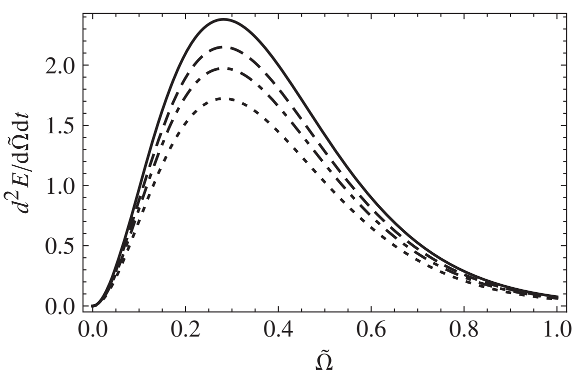

FIG. 4. Energy emission from black hole for the different values of k/M: Solid line corresponds to the vacuum case ($k/M = 0$), Dashed line corresponds to the case when $k/M = 0.4$, dot-dashed line corresponds to the case when $k/M = 0.6$, dotted line corresponds to the case when $k/M = 0.8$. Here we take the plasma frequency as $\omega_e/\omega_\xi = k/r$ and $\tilde{\Omega}$ is normalized to the T.

is shown in Fig. 4. One can see that with the increasing plasma parameter ω_e, the maximum value of the energy emission rate decreases, caused by a decrease in the radius of the shadow.

IV. CONCLUSIONS

In this paper we have studied the shadow and emission rate of an axial symmetric black hole in the presence of a plasma with radial power-law density. The obtained results can be summarized as follows:

(i) In the presence of a plasma the observed shape and size of the shadow changes depending on (i) the plasma parameters, (ii) the black hole spin, and (iii) the inclination angle between the observer plane and the axis of rotation of the black hole.

(ii) In order to extract the pure effect of the plasma's influence on the black hole image, the particular case of the Schwarzschild black hole has also been investigated. It is shown that under the influence of a plasma, the observed size of the shadow of the spherically symmetric black hole becomes smaller than that in the vacuum case. So it has been shown that the photon sphere around the spherically symmetric black hole is practically left unchanged under the plasma influence; however, the Schwarzschild black hole shadow size in a plasma is reduced due to the refraction of the electromagnetic radiation in the plasma environment of the black hole.

(iii) The study of the energy emission from the black hole in a plasma has shown that with the increase of the dimensionless plasma parameter, the maximum value of the energy emission rate from the black hole decreases due to the decrease of the size of the black hole shadow.

In the future work we plan to study the shadow and the related optical properties of different types of gravitational compact objects in the presence of a plasma in more detail and in more astrophysically relevant cases.

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