Masses of doubly and triply charmed baryons

Ke-Wei Wei (魏科伟)[†] and Bing Chen (陈兵)[‡]

College of Physics and Electrical Engineering, Anyang Normal University, Anyang 455000, China

Xin-Heng Guo (郭新恒)[*]

College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China

Until now, the first reported doubly charmed baryon $\Xi_{cc}^+(3520)$ is still a puzzle. It was discovered and confirmed by SELEX collaboration, but not confirmed by LHCb, *BABAR*, BELLE, FOCUS, or any other collaboration. In the present paper, by employing Regge phenomenology, we first express the mass of the ground state ($L = 0$) doubly charmed baryon Ω_{cc}^{*+} as a function of masses of the well established light baryons and singly charmed baryons. Inserting the recent experimental data, the mass of Ω_{cc}^{*+} is given to be 3809 ± 36 MeV, which is independent of any unobservable parameters. Then, with the quadratic mass relations, we calculate the masses of the ground state triply charmed baryon Ω_{ccc}^{++} and doubly charmed baryons $\Xi_{cc}^{(*)++}$, $\Xi_{cc}^{(*)+}$, and Ω_{cc}^+ [the mass of Ξ_{cc}^+ is determined as 3520^{+41}_{-40} MeV, which agrees with the mass of $\Xi_{cc}^+(3520)$]. The isospin splitting $M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^+} = 0.4 \pm 0.3$ MeV. After that, masses of the orbitally excited ($L = 1, 2, 3$) doubly and triply charmed baryons are estimated. The results are reasonable comparing with those extracted in many other approaches. We suggest more efforts to study doubly and triply charmed baryons both theoretically and experimentally, not only for the abundance of baryon spectra, but also for numerically examining whether the linear mass relations or the quadratic mass relations are realized in nature. Our predictions are useful for the discovery of unobserved doubly and triply charmed baryon states and the J^P assignment of these states.

I. INTRODUCTION

In the quark model, doubly and triply charmed baryons exist [1]. Many theoretical works have been focused on the mass spectra of doubly and triply charmed baryons in different approaches [2–45]. Experimentally, the doubly charmed baryon $\Xi_{cc}^+(3520)$ (ccd) was first reported in the charged decay mode $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ (SELEX 2002 [46]) and confirmed in another charged decay mode $\Xi_{cc}^+ \to pD^+K^-$ (SELEX 2005) [47]. However, the J^P number of $\Xi_{cc}^+(3520)$ has not been determined experimentally. Until recently, $\Xi_{cc}^+(3520)$ was the only doubly charmed state adopted by Particle Data Group with the average mass 3518.9 ± 0.9 MeV [1]. In our previous paper in 2008 [4], it was illustrated that if the state $\Xi_{cc}^+(3520)$ really exists, the mass of $\Xi_{cc}^+(3520)$ is too small to be assigned as the $\frac{3}{2}^+$ doubly charmed baryons, and its spin parity should be $J^P = \frac{1}{2}^+$. Under this assumption, masses of other doubly and triply charmed baryons (Ω_{cc}, Ξ_{cc}^*, Ω_{cc}^*, and Ω_{ccc}) were calculated in Ref. [4]. However, curiously, until now, $\Xi_{cc}^+(3520)$ has not been confirmed by any other collaboration (notably by LHCb [48], BELLE [49], *BABAR* [50],

and FOCUS [51]), even though they have more reconstructed charm baryons than SELEX. Thus, the existence of $\Xi_{cc}^+(3520)$ is doubtful. Therefore, the way to calculate the masses of the doubly and triply charmed baryons which is independent of $\Xi_{cc}^+(3520)$ is necessary. According to the latest "Review of Particle Physics" (RPP) [1], many light baryons and singly charmed baryons have been well established. They are labeled with three or four stars in the baryon summary table while $\Xi_{cc}^+(3520)$ is just labeled with one star. Therefore, we will attempt to avoid using the mass of $\Xi_{cc}^+(3520)$ and focus on searching mass relations which can express the mass of a doubly charmed baryon as a function of masses of the well established light baryons and singly charmed baryons. This is the most motivation of this paper. It is noted that the direct generalization of the Gell-Mann-Okubo formula [52] to the charmed and bottom hadrons cannot agree well with experimental data. The Regge trajectory *ansatz* is a simple and effective phenomenological model to study mass spectra [31,53–59] and mass relations [4,10,60,61] for baryons and mesons.

In the present paper, in Regge phenomenology, we will first express the mass of the ground state (the orbital quantum number $L = 0$) doubly charmed baryon Ω_{cc}^{*+} as a function of masses of light baryons (Σ^{*+}, Ξ^{*0}, Ω^-) and singly charmed baryons (Ξ_c^{*+}, Ω_c^{*0}) which are well established experimentally. With the help of the relation between the slope ratio and masses of baryons with different flavors,

[*]Corresponding author.
xhguo@bnu.edu.cn

[†]weikw@hotmail.com
[‡]chenbing@shu.edu.cn

we will extract the values of slopes for doubly and triply charmed baryons. Then, with the quadratic mass relations derived from Regge phenomenology, we will calculate the masses of the ground state triply charmed baryon Ω_{ccc}^{++} and doubly charmed baryons $\Xi_{cc}^{(*)++}$, $\Xi_{cc}^{(*)+}$, and Ω_{cc}^{+}. After that, masses of the orbitally excited ($L = 1, 2, 3$) doubly and triply charmed baryons will be estimated. The results will be compared with the existing experimental data and those suggested in many other approaches [2–45].

The remainder of this paper is organized as follows. In Sec. II, we first briefly introduce the Regge trajectory *ansatz*. Then, we express the quadratic mass of doubly charmed baryon Ω_{cc}^{*+} as a function of masses of light baryons and singly charmed baryons. After that, we calculate the slopes of doubly and triply charmed baryons and the masses of the doubly charmed baryons lying on the Ω_{cc}^{*+} trajectory. In Sec. III, the masses of baryons lying on the Ω_{ccc}^{++} and Ξ_{cc}^{*} trajectories are extracted. In Sec. IV, the masses of baryons lying on the Ξ_{cc} and Ω_{cc}^{+} trajectories are estimated. In Sec. V, we give a discussion and summary.

II. MASS OF THE DOUBLY CHARMED BARYON Ω_{cc}^{*+}

Regge theory is concerned with almost all aspects of strong interactions, including particle spectra, forces between particles, and the high energy behavior of scattering amplitudes [62]. It is known from Regge theory that all mesons and baryons are associated with Regge trajectories (Regge poles which move in the complex angular momentum plane as a function of energy) [63]. Hadrons lying on the same Regge trajectory which have the same internal quantum numbers are classified into the same family [62,64],

$$J = \alpha(M) = a(0) + \alpha' M^2, \tag{1}$$

where α' and $a(0)$ are respectively the intercept and slope of the Regge trajectory on which the particles lie. For a baryon multiplet, the Regge intercepts and Regge slopes for different flavors can be related by the following relations (see Ref. [4] and references therein):

$$a_{iik}(0) + a_{jjk}(0) = 2a_{ijk}(0), \tag{2}$$

$$\frac{1}{\alpha'_{iik}} + \frac{1}{\alpha'_{jjk}} = \frac{2}{\alpha'_{ijk}}, \tag{3}$$

where i, j, and k represent quark flavors. There is also a relation about the factorization of slopes, $\alpha'_{ijq}{}^{2} = \alpha'_{iiq} \times \alpha'_{jjq}$ [65]. This relation is consistent with the formal chiral limit, but fails in the heavy quark limit [66]. Charm quark is one of the heavy quarks. Therefore, we do not use this relation about the factorization of slopes in the present paper.

Using Eqs. (1)–(3), when the quark masses $m_j > m_i$, one can obtain

$$
\frac{\alpha'_{jjk}}{\alpha'_{iik}} = \frac{1}{2M_{jjk}^2} \times \Bigg[(4M_{ijk}^2 - M_{iik}^2 - M_{jjk}^2) \\
+ \sqrt{(4M_{ijk}^2 - M_{iik}^2 - M_{jjk}^2)^2 - 4M_{iik}^2 M_{jjk}^2} \Bigg]. \tag{4}
$$

Therefore, for the $\frac{3}{2}^+$ multiplet,

$$
\frac{\alpha'_{scc}}{\alpha'_{uus}} = \frac{1}{2M_{\Omega_{cc}^{*+}}^2} \times \Bigg[(4M_{\Xi_c^{*+}}^2 - M_{\Sigma^{*+}}^2 - M_{\Omega_{cc}^{*+}}^2) \\
+ \sqrt{(4M_{\Xi_c^{*+}}^2 - M_{\Sigma^{*+}}^2 - M_{\Omega_{cc}^{*+}}^2)^2 - 4M_{\Sigma^{*+}}^2 M_{\Omega_{cc}^{*+}}^2} \Bigg],
$$

$$
\frac{\alpha'_{sss}}{\alpha'_{uus}} = \frac{1}{2M_{\Omega^-}^2} \times \Bigg[(4M_{\Xi^{*0}}^2 - M_{\Sigma^{*+}}^2 - M_{\Omega^-}^2) \\
+ \sqrt{(4M_{\Xi^{*0}}^2 - M_{\Sigma^{*+}}^2 - M_{\Omega^-}^2)^2 - 4M_{\Sigma^{*+}}^2 M_{\Omega^-}^2} \Bigg],
$$

$$
\frac{\alpha'_{scc}}{\alpha'_{sss}} = \frac{1}{2M_{\Omega_{cc}^{*+}}^2} \times \Bigg[(4M_{\Omega_c^{*0}}^2 - M_{\Omega^-}^2 - M_{\Omega_{cc}^{*+}}^2) \\
+ \sqrt{(4M_{\Omega_c^{*0}}^2 - M_{\Omega^-}^2 - M_{\Omega_{cc}^{*+}}^2)^2 - 4M_{\Omega^-}^2 M_{\Omega_{cc}^{*+}}^2} \Bigg]. \tag{5}
$$

From Eq. (5), we can obtain the squared mass of Ω_{cc}^{*+} as a function of the squared masses of Σ^{*+}, Ξ^{*0}, Ω^-, Ξ_c^{*+} and Ω_c^{*0},

$$
M_{\Omega_{cc}^{*+}}^2 = \frac{2M_{\Omega_c^{*0}}^2(M_{\Omega^-}^2 - 4M_{\Xi^{*0}}^2 + 3M_{\Sigma^{*+}}^2) - M_{\Omega^-}^2(M_{\Omega^-}^2 - 6M_{\Xi_c^{*+}}^2 - 6M_{\Xi^{*0}}^2 + 2M_{\Sigma^{*+}}^2) - (4M_{\Xi^{*0}}^2 - M_{\Sigma^{*+}}^2)(2M_{\Xi_c^{*+}}^2 + 2M_{\Xi^{*0}}^2 - M_{\Sigma^{*+}}^2)}{2(M_{\Omega^-}^2 - 2M_{\Xi^{*0}}^2 + M_{\Sigma^{*+}}^2)} \\
+ \frac{\sqrt{M_{\Omega^-}^4 - 2M_{\Omega^-}^2(4M_{\Xi^{*0}}^2 + M_{\Sigma^{*+}}^2) + (M_{\Sigma^{*+}}^2 - 4M_{\Xi^{*0}}^2)^2}\sqrt{(M_{\Omega^-}^2 - 2M_{\Xi_c^{*+}}^2 - 2M_{\Xi^{*0}}^2 + M_{\Sigma^{*+}}^2)^2 - 4M_{\Omega_c^{*0}}^2(M_{\Omega^-}^2 + 2M_{\Xi_c^{*+}}^2 - 2M_{\Xi^{*0}}^2 + M_{\Sigma^{*+}}^2 - M_{\Omega_c^{*0}}^2)}}{2(M_{\Omega^-}^2 - 2M_{\Xi^{*0}}^2 + M_{\Sigma^{*+}}^2)}. \tag{6}
$$

From the latest RPP [1], $M_{\Sigma^{*+}} = 1382.80 \pm 0.35$ MeV, $M_{\Xi^{*0}} = 1531.80 \pm 0.32$ MeV, $M_{\Omega^-} = 1672.45 \pm 0.29$ MeV, $M_{\Xi_c^{*+}} = 2645.9_{-0.6}^{+0.5}$ MeV, $M_{\Omega_c^{*0}} = 2765.9 \pm 2.0$ MeV. Inserting these mass values into the relation (6), one can

get $M_{\Omega_{cc}^{*+}} = 3809 \pm 36$ MeV (where the uncertainty comes from the measurement errors of the input baryons). In this paper, all the masses of baryons used in the calculation are taken from RPP [1] and the mass differences between

TABLE I. The masses of the doubly charmed baryons lying on the Ω_{cc}^{*+} trajectory (in units of MeV). Our results are labeled with "This work" while experimental data from Ref. [1] are labeled with "RPP." The masses of the ground state Ω_{cc}^{*+} in other theoretical references vary in the range 3700–3880 MeV.

J^P	$\frac{3}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$	$\frac{9}{2}^-$
This work	3809 ± 36	4058^{+60}_{-59}	4294^{+81}_{-79}	4516^{+100}_{-98}
RPP [1]				
Ref. [2]	3876	4152	4230	
Ref. [3]	3730	4134	4204	
Ref. [4]	3808.4 ± 4.3		4313 ± 23	
Ref. [5]	3872	4303		
Ref. [6]	3760 ± 170			
Ref. [7]	3762 ± 17			
Ref. [8]	3765			
Ref. [9]	3764			
Ref. [10]	3850 ± 25			
Ref. [11]	3746			
Ref. [12]	3840 ± 60			
Ref. [13]	$3734 \pm 14 \pm 8 \pm 97$			
Ref. [14]	3820 ± 80			
Ref. [15]	3721			
Ref. [16]	3797			
Ref. [21]	$3822 \pm 20 \pm 22$			
Ref. [22]	3773 ± 38			
Ref. [24]	3758			
Ref. [25]	3780 ± 160			
Ref. [26]	$3765 \pm 43 \pm 17 \pm 5$			
Ref. [27]	3700			
Ref. [30]	$3735 \pm 33 \pm 18 \pm 43$			
Ref. [31]	3810 ± 60			
Ref. [34]	3651–3782			
Ref. [35]	3847			
Ref. [36]	3800			
Ref. [37]	3795			
Ref. [38]	3690			
Ref. [39]	3710			
Ref. [40]	3770			
Ref. [41]	3769			

isospin multiplet are also considered. Comparison of the masses of Ω_{cc}^{*+} extracted in the present paper and those given in other references is shown in Table I.

From Eq. (5), with the help of $\alpha'_{uus} = 2/(M^2_{\Sigma(2030)} - M^2_{\Sigma^{*+}})$, one can have the expression for α'_{scc} and get its

value ($\alpha'_{scc} = 0.509^{+0.053}_{-0.054} \, \text{GeV}^{-2}$). Similarly, using the masses of baryons presented in Eq. (5), with the aid of Eq. (3), one can have the expressions for α'_{ccc} and α'_{ucc} and get their values ($\alpha'_{ccc} = 0.423^{+0.055}_{-0.056} \, \text{GeV}^{-2}$, $\alpha'_{ucc} = 0.516^{+0.056}_{-0.057} \, \text{GeV}^{-2}$).

From Eq. (1), one has

$$M_{J+1} = \sqrt{M_J^2 + \frac{1}{\alpha'}}. \tag{7}$$

Then, with the expressions for $M_{\Omega_{cc}^{*+}}$ and α'_{scc} obtained above, from Eq. (7), the masses of the orbitally excited baryons ($L = 1, 2, 3$, while $J^P = \frac{5}{2}^-, \frac{7}{2}^+, \frac{9}{2}^-$) lying on the Ω_{cc}^{*+} trajectory can be expressed as functions of masses of light baryons and singly charmed baryons. The numerical results are also shown in Table I.

III. MASSES OF THE BARYONS LYING ON THE Ω_{ccc}^{++} AND Ξ_{cc}^* TRAJECTORIES

Based on Eqs. (2) and (3), we can introduce two parameters γ_x and λ_x, $\gamma_x = \frac{1}{\alpha_{uux}} - \frac{1}{\alpha_{uuu}}$, $\lambda_x = a_{uuu}(0) - a_{uux}(0)$ (x denotes the flavor of a quark). To evaluate the high-order effects, we introduce the parameter δ,

$$\delta_{ij,k} \equiv M_{iik}^2 + M_{jjk}^2 - 2M_{ijk}^2. \tag{8}$$

Combining with Eq. (1), we can have

$$\delta_{ij,k} = M_{iik}^2 + M_{jjk}^2 - 2M_{ijk}^2 = 2(\lambda_i - \lambda_j)(\gamma_i - \gamma_j). \tag{9}$$

From Eq. (9), we can see that $\delta_{ij,k}$ is independent of the k quark.

For the $\frac{3}{2}^+$ multiplet, noticing that $\delta^{\frac{3}{2}^+}_{ij,k}$ in the above relation (9) is independent of k, considering the difference of u-quark and d-quark, when $i = s$, $j = c$, $k = u, d, s, c$, Eq. (9) can be expressed as follows:

$$\begin{aligned}
\delta^{\frac{3}{2}^+}_{sc} &= M^2_{\Xi^{*0}} + M^2_{\Xi_{cc}^{*++}} - 2M^2_{\Xi_c^{*+}} \\
&= M^2_{\Xi^{*-}} + M^2_{\Xi_{cc}^{*+}} - 2M^2_{\Xi_c^{*0}} = M^2_{\Omega^-} + M^2_{\Omega_{cc}^{*+}} - 2M^2_{\Omega_c^{*0}} \\
&= M^2_{\Omega_c^{*0}} + M^2_{\Omega_{ccc}^{*++}} - 2M^2_{\Omega_{cc}^{*+}}.
\end{aligned} \tag{10}$$

With the expression for the mass of Ω_{cc}^{*+} [Eq. (6)], from the quadratic mass equations [Eq. (10)], we can obtain the expressions for the masses of Ξ_{cc}^{*++}, Ξ_{cc}^{*+}, and Ω_{ccc}^{++}. For example,

$$\begin{aligned}
M^2_{\Omega_{ccc}^{++}} =\;& \frac{2M^2_{\Omega^-}(9M^2_{\Xi_c^{*+}} + 7M^2_{\Xi^{*0}} - 2M^2_{\Sigma^{*+}}) - M^4_{\Omega^-} - 3(4M^2_{\Omega_c^{*0}}(M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}}) + (4M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}})(2M^2_{\Xi_c^{*+}} + 2M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}}))}{2(M^2_{\Omega^-} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}})} \\
&+ \frac{3\sqrt{M^4_{\Omega^-} + (M^2_{\Sigma^{*+}} - 4M^2_{\Xi^{*0}})^2 - 2M^2_{\Omega^-}(4M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}})}\sqrt{4M^4_{\Omega_c^{*0}} + (M^2_{\Omega^-} - 2M^2_{\Xi_c^{*+}} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}})^2 - 4M^2_{\Omega_c^{*0}}(M^2_{\Omega^-} + 2M^2_{\Xi_c^{*+}} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}})}}{2(M^2_{\Omega^-} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}})}.
\end{aligned} \tag{11}$$

Inserting the masses [1] of Σ^{*+}, Ξ^{*0}, Ω^-, Ξ_c^{*+}, and Ω_c^{*0} into Eq. (11), one has $M_{\Omega_{ccc}^{++}} = 4834^{+82}_{-81}$ MeV (truncated to the 1 MeV digit). Similarly, we can get $M_{\Xi_{cc}^{*++}} = 3696 \pm 33$ MeV,

$M_{\Xi_{cc}^{*+}} = 3695 \pm 35$ MeV. We use $M_{\Xi_{cc}^*}$ to note the averaged mass of Ξ_{cc}^{*++} and Ξ_{cc}^{*+}. Therefore, we can obtain the expression for $M_{\Xi_{cc}^*}$ and get its value to be

TABLE II. The masses of the triply charmed baryons lying on the Ω_{ccc}^{++} trajectory (in units of MeV). Our results are labeled with "This work" while experimental data from Ref. [1] are labeled with "RPP." The masses of the ground state Ω_{ccc}^{*+} in other theoretical references vary in the range 4700–4950 MeV.

J^P	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{7}{2}^+$	$\frac{7}{2}^-$
This work	4834^{+82}_{-81}	5073^{+109}_{-107}	5301^{+134}_{-131}	5520^{+156}_{-154}
RPP [1]				
Ref. [2]	4965	5160	5331	
Ref. [4]	4818.9 ± 6.8		5302 ± 21	
Ref. [6]	4990 ± 140	5110 ± 100		
Ref. [7]	4681 ± 28	5066 ± 48		
Ref. [8]	4773	5041		
Ref. [9]	4747			
Ref. [10]	4930 ± 45			
Ref. [11]	4803			
Ref. [12]	4925 ± 90			
Ref. [16]	4787			
Ref. [18]	4760 ± 60			
Ref. [19]	4790			
Ref. [20]	4632			
Ref. [21]	$4796 \pm 8 \pm 18$			
Ref. [22]	4794 ± 9			
Ref. [24]	4880			
Ref. [26]	$4761 \pm 52 \pm 61 \pm 6$			
Ref. [27]	4792			
Ref. [29]	$4676 \pm 46 \pm 30$			
Ref. [30]	$4734 \pm 12 \pm 11 \pm 9$			
Ref. [31]	4670 ± 150			
Ref. [34]	4728–4897			
Ref. [35]	4978			
Ref. [36]	4777			
Ref. [37]	4827			
Ref. [41]	4758	5060	5300	
Ref. [42]	4720 ± 120	4900 ± 100		
Ref. [43]	4761	5123	5396	5680
Ref. [44]	4900 ± 250			
Ref. [45]	4799			

TABLE III. The masses of the doubly charmed baryons lying on the Ξ_{cc}^* trajectory (in units of MeV). Our results are labeled with "This work" while experimental data from Ref. [1] are labeled with "RPP." The masses of the ground state Ξ_{cc}^* in other theoretical references vary in the range 3600–3750 MeV. The isospin splitting $M_{\Xi_{cc}^{*++}} - M_{\Xi_{cc}^{*+}} = 1.3^{+1.1}_{-1.2}$ MeV.

J^P	$\frac{3}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$	$\frac{9}{2}^-$
This work	3695 ± 34	3949^{+59}_{-58}	4187^{+81}_{-80}	4413^{+101}_{-100}
RPP [1]				
Ref. [2]	3753	4092	4097	
Ref. [3]	3610	4047	4089	
Ref. [4]	3684.4 ± 4.4		4192 ± 19	
Ref. [5]	3727	4155		
Ref. [6]	3610 ± 180			
Ref. [7]	3655 ± 20			
Ref. [8]	3723			
Ref. [9]	3630			
Ref. [10]	3735 ± 17			
Ref. [11]	3548			
Ref. [12]	3695 ± 60			
Ref. [13]	$3641 \pm 18 \pm 8 \pm 95$			
Ref. [14]	3740 ± 70			
Ref. [15]	3630			
Ref. [20]	3548 ± 24			
Ref. [21]	$3692 \pm 68 \pm 61$			
Ref. [22]	3627 ± 54			
Ref. [23]	3690 ± 12			
Ref. [24]	3623			
Ref. [25]	3690 ± 160			
Ref. [26]	$3648 \pm 42 \pm 18 \pm 7$			
Ref. [27]	3597			
Ref. [29]	3571 ± 25			
Ref. [30]	$3652 \pm 17 \pm 27 \pm 3$			
Ref. [31]	3900 ± 100			
Ref. [33]	3580 ± 50			
Ref. [34]	3537–3684			
Ref. [35]	3711			
Ref. [36]	3661			
Ref. [37]	3661			
Ref. [38]	3610			
Ref. [39]	3620			
Ref. [40]	3719			
Ref. [41]	3656			

3695 ± 34 MeV. We can also obtain the expression for the isospin splitting $M_{\Xi_{cc}^{*++}} - M_{\Xi_{cc}^{*+}}$ and get its value to be $1.3^{+1.1}_{-1.2}$ MeV. We can keep one digit after the decimal point in this result because all the input data in the expression have one or more digits after the decimal point. The masses of Ω_{ccc} and Ξ_{cc}^* extracted in the present paper and those given in other references are shown in Tables II and III, respectively.

Then, with the mass expressions for Ω_{ccc}^{++}, Ξ_{cc}^*, α'_{ccc} and α'_{ucc} obtained above, the masses of the baryons lying on the Ω_{ccc}^{++} and Ξ_{cc}^* trajectories can be expressed as functions of masses of light baryons and singly charmed baryons. The numerical results are also shown in Tables II and III, respectively. The wave function of a baryon is antisymmetry. According to the quantum number analysis, the odd-parity Ω_{ccc}^{++} baryons cannot have the total quark spin $S = \frac{3}{2}$. Therefore, the odd-parity

Ω_{ccc}^{++} baryons ($L = 1$, 3) have the spin parities $J^P = \frac{3}{2}^-$, $\frac{7}{2}^-$. Our calculation indicates that the isospin splittings for the orbital excited states are very small. Therefore, we only report the averaged mass of the two isospin partner in Table III.

IV. MASSES OF THE BARYONS LYING ON THE Ξ_{cc} AND Ω_{cc} TRAJECTORIES

For the $\frac{1}{2}^+$ multiplet, $\delta_{nc}^{\frac{1}{2}^+}$ can be expressed as (n denotes the quark u or d)

TABLE IV. The masses of the doubly charmed baryons lying on the Ξ_{cc} trajectory (in units of MeV). Our results are labeled with "This work" while experimental data from Ref. [1] are labeled with "RPP." The masses of the ground state Ξ_{cc} in other theoretical references vary in the range 3480–3650 MeV. In the previous paper [4], the mass of $\Xi_{cc}(3520)$ was taken as the input value of Ξ_{cc}. The isospin splitting $M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^{+}} = 0.4 \pm 0.3$ MeV.

J^P	$\frac{1}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{7}{2}^-$
This work	3520^{+41}_{-40}	3786^{+66}_{-65}	4034^{+89}_{-87}	4267^{+109}_{-107}
RPP [1]	3518.9 ± 0.9			
Ref. [2]	3676	3921	4047	
Ref. [3]	3478	3834	4050	
Ref. [4]	3518.9 ± 0.9		4047 ± 19	
Ref. [5]	3620	3959		
Ref. [6]	3570 ± 140			
Ref. [7]	3522 ± 16			
Ref. [8]	3642	3920		
Ref. [9]	3511			
Ref. [10]	3610 ± 3			
Ref. [11]	3510			
Ref. [12]	3635			
Ref. [13]	$3549 \pm 13 \pm 19 \pm 92$			
Ref. [14]	3660 ± 70			
Ref. [15]	3520			
Ref. [17]	3550 ± 80			
Ref. [20]	3524			
Ref. [21]	$3610 \pm 23 \pm 22$			
Ref. [22]	3558 ± 39			
Ref. [23]	3627 ± 12			
Ref. [24]	3532			
Ref. [26]	$3595 \pm 39 \pm 20 \pm 7$			
Ref. [27]	3527			
Ref. [28]	3520-3560			
Ref. [29]	$3513 \pm 23 \pm 24$			
Ref. [30]	$3568 \pm 14 \pm 19 \pm 1$			
Ref. [31]	4260 ± 190			
Ref. [32]	3470 ± 50			
Ref. [33]	3480 ± 50			
Ref. [34]	3468-3604			
Ref. [35]	3582			
Ref. [36]	3557			
Ref. [37]	3538			
Ref. [38]	3480			
Ref. [39]	3519			
Ref. [40]	3547			
Ref. [41]	3579			

TABLE V. The masses of the doubly charmed baryons lying on the Ω_{cc}^+ trajectory (in units of MeV). Our results are labeled with "This work" while experimental data from Ref. [1] are labeled with "RPP." The masses of the ground state Ω_{cc}^+ in other theoretical references vary in the range 3600–3800 MeV.

J^P	$\frac{1}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{7}{2}^-$
This work	3650 ± 40	3910^{+64}_{-63}	4153^{+86}_{-84}	4383^{+105}_{-103}
RPP [1]				
Ref. [2]	3815	4052	4202	
Ref. [3]	3594	3949		
Ref. [4]	3650.4 ± 6.3		4174 ± 26	
Ref. [5]	3778	4102		
Ref. [6]	3710 ± 140			
Ref. [7]	3637 ± 23			
Ref. [8]	3732	3986		
Ref. [9]	3664			
Ref. [10]	3804 ± 8			
Ref. [11]	3719			
Ref. [12]	3800			
Ref. [13]	$3663 \pm 11 \pm 17 \pm 95$			
Ref. [14]	3740 ± 80			
Ref. [15]	3619			
Ref. [16]	3737			
Ref. [17]	3650 ± 80			
Ref. [21]	$3738 \pm 20 \pm 20$			
Ref. [22]	3689 ± 38			
Ref. [24]	3667			
Ref. [26]	$3679 \pm 40 \pm 17 \pm 5$			
Ref. [27]	3598			
Ref. [28]	3620-3650			
Ref. [30]	$3658 \pm 11 \pm 16 \pm 50$			
Ref. [31]	4250 ± 200			
Ref. [34]	3566-3687			
Ref. [35]	3718			
Ref. [36]	3710			
Ref. [37]	3690			
Ref. [38]	3590			
Ref. [39]	3630			
Ref. [40]	3648			
Ref. [41]	3697			

$$\delta^{\frac{3}{2}^+}_{uc,d} = \delta^{\frac{3}{2}^+}_{uc,s} = M^2_{\Sigma^{*+}} + M^2_{\Omega_{cc}^{*+}} - 2M^2_{\Xi_c^{*+}},$$

$$\delta^{\frac{3}{2}^+}_{dc,u} = \delta^{\frac{3}{2}^+}_{dc,s} = M^2_{\Sigma^{*-}} + M^2_{\Omega_{cc}^{*+}} - 2M^2_{\Xi_c^{*0}}. \tag{13}$$

Considering the isospin breaking effects, Eq. (61) in Ref. [4] can be expressed as

$$(M^2_{\Omega_{cc}^+} - M^2_{\Xi_{cc}^{++}}) + (M^2_{\Xi^0} - M^2_{\Sigma^+}) = (M^2_{\Omega_c^0} - M^2_{\Sigma_c^{++}}),$$

$$(M^2_{\Omega_{cc}^+} - M^2_{\Xi_{cc}^+}) + (M^2_{\Xi^-} - M^2_{\Sigma^-}) = (M^2_{\Omega_c^0} - M^2_{\Sigma_c^0}). \tag{14}$$

The linear forms in Eq. (14) were given by Verma and Khanna considering the second-order effects arising from the $\underline{84}$ representation of SU(4) [67] and by Singh *et al.* studying SU(4) second-order mass-breaking effects with a

$$\delta^{\frac{1}{2}^+}_{uc,d} + \delta^{\frac{1}{2}^+}_{dc,u} = M^2_{N^+} + M^2_{\Xi_{cc}^+} - 2\left(\frac{3M^2_{\Lambda_c^+} + M^2_{\Sigma_c^+}}{4}\right) + M^2_{N^0}$$

$$+ M^2_{\Xi_{cc}^{++}} - 2\left(\frac{3M^2_{\Lambda_c^+} + M^2_{\Sigma_c^+}}{4}\right). \tag{12}$$

Based on Eq. (9), when $i = u(d)$, $j = c$, $k = d(u), s$, $\delta^{\frac{3}{2}^+}_{nc}$ can be expressed as

dynamical consideration [68]. The linear forms in Eq. (14) can satisfy the instanton model [69] and the SU(8) symmetry [70].

As done in Ref. [4], assuming that $\delta_{nc}^{\frac{1}{2}^+} = \delta_{nc}^{\frac{3}{2}^+}$, inserting the masses of N^+, N^0, Σ^+, Σ^-, Ξ^0, Ξ^-, Λ_c^+, Σ_c^{++}, Σ_c^+, Σ_c^0, Ω_c^0, Ξ_c^{*+}, Ξ_c^{*0} from RPP [1] and the expression for $M_{\Omega_{cc}^{*+}}$ obtained in Sec. II into Eqs. (12)–(14), we can have the expressions for $M_{\Omega_{cc}^+}$, $M_{\Xi_{cc}^{++}}$, and Ξ_{cc}^+. Then, we can get their values: $M_{\Omega_{cc}^+} = 3650 \pm 40$ MeV, $M_{\Xi_{cc}^{++}} = 3521^{+41}_{-40}$ MeV, $\Xi_{cc}^+ = 3520^{+41}_{-40}$ MeV (truncated to the 1 MeV digit). We use $M_{\Xi_{cc}}$ to note the averaged mass of Ξ_{cc}^{++} and Ξ_{cc}^+. Therefore, we can obtain the expression for $M_{\Xi_{cc}}$ and get its value to be 3520^{+41}_{-40} MeV. We can also obtain the expression for the isospin splitting $M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^+}$ and get its value to be 0.4 ± 0.3 MeV (where the uncertainties come from the errors of the input data). In the calculation, we avoid using the masses of Δ^{++}, Δ^+, Δ^0, and Δ^- because only the charge-mixed states of $\Delta(1232)$ were reliably measured, as indicated in RPP [1]. Comparison of the masses of Ξ_{cc} and Ω_{cc}^+ extracted in the present paper and those given in other references is shown in Tables IV and V, respectively.

Then, with the mass expressions for Ξ_{cc}, Ω_{cc}^+, α'_{scc} and α'_{ucc} obtained above, the masses of the orbitally excited baryons ($\frac{3}{2}^-$, $\frac{5}{2}^+$, and $\frac{7}{2}^-$) lying on the Ξ_{cc} and Ω_{cc}^+ trajectories can be expressed as functions of masses of light baryons and singly charmed baryons. The numerical results are also shown in Tables IV and V.

V. DISCUSSION AND SUMMARY

In the present paper, we focused on studying masses of doubly and triply charmed baryons which do not rely on unobservable parameters and distrustful resonances. Under the Regge phenomenology, we first expressed the mass of the ground state ($L = 0$) doubly charmed baryon Ω_{cc}^{*+} as a function of masses of light baryons (Σ^{*+}, Ξ^{*0}, Ω^-) and singly charmed baryons (Ξ_c^{*+}, Ω_c^{*0}) which are well established experimentally [1]. Then, with the quadratic mass relations derived from Regge phenomenology, we calculated the masses of the ground state triply charmed baryon Ω_{ccc}^{++} and doubly charmed baryons $\Xi_{cc}^{(*)++}$, $\Xi_{cc}^{(*)+}$, and Ω_{cc}^+. After that, masses of the orbitally excited (the orbital quantum number $L = 1, 2, 3$) doubly and triply charmed baryons were estimated. In this paper, all the input masses of baryons used in the calculation were taken from the Particle Data Group's latest "Review of Particle Physics" [1] and the isospin splittings were also considered. The uncertainties of the results only come from the errors of the input data. Regge slopes used in this paper were also estimated from light and singly charmed baryons. No systematic error due to any small deviations from the Regge trajectories has been taken into account in this paper.

From Tables I–V, we can see that the masses of ground state and orbitally excited doubly and triply charmed baryons predicted here are reasonable comparing with the existing experimental data and those given in many other different approaches. The mass relations and the predictions may be useful for the discovery of the unobserved doubly and triply charmed baryon states and the J^P assignment of these baryon states when they are observed in the near future.

In Ref. [12], Bjorken pointed out $\frac{M_{\Omega_{ccc}}}{M_\psi} = 1.59 \pm 0.03$ and gave the mass of Ω_{ccc} to be 4925 ± 90 MeV, which agrees with our result $M_{\Omega_{ccc}} = 4834^{+82}_{-81}$ MeV shown in Table II. In the present paper, the central value of mass splittings ($M_{\Omega_{cc}^{*+}} - M_{\Xi_{cc}^*} = 3809 - 3695 = 114$ MeV and $M_{\Omega_{cc}^+} - M_{\Xi_{cc}} = 3650 - 3520 = 130$ MeV) are reasonable. The central value of mass splittings ($M_{\Omega_{cc}^{*+}} - M_{\Omega_{cc}^+} = 3809 - 3650 = 159$ MeV and $M_{\Xi_{cc}^*} - M_{\Xi_{cc}} = 3695 - 3520 = 175$ MeV) are a little big. In Ref. [40], $M_{\Xi_{cc}^*} - M_{\Xi_{cc}} = 3719 - 6547 = 172$ MeV agrees with our present results. The mass splitting obtained in the framework of non-relativistic effective field theories of QCD is $M_{\Xi_{cc}^*} - M_{\Xi_{cc}} = 120 \pm 40$ MeV (see Ref. [71] and references therein). The isospin splitting in the present paper, $M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^+} = 0.4 \pm 0.3$ MeV, is comparable with 1.5 ± 2.7 MeV in Ref. [72] and 2.3 ± 1.7 MeV in Ref. [73]. These can be tested by experiments in the future. In Ref. [72] strong and electromagnetic sources of isospin breaking are handled differently. Regge theory appears to be a pure QCD emergent phenomenon. In this paper, we do not consider the electromagnetic corrections separately because the electromagnetic corrections cancel out in Eqs. (10) and (14). (We would like to thank the anonymous referee for his/her valuable suggestion.)

The doubly charmed baryon $\Xi_{cc}^+(3520)$ (ccd) was first reported in the charged weak decay mode $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ (SELEX 2002 [46]), with mass $M = 3519 \pm 1$ MeV. $\Xi_{cc}^+(3520)$ was confirmed in another charged weak decay mode $\Xi_{cc}^+ \to p D^+ K^-$ (SELEX 2005 [47]), with mass $M = 3518 \pm 3$ MeV. These reports were adopted by Particle Data Group [1] with the average mass 3518.9 ± 0.9 MeV. However, the J^P number has not been determined experimentally. Moreover, it has not been confirmed by other experiments (notably by LHCb [48], BELLE [49], *BABAR* [50], and FOCUS [51]) even though they have more reconstructed charm baryons than SELEX.

In the present paper, in Sec. IV, the mass of the $\frac{1}{2}^+$ doubly charmed baryon Ξ_{cc}^+ (ccd) was predicted to be 3520^{+41}_{-40} MeV. In the previous paper [4], we proved that the mass of $\Xi_{cc}^+(3520)$ is too small to be assigned as the $\frac{3}{2}^+$ doubly charmed baryon. The mass of Ξ_{cc}^+ obtained in the present paper agrees with the mass of $\Xi_{cc}^+(3520)$. Therefore, we support that the state $\Xi_{cc}^+(3520)$ really exists. We suggest that the J^P of $\Xi_{cc}^+(3520)$ is $\frac{1}{2}^+$. This assignment coincides with the fact that $\Xi_{cc}^+(3520)$ is observed to decay

only weakly [46,47] (if the J^P of $\Xi_{cc}^+(3520)$ were $\frac{3}{2}^+$, it should decay electromagnetically [74]) and agrees with many theoretical discussions [7,11,39,75].

In this paper, we took squared mass relations rather than linear mass relations taken in Refs. [67–70]. In the light quark sector, the linear mass relations and the quadratic mass relations lead to the similar results. However, they do lead to different values when mass relations include light, charmed, and doubly charmed baryons such as Eqs. (10) and (14). Searching for doubly and triply charmed baryons is helpful not only for the abundance of baryon spectra, but also for numerically examining whether the linear mass relations or the quadratic mass relations are realized in nature. The triply charmed baryon Ω_{ccc} is of considerable theoretical interest [18,76]. Therefore, more efforts should be given to study doubly and triply charmed baryons both theoretically and experimentally.

The approach presented in the present paper can be used to calculate the masses of the doubly and triply charmed baryons based on the well established light and singly charmed baryons while the approach in the previous paper [4] needs the mass of one doubly heavy baryon to predict the masses of other doubly heavy baryons. We will estimate masses of doubly and triply bottom baryons in our next paper.

ACKNOWLEDGMENTS

[1] K. A. Olive *et al.* (Particle Data Group), Review of Particle Physics, Chin. Phys. C **38**, 090001 (2014).

[2] W. Roberts and M. Pervin, Heavy baryons in a quark model, Int. J. Mod. Phys. A **23**, 2817 (2008).

[3] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, Spectroscopy of doubly heavy baryons, Phys. Rev. D **62**, 054021 (2000); V. V. Kiselev, A. K. Likhoded, O. N. Pakhomova, and V. A. Saleev, Mass spectra of doubly heavy $\Omega_{QQ'}$ baryons, Phys. Rev. D **66**, 034030 (2002).

[4] X.-H. Guo, K.-W. Wei, and X.-H. Wu, Some mass relations for mesons and baryons in Regge phenomenology, Phys. Rev. D **78**, 056005 (2008).

[5] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, Mass spectra of doubly heavy baryons in the relativistic quark model, Phys. Rev. D **66**, 014008 (2002); Yad. Fiz.**68**, 817 (2005) [Properties of doubly heavy baryons in the relativistic quark model, Phys. At. Nucl. **68**, 784 (2005)].

[6] Z. G. Wang, Analysis of the $\frac{3}{2}^+$ heavy and doubly heavy baryon states with QCD sum rules, Eur. Phys. J. C **68**, 459 (2010); Analysis of the $\frac{1}{2}^+$ doubly heavy baryon states with QCD sum rules, Eur. Phys. J. A **45**, 267 (2010); Analysis of the triply heavy baryon states with QCD sum rules, Commun. Theor. Phys. **58**, 723 (2012).

[7] T. W. Chiu and T. H. Hsieh, Baryon masses in lattice QCD with exact chiral symmetry, Nucl. Phys. **A755**, 471 (2005).

[8] S. Migura, D. Merten, B. C. Metsch, and H. R. Petry, Charmed baryons in a relativistic quark model, Eur. Phys. J. A **28**, 41 (2006).

[9] W. A. Ponce, Heavy quarks in a spherical bag, Phys. Rev. D **19**, 2197 (1979).

[10] L. Burakovsky, T. Goldman, and L. P. Horwitz, New quadratic baryon mass relations, Phys. Rev. D **56**, 7124 (1997).

[11] A. P. Martynenko, Ground-state triply and doubly heavy baryons in a relativistic three-quark model, Phys. Lett. B **663**, 317 (2008).

[12] J. D. Bjorken, Is the CCC a new deal for baryon spectroscopy?, AIP Conf. Proc. **132**, 390 (1985).

[13] J. M. Flynn, F. Mescia, and A. S. B. Tariq (UKQCD Collaboration), Spectroscopy of Doubly-Charmed Baryons in Lattice QCD, J. High Energy Phys. 07 (2003) 066.

[14] R. Roncaglia, D. B. Lichtenberg, and E. Predazzi, Predicting the masses of baryons containing one or two heavy quarks, Phys. Rev. D **52**, 1722 (1995).

[15] D.-H. He, K. Qian, Y.-B. Ding, X.-Q. Li, and P.-N. Shen, Evaluation of spectra of baryons containing two heavy quarks in bag model, Phys. Rev. D **70**, 094004 (2004).

[16] A. Martin and J.-M. Richard, Ω_c and other charmed baryons revisited, Phys. Lett. B **355**, 345 (1995).

[17] V. V. Kiselev and A. E. Kovalsky, Doubly heavy baryons $\Omega_{QQ'}$ versus $\Xi_{QQ'}$ in sum rules of nonrelativistic QCD, Phys. Rev. D **64**, 014002 (2001).

[18] Y. Jia, Variational study of weakly coupled triple heavy baryons, J. High Energy Phys. 10 (2006) 073.

[19] P. Hasenfratz, R. R. Horgan, J. Kuti, and J. M. Richard, Heavy baryon spectroscopy in the QCD bag model, Phys. Lett. B **94**, 401 (1980).

[20] J. Vijande, H. Garcilazo, A. Valcarce, and F. Fernandez, Spectroscopy of doubly charmed baryons, Phys. Rev. D **70**, 054022 (2004).

[21] Z. S. Brown, W. Detmold, S. Meinel, and K. Orginos, Charmed bottom baryon spectroscopy from lattice QCD, Phys. Rev. D **90**, 094507 (2014).

[22] Z. S. Brown, W. Detmold, S. Meinel, and K. Orginos, Charmed bottom baryon spectroscopy, *Proc. Sci.*, LATTICE2013 (2014) 248.

[23] M. Karliner and J. L. Rosner, Baryons with two heavy quarks: Masses, production, decays, and detection, Phys. Rev. D **90**, 094007 (2014).

[24] Z. Ghalenovi, A. A. Rajabi, S. x. Qin, and D. H. Rischke, Ground-state masses and magnetic moments of heavy baryons, Mod. Phys. Lett. A **29**, 1450106 (2014).

[25] T. M. Aliev, K. Azizi, and M. Savci, The masses and residues of doubly heavy spin-3/2 baryons, J. Phys. G **40**, 065003 (2013).

[26] R. A. Briceno, H. W. Lin, and D. R. Bolton, Charmed-baryon spectroscopy from lattice QCD with $N_f = 2 + 1 + 1$ flavors, Phys. Rev. D **86**, 094504 (2012).

[27] S. M. Gerasyuta and D. V. Ivanov, Charmed baryons in bootstrap quark model, Nuovo Cimento A **112**, 261 (1999).

[28] M.-H. Weng, X.-H. Guo, and A. W. Thomas, Bethe-Salpeter equation for doubly heavy baryons in the covariant instantaneous approximation, Phys. Rev. D **83**, 056006 (2011).

[29] C. Alexandrou, J. Carbonell, D. Christaras, V. Drach, M. Gravina, and M. Papinutto, Strange and charm baryon masses with two flavors of dynamical twisted mass fermions, Phys. Rev. D **86**, 114501 (2012).

[30] C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis, and G. Koutsou, Baryon spectrum with $N_f = 2 + 1 + 1$ twisted mass fermions, Phys. Rev. D **90**, 074501 (2014).

[31] J. R. Zhang and M. Q. Huang, Doubly heavy baryons in QCD sum rules, Phys. Rev. D **78**, 094007 (2008); Deciphering triply heavy baryons in terms of QCD sum rules, Phys. Lett. B **674**, 28 (2009).

[32] V. V. Kiselev and A. I. Onishchenko, Doubly heavy baryons in sum rules of NRQCD, Nucl. Phys. **B581**, 432 (2000).

[33] E. Bagan, M. Chabab, and S. Narison, Baryons with two heavy quarks from QCD spectral sum rules, Phys. Lett. B **306**, 350 (1993).

[34] B. Patel, A. Majethiya, and P. C. Vinodkumar, Masses and magnetic moments of triply heavy flavor baryons in hypercentral model, Pramana **72**, 679 (2009); B. Patel, A. K. Rai, and P. C. Vinodkumar, Heavy flavor baryons in hypercentral model, Pramana **70**, 797 (2008).

[35] Z. Ghalenovi, A. A. Rajabi, and M. Hamzavi, The heavy baryon masses in variational approach and spin-isospin dependence, Acta Phys. Pol. B **42**, 1849 (2011).

[36] A. Bernotas and V. Simonis, Heavy hadron spectroscopy and the bag model, Lith. J. Phys. **49**, 19 (2009).

[37] R. L. Jaffe and J. E. Kiskis, Spectra of new hadrons, Phys. Rev. D **13**, 1355 (1976).

[38] V. V. Kiselev and A. K. Likhoded, Baryons with two heavy quarks, Usp. Fiz. Nauk **172**, 497 (2002) [Phys. Usp. **45**, 455 (2002)].

[39] L. Tang, X. H. Yuan, C. F. Qiao, and X. Q. Li, Study of doubly heavy baryon spectrum via QCD sum rules, Commun. Theor. Phys. **57**, 435 (2012).

[40] F. Giannuzzi, Doubly heavy baryons in a Salpeter model with AdS/QCD inspired potential, Phys. Rev. D **79**, 094002 (2009).

[41] A. Valcarce, H. Garcilazo, and J. Vijande, Towards an understanding of heavy baryon spectroscopy, Eur. Phys. J. A **37**, 217 (2008); J. Vijande, A. Valcarce, and H. Garcilazo, Heavy-baryon quark model picture from lattice QCD, Phys. Rev. D **90**, 094004 (2014).

[42] T. M. Aliev, K. Azizi, and M. Savci, Properties of triply heavy spin-3/2 baryons, J. Phys. G **41**, 065003 (2014).

[43] M. Padmanath, R. G. Edwards, N. Mathur, and M. Peardon, Spectroscopy of triply-charmed baryons from lattice QCD, Phys. Rev. D **90**, 074504 (2014).

[44] F. J. Llanes-Estrada, O. I. Pavlova, and R. Williams, A first estimate of triply heavy baryon masses from the pNRQCD perturbative static potential, Eur. Phys. J. C **72**, 2019 (2012).

[45] J. M. Flynn, E. Hernandez, and J. Nieves, Triply heavy baryons and heavy quark spin symmetry, Phys. Rev. D **85**, 014012 (2012).

[46] M. Mattson et al. (SELEX Collaboration), First Observation of the Doubly Charmed Baryon Ξ_{cc}^+, Phys. Rev. Lett. **89**, 112001 (2002).

[47] A. Ocherashvili et al. (SELEX Collaboration), Confirmation of the doubly charmed baryon $\Xi_{cc}^+(3520)$ via its decay to pD^+K^-, Phys. Lett. B **628**, 18 (2005).

[48] R. Aaij et al. (LHCb Collaboration), Search for the doubly charmed baryon Ξ_{cc}^+, J. High Energy Phys. 12 (2013) 090.

[49] R. Chistov et al. (BELLE Collaboration), Observation of New States Decaying into $\Lambda_c^+ K^- \pi^+$ and $\Lambda_c^+ K_S^0 \pi^-$, Phys. Rev. Lett. **97**, 162001 (2006).

[50] B. Aubert et al. (BABAR Collaboration), Search for doubly charmed baryons Ξ_{cc}^+ and Ξ_{cc}^{++} in BABAR Phys. Rev. D **74**, 011103 (2006).

[51] S. P. Ratti (FOCUS Collaboration), New results on c-baryons and a search for cc-baryons in FOCUS, Nucl. Phys. B, Proc. Suppl. **115**, 33 (2003); K. Stenson et al. (FOCUS Collaboration), FOCUS double charm baryon search, http://www-focus.fnal.gov/xicc/xicc_focus.html.

[52] M. Gell-Mann, Symmetries of Baryons and Mesons, Phys. Rev. **125**, 1067 (1962); A schematic model of baryons and mesons, Phys. Lett. **8**, 214 (1964); S. Okubo, Note on unitary symmetry in strong interactions, Prog. Theor. Phys. **27**, 949 (1962); Note on unitary symmetry in strong interaction. II excited states of baryons, Prog. Theor. Phys. **28**, 24 (1962).

[53] D. Ebert, R. N. Faustov, and V. O. Galkin, Spectroscopy and Regge trajectories of heavy baryons in the relativistic quark-diquark picture, Phys. Rev. D **84**, 014025 (2011).

[54] K.-W. Wei and X.-H. Guo, Mass spectra of doubly heavy mesons in Regge phenomenology, Phys. Rev. D **81**, 076005 (2010).

[55] D.-M. Li, B. Ma, Y.-X. Li, Q.-K. Yao, and H. Yu, Meson spectrum in Regge phenomenology, Eur. Phys. J. C **37**, 323 (2004).

[56] A. V. Anisovich, V. V. Anisovich, and A. V. Sarantsev, Systematics of $q\bar{q}$ states in the (n, M^2) and (J, M^2) planes, Phys. Rev. D **62**, 051502(R) (2000); V. V. Anisovich, Usp. Fiz. Nauk **174**, 49 (2004) [Systematics of quark–antiquark states and scalar exotic mesons, Phys. Usp. **47**, 45 (2004)].

[57] A. Zhang, Charmonium spectrum and new observed states, Phys. Lett. B **647**, 140 (2007).

[58] M. M. Brisudova, L. Burakovsky, and T. Goldman, Effective functional form of Regge trajectories, Phys. Rev. D **61**, 054013 (2000).

[59] P. Masjuan, E. R. Arriola, and W. Broniowski, Systematics of radial and angular-momentum Regge trajectories of light non-strange $q\bar{q}$-states, Phys. Rev. D **85**, 094006 (2012).

[60] L. Burakovsky, T. Goldman, and L. P. Horwitz, New mass relations for heavy quarkonia, Phys. Rev. D **56**, 7119 (1997); New mass relation for meson 25-plet, J. Phys. G **24**, 771 (1998).

[61] A. B. Kaidalov, Hadronic mass-relations from topological expansion and string model, Z. Phys. C **12**, 63 (1982).

[62] P. D. Collins, *An Introduction to Regge Theory and High Energy Physics* (Cambridge University Press, Cambridge, 1977).

[63] T. Regge, Introduction to complex orbital momenta, Nuovo Cimento **14**, 951 (1959); Bound states, shadow states and mandelstam representation, Nuovo Cimento **18**, 947 (1960).

[64] G. F. Chew and S. C. Frautschi, Principle of Equivalence for all Strongly Interacting Particles within the S-Matrix Framework, Phys. Rev. Lett. **7**, 394 (1961); Regge Trajectories and the Principle of Maximum Strength for Strong Interactions, Phys. Rev. Lett. **8**, 41 (1962).

[65] K. Igi, Factorization of Regge slopes for ordinary and new hadrons, Phys. Lett. B **66**, 276 (1977); Factorization of Regge slopes for ordinary and new hadrons and their spectroscopy, Phys. Rev. D **16**, 196 (1977).

[66] L. Burakovsky and T. Goldman, On the Regge slopes intramultiplet relation, Phys. Lett. B **434**, 251 (1998).

[67] R. C. Verma and M. P. Khanna, Higher-order mass formulas in SU(4), Phys. Rev. D **18**, 828 (1978).

[68] C. P. Singh, R. C. Verma, and M. P. Khanna, Second-order mass-breaking effects on hadron masses, Phys. Rev. D **21**, 1388 (1980).

[69] S. Iwao, Mass formula for hadrons with zero pontryagin index, Prog. Theor. Phys. **87**, 1443 (1992).

[70] A. W. Hendry and D. B. Lichtenberg, Hadron mass relations in a charmed quark model with broken SU(8) symmetry, Phys. Rev. D **12**, 2756 (1975); R. C. Verma and M. P. Khanna, SU (8) mass relations among $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons, Pramana **8**, 524 (1977).

[71] N. Brambilla, A. Vairo, and T. Rosch, Effective field theory Lagrangians for baryons with two and three heavy quarks, Phys. Rev. D **72**, 034021 (2005); M. Padmanath, R. G. Edwards, N. Mathur, and M. Peardon, Spectroscopy of doubly-charmed baryons from lattice QCD, Phys. Rev. D **91**, 094502 (2015).

[72] S. J. Brodsky, F.-K. Guo, C. Hanhart, and U. G. Meissner, Isospin splittings of doubly heavy baryons, Phys. Lett. B **698**, 251 (2011).

[73] C.-W. Hwang and C.-H. Chung, Isospin mass splittings of heavy baryons in HQS, Phys. Rev. D **78**, 073013 (2008).

[74] J. Hu and T. Mehen, Chiral Lagrangian with heavy quark-diquark symmetry, Phys. Rev. D **73**, 054003 (2006).

[75] T. D. Cohen and P. M. Hohler, Doubly heavy hadrons and the domain of validity of doubly heavy diquark-antiquark symmetry, Phys. Rev. D **74**, 094003 (2006); A. J. Edwards, A study of double-charm and charm-strange baryons in electron-positron annihilations, Ph.D. thesis, Stanford University [Report No. SLAC-R-883, 2007].

[76] N. Brambilla, F. Karbstein, and A. Vairo, Symmetries of the three-heavy-quark system and the color-singlet static energy at next-to-next-to-leading logarithmic order, Phys. Rev. D **87**, 074014 (2013).