

Renormalization group invariants and sum rules in the deflected mirage mediation supersymmetry breaking

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We examine the deflected mirage mediation supersymmetry breaking (DMMSB) scenario, which combines three supersymmetry breaking scenarios, namely anomaly mediation, gravity mediation and gauge mediation using the one-loop renormalization group invariants (RGIs). We examine the effects on the RGIs at the threshold where the gauge messengers emerge, and derive the supersymmetry breaking parameters in terms of the RGIs. We further discuss whether the supersymmetry breaking mediation mechanism can be determined using a limited set of invariants, and derive sum rules valid for DMMSB below the gauge messenger scale. In addition we examine the implications of the measured Higgs mass for the DMMSB spectrum.

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I. INTRODUCTION

Supersymmetry remains at present a favored solution to the hierarchy problem of the standard model (SM). Supersymmetry is obviously a broken symmetry, as none of the superpartners of the SM particles have been experimentally observed so far. The idea of weak scale supersymmetry, as embodied, for example, in the minimal supersymmetric standard model (MSSM), as a solution to the gauge hierarchy problem has already been partly tested at the Large Hadron Collider (LHC) [1,2]. The most minimal constrained model is found to be disfavored, and the emphasis in the studies has moved to other well-motivated models.

The mechanism of supersymmetry breaking, which is crucial for determining the masses of the superpartners of the SM particles, is not known at present. There are different viable models of supersymmetry breaking in which supersymmetry breaking is mediated by a specific interaction in a hidden sector and is communicated to the visible sector via some mediator fields. Well-known examples of hidden sector supersymmetry breaking include gravity mediated supersymmetry breaking [3], gauge mediated supersymmetry breaking [4], and anomaly mediated supersymmetry breaking [5], respectively. It is usually assumed that one mediation mechanism dominates, which depends on the type of problem that one wishes to address in the context of MSSM. In fact, it may well be that a single mediation mechanism does not dominate. In some situations the anomaly mediated and gravity mediated contributions to the supersymmetry breaking can coexist and

manifest at a comparable value, leading to what is known as mirage mediation, a situation that naturally arises in some string theories [6]. The name derives from the fact that soft gaugino masses unify at a scale (the mirage scale) which is lower than the grand unification or GUT scale. Models based on mirage mediation can solve the tachyonic slepton mass problem of the anomaly mediation models. The mirage mediation models have been studied extensively in the literature [7–10].

However, in the absence of any definite experimental indication regarding the mass spectrum of the sparticles, it is important to consider the case where all the three types of supersymmetry breaking mechanisms contribute to the soft masses, and ultimately determine the mass spectrum of the supersymmetric partners of the SM states. Such a scenario has been dubbed as deflected mirage mediation [11]. In the deflected mirage mediation the gaugino mass unification is deflected by the threshold effect from the messenger fields associated with the gauge mediation contribution. The messenger fields are included at scales $\mu > \mu_{\text{mess}}$, and the renormalization group running is affected by the threshold corrections at this scale. It is worthwhile to note that in a broad class of supergravity models, which might be realized in string theory, the contributions to soft supersymmetry breaking masses from the three different mediation mechanisms are comparable [12]. This makes the study of deflected mirage mediation models rather compelling.

As discussed above, in order to predict the masses of the superpartners of the SM particles, it is essential to understand the nature of supersymmetry breaking. The usual approach to this has been to consider a particular model of supersymmetry breaking, e.g. the gravity mediated supersymmetry breaking model (usually referred to as mSUGRA) with a limited number of parameters at the

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high scale (possibly the GUT scale), and evolve these, using renormalization group equations, to the electroweak scale and fit them to the data [13–19]. This method, the so-called “top-down” approach has some drawbacks, namely that the reliability of the fit decreases with the increase of the number of parameters at the high scale. This approach is also sensitive to uncertainties of the measured quantities, which includes gauge couplings at the electroweak scale and the masses of the SM particles.

An alternative approach, which is complementary to the “top-down” approach, that has been advocated involves the measured masses at the electroweak scale and evolve these to the high scale where supersymmetry is broken [20–23]. The resulting structure is then analyzed and conclusions about the underlying theory at high scales obtained. This approach, which can be called the “bottom-up” approach, has uncertainties resulting from the present experimental uncertainties in the measurement of gauge and Yukawa couplings.

Another alternative to obtaining information on the nature of supersymmetry breaking is to obtain specific relations among the masses of the superpartners of the SM particles which result from the structure of the underlying theory at the high scale. In this approach the Yukawa couplings of the first- and second-generations are usually ignored and the renormalization group is used to evolve the parameters from high scale to the low scale, and then specific relations among sparticle masses are derived based on the particular theory at the high scale [24–31]. In this context it has also been pointed out that there are a set of combinations of parameters of a supersymmetric model that are renormalization group invariant (RGI) at the 1-loop level [32–38]. Although the argument regarding RGI holds only at the leading-log order, 2-loop corrections are expected to be small and are likely to be negligible compared with the experimental uncertainties. Thus, these higher order effects can either be neglected or absorbed into a shift of the measured values of the renormalization group invariants. It has been argued that these RGIs can be used to extract the parameters of different supersymmetry breaking models at the high scale, and thereby establish the mechanism of supersymmetry breaking. RGIs can also be used for constructing sum rules relating the particle masses, and they are discussed in this context in [25,26,28,37–39].

In this paper we derive the renormalization group invariants for deflected mirage mediation and examine how the appearance of messenger particles at the specific energy scale lower than the GUT-scale affects the invariants. We investigate the GUT-scale parameters of the deflected mirage mediation in terms of the RGIs, and examine validity of sum rules derived previously of the RGIs and derive new ones specific to the deflected mirage mediation. In particular, we derive soft supersymmetry breaking parameters in terms of the RGIs. We also examine

the Higgs mass in the deflected mirage mediation scenario to constrain the parameters of the model.

The plan of this paper is as follows. In Sec. II, we describe the deflected mirage mediation mechanism that we consider in this paper. Here we analyze the soft supersymmetry breaking gaugino and scalar masses of the model, and the boundary conditions on these parameters. Phenomenological implications, which include the Higgs mass, are studied in this section. In Sec. III we describe the renormalization group invariants and calculate these invariants for the deflected mirage mediation supersymmetry breaking model. We then use these invariants to solve for the parameters of the model. In Sec. IV we obtain sum rules for the deflected mirage mediation scenario and discuss some special cases. In Sec. V we proceed to compare the predictions of the pure mirage mediation scenario with those of the gravity, anomaly mediated supersymmetry breaking (AMSB), and the gauge mediation of supersymmetry breaking models, respectively. Finally, in Sec. VI we present our conclusions.

II. DEFLECTED MIRAGE MEDIATION

The deflected mirage mediation mechanism for supersymmetry breaking mechanism involves contributions of comparable scale from gravity mediation, anomaly mediation and gauge mediation as opposed to the mirage mediation which excludes the contribution from gauge mediation. The quantity

$$\alpha_m = m_{3/2}/(M_0 \log M_P/m_{3/2}), \quad (2.1)$$

parametrizes the anomaly to gravity mediation ratio, while M_0 describes the mass scale of soft supersymmetry breaking terms [40]. Here $m_{3/2}$ is the gravitino mass and M_P the reduced Planck mass. The ratio of the gauge mediated contribution to its anomaly mediated counterpart is parametrized by α_g . It is related to the messenger fields by the equation

$$|\alpha_g| = \Lambda/m_{3/2}, \quad (2.2)$$

where Λ is a mass scale associated with the messenger fields. The absolute value allows α_g to have negative values. The parameters α_m and α_g can be considered continuous but in string motivated scenarios they usually have discrete values of the order one [12]. The messenger sector is assumed to come in complete GUT representations in order to preserve gauge coupling unification. N represents the number of copies of $\mathbf{5}, \bar{\mathbf{5}}$ representations under $SU(5)$. The original Kaluza-Klein compactification is obtained with $\alpha_m = 1$ and $N = 0$. Both positive and negative values are possible for α_g . Phenomenological implications of various values of the parameters are discussed in e.g. [12,41], especially regarding the Higgs mass.

Above the messenger scale the renormalization group equations are modified from the MSSM form by adding the number of messenger pairs to the β -function coefficients b_a [40]. Thus,

$$b'_a = b_a + N, \quad (2.3)$$

where $\{b_1, b_2, b_3\} = \{33/5, 1, -3\}$. At the GUT scale μ_{GUT} , the gaugino mass boundary conditions can be written as [11]:

$$\begin{aligned} M_a(\mu_{\text{GUT}}) &= M_0(1 + g_a \ln(M_P/m_{3/2})b_a g_a \alpha_m) \\ &= M_0 + g_a^2(\mu_{\text{GUT}}) \frac{b'_a}{16\pi^2} m_{3/2}, \\ &(a = 1, 2, 3). \end{aligned} \quad (2.4)$$

Here μ_{GUT} is the high scale which we take to be the GUT scale. Similarly the scalar masses can be written as

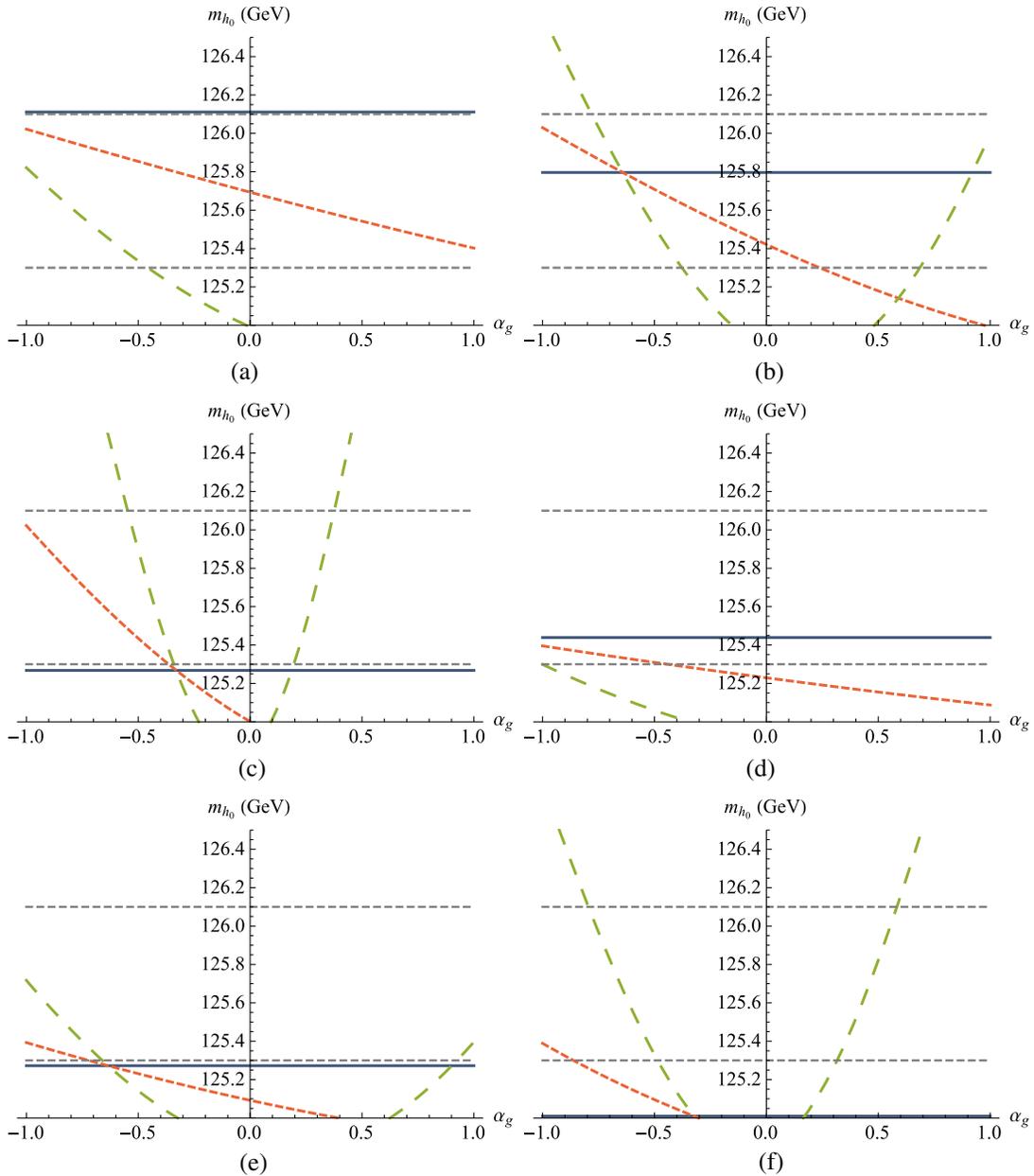


FIG. 1 (color online). Higgs mass as a function of α_g , $n_Q = n_U = n_D = n_L = n_E = 1/2$, and $\mu_{\text{mess}} = 10^{12}$ GeV. Thick lines in the order of increasing dash length correspond to $N = 0$ (solid), 3, 10. The Higgs mass is within current experimental limits between the dashed horizontal lines. (a) $\alpha_m = 0.5$, $M_0 = 3$ TeV, (b) $\alpha_m = 1$, $M_0 = 3$ TeV, (c) $\alpha_m = 2$, $M_0 = 3$ TeV, (d) $\alpha_m = 0.5$, $M_0 = 2$ TeV, (e) $\alpha_m = 1$, $M_0 = 2$ TeV, (f) $\alpha_m = 2$, $M_0 = 2$ TeV.

$$\begin{aligned}
 m_i^2(\mu_{\text{GUT}}) &= M_0^2 \left[(1 - n_i) - \frac{\theta_i}{16\pi^2} \alpha_m \ln(M_P/m_{3/2}) \right. \\
 &\quad \left. - \frac{\dot{\gamma}'_i}{(16\pi^2)^2} (\alpha_m \ln(M_P/m_{3/2}))^2 \right] \\
 &= M_0^2 (1 - n_i) - \frac{\theta_i}{16\pi^2} m_{3/2} M_0 - \frac{\dot{\gamma}'_i}{(16\pi^2)^2} m_{3/2}^2,
 \end{aligned} \quad (2.5)$$

where n_i are the modular weights for the scalar masses, γ_i are the anomalous dimensions,

$$\gamma_i = 2 \sum_a g_a^2 c_a(\Phi_i) - \frac{1}{2} \sum_{lm} |y_{ilm}|^2, \quad (2.6)$$

in which c_a is the quadratic Casimir operator for the field Ψ_i , and y_{ilm} are the normalized Yukawa couplings. $\dot{\gamma}$, and θ_i are defined as

$$\dot{\gamma}_i = 2 \sum_a g_a^4 b_a c_a(\Phi_i) - \sum_{lm} |y_{ilm}|^2 b_{y_{ilm}}, \quad (2.7)$$

$$\theta_i = 4 \sum_a g_a^2 c_a(\Psi_i) - \sum_{j,k} |y_{ijk}|^2 (p - n_i - n_j - n_k), \quad (2.8)$$

in which $b_{y_{ilm}}$ is the beta function for the Yukawa coupling y_{ilm} . $\dot{\gamma}'_i$ is obtained by replacing b_a with $b'_a = b_a + N$. For explicit values of θ'_i , $\dot{\gamma}'_i$ see [40]. One-loop renormalization group equations give the boundary condition at the messenger scale μ_{mess} for the soft gaugino mass parameters:

$$\begin{aligned}
 M_a &= g_a^2 \frac{b'_a}{16\pi^2} m_{3/2} + M_0 \left[1 - g_a^2 \frac{b'_a}{8\pi^2} \log \left(\frac{\mu_{\text{GUT}}}{\mu_{\text{mess}}} \right) \right] \\
 &\quad + \Delta M_a \quad (a = 1, 2, 3),
 \end{aligned} \quad (2.9)$$

where

$$\Delta M_a = -N M_0 \frac{g_a^2}{16\pi^2} \alpha_m (1 + \alpha_g) \ln \frac{M_P}{m_{3/2}} \quad (2.10)$$

is a threshold contribution that arises when the messenger fields are integrated out. Similarly, scalar masses receive a threshold correction,

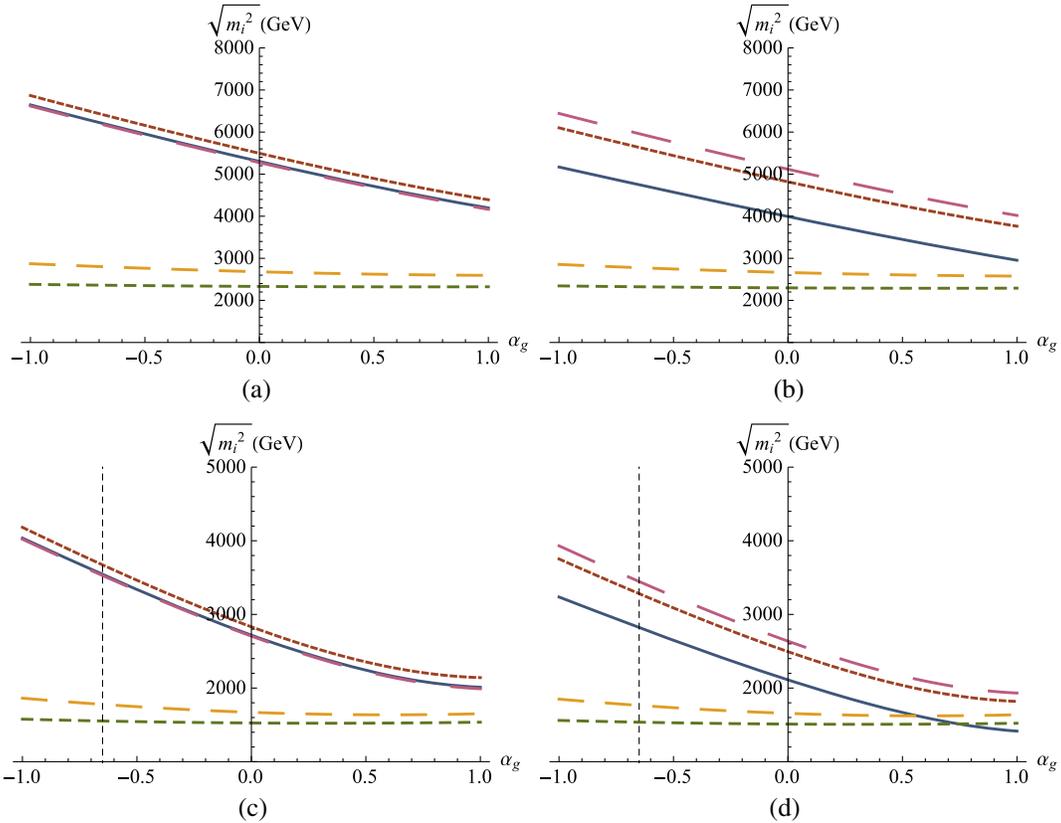


FIG. 2 (color online). The first and the third generation soft scalar mass parameters plotted as a function of α_g . In the order of increasing dash length: $m_{\bar{U}}$ (solid), $m_{\bar{Q}_1}$, $m_{\bar{L}}$, $m_{\bar{D}}$, and $m_{\bar{E}}$. $n_Q = n_U = n_D = n_L = n_E = 1/2$, $\mu_{\text{mess}} = 10^{12}$ GeV, and $N = 3$. In (c) and (d) Higgs mass is in the allowed range on the left-hand side of the dashed vertical line. (a, b) 1. generation. $\alpha_m = 0.5$, $M_0 = 3$ TeV, (c, d) 1. generation. $\alpha_m = 1.0$, $M_0 = 2$ TeV.

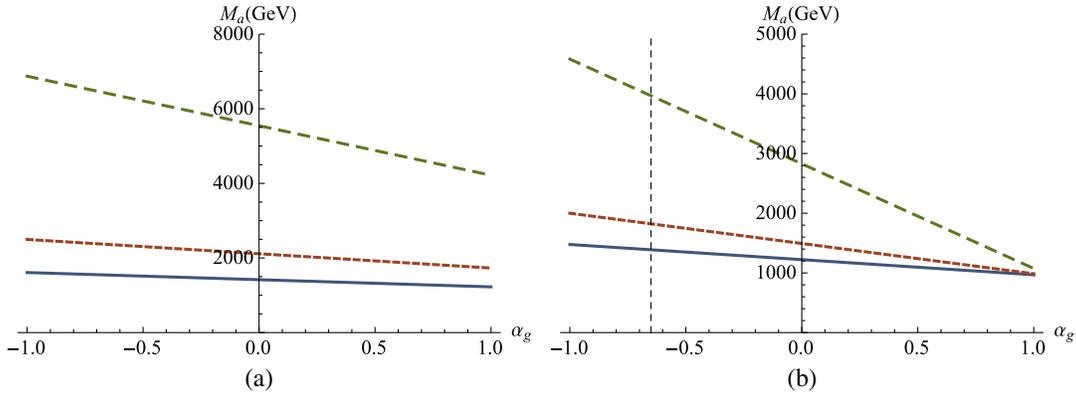


FIG. 3 (color online). The gaugino masses plotted as a function of α_g . We have set $\mu_{\text{mess}} = 10^{12}$ GeV, $n_Q = n_U = n_D = n_L = n_E = 1/2$, and $N = 3$. In the order of increasing dash length the lines correspond to M_1 (solid), M_2 , M_3 . In (b) Higgs mass is in the allowed range on the left-hand side of the dashed vertical. (a) $\alpha_m = 0.5$, $M_0 = 3$ TeV, (b) $\alpha_m = 1.0$, $M_0 = 2$ TeV.

$$\Delta m_i^{2j} = M_0^2 \sum_a 2c_a(\Psi_i) N \frac{g_a^4(\mu_{\text{mess}})}{(16\pi^2)^2} \left[\alpha_m (1 + \alpha_g) \ln \frac{M_P}{m_{3/2}} \right]^2 \delta_i^j. \quad (2.11)$$

The gaugino masses unify at the mirage scale [40]

$$\mu_{\text{mirage}} = \mu_{\text{GUT}} \left(\frac{m_{3/2}}{M_P} \right)^{\alpha_m \rho / 2}, \quad (2.12)$$

in which

$$\rho = \frac{1 + \frac{2N g_0^2}{16\pi^2} \ln \frac{M_{\text{GUT}}}{\mu_{\text{mess}}}}{1 - \frac{\alpha_m \alpha_g N g_0^2}{16\pi^2} \ln \frac{M_P}{m_{3/2}}}. \quad (2.13)$$

When $\rho = 1$, this reduces to the mirage scale of pure mirage mediation as the deflection is removed. We note that even if gauge mediation is turned off by setting $\alpha_g = 0$, mirage mediation is not recovered. This is achieved only by removing the messenger fields by setting $N = 0$. This is due to the messenger particles affecting the beta functions and thus anomaly mediation at high scales.

To examine the sparticle spectrum of the deflected mirage mediation we have calculated the soft scalar and gaugino masses using one-loop renormalization group equations and the lightest Higgs mass using the one-loop radiative corrections presented in [42]. In Fig. 1 we have plotted the lightest Higgs mass for several values for the parameters α_m , M_0 and N as a function of α_g . The current experimental limits for the Higgs mass are represented as horizontal dashed lines [43]. Although not all the parameter combinations satisfy the experimental bounds on the Higgs mass, for any two values of the studied parameters α_g , α_m , M_0 , N , a viable set can be found. We note that larger values for M_0 and negative values for α_g are favored. Smaller M_0 requires small α_g in order to have the Higgs mass in the allowed range.

In Figs. 2 and 3 we have plotted the first and the third generation scalar mass parameters and the gaugino mass parameters as a function of α_g for $N = 3$. We note that the difference of the mass scale of $m_{\tilde{e}_1}$ and $m_{\tilde{L}_1}$ to the rest of the scalars is a good indicator for the magnitude of α_g , with large difference implying a larger absolute value for α_g . For $\alpha_m = 1$ and $M_0 = 2$ TeV only α_g close to -1 is allowed by the Higgs mass limit (represented here by the vertical dashed line). In such a case the squark masses turn out to be several TeVs while slepton masses can be of the order of one TeV. Similarly, the ratios of the gaugino mass parameters M_3 to M_1 and M_3 to M_2 correspond the value of α_g with a large ratios implying a α_g close to -1 .

III. RENORMALIZATION GROUP INVARIANTS IN DEFLECTED MIRAGE MEDIATION

Renormalization group invariants are linear combinations of the soft mass parameters that remain constant under one-loop renormalization group running. Complete renormalization group invariants for the MSSM together with corresponding sum rules have been derived in [34]. The RGIs are derived under several assumptions, including the vanishing of first and second generation Yukawas [20]. We have derived the invariants for general b_i in order to determine their values above the messenger scale (Table I). Since in deflected mirage mediation new particles are introduced at a certain renormalization scale and integrated out below, one should verify whether the RGIs are affected by the modified spectrum. By considering the renormalization group beta functions [with $\beta(p) \equiv 16\pi^2 \frac{dp}{dt}$ and $t = \log(\mu/\mu_0)$] for the gauginos and for the coupling constants ($a = 1, 2, 3$),

$$\beta(g_a) = b_a g_a^3, \quad (3.1)$$

$$\beta(M_a) = 2b_a g_a^2 M_a, \quad (3.2)$$

TABLE I. The renormalization group invariants I_A involving scalar masses, gaugino masses and coupling constants. The second column defines the invariant in terms of soft masses and couplings without messenger fields present. The third column describes the difference of the value of the invariant above and below messenger scale; the masses and the couplings are to be evaluated at the messenger scale. The fourth column describes the value of the invariant at the GUT scale; the couplings are to be evaluated at the GUT scale. The quantity Δm^2 is defined as $N/(16\pi^4)(M_0\alpha_m(1+\alpha_g)\ln\frac{M_P}{m_{3/2}})^2$. The combinations of modular weights n_{e_a} are defined in (3.65)–(3.67).

Invariant	Definition	Correction at the messenger scale	Value at the GUT scale
I_{B_r}	M_r/g_r^2	$\Delta M_r/g_r^2$	$M_0/g^2 + \frac{b'_a}{16\pi^2} m_{3/2}$
I_{M_1}	$M_1^2 - \frac{5b_1}{8}(m_{d_1}^2 - m_{u_1}^2 - m_{e_1}^2)$	$-2M_1\Delta M_1 - \Delta M_1^2 + \frac{5N}{8}(m_{d_1}^2 - m_{u_1}^2 - m_{e_1}^2) - \frac{5b_1}{80}\Delta m^2 g_1^4$	$M_0^2(1 + n_{e_1} b'_1)$
I_{M_2}	$M_2^2 + \frac{b_2}{24}(9(m_{d_1}^2 - m_{u_1}^2) + 16m_{L_1}^2 - m_{e_1}^2)$	$-2M_2\Delta M_2 - \Delta M_2^2 - \frac{N}{24}(9(m_{d_1}^2 - m_{u_1}^2) + 16m_{L_1}^2 - m_{e_1}^2) + \frac{3b_2}{48}\Delta m^2 g_2^4$	$M_0^2(1 + n_{e_2} b'_2)$
I_{M_3}	$M_3^2 + \frac{b_3}{16}(5m_{d_1}^2 + m_{u_1}^2 - m_{e_1}^2)$	$2M_3\Delta M_3 + \Delta M_3^2 - \frac{N}{16}(5m_{d_1}^2 + m_{u_1}^2 - m_{e_1}^2) + \frac{b_3}{16}\Delta m^2 g_3^4$	$M_0^2(1 + n_{e_3} b'_3)$
I_{g_2}	$1/g_1^2 - (b_1/b_2)g_2^{-2}$	$28N/(5g_2^2(1+N))$	$1/g^2(1 - b'_1/b'_2)$
I_{g_3}	$1/g_1^2 - (b_1/b_3)g_3^{-2}$	$-16N/(5g_3^2(3-N))$	$1/g^2(1 - b'_1/b'_3)$

and by noting that $\beta(\frac{M_r}{g_r}) = 0$, we can define a quantity that is constant under renormalization group evolution,

$$I_{B_r} \equiv \frac{M_r}{g_r^2}. \quad (3.3)$$

Similarly from the full set of MSSM renormalization group equations one can define 12 other invariants that we have enumerated in Tables I and II. If the messenger fields would not enter the theory at an energy scale different of the GUT-scale, the invariants would remain unchanged at all scales. Since the appearance of the messengers modifies the beta functions and contributes to gaugino and scalar masses at the messenger scale μ_{mess} , the invariant can have a different value above and below the scale.

Just above the messenger scale the value is equal to the GUT-scale value

$$I_{B_r}(\mu_{\text{mess}}^+) = \frac{M_r(\mu_{\text{mess}}^+)}{g_r^2(\mu_{\text{mess}})} = I_{B_r}(\mu_{\text{GUT}}). \quad (3.4)$$

We define μ_{mess}^+ as evaluation at μ_{mess} with modified coefficients b'_a and without the threshold corrections added

to gaugino and scalar masses, and μ_{mess}^- as evaluation at μ_{mess} with the usual MSSM coefficients b_a and with threshold corrections added. Below the messenger scale the gaugino masses receive the threshold correction (2.10). Consequently just below the messenger scale

$$\begin{aligned} I_{B_r}(\mu_{\text{mess}}^-) &= \frac{M_r(\mu_{\text{mess}}^-)}{g_r^2(\mu_{\text{mess}})} = \frac{M_r(\mu_{\text{mess}}^+)}{g_r^2(\mu_{\text{mess}})} + \frac{\Delta M_r}{g_r^2(\mu_{\text{mess}})} \\ &= I_{B_r}(\mu_{\text{GUT}}) + \Delta I_{B_r}, \end{aligned} \quad (3.5)$$

where ΔM_r is as in (2.10) and we have defined

$$\begin{aligned} \Delta I_{B_r} &\equiv \Delta M_r/g_r^2(\mu_{\text{mess}}) \\ &= -NM_0/(16\pi^2)(1+\alpha_g)\alpha_m \ln \frac{M_P}{m_{3/2}}. \end{aligned} \quad (3.6)$$

We evolve the couplings down from the GUT scale and obtain

TABLE II. The invariants D_I and their GUT-scale values parametrized as $D_I(\mu_{\text{GUT}}) = \gamma_I m_{3/2}^2 + \delta_I M_0^2$. See (3.19)–(3.27).

I	Definition	γ_I	δ_I
D_Z	$3(m_{d_3}^2 - m_{d_1}^2) + 2(m_{L_3}^2 - m_{H_d}^2)$	$Y_{Za} + Y_{Zb}g^2$	$2n_\alpha$
D_{χ_1}	$3(3m_{d_1}^2 - 2(m_{Q_1}^2 - m_{L_1}^2) - m_{u_1}^2) - m_{e_1}^2$	0	n_β
$D_{L_{13}}$	$2(m_{L_1}^2 - m_{L_3}^2) - m_{e_1}^2 + m_{e_3}^2$	0	0
$D_{B_{13}}$	$2(m_{Q_1}^2 - m_{Q_3}^2) - m_{u_1}^2 + m_{u_3}^2 - m_{d_1}^2 + m_{d_3}^2$	$Y_{B_{13}a} + Y_{B_{13}b}g^2$	0
$D_{Y_{13H}}$	$m_{Q_1}^2 - 2m_{u_1}^2 + m_{d_1}^2 - m_{L_1}^2 + m_{e_1}^2 - \frac{10}{13}(m_{Q_3}^2 - 2m_{u_3}^2 + m_{d_3}^2 - m_{L_3}^2 + m_{e_3}^2 + m_{H_u}^2 - m_{H_d}^2)$	$\frac{10}{13}(-Y_{\alpha 1} - Y_{\alpha 2}g^2)$	$-\frac{1}{13}n_\gamma$
$D_{Y\alpha}$	$(m_{H_u}^2 - m_{H_d}^2 + \sum_{\text{gen}}(m_{Q_i}^2 - 2m_{u_i}^2 + m_{d_i}^2 - m_{L_i}^2 + m_{e_i}^2))/g^2$	$\frac{1}{g^2}(Y_{\alpha 1} + Y_{\alpha 2})$	$-\frac{1}{g^2}n_\delta$

$$\begin{aligned}
g_1(\mu_{\text{mess}}) &= \frac{2\sqrt{10}\pi}{\sqrt{40\pi^2/g^2 + (33 + 5N)t_{\text{mess}}}}, \\
g_2(\mu_{\text{mess}}) &= \frac{2\sqrt{2}\pi}{\sqrt{8\pi^2/g^2 + (1 + N)t_{\text{mess}}}}, \\
g_3(\mu_{\text{mess}}) &= \frac{2\sqrt{2}\pi}{\sqrt{8\pi^2/g^2 + (N - 3)t_{\text{mess}}}}, \quad (3.7)
\end{aligned}$$

where $t_{\text{mess}} = \ln \mu_{\text{GUT}}/\mu_{\text{mess}}$. Thus while the invariants remain invariant from messenger scale to GUT scale as well as from the electroweak scale to messenger scale, there is a discontinuity at the messenger scale, which must be taken into account, unless the threshold contributions are cancelled out.

The invariants designated D_I are linear combinations of the squared scalar masses with the GUT-scale value of the form

$$D_I = \gamma_I m_{3/2}^2 + \delta_I M_0^2. \quad (3.8)$$

The invariants are constructed in such a way that threshold corrections (2.11) cancel at the messenger scale, thus $D_I(\mu_{\text{TeV}}) = D_I(\mu_{\text{GUT}})$, and they are ‘‘true’’ invariants in models that include gauge mediated supersymmetry breaking such as the deflected mirage mediation. We present D_{χ_1} as an example with the GUT-scale value

$$\begin{aligned}
D_{\chi_1} &\equiv 3[3m_{d_1}^2 - 2(m_{Q_1}^2 - m_{L_1}^2) - m_{u_1}^2] - m_{e_1}^2 \\
&= M_0^2(5 + 3n_U - 9n_D - 6n_L + n_E + 6n_Q). \quad (3.9)
\end{aligned}$$

We use the scalar mass based invariants D_I to derive the high energy parameters of the deflected mirage mediation in terms of the scalar masses. The three invariants I_{M_a} are linear combinations of the squares of both scalar and gaugino masses, and are also explicitly dependent on b_a , e.g.

$$I_{M_1} = M_1^2 - \frac{5b_1}{8}(m_{d_1}^2 - m_{u_1}^2 - m_{e_1}^2), \quad (3.10)$$

where b_1 is replaced by $b'_1 = b_1 + N$ above the messenger scale. Thus the shift at the messenger scale has a complex form,

$$\begin{aligned}
\Delta I_{M_1} &\equiv I_{M_1}(\mu_{\text{mess}}^+) - I_{M_1}(\mu_{\text{mess}}^-) \\
&= (M_1 + \Delta M_1)^2 - \frac{5b_1}{8}((m_{d_1}^2 + \Delta m_{d_1}^2) - (m_{u_1}^2 + \Delta m_{u_1}^2) \\
&\quad - (m_{e_1}^2 + \Delta m_{e_1}^2)) - M_1^2 + \frac{5b'_1}{8}(m_{d_1}^2 - m_{u_1}^2 - m_{e_1}^2) \\
&= -2M_1 \Delta M_1 - \Delta M_1^2 + \frac{5N}{8}(m_{d_1}^2 - m_{u_1}^2 - m_{e_1}^2) \\
&\quad - \frac{5b_1}{8}(\Delta m_{d_1}^2 - \Delta m_{u_1}^2 - \Delta m_{e_1}^2), \quad (3.11)
\end{aligned}$$

where the gaugino mass and the scalar squareds are evaluated at μ_{mess}^+ and ΔM_r and Δm_i are defined in (2.10) and (2.11). Since ΔI_{M_i} depends on masses at the messenger scale, accessing GUT-scale values from the TeV scale measurements is not as straightforward as with D_I . As with other invariants, ΔI_{M_r} is generated by the messengers and vanishes when the messengers are removed with $N = 0$.

We have listed the correction at the messenger scale and the value at the GUT-scale for each invariant in Table I, except for the D_I invariants which are listed in Table II. Figure 4. shows the values of the invariants I_{B_a} and the square roots of I_{M_a} , and D_I above and below the messenger scale at the point $M_0 = 3$ TeV, $N = 3$, $\alpha_m = 1$, $\alpha_g = -0.5$, $t_{\text{MESS}} = -10$, $n_u = 1/2$, and $n_h = 1$.

A. Solving parameters using the invariants

We will now attempt to utilize the RGIs to solve high scale parameters $m_{3/2}$, M_0 , the messenger scale μ_{mess} , and the number of messenger pairs N in terms of low energy masses and mass parameters.

Following (3.5) we can write three equations involving the invariants I_{B_r} by setting the low energy scale value of I_{B_r} equal to the value at the GUT-scale corrected by the difference at the messenger scale,

$$I_{B_a}(\mu_{\text{GUT}}) = I_{B_a}(\mu_{\text{TeV}}) - \Delta I_{B_a}, \quad (3.12)$$

where I_{B_r} is defined in (3.3) and ΔI_{B_r} in (3.6). We note that ΔI_{B_r} vanishes if $\alpha_g = -1$. Thus the equivalence of the TeV-scale value of I_{B_a} to its GUT-scale value cannot be taken as proof of the absence of gauge messengers.

Equations (3.12) provide three independent solutions for $m_{3/2}$, and α_g , which we distinguish from each other by designating with the subindex (a),

$$m_{3/2(a)} = 16\pi^2 \frac{\sum_{b,c=1}^3 \epsilon_{abc} I_{B_c}}{\sum_{d,e=1}^3 \epsilon_{ade} b_e}, \quad (a = 1, 2, 3), \quad (3.13)$$

$$\alpha_{g(a)} = \frac{b_a g^2 m_{3/2} - 16\pi^2 g^2 I_{B_a} + 16\pi^2 M_0}{g^2 m_{3/2} N}, \quad (a = 1, 2, 3). \quad (3.14)$$

It is easy to verify by evolving g_i that the deflected mirage mediation coupling constant at the GUT scale, g , is related to g_{GUT} , which is the coupling constant at the GUT-scale with $N = 0$, by

$$\frac{1}{g} = \sqrt{\frac{1}{g_{\text{GUT}}^2} - \frac{N t_{\text{mess}}}{8\pi^2}}. \quad (3.15)$$

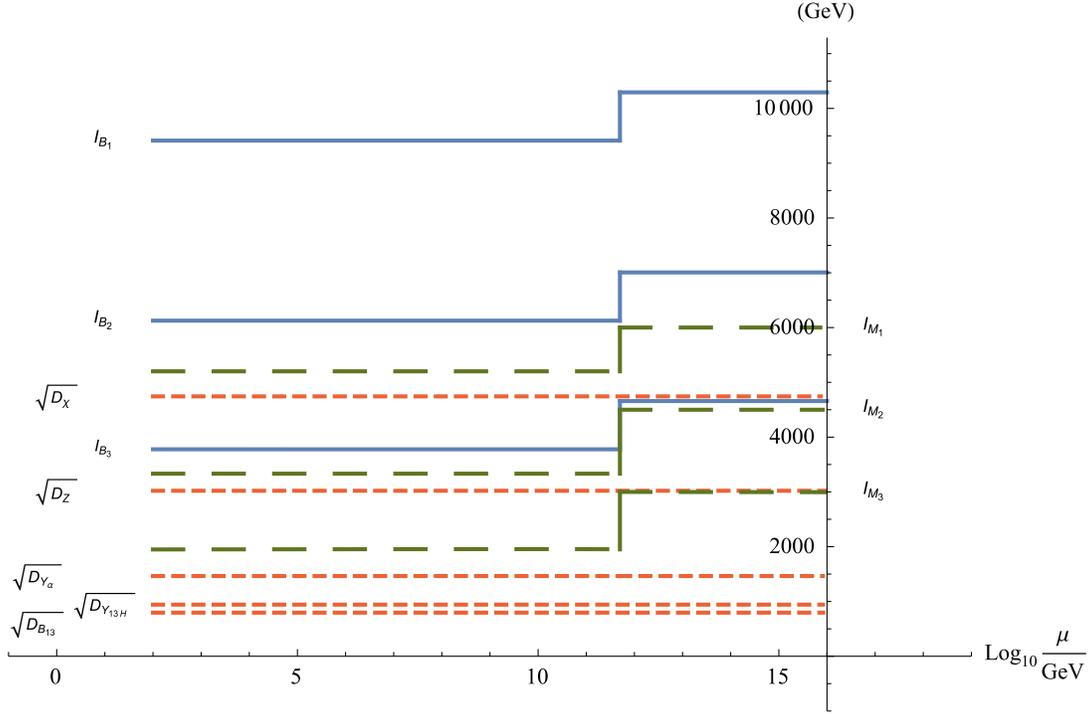


FIG. 4 (color online). The renormalization group invariants I_{B_a} (blue, solid) and the square roots of the absolute value of the invariants D_I (red, small dash) and I_{M_a} (green, large dash). Here $n_u = 1/2$, $n_h =$, $M_0 = 3$ TeV $\alpha_m = 1$, $\alpha_g = -0.5$, $\mu_{\text{MESS}} = 10^{12}$ GeV, and $N = 3$. $D_{L_{13}} = 0$ is not shown.

A different set of parameters to eliminate could, of course, be chosen, but this choice proves to be most convenient for solving all parameters, as M_0 is readily solved from the scalar mass involving invariants D_I which do not allow the determination of α_g .

The invariants D_{χ_1} , $D_{B_{13}}$, $D_{Y_{13H}}$, D_Z , and $D_{Y\alpha}$ are composed of linear combinations of soft scalar mass parameters and have identical values above and below the messenger scale. This follows from their form of their boundary condition which ensures the canceling out of threshold corrections (2.11) (for the proof see the Appendix). The invariants have the schematic form

$$D_I = \gamma_I m_{3/2}^2 + \delta_I M_0^2, \quad (3.16)$$

at the GUT scale, where the coefficients γ_I , δ_I are determined by the Yukawa couplings y_τ , y_t and y_b and g

at the GUT scale We have listed the values in Table II, where

$$n_\alpha \equiv n_{H_d} - n_L, \quad (3.17)$$

$$n_\beta \equiv 5 + 3n_U - 9n_D - 6n_L + n_E + 6n_Q, \quad (3.18)$$

$$n_\gamma \equiv 3n_D + 3n_E + 10n_{H_d} - 10n_{H_u} - 3n_L + 3n_Q - 6n_U, \quad (3.19)$$

$$n_\delta \equiv 3n_D + 3n_E - n_{H_d} + n_{H_u} - 3n_L + 3n_Q - 6n_U, \quad (3.20)$$

$$n_e \equiv n_{H_u} + n_Q + n_U - 3, \quad (3.21)$$

$$Y_{B_{13}a} \equiv \frac{1}{10240\pi^4} [1440y_b^6 + 240y_b^4y_\tau^2 + 240y_b^4y_t^2 - 1440y_b^4 - 240y_b^2y_\tau^2 + 240y_b^2y_t^4 - 480y_b^2y_t^2 + 1440y_t^6 - 1440y_t^4], \quad (3.22)$$

$$Y_{B_{13}b} \equiv \frac{1}{30720\pi^4} [-2105y_b^4 + 2105y_b^2 - 2208y_t^4 + 2208y_t^2], \quad (3.23)$$

$$Y_{Za} \equiv \frac{1}{10240\pi^4} [1440y_b^6 + 240y_b^4y_\tau^2 + 240y_b^4y_t^2 + 1440y_b^4 - 720y_b^2y_\tau^2 - 240y_b^2y_t^2 - 640y_\tau^4], \quad (3.24)$$

$$Y_{Zb} \equiv \frac{1}{10240\pi^4} [-2105y_b^4 + 2105y_b^2 + 768y_\tau^2], \quad (3.25)$$

$$Y_{\alpha 1} \equiv \frac{1}{30720\pi^4} [1440y_b^6 + 240y_b^4 y_\tau^2 + 240y_b^4 y_t^2 - 1440y_b^4 - 960y_b^2 y_\tau^2 - 480y_b^2 y_t^4 + 240y_b^2 y_t^2 - 960y_\tau^4 - 2880y_t^6 + 2880y_t^4], \quad (3.26)$$

$$Y_{\alpha 2} \equiv \frac{1}{30720\pi^4} [-2105y_b^4 + 2105y_b^2 + 1152y_\tau^2 + 4416y_t^4 - 4416y_t^2]. \quad (3.27)$$

The parameter M_0 is easily solved using the GUT-scale value of D_{χ_1} ,

$$D_{\chi_1} = 3(3m_{d_1}^2 - 2(m_{Q_1}^2 - m_{L_1}^2) - m_{u_1}^2) - m_{e_1}^2 = n_\beta M_0, \quad (3.28)$$

to obtain

$$M_0 = \sqrt{\frac{D_{\chi_1}}{n_\beta}}, \quad \text{for } n_\beta \neq 0. \quad (3.29)$$

For solving g we use the GUT-scale values of the two invariants $D_{Y_{13H}}$, $D_{Y\alpha}$, defined as

$$\begin{aligned} D_{Y_{13H}} &= m_{Q_1}^2 - 2m_{u_1}^2 + m_{d_1}^2 - m_{L_1}^2 + m_{e_1}^2 - \frac{10}{13}(m_{Q_3}^2 - 2m_{u_3}^2 + m_{d_3}^2 - m_{L_3}^2 + m_{e_3}^2 + m_{H_u}^2 - m_{H_d}^2) \\ &= \frac{10}{13}(-Y_{\alpha 1} - Y_{\alpha 2}g^2)m_{3/2}^2 - \frac{1}{13}n_\gamma M_0^2, \end{aligned} \quad (3.30)$$

$$D_{Y\alpha} = \left(m_{H_u}^2 - m_{H_d}^2 + \sum_{\text{gen}} (m_Q^2 - 2m_u^2 + m_d^2 - m_L^2 + m_e^2) \right) / g^2 = \frac{1}{g^2}(Y_{\alpha 1} + g^2 Y_{\alpha 2})m_{3/2}^2 - \frac{1}{g^2}n_\delta M_0^2, \quad (3.31)$$

Yukawa-dependent terms $Y_{\alpha 1}$ and $Y_{\alpha 2}$ can be eliminated by forming the linear combination

$$D_{Y_{13H}} + \frac{10g^2}{13}D_{Y\alpha} = -\frac{10}{13}M_0^2(n_\delta + n_\gamma). \quad (3.32)$$

We obtain the solution

$$g^2 = -\frac{\mathcal{Y}(n_{\delta\gamma})}{10D_{Y\alpha}}, \quad (3.33)$$

where we have defined

$$n_{\delta\gamma} = n_\delta + n_\gamma = 2n_\gamma - 11n_{H_d} + 11n_{H_u}, \quad (3.34)$$

$$\mathcal{Y}(n_A) = 13D_{Y_{13H}} + 10M_0^2 n_A. \quad (3.35)$$

Using the solution (3.29) for M_0 , we write

$$g^2 = \frac{-10D_{\chi_1}(n_\delta + n_\gamma) - 13D_{Y_{13H}}n_\beta}{10D_{Y\alpha}\sqrt{n_\beta}}, \quad (3.36)$$

$$\mathcal{Y}(n_A) = 13D_{Y_{13H}} + \frac{10n_A}{n_\beta}D_{\chi_1}. \quad (3.37)$$

Since from (3.15) we have

$$t_{\text{MESS}} = \frac{8\pi^2(g^2 - g_{\text{GUT}}^2)}{g^2 g_{\text{GUT}}^2 N}, \quad (3.38)$$

the relation of t_{MESS} and N is fixed once g^2 is measured. In addition to (3.33), equations

$$\begin{cases} D_{Y_{13H}} = \frac{10}{13}(-Y_{\alpha 1} - Y_{\alpha 2}g^2)m_{3/2}^2 - \frac{1}{13}n_\gamma M_0^2, \\ D_{Y\alpha} = \frac{1}{g^2}(Y_{\alpha 1} + Y_{\alpha 2})m_{3/2}^2 - \frac{1}{g^2}n_\delta M_0^2, \end{cases} \quad (3.39)$$

produce a solution to $m_{3/2}$,

$$m_{3/2} = \frac{32\sqrt{30}\pi^2 \sqrt{D_{Y\alpha}\mathcal{Y}(n_\gamma)}}{\sqrt{Y_{\alpha 2}\mathcal{Y}(n_{\delta\gamma}) - 10D_{Y\alpha}Y_{\alpha 1}}}. \quad (3.40)$$

By solving simultaneously the equations

$$\begin{cases} D_{B_{13}} = (Y_{B_{13}a} + Y_{B_{13}b}g^2)m_{3/2}^2 \\ D_Z = 2n_\alpha M_0^2 + (Y_{Za} + Y_{Zb}g^2)m_{3/2}^2, \end{cases} \quad (3.41)$$

we find alternative solutions to \mathcal{M}_0 and $m_{3/2}$ that are dependent on the Yukawa couplings

$$m_{3/2} = \frac{32\sqrt{30}\sqrt{D_{B_{13}}}}{\sqrt{g^2 Y_{B_{13}b} + Y_{B_{13}a}}} = \frac{320\sqrt{3}\sqrt{D_{B_{13}}}\sqrt{D_{Y\alpha}}\sqrt{n_\beta}}{\sqrt{10D_{Y\alpha}n_\beta Y_{B_{13}a} - Y_{B_{13}b}\mathcal{Y}(n_{\delta\gamma})}}, \quad (3.42)$$

$$M_0 = \frac{\sqrt{3D_{B_{13}}(g^2 Y_{Zb} + Y_{Za}) - \pi^4 D_Z(g^2 Y_{B_{13}b} + Y_{B_{13}a})}}{\sqrt{2}\pi^2 \sqrt{-n_\alpha(g^2 Y_{B_{13}b} + Y_{B_{13}a})}} \\ = \sqrt{\frac{(\pi^4 D_Z Y_{B_{13}b} - 3D_{B_{13}} Y_{Zb})\mathcal{Y}(n_{\delta\gamma}) - 10D_{Y\alpha}n_\beta(\pi^4 D_Z Y_{B_{13}a} - 3D_{B_{13}} Y_{Za})}{2\pi^4 n_\alpha(Y_{B_{13}b}\mathcal{Y}(n_{\delta\gamma}) - 10D_{Y\alpha}n_\beta Y_{B_{13}a})}}. \quad (3.43)$$

1. Yukawa couplings in terms of invariants

Instead of the mass parameters it is possible to use D_I to determine the values of the Yukawas at the GUT scale, although the necessary parameters for running the Yukawas up from the low energy scale are solvable independently of the Yukawas. We assume a small y_b and y_τ compared to y_t and small enough y_t to neglect terms with y_t^6 and higher order, and y_b^4 , y_τ^4 and higher order. Equations (3.22)–(3.27) are then reduced to

$$Y_{B_{13}a} = -1440y_t^4, \quad (3.44)$$

$$Y_{B_{13}b} = -2105y_b^2 - 2208y_t^4 + 2208y_\tau^2, \quad (3.45)$$

$$Y_{Za} = 0, \quad (3.46)$$

$$Y_{Zb} = 2105y_b^2, \quad (3.47)$$

$$Y_{\alpha 1} = 2880y_t^4, \quad (3.48)$$

$$Y_{\alpha 2} = 2105y_b^2 + 1152y_\tau^2 + 4416y_t^4 - 4416y_\tau^2. \quad (3.49)$$

From (3.39), and (3.41) we can then solve

$$y_\tau = \frac{4\sqrt{5}\sqrt{-D_{Y\alpha}}\sqrt{60D_{B_{13}} - \pi^4(13\mathcal{Y}(n_\gamma) - 10D_Z + 20M_0^2 n_\alpha)}}{3\sqrt{m_{3/2}^2 \mathcal{Y}(n_{\delta\gamma})}}, \quad (3.50)$$

$$y_b = \frac{64\sqrt{\frac{5}{421}}\pi^2\sqrt{-D_{Y\alpha}(D_Z - 2M_0^2 n_\alpha)}}{\sqrt{m_{3/2}^2 \mathcal{Y}(n_{\delta\gamma})}}, \quad (3.51)$$

$$y_t = \frac{1}{6m_{3/2}^2(23\mathcal{Y}(n_{\delta\gamma}) - 150D_{Y\alpha})} \left\{ 4761m_{3/2}^4 \mathcal{Y}(n_{\delta\gamma})^2 + 69\mathcal{Y}(n_{\delta\gamma}) \right. \\ \left. + \sqrt{38400D_{Y\alpha}m_{3/2}^2(3D_{B_{13}} + \pi^4(D_Z - 2M_0^2 n_\alpha))(23\mathcal{Y}(n_{\delta\gamma}) - 150D_{Y\alpha})} \right\}. \quad (3.52)$$

Note that while the solution D_{χ_1}/n_β can be substituted for M_0^2 , and $16\pi^2(I_{B_1} - I_{B_3})/(b_1 - b_3)$ for $m_{3/2}$ here, the solution (3.43) for M_0 and the solutions (3.40) and (3.42) for $m_{3/2}$ derived above, are not independent from (3.50)–(3.52) and thus cannot be used.

2. Solving N

For solving N we use the remaining three invariants composed of the mass parameters,

$$I_{M_1} = M_1^2 - \frac{5b'_1}{8}(m_{d_1}^2 - m_{u_1}^2 - m_{e_1}^2), \quad (3.53)$$

$$I_{M_2} = M_2^2 + \frac{b'_2}{24}(9(m_{d_1}^2 - m_{u_1}^2) + 16m_{L_1}^2 - m_{e_1}^2), \quad (3.54)$$

$$I_{M_3} = M_3^2 + \frac{b'_3}{16}(5m_{d_1}^2 + m_{u_1}^2 - m_{e_1}^2). \quad (3.55)$$

We remind the reader that parameter $b'_a = b_a + N$ is to be replaced with b_a below the messenger scale. In order to connect TeV scale measurements to the GUT scale parameter values, we examine I_{M_1} above the messenger scale:

$$I_{M_1}(\mu_{\text{MESS}}^+) = M_1^2(\mu_{\text{MESS}}^+) - \frac{5b'_1}{8}(m_{d_1}^2(\mu_{\text{MESS}}^+) - m_{u_1}^2(\mu_{\text{MESS}}^+) - m_{e_1}^2(\mu_{\text{MESS}}^+)) = I_{M_1}(\mu_{\text{GUT}}). \quad (3.56)$$

By using the relations

$$\begin{aligned} I_{B_1}(\mu_{\text{GUT}}) &= I_{B_1}(\mu_{\text{MESS}}^+) = M_1(\mu_{\text{MESS}}^+)/g_1^2(\mu_{\text{MESS}}^+), \\ m_i^2(\mu_{\text{MESS}}^+) &= m_i^2(\mu_{\text{MESS}}^-) - \Delta m_i^2, \end{aligned} \quad (3.57)$$

we eliminate the mass parameters measured above messenger scale and obtain

$$I_{M_1}(\mu_{\text{GUT}}) = I_{B_1}^2(\mu_{\text{GUT}})g_1^4(\mu_{\text{MESS}}) - \frac{5b'_1}{8}(m_{d_1}^2(\mu_{\text{MESS}}^-) - m_{u_1}^2(\mu_{\text{MESS}}^-) - m_{e_1}^2(\mu_{\text{MESS}}^-)) + \frac{5b'_1}{8}(\Delta m_{d_1}^2 + \Delta m_{u_1}^2 - \Delta m_{e_1}^2). \quad (3.58)$$

Directly from (3.53) and (3.57) we see that below the messenger scale $5b_1/8(m_{d_1}^2 - m_{u_1}^2 - m_{e_1}^2) = I_{M_1} - M_1^2 = I_{M_1} - I_{B_1}^2g_1^4(\mu_{\text{MESS}})$. Thus

$$I_{M_1}(\mu_{\text{GUT}}) = I_{B_1}^2(\mu_{\text{GUT}})g_1^4(\mu_{\text{MESS}}) + \frac{b'_1}{b_1}(I_{M_1} - I_{B_1}^2g_1^4(\mu_{\text{MESS}})) + \frac{5b'_1}{8}(\Delta m_{d_1}^2 + \Delta m_{u_1}^2 - \Delta m_{e_1}^2). \quad (3.59)$$

We have dropped the argument μ_{MESS}^- so that the low scale value of the invariants are meant unless otherwise specified. Analogously

$$I_{M_2}(\mu_{\text{GUT}}) = I_{B_2}^2(\mu_{\text{GUT}})g_2^4(\mu_{\text{MESS}}) + \frac{b'_1}{b_1}(I_{M_1} - I_{B_2}^2g_2^4(\mu_{\text{MESS}})) - \frac{b'_2}{24}(9(\Delta m_{d_1}^2 - \Delta m_{u_1}^2) + 16\Delta m_{L_1}^2 - \Delta m_{e_1}^2), \quad (3.60)$$

$$I_{M_3}(\mu_{\text{GUT}}) = I_{B_3}^2(\mu_{\text{GUT}})g_3^4(\mu_{\text{MESS}}) + \frac{b'_1}{b_1}(I_{M_3} - I_{B_3}^2g_3^4(\mu_{\text{MESS}})) - \frac{b'_3}{16}(5\Delta m_{d_1}^2 + \Delta m_{u_1}^2 - \Delta m_{e_1}^2). \quad (3.61)$$

After substituting the GUT scale values $I_{B_1}(\mu_{\text{GUT}})$ and $I_{M_1}(\mu_{\text{GUT}})$, the corrections to the scalar masses (2.11) with α_g from (3.14), $g_a(\mu_{\text{MESS}})$ from (3.7), and $\alpha_{g(a)}$, Eqs. (3.59)–(3.61) can be collectively written as

$$\begin{aligned} 256\pi^4 \{ &-b_a^2(g^2 - g_{\text{GUT}}^2)^2(\mathcal{I}_{M_a} - M_0^2) + b_a g^2 N(g^2(M_0^2 - 2\mathcal{I}_{M_a})g_{\text{GUT}}^2\mathcal{I}_{M_a}) - g^4\mathcal{I}_{M_a}N^2\} \\ &+ g^4\mathcal{I}_{B_a}^2 N(b_a + N) + 32\pi^2 b_a g^2 g_{\text{GUT}}^2 \mathcal{I}_{B_a} M_0 N = 0, \quad (a = 1, 2, 3), \end{aligned} \quad (3.62)$$

where we have defined

$$\mathcal{I}_{B_a} \equiv (b_a m_{3/2} - 16\pi^2 I_{B_a})g_{\text{GUT}}^2, \quad (3.63)$$

$$\mathcal{I}_{M_a} \equiv I_{M_a} - b_a M_0^2 n_{e_a}, \quad (3.64)$$

where

$$n_{e_1} \equiv \frac{5}{8}(1 + n_D - n_e - n_U), \quad (3.65)$$

$$n_{e_2} \equiv \frac{1}{24}(15 - 9n_D + n_e + 9n_U - 16n_L), \quad (3.66)$$

$$n_{e_3} \equiv \frac{1}{16}(5 - 5n_D + n_e - n_U). \quad (3.67)$$

All $m_{3/2}$ -dependence in (3.62) is now contained in \mathcal{I}_{B_a} , and can be eliminated with the solution to $m_{3/2(a)}$ from (3.13) to obtain

$$\mathcal{I}_{B_a} = 16\pi^2 g_{\text{GUT}}^2 \frac{\sum_{b,c} \epsilon_{abc} (b_a I_{B_c} - b_c I_{B_a})}{\sum_{d,e} \epsilon_{ade} b_e}. \quad (3.68)$$

A solution for N can be obtained from each of the equations (3.62),

$$N_{(a)} = -\frac{b_a}{2g^2(\mathcal{I}_{B_a}^2 - 256\pi^4\mathcal{I}_{M_a})} \left\{ g^2(\mathcal{I}_{B_a}^2 + 256\pi^4(-2\mathcal{I}_{M_a} - M_0^2)) + 32g_{\text{GUT}}^2(\pi^2\mathcal{I}_{B_a}M_0 + 16\pi^4\mathcal{I}_{M_a}) \right. \\ \left. \pm (\mathcal{I}_{B_a} + 16\pi^2M_0) \sqrt{g^4(\mathcal{I}_{B_a} - 16\pi^2M_0)^2 + 64\pi^2g^2g_{\text{GUT}}^2(\mathcal{I}_{B_a}M_0 - 16\pi^2\mathcal{I}_{M_a}) + 1024\pi^4g_{\text{GUT}}^4\mathcal{I}_{M_a}} \right\}. \quad (3.69)$$

Solutions can also be obtained for M_0 , n_{ϵ_a} , $m_{3/2}$, or g :

$$M_{0(a)} = \frac{Ng^2 + b_ag'^2}{n_{\epsilon_a}(Ng^2 + bg'^2)^2 + bg'^4 + Ng^4} \left\{ \pm \sqrt{(n_{\epsilon_a}(Ng^2 + bg'^2)^2 + bg'^4 + Ng^4)I_{M_a} - b_ag^4N(n_{\epsilon_a}b'_a + 1)\mathcal{I}_{B_a}^2} \right. \\ \left. - \frac{Nb_ag^2(g^2 - g'^2)\mathcal{I}_{B_a}}{16\pi^2(Ng^2 + b_ag'^2)} \right\}, \quad (3.70)$$

$$n_{\epsilon_a} = \frac{I_{M_a}}{b_aM_0^2} - \frac{(Ng^2\mathcal{I}_{B_a} - 16\pi^2b_ag'^2M_0)^2 + b_aNg^4(\mathcal{I}_{B_a} + 16\pi^2M_0)^2}{256\pi^4b_aM_0^2(Ng^2 + b_ag'^2)^2}, \quad (3.71)$$

$$m_{3/2(a)} = 16\pi^2 \left(\frac{I_{B_a}}{b_a} - \frac{M_0}{g^2b'_a} \pm \frac{(Ng^2 + b_ag'^2)\sqrt{N(b'_a\mathcal{I}_{M_a} - b_aM_0^2)}}{Nb'_ab_ag^2(g^2 - g'^2)} \right), \quad (3.72)$$

$$g_{(a)}^2 = \pm \frac{16\pi^2g_{\text{GUT}}^2b_a(\mathcal{I}_{M_a} - M_0^2)}{16\pi^2(b'_a\mathcal{I}_{M_a} - b_aM_0^2) + \mathcal{I}_{B_a}M_0N \pm (\mathcal{I}_{B_a} + 16\pi^2M_0)\sqrt{N(b'_a\mathcal{I}_{M_a} - b_aM_0^2)}}, \quad (3.73)$$

where

$$b'_a = b_a + N, \quad (3.74)$$

$$g^2 = g^2 - g_{\text{GUT}}^2. \quad (3.75)$$

The correct signs have to be determined by other means such as by comparing the solutions and discarding negative values for the parameters. Only the solutions with a different index are independent; i.e. we cannot use the solution $N = N_{(1)}$ to solve $M_{0(1)}$ etc. Three linearly independent solutions can be “picked,” each with a different index, e.g. $M_{0(1)}$, $N_{(2)}$, $g_{(3)}$, and the supplemented with the solutions obtained from D_I and I_{B_a} .

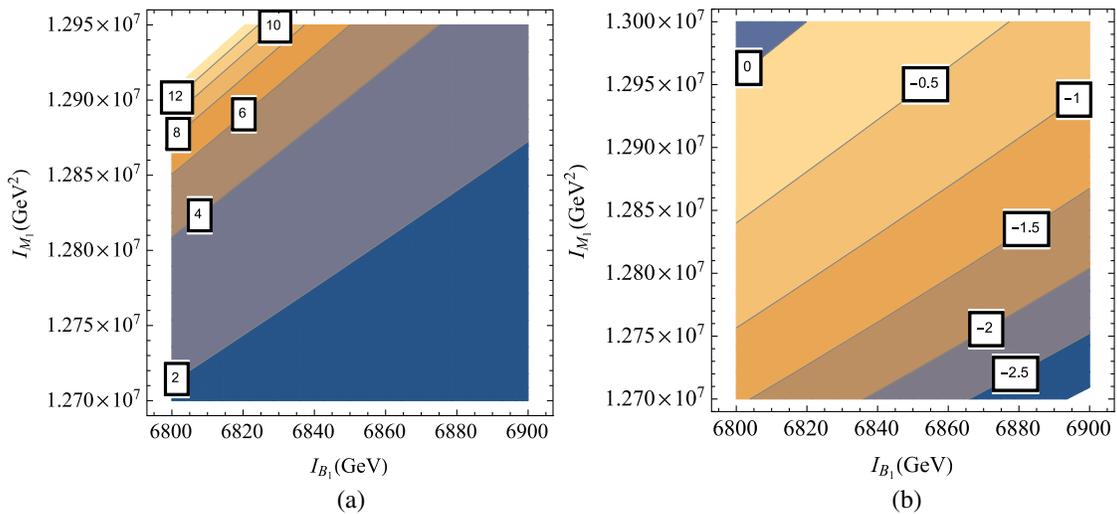


FIG. 5 (color online). The number of messenger field pairs N , and α_g as predicted by (3.69) and (3.14). $M_0 = 2.0$ TeV, $\alpha_m = 1$, and $g = -\frac{\gamma(n_{\epsilon_a})}{10D_{Y_a}} = 0.80$, and $n_u = 1/2$. (a) N , (b) α_g .

TABLE III. Summary of the solutions for the parameters derived in Sec. III.

Condition	$I_{B_a}(\text{TeV}) = I_{B_a}(\text{GUT}) + \Delta I_{B_a}$	$D_I(\text{TeV}) = D_I(\text{GUT})$	$I_{M_a}(\text{TeV}) = I_{M_a}(\text{GUT}) + \Delta I_{M_a}; (\text{III.14})$
		$\{ M_0 \} \quad (\text{III.30})$	$3 \times \{ N(M_0, m_{3/2}, g) \} \quad (\text{III.73})$
		$\begin{cases} m_{3/2}(M_0) & (\text{III.43}) \\ g(M_0) & (\text{III.34}) \end{cases}$	$3 \times \{ M_0(N, m_{3/2}, g) \} \quad (\text{III.74})$
Solution	$3 \times \{ m_{3/2} \} \quad (\text{III.13})$ $3 \times \{ \alpha_g(M_0, N, m_{3/2}) \} \quad (\text{III.14})$	$\begin{cases} m_{3/2}(g) & (\text{III.43}) \\ M_0(g) & (\text{III.34}) \end{cases}$	$3 \times \{ g(N, M_0, m_{3/2},) \} \quad (\text{III.77})$
		$\begin{cases} g(M_0) & (\text{III.34}) \\ y_t(M_0, m_{3/2}) & (\text{III.55}) \\ y_\tau(M_0, m_{3/2}) & (\text{III.53}) \\ y_b(M_0, m_{3/2}) & (\text{III.54}) \end{cases}$	$3 \times \{ m_{3/2}(N, M_0, g,) \} \quad (\text{III.76})$

Figure 5 illustrates the behavior of N and α_g as a function of I_{B_1} and I_{M_1} as predicted by (3.69) and (3.14). We assume $n_u = 1/2$, and $D_{Y\alpha}$, $D_{Y_{13H}}$, D_{χ_1} , and two of the I_{B_a} to have been measured to fix $g = 0.80$, $\alpha_m = 1$, and $M_0 = 2$ TeV. Thus I_{B_1} and I_{M_1} determine the values of N and α_g . Small variation in I_{M_1} and I_{B_1} point to significantly different values of N and α_g ; if $I_{M_1} = 1.29 \times 10^7$ GeV², a difference of 50 GeV in I_{B_1} separates $N = 4$ and $N = 12$. Table III summarizes the solutions derived above.

3. The modular weights

In terms of universal modular weights $n_h = n_{H_d} = n_{H_u}$ and $n_u = n_U = n_D = n_L = n_E = n_Q$, for which

$$n_\alpha = n_h - n_u, \quad (3.76)$$

$$n_\beta = 5(1 - n_u), \quad (3.77)$$

$$n_\gamma = n_\delta = n_{\delta\gamma} = 0, \quad (3.78)$$

$$n_{e_1} = \frac{5}{8}(1 - n_u), \quad (3.79)$$

$$n_{e_2} = \frac{5}{8}(1 - n_u), \quad (3.80)$$

$$n_{e_3} = \frac{5}{16}(1 - n_u), \quad (3.81)$$

the M_0 -term is removed from the solution of g , thus (3.33) is simplified to

$$g^2 = -\frac{13D_{Y_{13H}}}{10D_{Y\alpha}}. \quad (3.82)$$

Determining an analytical solution for n_u requires solving two of the three equations (3.62) simultaneously for N and n_u . While the solution exists, it is too complex to be

presented explicitly. A simpler way may be to determine n_u numerically or by guessing from the equation

$$N_{(1)} = N_{(2)}. \quad (3.83)$$

The third independent equation $N_{(3)}$ then remains for determining N . M_0 , $m_{3/2}$, μ_{MESS} , and N are now known, thus the Yukawas at the GUT scale can be evaluated through conventional method of the renormalization group running. Then n_h can be determined from

$$D_Z = 2M_0^2(n_h - n_u) + (Y_{Za} + Y_{Zb}g^2)m_{3/2}^2. \quad (3.84)$$

A measurement of $D_{\chi_1} = 0$ would present the problem of determining whether $M_0 = 0$ or $n_u = 1$. If we assume the latter, two of the equations (3.62) could be solved simultaneously for M_0 and N . Then the value of $m_{3/2}$ from the remaining independent equation (3.72) can be evaluated and checked for consistency with (3.13), to verify the hypothesis.

In the general case it is not possible to determine all seven combinations of the modular weights since after solving the five parameters only five independent RGIs remain. Some of n_{e_a} can be solved from (3.71) and sum rules discussed in Sec. IV could be helpful in determining n_β , n_α , n_γ , and n_δ .

IV. SUM RULES IN DEFLECTED MIRAGE MEDIATION

Renormalization group invariants can be used to construct sum rules by applying various conditions to reduce variables, e.g. mass unification. As a generic example we assume gaugino mass unification at some scale and write $M_1 = M_2 = M_3 = M_{1/2}$. From (3.3),

$$I_{B_a} = \frac{M_{1/2}}{g_a^2}. \quad (4.1)$$

By combining this to the definitions of the invariants I_{g_2} and I_{g_3} ,

TABLE IV. Sum rules derived from the condition of gauge coupling unification, gaugino mass unification and scalar mass unification. Third and fourth rows describe whether the sum rule is valid above and below the messenger scale, respectively. The bottom three sum rules involving scalar masses are valid only above messenger scale and with the condition of universal modular weights $n_Q = n_U = n_D = n_E = n_L = n_u$. Above the messenger scale b'_a is to be substituted for b_a .

Sum rule	$\mu < \mu_{\text{mess}}$	$\mu > \mu_{\text{mess}}$
$I_{g_1} - I_{g_2}(1 - b_1/b_2)/(1 - b_1/b_3) = 0$	OK	OK
$(I_{B_1} - (b_1/b_3)I_{B_3})I_{g_2} = (I_{B_1} - (b_1/b_2)I_{B_2})I_{g_3}$	OK	OK
$I_{g_2} = (I_{M_1} - \frac{b_1}{8}D_{\chi_1})^{-1/2}I_{B_1} - \frac{b_1}{b_2}(I_{M_2} - \frac{b_2}{8}D_{\chi_1})^{-1/2}I_{B_2}$	X	$n_i = n_u$
$I_{g_3} = (I_{M_1} - \frac{b_1}{8}D_{\chi_1})^{-1/2}I_{B_1} + \frac{b_1}{b_3}(I_{M_3} + \frac{b_3}{16}D_{\chi_1})^{-1/2}I_{B_3}$	X	$n_i = n_u$
$I_{M_1} - (2b_1 + b_3)/(2b_2 + b_3)I_{M_2} + 2(b_1 - b_2)/(2b_2 + b_3)I_{M_3} = 0$	X	$n_i = 1$

$$I_{g_2} = 1/g_1^2 - (b_1/b_2)g_2^{-2}, \quad (4.2)$$

$$I_{g_3} = 1/g_1^2 - (b_1/b_3)g_3^{-2}, \quad (4.3)$$

we can eliminate $M_{1/2}$ from the resulting group of equations to obtain the sum rule

$$[I_{B_1} - (b_1/b_3)I_{B_3}]I_{g_2} = [I_{B_1} - (b_1/b_2)I_{B_2}]I_{g_3}. \quad (4.4)$$

If we assume the gaugino mass unification to occur in conjunction with a scalar mass unification (with the common scalar mass squared value m_0^2) at the same scale, the RGIs I_{M_a} have the values

$$I_{M_1} = \frac{33m_0^2}{8} + M_{1/2}^2, \quad (4.5)$$

$$I_{M_2} = \frac{5m_0^2}{8} + M_{1/2}^2, \quad (4.6)$$

$$I_{M_3} = M_{1/2}^2 - \frac{15m_0^2}{16}. \quad (4.7)$$

This allows us to write the sum rule

$$81I_{M_2} - 56I_{M_3} - 25I_{M_1} = 0. \quad (4.8)$$

Similarly from the assumption of gauge coupling unification at the GUT scale one can derive

$$I_{g_1} - I_{g_2}(1 - b_1/b_2)/(1 - b_1/b_3) = 0. \quad (4.9)$$

In the specific case of deflected mirage mediation we can verify the formula easily for renormalization scales above the messenger scale by substituting b'_a for b_a and plugging in the GUT-scale values for the gaugino masses (2.4). To examine the equation below the messenger scale we restore b_a and

write the invariants at the scale μ_{mess}^- . Using (4.3), (3.7), and (3.5) it is straightforward to verify that (4.4) is valid below messenger scale as well. Similarly one can derive sum rules based on scalar mass unification and scalar mass unification combined to gaugino mass unification at the same unification scale [37]. Validity of the sum rules derived in [37] at regions above and below the messenger scale is listed in Table IV for deflected mirage mediation.

We will attempt to derive sum rules valid at all energies using the RGIs, that are not dependent on other parameters than n_i and N . From Table II we see that $D_{L_{13}} = 0$ and thus we immediately have

$$2(m_{L_1}^2 - m_{L_3}^2) - m_{e_1}^2 + m_{e_3}^2 = 0. \quad (4.10)$$

More sum rules can be constructed by combining the solutions to the mass parameters and g . For instance by equating the two expressions for $m_{3/2}$; (3.13), and (3.72) with $a = 1$, we obtain

$$\begin{aligned} & \frac{165}{48}g^4g_{\text{GUT}}^4N(5N+33)(I_{B_1} - I_{B_3}) \\ &= \sqrt{5}g^2g_{\text{GUT}}^2(g^2(5N+33) - 33g_{\text{GUT}}^2) \\ & \times \sqrt{N\left(\mathcal{I}_{M_1}n_\beta(5N+33) - \frac{33}{n_\beta}D_{\chi_1}\right)} \\ &+ 5g^4g_{\text{GUT}}^4I_{B_1}N(5N+33) - 165g^2g_{\text{GUT}}^4N\sqrt{\frac{D_{\chi_1}}{n_\beta}}, \end{aligned} \quad (4.11)$$

where we have again used $M_0^2 = \frac{D_{\chi_1}}{n_\beta}$ and $g^2 = \frac{-\mathcal{Y}(n_{\delta\gamma})}{10D_{Y\alpha}}$. Similarly from the solutions of g^2 ; (3.36) and (3.73), we obtain

$$-10n_{\delta\gamma}D_{\chi_1} - 13D_{Y_{13H}}n_\beta = \frac{160\pi^2g_{\text{GUT}}^2D_{Y\alpha}(n_\beta\mathcal{I}_{M_2} - D_{\chi_1})}{16\pi^2\left((1+N)\mathcal{I}_{M_2} - \frac{aD_{\chi_1}}{n_\beta}\right) + \mathcal{I}_{B_2}\sqrt{\frac{D_{\chi_1}}{n_\beta}}N - \left(\mathcal{I}_{B_2} + 16\pi^2\sqrt{\frac{D_{\chi_1}}{n_\beta}}\right)\sqrt{N\left((1+N)\mathcal{I}_{M_a} - \frac{D_{\chi_1}}{n_\beta}\right)}}. \quad (4.12)$$

To construct sum rules independent on N , one can combine to solutions (3.69) to form three sum rules

$$N_{(1)} = N_{(2)} = N_{(3)}. \quad (4.13)$$

We leave out the explicit formula for brevity. The sum rules provide a possible way to determine some of the modular weights. Additional sum rules can be formed in a similar manner from (3.40), (3.42), (3.43), and (3.70) or by combining other solutions summarized in Table III.

A. Special cases

In order to draw distinctions between the supersymmetry breaking scenarios and identify dominating contributions we consider sum rules in some special cases obtained from the deflected mirage mediation boundary conditions.

1. The case $N = 0$

In the absence of messenger fields, $\Delta I_{B_a} = \Delta I_{M_a} = 0$, and all the invariants have equal values at the TeV scale and the GUT scale. As noted before, since the threshold correction for I_{B_a} ,

$$\Delta I_{B_a} = -NM_0/(16\pi^2)(1 + \alpha_g)\alpha_m \ln \frac{M_P}{m_{3/2}}, \quad (4.14)$$

vanishes for $\alpha_g = -1$ as well as for $N = 0$, we cannot deduce the absence of the messenger fields from the condition $I_{B_a}(\mu_{\text{TeV}}) = I_{B_a}(\mu_{\text{GUT}})$ alone. On the other hand, from (3.11), the threshold correction for I_{M_1} reads

$$\begin{aligned} \Delta I_{M_1} = & -2M_1\Delta M_1 - \Delta M_1^2 + \frac{5N}{8}(m_{d_1}^2(\mu_{\text{MESS}}^+) \\ & - m_{\tilde{u}_1}^2(\mu_{\text{MESS}}^+) - m_{\tilde{e}_1}^2(\mu_{\text{MESS}}^+)) \\ & - \frac{5b_1}{8}(\Delta m_{d_1}^2 - \Delta m_{\tilde{u}_1}^2 - \Delta m_{\tilde{e}_1}^2). \end{aligned} \quad (4.15)$$

If $\alpha_g = -1$, the corrections to scalar and gaugino masses vanish, leaving

$$\begin{aligned} \Delta I_{M_1} = & \frac{5N}{8}(m_{d_1}^2(\mu_{\text{MESS}}^+) - m_{\tilde{u}_1}^2(\mu_{\text{MESS}}^+) \\ & - m_{\tilde{e}_1}^2(\mu_{\text{MESS}}^+)), \quad \text{for } \alpha_g = -1. \end{aligned} \quad (4.16)$$

As one can see, $\Delta I_{M_1} = 0$ only if $N = 0$ (or $m_{d_1}^2 - m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2 = 0$, which would imply a constant M_1 above the messenger scale). A similar condition obviously applies for I_{M_2} and I_{M_3} . Consequently, the equation $I_{M_a}(\mu_{\text{GUT}}) = I_{M_a}(\mu_{\text{TeV}})$ provides us with a better condition for the absence of messengers. Thus we write,

$$I_{M_a}(\mu_{\text{TeV}}) = I_{M_a}(\mu_{\text{GUT}}) = M_0^2(1 + b_a n_{e_a}), \quad \text{for } N = 0. \quad (4.17)$$

From (3.65)–(3.67) we see that in the case of universal modular weights $n_{e_1} = n_{e_2} = 2n_{e_3} = 5/8(1 - n_u)$. Now the n_{e_a} can be eliminated from (4.17) and three conditions obtained,

$$M_0 = \frac{\sqrt{3I_{M_2} + 2I_{M_3}}}{\sqrt{5}}, \quad (4.18)$$

$$5/8(1 - n_u) = \frac{2(I_{M_2} - I_{M_3})}{3I_{M_2} + 2I_{M_3}}, \quad (4.19)$$

$$81I_{M_2} - 56I_{M_3} - 25I_{M_1} = 0. \quad (4.20)$$

Equation (4.20) can in fact also be derived from the assumptions of scalar mass and gaugino mass unification at the same scale without additional assumptions about the supersymmetry breaking scenario [37]. We then use the solution $M_0^2 = \frac{D_{\chi_1}}{5(1-n_u)}$ from (3.29), and the first two equations yield (after eliminating n_u and M_0),

$$D_{\chi_1} = 16/5(I_{M_2} - I_{M_3}), \quad \text{for } N = 0, \quad n_u \neq 1. \quad (4.21)$$

Equations (4.20) and (4.21) now provide a simple test for determining the existence of the messenger sector, with the restriction that universal modular weights are required.

2. The cases $M_0 = 0$, $m_{3/2} = 0$, $g = g_{\text{GUT}}$

In the case of zero M_0 and $m_{3/2}$ we obtain from (3.29), (3.13),

$$D_{\chi_1} = 0, \quad \text{for } M_0 = 0, \quad (4.22)$$

$$\sum_{b,c=1}^3 \epsilon_{abc} I_{B_c} = 0, \quad (a = 1, 2, 3), \quad \text{for } m_{3/2} = 0. \quad (4.23)$$

The case $g = g_{\text{GUT}}$ implies either $\mu_{\text{MESS}} = \mu_{\text{GUT}}$ or $N = 0$. From (3.33)

$$13D_{Y_{13H}} + 10M_0^2(n_\gamma + n_\delta) = 10g_{\text{GUT}}^2 D_{Y_\alpha}. \quad (4.24)$$

If we assume universal modular weights this is simplified to

$$13D_{Y_{13H}} = 10g_{\text{GUT}}^2 D_{Y_\alpha}. \quad (4.25)$$

V. COMPARISON WITH SUGRA, AMSB, GAUGE MEDIATION AND PURE MIRAGE MEDIATION

Deflected mirage mediation includes contributions from three separate supersymmetry breaking mechanisms, namely gravity mediation (SUGRA) [3,44–47], gauge mediation (GMSB) [48–50], and anomaly mediation

(AMSB) [51,52]. The boundary conditions for the scalar and the gaugino masses can be parametrized as

$$\text{SUGRA: } m_i^2(\mu_{\text{GUT}}) = (1 - n_i)M_0^2; \quad M_a(\mu_{\text{GUT}}) = M_0, \quad (5.1)$$

$$\begin{aligned} \text{AMSB: } m_i^2(\mu_{\text{GUT}}) &= -\frac{\dot{\gamma}'_i}{(16\pi^2)^2} m_{3/2}^2; \\ M(\mu_{\text{GUT}}) &= g^2 \frac{b_a}{16\pi^2} m_{3/2}, \end{aligned} \quad (5.2)$$

$$\begin{aligned} \text{GMSB: } m_i^2(\mu_{\text{MESS}}) &= \frac{N\Lambda^2}{(16\pi^2)^2} \sum_{a=1}^3 c_a(\Psi_i) g_a^4; \\ M(\mu_{\text{MESS}}) &= \frac{N g_a^2}{16\pi^2} \Lambda. \end{aligned} \quad (5.3)$$

Note that the GMSB boundary conditions are defined on the messenger scale possibly different from the GUT scale while the AMSB and SUGRA boundary conditions are defined at the GUT scale. Additionally, two combinations of the above exist: mirage mediation [6] is obtained from DMMSB by removing the messengers and deflected anomaly mediation (DAMSB) [53–55] is obtained by setting M_0 to zero.

The boundary conditions for the five models can be extracted from (2.4), (2.5), (2.11), and (2.10) with the following prescriptions:

$$\text{SUGRA: } m_{3/2} = 0; \quad N = 0, \quad (5.4)$$

$$\text{AMSB: } M_0 = 0; \quad N = 0, \quad (5.5)$$

$$\text{GMSB: } M_0 = 0; \quad m_{3/2} = 0, \quad (5.6)$$

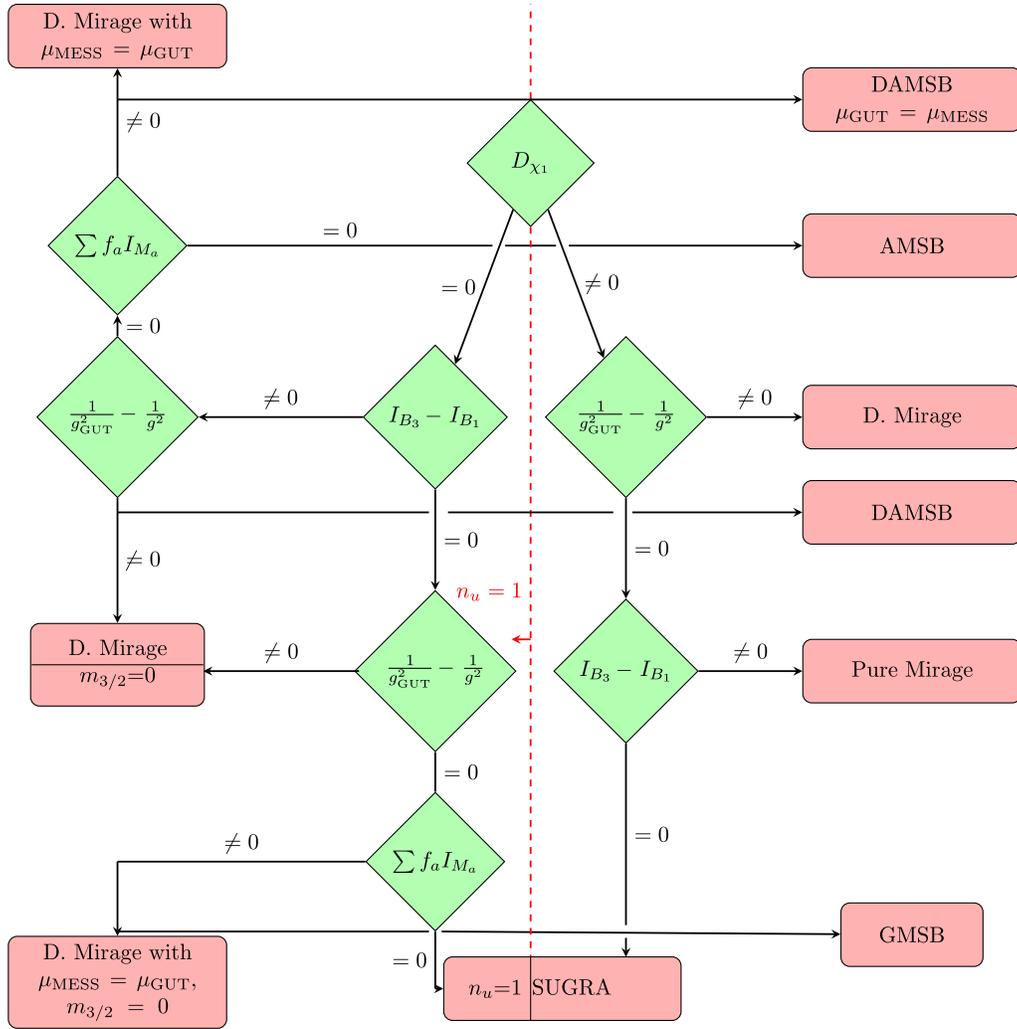


FIG. 6 (color online). Implications of measurement of RGIs for the supersymmetry breaking mechanism with the assumption of $n_h = n_{H_d} = n_{H_u}$ and $n_u = n_U = n_D = n_L = n_E = n_Q$, assuming that one or two of the mechanisms dominate. On the left side of the red dotted line all endpoints have $n_u = 1$. $\frac{1}{g^2} = -\frac{10D_{Y_a}}{13D_{Y_{13H}}}$ and $\sum f_a I_{M_a} = 81I_{M_2} - 56I_{M_3} - 25I_{M_1}$.

$$\text{Pure mirage: } N = 0, \quad (5.7)$$

$$\text{DAMSB: } M_0 = 0. \quad (5.8)$$

Here we assume universal modular weights for all applicable models and the parameters not specified to be nonzero. In the case that one of the mechanisms clearly

dominates the supersymmetry breaking scenario can then in principle be resolved or narrowed by measuring the parameters in terms of RGIs. By comparing to the solutions for the parameters in terms of invariants (3.13), (3.29), and (3.38), along with the sum rules (4.20)–(4.25), we can deduce from the measured invariants $I_{B_1}, I_{B_3}, I_{M_a}, D_{Y\alpha}$, and $D_{Y_{13H}}$ the following:

$$I_{B_1} - I_{B_3} \propto m_{3/2} \begin{cases} 0, & \text{for mSUGRA, GMSB} \\ >0, & \text{for AMSB, Mirage, DAMSB} \end{cases}, \quad (5.9)$$

$$D_{\chi_1} = (1 - n_u) M_0^2 \begin{cases} =0, & \text{for AMSB, GMSB} \\ \propto(1 - n_u), & \text{for mSUGRA, Pure Mirage, DAMSB} \end{cases}, \quad (5.10)$$

$$\frac{10D_{Y\alpha}}{13D_{Y_{13H}}} + \frac{1}{g_{\text{GUT}}^2} = \frac{1}{g_{\text{GUT}}^2} - \frac{1}{g^2} \begin{cases} =0 & \text{for mSUGRA, AMSB, Pure Mirage} \\ \propto t_{\text{MESS}} N, & \text{for GMSB, DAMSB} \end{cases}, \quad (5.11)$$

$$81I_{M_2} - 56I_{M_3} - 25I_{M_1} \begin{cases} =0, & \text{for AMSB, mSUGRA, Pure Mirage} \\ \neq 0, & \text{for Deflected Mirage, GMSB, DAMSB} \end{cases}. \quad (5.12)$$

Thus e.g. observing $D_{\chi_1} = 0$ would exclude mediation mechanism with a gravity mediated contribution with $n_u \neq 1$, but deflected or pure mirage with $n_u = 1$ cannot be ruled out. On the other hand a nonzero $\frac{10D_{Y\alpha}}{13D_{Y_{13H}}} + \frac{1}{g_{\text{GUT}}^2}$ implicates a gauge mediated contribution, with the messenger scale different from the GUT scale. We have illustrated the implications of different values of (5.12) in Fig. 6 by starting from the measurement of D_{χ_1} .

VI. CONCLUSIONS

Deflected mirage mediation is the most general type of mechanism for supersymmetry breaking in the sense that it includes contributions from three supersymmetry breaking mechanisms, namely gravity-, anomaly-, and gauge mediation. The renormalization group invariants provide a way of determining the values of the supersymmetry breaking parameters, but in the case of DMMSB, the emergence of gauge messenger fields at a scale possibly different from the GUT scale complicates their use by inducing corrections to the gaugino and the scalar masses and modifying the beta functions at this threshold. Thus the invariants have differing values above and below the messenger scale. In order to connect the TeV scale measurements of the particle masses to the GUT-scale parameters we have derived the threshold corrections to the RGIs and derived the RGIs for arbitrary b_a -coefficients of the beta functions. It is shown that the high scale parameters which include $N, \mu_{\text{MESS}}, m_{3/2}, M_0$, and α_g can be analytically solved in terms of the RGIs, and the explicit formulas are provided. We have then

examined various limits of DMMSB to see how any of the contributing three pure supersymmetry breaking scenarios can be detected by measuring the RGIs at the TeV scale.

We have also discussed how the solutions to the supersymmetry breaking parameters can be used to construct sum rules that would allow further testing of the theory and determine the modular weights for the scalar masses. The lightest Higgs mass measurement allows the restriction of the parameter space of DMMSB. We have discussed the implications of the measured Higgs mass of 125.7 GeV to the mass spectrum and the parameters of DMMSB.

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APPENDIX: PROOF OF $\Delta D_I = 0$

We show that the absence of $m_{3/2}M_0$ mix terms at the input scale for a linear combination of squares of scalar

masses implies the vanishing of threshold correction for the said combination. Let us assume that the linear combination $D_A = \sum_{i=1}^n \alpha_i m_{k(i)}^2$ has at an input scale $\mu_{UV} > \mu_{\text{MESS}}$ a boundary condition of the form

$$D_A(\mu_{UV}) = \gamma_A m_{3/2}^2 + \delta_A M_0^2, \quad (\text{A1})$$

i.e. does not contain terms dependent on $m_{3/2} M_0$. The boundary condition according to (2.5) is

$$D_A(\mu_{UV}) = M_0^2 \sum_{i=1}^n \alpha_i (1 - n_{k(i)}) - \frac{1}{16\pi^2} M_0 m_{3/2} \sum_{i=1}^n \alpha_i \theta_{k(i)} - \frac{1}{(16\pi^2)^2} m_{3/2}^2 \sum_{i=1}^n \alpha_i \dot{\gamma}_{k(i)}. \quad (\text{A2})$$

For the coefficient of $M_0 m_{3/2}$ -term to vanish, we require

$$\sum_{i=1}^n \alpha_i \theta_{k(i)} = 4 \sum_{a=1}^3 g_a^2(\mu_{UV}) \sum_{i=1}^n \alpha_i c_a(\Psi_{k(i)}) - \sum_{i,l,m=1}^n \alpha_i |y_{k(i)lm}|^2 (p - n_{k(i)} - n_l - n_m) = 0, \quad (\text{A3})$$

where we have used the explicit formula (2.8) for θ_i . The parametrization (A1) is to be valid for arbitrary values of Yukawa couplings, thus the two terms of (A3) must vanish

independently. Furthermore, in a general case we do not require coupling constants to unify at the boundary and indeed expect for (A3) to be satisfied for arbitrary values of g_a . Each addend of the first sum should therefore vanish separately. From the first term we deduce

$$\sum_{i=1}^n \alpha_i c_a(\Psi_{k(i)}) = 0, \quad \text{for } a = 1, 2, 3. \quad (\text{A4})$$

On the other hand the threshold correction for D_A reads

$$\begin{aligned} \Delta D_A &= \sum_{i=1}^n \alpha_i \Delta m_{k(i)}^2 \\ &= \frac{2M_0^2 N}{(16\pi^2)^2} \left[\alpha_m (1 + \alpha_g) \ln \frac{M_P}{m_{3/2}} \right]^2 \\ &\quad \times \sum_{a=1}^3 g_a^4(\mu_{\text{mess}}) \sum_{i=1}^n \alpha_i c_a(\Psi_{k(i)}), \end{aligned} \quad (\text{A5})$$

where we have used the formula (2.11) for scalar mass correction. From (A4) and (A5) it is obvious that the threshold correction vanishes. We conclude that

$$\Delta D_A = 0, \quad (\text{A6})$$

for any linear combination of squares of scalar masses that has a boundary condition of the form (A1).

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