Stealth dark matter: Dark scalar baryons through the Higgs portal

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We present a new model of stealth dark matter: a composite baryonic scalar of an $SU(N_D)$ strongly coupled theory with even $N_D \geq 4$. All mass scales are technically natural, and dark matter stability is automatic without imposing an additional discrete or global symmetry. Constituent fermions transform in vectorlike representations of the electroweak group that permit both electroweak-breaking and electroweak-preserving mass terms. This gives a tunable coupling of stealth dark matter to the Higgs boson independent of the dark matter mass itself. We specialize to $SU(4)$, and investigate the constraints on the model from dark meson decay, electroweak precision measurements, basic collider limits, and spin-independent direct detection scattering through Higgs exchange. We exploit our earlier lattice simulations that determined the composite spectrum as well as the effective Higgs coupling of stealth dark matter in order to place bounds from direct detection, excluding constituent fermions with dominantly electroweak-breaking masses. A lower bound on the dark baryon mass $m_B \gtrsim 300$ GeV is obtained from the indirect requirement that the lightest dark meson not be observable at LEP II. We briefly survey some intriguing properties of stealth dark matter that are worthy of future study, including collider studies of dark meson production and decay; indirect detection signals from annihilation; relic abundance estimates for both symmetric and asymmetric mechanisms; and direct detection through electromagnetic polarizability, a detailed study of which will appear in a companion paper.

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I. INTRODUCTION

Composite dark matter, made up of electroweak-charged constituents, provides a straightforward mechanism for obtaining viable electrically-neutral particle dark matter that can yield the correct cosmological abundance while surviving direct and indirect detection search limits, e.g., [1–3]. In this paradigm, the dark sector consists of fermions that transform under the electroweak group and a new, strongly coupled non-Abelian dark force. This was considered long ago in the context of technicolor theories, where the strong dynamics was doing double duty to both break electroweak symmetry and provide a dark matter candidate [4–7].

In this paper, electroweak symmetry breaking is accomplished through the weakly coupled Standard Model Higgs mechanism, while the new strongly coupled sector is reserved solely for providing a viable dark matter candidate. This dark sector is not easy to detect in dark matter detection experiments or in collider experiments, and so we give it the name "stealth dark matter." Earlier work in this direction includes [4–28], and except for [29–36] was often limited by the inability to perturbatively calculate the spectrum and form factors due to strong coupling.

The proposed dark matter candidate is a scalar baryon of $SU(N_D)$ and, hence, N_D must be even.[1] We take the dark

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[1]Fermionic baryons arising from odd N_D were considered in Ref. [31] where the limit $M \gtrsim 10$ TeV was found to avoid the direct detection constraints from the magnetic dipole interaction.

fermions to be in vectorlike representations of the electroweak group. Hence, the constituent dark fermions can acquire bare mass terms (fermion masses that do not require electroweak symmetry breaking) while also permitting Yukawa interactions that marry dark fermion electroweak doublets with singlets. This yields a theory in which dark matter couples to the Higgs boson in a tunable way that is essentially independent of the dark matter mass itself. This is somewhat analogous to dark sector models with a dark U(1) portal (e.g., [37–41]), where the coupling to the Standard Model is tunable through an otherwise arbitrary parameter—the kinetic mixing between the dark U(1) and hypercharge.

The existence of both electroweak-preserving and electroweak-breaking masses for the dark fermions provides two main benefits. First, given that the Higgs boson couples electroweak doublets with singlets, the global flavor symmetries of the dark fermions can be completely broken to just dark baryon number. All mesons can decay through an electroweak process (e.g., electrically charged mesons through W exchange) or through the usual chiral anomaly (e.g., the lightest neutral meson). Ensuring that these particles decay before big bang nucleosynthesis sets a weak lower bound on the Higgs interaction strength. (This is in contrast with [20,42] where additional interactions were required to ensure mesons decay, e.g., through higher-dimensional operators). The second reason is related to the orientation of the chiral condensate after the dark force confines. Large vectorlike masses for the dark fermions ensure that the condensate can be aligned toward the electroweak-preserving direction, and thus the dark sector leads to only small corrections to electroweak precision measurements. We estimate the size of these corrections in this paper.

There are many appealing features of an electroweak-neutral composite dark matter candidate made up from fermions transforming under the electroweak group, including the following:

(i) All of the dimensionful scales are technically natural, since they arise from fermion masses (vectorlike and electroweak breaking) and the confinement of a strong-coupled dark force.

(ii) Dark matter stability is an automatic consequence of dark baryon number conservation. No additional global discrete or continuous symmetries are required. For $N_D \geq 3$, operators involving dark baryon decay are necessarily dimension six or higher, and thus safe from GUT-scale or Planck-scale suppressed violations of dark baryon number.

(iii) There are no dimension-four interactions of the composite dark matter particle with the Standard Model except with the Higgs boson. The direct detection scattering cross section is thus automatically suppressed compared with a generic elementary WIMP candidate.

(iv) Higher-dimensional interactions of the dark matter with the Standard Model are suppressed, in the nonrelativistic limit, by several powers of the dark matter mass. For a composite scalar, the leading operators are charge radius (at dimension six) and polarizability (at dimension seven). The impact of these (and other) operators on the dark matter scattering cross section in direct detection experiments has been studied in [9–11,16,43–50].

(v) Interactions of the dark baryon through the neutral weak current, the charge radius interaction, as well as the contributions to the electroweak precision T parameter, are simultaneously eliminated if the fermion interactions obey a global custodial $SU(2)$ symmetry. Additionally, as we will see, dark matter electric neutrality also follows from custodial $SU(2)$. To simplify our analysis, here we will primarily study the subset of stealth dark matter parameter space in which the custodial $SU(2)$ is preserved. (This simplification is very familiar from composite Higgs theories, e.g. [51]).

(vi) The abundance of a strongly coupled dark scalar baryon could arise through several mechanisms: an asymmetric abundance (such as through electroweak sphalerons [6,8] or other mechanisms [52]), when the mass is not too large $\lesssim$ few TeV, or a symmetric abundance, when the mass is large (perhaps $\sim\mathcal{O}(100)$ TeV) [20,53,54].

We focus mainly on a confining $SU(4)$ gauge theory dark sector with dark fermions transforming non-trivially under the electroweak group. We apply our recent results [33] using lattice simulations for the spectrum and effective Higgs interaction for $SU(4)$. As emphasized in [31,33], this theory is well suited for lattice calculations since we are not interested in the chiral limit of vanishing dark fermion masses. Indeed, lattice simulations can efficiently simulate the parameter region where the dark fermion masses are comparable to the confinement scale, exactly where the perturbative estimates are least useful.

The organization of the paper is as follows. In Sec. II we discuss the assumptions and requirements to construct our stealth dark matter model. In Sec. III we detail the dark fermion interactions and masses. In addition, we write the electroweak currents in terms of the dark fermion mass eigenstates of the theory, detailed in the Appendix. Until this point, the discussion of the model is general. In Sec. IV, we simplify the parameter space for phenomenological and calculational purposes, applying a global custodial $SU(2)$ symmetry and taking the approximately symmetric dark fermion mass matrix limit. Then in Sec. V we discuss the light nonsinglet mesons in the theory, in particular their decay rates and constraints from nonobservation at LEP II. In Sec. VI we discuss the stealth dark matter contributions to the S parameter, and demonstrate the parametric suppression that happens in several regimes. In Sec. VII we obtain the Higgs boson coupling to the dark fermions. Then in Sec. VIII we apply our previous model-independent results on the $SU(4)$ spectrum and effective Higgs coupling to stealth dark matter. We obtain the bounds on the parameter space from the nonobservation of a spin-independent direct detection signal at LUX. We briefly discuss the relic abundance of stealth dark matter in Sec. IX. Finally we conclude with a discussion in Sec. X.

II. CONSTRUCTING A VIABLE MODEL

A. Basic assumptions

We assume that the dark matter candidate is a composite particle of a non-Abelian, confining gauge theory based on the group $SU(N_D)$ with N_f flavors of fermions transforming in the fundamental representation. The number N_f is restricted by only the condition of confinement. For reasons outlined in the introduction (abundance, detectability), the dark fermions carry electroweak charges. Our model includes a tunable Higgs "portal" coupling between the dark sector and the Standard Model via dimension-four Higgs couplings.[2] We do not consider QCD-colored dark fermions since with $N_D \neq 3$, dark baryons would not generally be color singlets.[3]

B. Requirements

We require dark matter stability to be automatic, arising from a global symmetry. This motivates considering the dark baryon of the non-Abelian dark sector to be the dark matter [4–6]. In the presence of GUT-scale or Planck-scale suppressed operators, the stability of the dark baryon should be sufficient to avoid cosmological constraints.

The requirement of a sufficiently preserved accidental baryon number disfavors a dark $SU(2)$ group. First, there is no automatic baryon number in $SU(2)$ because there is no fundamental distinction between mesons and baryons. Imposing a global $U(1)$ baryon number is possible (e.g. see [16]) but in addition baryon number violating dimension-five Planck-suppressed operators such as $f_{\mathrm{dark}} f_{\mathrm{dark}} H^\dagger H/M_{\mathrm{Pl}}$ must be absent, where f_{dark} is the dark fermion. (Otherwise, the dark $SU(2)$ baryon would decay on a timescale much shorter than the age of the Universe.)

For $N_D \geq 3$, operators involving dark baryon decay are necessarily dimension six or higher and, thus, safe from GUT-scale or Planck-scale suppressed violations of dark baryon number. $SU(N_D)$ with odd N_D is a perfectly interesting theory, having been studied before for $N_D = 3$ by our collaboration [31]. There it was found that a fermionic dark baryon has a magnetic dipole interaction that leads to a significant contribution to spin-independent scattering. Constraints from the XENON100 experiment were satisfied only when the dark matter mass $M \gtrsim 10$ TeV [31]. This strong constraint on the mass scale implies the model is difficult to test at near-future colliders.

The magnetic dipole interaction (and other higher-dimensional operators that require spin) are absent when the dark baryon is a scalar. We are thus naturally led to $SU(N_D)$ with even $N_D \geq 4$, for which the otherwise strong constraints from direct detection are weakened, lowering the scales of interest into a regime that can be probed by colliders and other detection strategies.

We assume the dark fermions have masses M_f on the order of the $SU(N_D)$ confinement scale Λ_D. If the masses were much smaller, the dark sector would contain light pseudo-Goldstone pions that transform under the electroweak group, which are strongly constrained by collider experiments. A dark sector with purely vectorlike fermion masses has approximately stable electrically charged mesons due to dark flavor symmetries. Conversely, a dark sector with purely electroweak breaking fermion masses has a dark matter candidate that is ruled out by spin-independent direct detection through single Higgs exchange. (For example, quirky dark matter [16] is now completely ruled out by Higgs exchange, given the direct detection bounds from LUX [3] combined with the relatively light Higgs mass [56,57].) Fermions with both vectorlike and (small) electroweak breaking contributions to their masses can avoid both problems.

We require the lightest dark baryon to be electrically neutral. We also require Higgs couplings at dimension four to pairs of dark fermions. These two requirements impose restrictions on the electroweak charges of the dark fermions.

One solution is familiar from old technicolor theories (e.g. [58,59]): requiring the dark fermion charges to roughly satisfy $|Y| \lesssim |T_3|$ where T_3 is the $SU(2)_L$ isospin.

TABLE I. Dark fermion particle content of the stealth dark matter model. All fields are two-component (Weyl) spinors. $SU(2)_L$ refers to the Standard Model electroweak gauge group, and Y is the hypercharge. In the broken phase of the electroweak theory, the dark fermions have the electric charge $Q = T_3 + Y$ as shown.

Field	$SU(N_D)$	$(SU(2)_L, Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	$\mathbf{N}$	$(\mathbf{2}, 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\bar{\mathbf{N}}$	$(\mathbf{2}, 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
F_3^u	$\mathbf{N}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_3^d	$\mathbf{N}$	$(\mathbf{1}, -1/2)$	$-1/2$
F_4^u	$\bar{\mathbf{N}}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_4^d	$\bar{\mathbf{N}}$	$(\mathbf{1}, -1/2)$	$-1/2$

[2]Other portals, such as a dark gauged U(1) group that kinetically mixes with hypercharge, are neither present nor required here.

[3]The obvious exception, when $N_D = N_c = 3$, is discussed in [31,55], which is not a focus for us due to the baryons being fermions. Construction of QCD-singlet dark baryons with $N_D = 6, 12, 18, \ldots$ may be possible, but we do not study this possibility further here.

Choosing doublets ($|T_3| = 1/2$ under $SU(2)_L$) then gives a finite number of discrete possibilities.

A simple model that satisfies all of these requirements is shown in Table I. The electric charges of the dark fermions in the broken electroweak phase are $Q = \pm 1/2$, ensuring all hadrons have integer electric charges. So long as the lightest $Q = 1/2$ and $Q = -1/2$ dark fermions are close in mass, the lightest baryon will be a scalar and electrically neutral. Finally, with the assignments shown in Table I, all gauge (and global) anomalies vanish, which is automatic with fermions that transform under vectorlike representations of the $SU(N_D)$ and electroweak groups.

III. DARK FERMION INTERACTIONS AND MASSES

The fermions $F_i^{u,d}$ transform under a global $U(4) \times U(4)$ flavor symmetry with $[SU(2) \times U(1)]^4$ surviving after the weak gauging of the electroweak symmetry. From this large global symmetry, one $SU(2)$ (diagonal) subgroup will be identified with $SU(2)_L$, one $U(1)$ subgroup will be identified with $U(1)_Y$, and one $U(1)$ will be identified with dark baryon number. The total fermionic content of the model is, therefore, eight Weyl fermions that pair up to become four Dirac fermions in the fundamental or antifundamental representation of $SU(N_D)$ with electric charges of $Q \equiv T_{3,L} + Y = \pm 1/2$. We use the notation where the superscript u or d (as in F^u, F^d and later ψ^u, ψ^d, Ψ^u, Ψ^d) denotes a fermion with electric charge of $Q = 1/2$ or $Q = -1/2$, respectively.

The fermion kinetic terms in the Lagrangian are given by

$$\mathcal{L} \supset \sum_{i=1,2} i F_i^\dagger \bar{\sigma}^\mu D_{i,\mu} F_i + \sum_{i=3,4;j=u,d} i F_i^{j\dagger} \bar{\sigma}^\mu D_{i,\mu}^j F_i^j, \tag{1}$$

where the covariant derivatives are

$$D_{1,\mu} \equiv \partial_\mu - igW_\mu^a \sigma^a/2 - ig_D G_\mu^b t^b \tag{2}$$

$$D_{2,\mu} \equiv \partial_\mu - igW_\mu^a \sigma^a/2 + ig_D G_\mu^b t^{b*} \tag{3}$$

$$D_{3,\mu}^j \equiv \partial_\mu - ig' Y^j B_\mu - ig_D G_\mu^b t^b \tag{4}$$

$$D_{4,\mu}^j \equiv \partial_\mu - ig' Y^j B_\mu + ig_D G_\mu^b t^{b*} \tag{5}$$

with the interactions among the electroweak group and the new $SU(N_D)$. Here $Y^u = 1/2$, $Y^d = -1/2$ and t^b are the representation matrices for the fundamental of $SU(N_D)$.

The vectorlike mass terms allowed by the gauge symmetries are

$$\mathcal{L} \supset M_{12} \epsilon_{ij} F_1^i F_2^j - M_{34}^u F_3^u F_4^d + M_{34}^d F_3^d F_4^u + \text{H.c.}, \tag{6}$$

where $\epsilon_{12} \equiv \epsilon_{ud} = -1 = -\epsilon^{12}$ and the relative minus signs between the mass terms have been chosen for later convenience. The mass term M_{12} explicitly breaks an $[SU(2) \times U(1)]^2$ global symmetry down to the diagonal $SU(2)_{\text{diag}} \times U(1)$ where the $SU(2)_{\text{diag}}$ is identified with $SU(2)_L$. The mass terms $M_{34}^{u,d}$ explicitly break the remaining $[SU(2) \times U(1)]^2$ down to $U(1) \times U(1)$ where one of the $U(1)$'s is identified with $U(1)_Y$. (In the special case when $M_{34}^u = M_{34}^d$, the global symmetry is enhanced to $SU(2) \times U(1)$, where the global $SU(2)$ acts as a custodial symmetry.) Thus, after weakly gauging the electroweak symmetry and writing arbitrary vectorlike mass terms, the unbroken flavor symmetry is $U(1) \times U(1)$.

Electroweak symmetry breaking mass terms arise from coupling to the Higgs field H that we take to be in the $(\mathbf{2}, +1/2)$ representation. They are given by

$$\begin{aligned}\mathcal{L} \supset\ & y_{14}^u \epsilon_{ij} F_1^i H^j F_4^d + y_{14}^d F_1 \cdot H^\dagger F_4^u \\ & - y_{23}^d \epsilon_{ij} F_2^i H^j F_3^d - y_{23}^u F_2 \cdot H^\dagger F_3^u + \text{H.c.},\end{aligned} \tag{7}$$

where again the relative minus signs are chosen for later convenience. After electroweak symmetry breaking, $H = (0 \quad v/\sqrt{2})^T$, with $v \simeq 246$ GeV. Replacing the Higgs field by its VEV in Eq. (7), we obtain mass terms for the fermions, in 2-component notation,

$$\begin{aligned}\mathcal{L} \supset & -(F_1^u \quad F_3^u) M^u \begin{pmatrix} F_2^d \\ F_4^d \end{pmatrix} - (F_1^d \quad F_3^d) M^d \begin{pmatrix} F_2^u \\ F_4^u \end{pmatrix} \\ & + \text{H.c.},\end{aligned} \tag{8}$$

with the mass matrices given by

$$M^u \equiv \begin{pmatrix} M_{12} & y_{14}^u v/\sqrt{2} \\ y_{23}^u v/\sqrt{2} & M_{34}^u \end{pmatrix} \tag{9}$$

$$M^d \equiv -\begin{pmatrix} M_{12} & y_{14}^d v/\sqrt{2} \\ y_{23}^d v/\sqrt{2} & M_{34}^d \end{pmatrix}. \tag{10}$$

These Yukawa couplings break the remaining $U(1) \times U(1)$ flavor symmetry to $U(1)_D$ dark baryon number. The mass matrices M^u and M^d correspond to the masses of two sets of fermions with electric charge $Q = +1/2$ and $Q = -1/2$, respectively, in the fundamental representation of $SU(N_D)$. The two biunitary mass matrices can be diagonalized by four independent rotation angles,

$$\begin{pmatrix} M_1^u & 0 \\ 0 & M_2^u \end{pmatrix} = R(\theta_1^u)^{-1} M^u R(\theta_2^u) \tag{11}$$

$$\begin{pmatrix} M_1^d & 0 \\ 0 & M_2^d \end{pmatrix} = R(\theta_1^d)^{-1} M^d R(\theta_2^d), \tag{12}$$

where the rotation matrices are defined by

$$R(\theta_i^j) \equiv \begin{pmatrix} \cos\theta_i^j & -\sin\theta_i^j \\ \sin\theta_i^j & \cos\theta_i^j \end{pmatrix}. \tag{13}$$

The 2-component mass eigenstate spinors are, thus,

$$\begin{pmatrix} \psi_1^u \\ \psi_2^u \end{pmatrix} = R(\theta_1^u) \begin{pmatrix} F_1^u \\ F_3^u \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} \psi_1^d \\ \psi_2^d \end{pmatrix} = R(\theta_2^u) \begin{pmatrix} F_2^d \\ F_4^d \end{pmatrix} \tag{15}$$

$$\begin{pmatrix} \chi_1^d \\ \chi_2^d \end{pmatrix} = iR(\theta_1^d) \begin{pmatrix} F_1^d \\ F_3^d \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} \chi_1^u \\ \chi_2^u \end{pmatrix} = iR(\theta_2^d) \begin{pmatrix} F_2^u \\ F_4^u \end{pmatrix}, \tag{17}$$

where the extra phase in Eqs. (16) and (17) ensures the $Q = -1/2$ fermions will have positive mass eigenvalues.

The Lagrangian for the fermion mass eigenstates becomes

$$\mathcal{L} \supset -\sum_{i=1}^{2} (M_i^u \psi_i^u \psi_i^d + M_i^d \chi_i^d \chi_i^u + \text{H.c.}) \tag{18}$$

where the mass eigenvalues are $M_{1,2}^u$ for $Q = 1/2$, and the distinction between fermions ψ and χ allows us to write the $Q = -1/2$ fermion masses as $M_{1,2}^d$. The Dirac spinor mass eigenstates are constructed from the 2-component Weyl spinor mass eigenstates in the usual way,

$$\Psi_i^u \equiv \begin{pmatrix} \psi_i^u \\ \psi_i^{d\dagger} \end{pmatrix} \qquad i = 1,2 \tag{19}$$

$$\Psi_i^d \equiv \begin{pmatrix} \chi_i^d \\ \chi_i^{u\dagger} \end{pmatrix} \qquad i = 1,2 \tag{20}$$

giving the Dirac fermion masses

$$\mathcal{L} \supset -\sum_{i=1}^{2} (M_i^u \overline{\Psi}_i^u \Psi_i^u + M_i^d \overline{\Psi}_i^d \Psi_i^d). \tag{21}$$

The fermion masses themselves are obtained from a straightforward diagonalization of the mass matrices,

$$M_{1,2}^u = \frac{M_{12} + M_{34}^u}{2} \mp \left[\left(\frac{M_{12} - M_{34}^u}{2} \right)^2 + \frac{y_{14}^u y_{23}^u v^2}{2} \right]^{1/2}, \tag{22}$$

with mixing angles

$$\tan 2\theta_1^u = \frac{2\sqrt{2}v(M_{12} y_{23}^u + M_{34}^u y_{14}^u)}{2M_{12}^2 - 2(M_{34}^u)^2 + (y_{14}^u v)^2 - (y_{23}^u v)^2} \tag{23}$$

$$\tan 2\theta_2^u = \frac{2\sqrt{2}v(M_{12} y_{14}^u + M_{34}^u y_{23}^u)}{2M_{12}^2 - 2(M_{34}^u)^2 - (y_{14}^u v)^2 + (y_{23}^u v)^2}, \tag{24}$$

with identical expressions for $M_{1,2}^d$ and $\tan 2\theta_{1,2}^d$ with the replacement $u \leftrightarrow d$ everywhere.

It is important to note that the electroweak currents $(j_+^\mu,\ j_-^\mu,\ j_3^\mu,\ j_Y^\mu)$ play an important role in the upcoming phenomenological discussions. Due to the extended expressions for these quantities in terms of our Dirac spinors, we have relegated a detailed derivation of the electroweak currents to the Appendix.

IV. SIMPLIFICATIONS

Our main interest is the more specialized case where the lightest $Q = +1/2$ and $Q = -1/2$ fermions are degenerate in mass to a very good approximation. This leads to a neutral scalar baryon with a vanishing charge radius. While there are several ways this could be accomplished, we can simply impose a custodial $SU(2)$ global symmetry on the Lagrangian. In order to simplify notation, we define $c_i^j \equiv \cos\theta_i^j$, $s_i^j \equiv \sin\theta_i^j$ and $P_{L,R} = (1 \mp \gamma_5)/2$. In the custodial $SU(2)$ symmetric theory, $c_i^u = c_i^d$ and $s_i^u = s_i^d$.

A. Custodial SU(2)

An exact custodial $SU(2)$ symmetry implies the masses and interactions are symmetric with respect to the interchange $u \leftrightarrow d$. This means the Lagrangian parameters satisfy

$$y_{14}^u = y_{14}^d \equiv y_{14}, \qquad y_{23}^u = y_{23}^d \equiv y_{23},$$
$$M_{34}^u = M_{34}^d \equiv M_{34}. \tag{25}$$

Defining the overall vectorlike mass scale M and difference Δ to be

$$M \equiv \frac{M_{12} + M_{34}}{2} \qquad \Delta \equiv \left| \frac{M_{12} - M_{34}}{2} \right|, \tag{26}$$

the dark fermion mass eigenvalues are

$$M_{1,2} = M \mp \sqrt{\Delta^2 + \frac{y_{14} y_{23} v^2}{2}}. \tag{27}$$

We assume $\Delta < M$, such that fermion masses remain positive, to avoid further fermion field rephasings. No u or d labels are necessary since custodial $SU(2)$ symmetry implies that there is one pair of Dirac fermions with electric charge $Q = (+1/2, -1/2)$ with mass M_1 (the lightest pair), as well as a second pair of Dirac fermions with electric

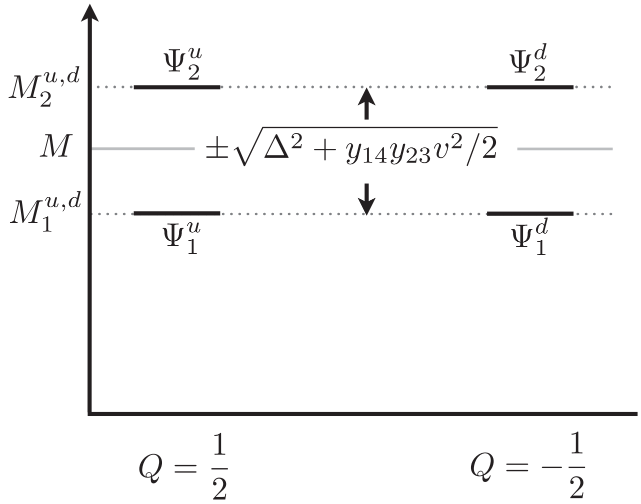

FIG. 1. Illustration of the fermion mass spectra considered in the paper. Four Dirac fermions (Ψ_1^u, Ψ_1^d, Ψ_2^u, Ψ_2^d) have masses (M_1^u, M_1^d, M_2^u, M_2^d). The u (d) fermions have electric charge $Q = +1/2$ ($Q = -1/2$); we assume an exact custodial $SU(2)$ global symmetry that ensures each $Q = +1/2$ fermion is accompanied by a $Q = -1/2$ fermion with equal mass as shown in the figure. If $\Delta \ll \sqrt{y_{14}y_{23}}v$ ($\Delta \gg \sqrt{y_{14}y_{23}}v$) the mass splitting is dominated by electroweak breaking (preserving) masses that we call the "linear (quadratic) case." See the text for details.

charge $Q = (+1/2, -1/2)$ with mass M_2 (the heavier pair). The spectrum is illustrated in Fig. 1.

In the limit $y_{14}, y_{23} \to 0$, the fermions acquire purely vectorlike masses, and thus the chiral condensate of the dark force is aligned to a purely electroweak-preserving direction. In order that the chiral condensate's electroweak-preserving orientation is not significantly disrupted, we consider small electroweak breaking masses, $y_{14}v, y_{23}v \ll M$.

This leaves two distinct regimes for the spectrum, depending on the relative sizes of $\sqrt{y_{14}y_{23}}v$ and Δ.

B. Approximately symmetric mass matrices

A second simplification, useful to analytically and numerically evaluate our results, is to take $y_{14} \simeq y_{23}$. The mass matrices Eqs. (9) and (10) are approximately symmetric. Specifically, we can write

$$y_{14} = y + \epsilon_y, \qquad y_{23} = y - \epsilon_y, \qquad |\epsilon_y| \ll |y|. \tag{28}$$

and expand in powers of ϵ_y. For example, the dark fermion masses become simply

$$M_{1,2} = M \mp \sqrt{\Delta^2 + \frac{y^2v^2}{2}}. \tag{29}$$

to leading order in $O(\epsilon_y)$.

The distinct regimes are thus $yv \gg \Delta$ and $yv \ll \Delta$. In the linear case $yv \gg \Delta$, electroweak symmetry breaking is (dominantly) responsible for the mass *splitting* between $\Psi_1^{u,d}$ and $\Psi_2^{u,d}$. In the quadratic case $yv \ll \Delta$, the splitting is dominantly attributed to the vectorlike mass splitting Δ. As we shall see, the primary distinction between these two cases is in the Higgs coupling to the fermion mass eigenstates: proportional to y for the linear case and y^2 for the quadratic case, hence, the case names. A similar observation was also found in Ref. [60].

From this point forward unless noted otherwise, we assume the fermion mass parameters satisfy an exact custodial $SU(2)$ and the mass matrices are approximately symmetric.

V. LIGHT NONSINGLET MESON PHENOMENOLOGY

Theories with new fermions that transform under vectorlike representations of the electroweak group generically have enlarged global flavor symmetries that can prevent decay of the lightest nonsinglet mesons and baryons. In the case of dark baryons, this is a feature, providing the rationale for the stability of the lightest dark baryon of the theory.

In the case of the lightest nonsinglet mesons, this can be problematic, since some of these mesons carry electric charge.[4] Stable integer charged mesons are strongly constrained from collider searches as well as cosmology. One solution is to postulate additional higher-dimensional operators that connect a dark fermion pair with a Standard Model fermion pair [20,42]. This must be carefully done to avoid also writing operators that violate the approximate global symmetries protecting the stability of the dark matter. In the stealth dark matter model, however, electroweak symmetry breaking can provide the source of global flavor symmetry breaking, leading to the decay of the lightest charged mesons. (We will not discuss the lightest neutral mesons, but they are generically more difficult to produce in colliders, and they will decay through essentially the same mechanism as we describe for the charged mesons.)

The lightest electrically charged mesons are composed dominantly of the dark fermion pairs $\Pi^+ = (\overline{\Psi_1^d}\Psi_1^u)$ and $\Pi^- = (\overline{\Psi_1^u}\Psi_1^d)$. We can estimate the lightest meson lifetime by generalizing pion decay of QCD to our model. The relevant matrix element is (see, e.g., [61])

$$\langle 0|j^\mu_{\pm,\mathrm{axial}}|\Pi^\pm\rangle = if_\Pi p^\mu, \tag{30}$$

where f_Π is the "pion decay constant" associated with the dark force in this paper. The axial part of the electroweak current can be read off from the electroweak currents given in Eqs. (A5) and (A6)

[4]We use the term "lightest mesons" and not "pions" since the would-be global symmetry that protects pion masses is completely broken by the dark fermion vectorlike masses. Nevertheless, we use the symbol Π to denote the corresponding fields.

$$j^{\mu}_{+,\mathrm{axial}} \supset c_{\mathrm{axial}} \bar{\Psi}^u_1 \gamma^\mu \gamma_5 \Psi^d_1, \tag{31}$$

where

$$c_{\mathrm{axial}} = \frac{c^u_1 c^d_1 - c^u_2 c^d_2}{\sqrt{2}} \tag{32}$$

and $j^{\mu}_{-,\mathrm{axial}}$ is identical upon $u \leftrightarrow d$. In the custodial limit, Eq. (25), the axial coefficient is

$$c_{\mathrm{axial}} = \frac{(y_{14}^2 - y_{23}^2) v^2}{\sqrt{2(8M^2 + (y_{14} - y_{23})^2 v^2)(8\Delta^2 + (y_{14} + y_{23})^2 v^2)}}. \tag{33}$$

Some insight can be gained using approximately symmetric mass matrices, Eq. (28). We then obtain

$$\begin{aligned} c_{\mathrm{axial}} &= \frac{\epsilon_y y v^2}{2M\sqrt{2\Delta^2 + y^2 v^2}} \\ &\simeq \frac{\epsilon_y v}{2M} \times \begin{cases} 1 & \text{linear case} \\ yv/(\sqrt{2}\Delta) & \text{quadratic case.} \end{cases} \end{aligned} \tag{34}$$

The decay width can be obtained from the pion decay of QCD by replacing V_{ud} in the Standard Model with c_{axial} for the dark mesons. Since the charged dark mesons of this model are much heavier than the QCD pions, there are many possible decay modes. For a general decay into a Standard Model doublet (ff'), assuming $m_f \gg m_{f'}$, the decay width is

$$\Gamma(\Pi^+ \to f\bar{f}') = \frac{G_F^2}{4\pi} f_\Pi^2 m_f^2 m_\Pi c^2_{\mathrm{axial}} \left(1 - \frac{m_f^2}{m_\Pi^2}\right). \tag{35}$$

If $m_\Pi > m_t + m_b$, the dominant decay mode is expected to be $\Pi^+ \to t\bar{b}$, otherwise $\Pi^+ \to \tau^+ \nu_\tau$ and $\Pi^+ \to \bar{s}c$, with branching ratios of roughly 70% and 30%, respectively. Note that the decay width has several enhancement factors relative to the QCD pion decay width

$$\frac{\Gamma(\Pi^+ \to f\bar{f}')}{\Gamma(\pi \to \mu^+ \nu_\mu)} \simeq \frac{c^2_{\mathrm{axial}}}{|V_{ud}|^2} \left(\frac{f_\Pi}{f_\pi}\right)^2 \left(\frac{m_f}{m_\mu}\right)^2 \left(\frac{m_\Pi}{m_\pi}\right), \tag{36}$$

where for simplicity we have neglected kinematic suppression. As an example, if $f_\Pi \simeq m_\Pi \simeq v$, we find the lightest charged dark mesons decay faster than QCD charged pions so long as $c_{\mathrm{axial}} \gtrsim 10^{-8}$. This is easy to satisfy with small Yukawa couplings and dark fermion masses at or beyond the electroweak scale.

We can now make some comments about existing collider constraints on nonsinglet mesons. The lightest charged mesons $\Pi^\pm$ can be pair produced in particle colliders through the Drell-Yan process, and will decay through annihilation of the constituent fermions into a W boson. Because the Drell-Yan production is mediated by a photon and the mesons have unit electric charge, the production cross-section is substantial, leading to robust bounds from LEP-II. For charged states near the LEP-II energy threshold, the dominant decay mode is expected to be $\Pi^+ \to \tau^+ \nu_\tau$ as noted above. Reinterpreting the LEP-II bound from the pair production of supersymmetric partners to the tau (with the stau decaying into a tau and a nearly massless gravitino), we find $m_\Pi \gtrsim 86.6$ GeV [62–66]. Stronger bounds from the LHC may be possible, although existing searches do not yet give any significant constraints on the charged mesons [20]; we briefly highlight the signals in the discussion.

Using our lattice results from Ref. [33], we can translate the experimental bound on the mass of the pseudoscalar meson into a bound on the baryon mass, $m_B > 245, 265, 320$ GeV when the ratio of the pseudoscalar mass to the vector meson mass is $m_\Pi / m_V = 0.77, 0.70, 0.55$.

VI. CONTRIBUTIONS TO ELECTROWEAK PRECISION OBSERVABLES

Stealth dark matter contains dark fermions that acquire electroweak symmetry breaking contributions to their masses. Consequently, there are contributions to the electroweak precision observables of the Standard Model, generally characterized by S and T [67,68]. In the custodial $SU(2)$ limit, Eq. (25), the contribution to T vanishes. There is a contribution to S, controllable through the relative size of the electroweak breaking and electroweak preserving masses of the dark fermions.

The S parameter is defined in terms of momentum derivatives of current-current correlators [67,68],

$$\begin{aligned} S &\equiv 16\pi \Pi'_{3Y}(0) \\ &= \frac{d}{dq^2}\left[\frac{16\pi}{3}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) X^{\mu\nu}(q^2)\right]_{q^2=0} \end{aligned} \tag{37}$$

$$X^{\mu\nu}(q^2) \equiv \int d^4x e^{-iqx} \langle j^\mu_3(x) j^\nu_Y(0)\rangle, \tag{38}$$

where the currents $j^\mu_3(x)$ and $j^\nu_Y(x)$ for the stealth dark matter model are defined in Eqs. (A7) and (A8). After some algebra and identifications of symmetric contractions, these definitions of the currents in terms of 4-component fermion fields lead to the current-current correlator. In the custodial limit, we obtain

$$\begin{aligned} 2\langle j^\mu_3(x) j^\nu_Y(0)\rangle = {} & c_1^2 s_1^2 ({}^{11}G^{\mu\nu}_{LL} + {}^{22}G^{\mu\nu}_{LL} - {}^{12}G^{\mu\nu}_{LL} - {}^{21}G^{\mu\nu}_{LL}) \\ & + c_2^2 s_2^2 ({}^{11}G^{\mu\nu}_{RR} + {}^{22}G^{\mu\nu}_{RR} - {}^{12}G^{\mu\nu}_{RR} - {}^{21}G^{\mu\nu}_{RR}) \\ & + c_1^2 s_2^2 ({}^{11}G^{\mu\nu}_{LR} + {}^{22}G^{\mu\nu}_{RL}) \\ & + c_2^2 s_1^2 ({}^{11}G^{\mu\nu}_{RL} + {}^{22}G^{\mu\nu}_{LR}) \\ & - c_1 c_2 s_1 s_2 ({}^{12}G^{\mu\nu}_{LR} + {}^{12}G^{\mu\nu}_{RL} + {}^{21}G^{\mu\nu}_{LR} + {}^{21}G^{\mu\nu}_{RL}), \end{aligned} \tag{39}$$

where the connected contributions to the correlation functions are given by

$$ {}^{ij}G^{\mu\nu}_{AB} \equiv \langle \bar{\Psi}^u_i \gamma^\mu P_A \Psi^u_j \bar{\Psi}^u_j \gamma^\nu P_B \Psi^u_i \rangle|_{\mathrm{connected}}. \tag{40} $$

Here, $A, B = L, R$ and the flavor indices $i, j = 1, 2$, where it is understood that the flavors labeled 2 have larger fermion masses than the flavors labeled 1. Since the u, d flavors have the same mass, the u and d labels are interchangeable (i.e. everything is written in terms of the u flavors).

We can obtain expressions for the mixing angle coefficients. Like the case of light meson decay, if we consider an approximately symmetric mass matrix, with Yukawa couplings given by Eq. (28), all of the mixing angle coefficients are approximately equal to each other, differing only at first order in ϵ_y, i.e.,

$$ \begin{aligned} c_1^2 s_1^2 &\simeq c_2^2 s_2^2 \simeq c_1^2 s_2^2 \simeq c_2^2 s_1^2 \simeq c_1 c_2 s_1 s_2 \\ &= \frac{1}{4} \frac{y^2 v^2}{y^2 v^2 + 2\Delta^2} [1 + O(\epsilon_y) \ldots] \\ &\simeq \frac{1}{4} \times \begin{cases} 1 & \text{linear case} \\ y^2 v^2 / (2\Delta^2) & \text{quadratic case.} \end{cases} \end{aligned} \tag{41} $$

In the linear case, the mixing angles are approximately equal $c_1 \simeq s_1 \simeq c_2 \simeq s_2 \simeq 1/\sqrt{2}$. In the quadratic case, all of the contributions to the S parameter are suppressed by $(yv/\Delta)^2$. To calculate the S parameter in general requires lattice methods, paying close attention to the heavy-light splitting of the fermions, $M_2 - M_1$. To a first approximation we expect that in the limit of small mass splitting, $M_2 - M_1 \ll M$,

$$ G^{\mu\nu}_{AB} \equiv {}^{11}G^{\mu\nu}_{AB} \simeq {}^{22}G^{\mu\nu}_{AB} \simeq {}^{12}G^{\mu\nu}_{AB} \simeq {}^{21}G^{\mu\nu}_{AB}. \tag{42} $$

This gives for the current–current correlator

$$ \begin{aligned} 2\langle j^\mu_3(x) j^\nu_Y(0) \rangle &\simeq [c_1^2 s_2^2 + c_2^2 s_1^2 - 2 c_1 c_2 s_1 s_2] G^{\mu\nu}_{LR} \\ &\simeq \frac{\epsilon_y^2 v^2}{2M^2} G^{\mu\nu}_{LR}, \end{aligned} \tag{43} $$

where all of the G_{LL} and G_{RR} contributions self-cancel. Hence, we see that the contribution to the S parameter is suppressed as $M \gg v$ or $\epsilon_y \ll 1$, as expected.

VII. FERMION COUPLINGS TO THE HIGGS BOSON

In terms of the gauge-eigenstate fields, the interactions of the Higgs boson with the dark-sector fermions are, in matrix notation,

$$ \begin{aligned} \mathcal{L} \supset &-\frac{h}{\sqrt{2}} (\, F^u_1 \quad F^u_3 \,) \begin{pmatrix} 0 & y^u_{14} \\ y^u_{23} & 0 \end{pmatrix} \begin{pmatrix} F^d_2 \\ F^d_4 \end{pmatrix} \\ &+ \frac{h}{\sqrt{2}} (\, F^d_1 \quad F^d_3 \,) \begin{pmatrix} 0 & y^d_{14} \\ y^d_{23} & 0 \end{pmatrix} \begin{pmatrix} F^u_2 \\ F^u_4 \end{pmatrix} \\ &+ \mathrm{H.c.} \end{aligned} \tag{44} $$

These matrices are not simultaneously diagonalizable with the mass matrices, Eqs. (9) and (10). This means that the Higgs boson in general has off-diagonal, "dark flavor-changing" interactions with the mass eigenstate fields. Explicitly, we find in terms of the mixing angles

$$ \begin{aligned} \mathcal{L} \supset &\frac{h}{\sqrt{2}} (\, \bar{\Psi}^u_1 \quad \bar{\Psi}^u_2 \,) \\ &\times \begin{pmatrix} c^u_1 s^u_2 y^u_{14} + s^u_1 c^u_2 y^u_{23} & c^u_1 c^u_2 y^u_{14} - s^u_1 s^u_2 y^u_{23} \\ c^u_1 c^u_2 y^u_{23} - s^u_1 s^u_2 y^u_{14} & -s^u_1 c^u_2 y^u_{14} - c^u_1 s^u_2 y^u_{23} \end{pmatrix} \begin{pmatrix} \Psi^u_1 \\ \Psi^u_2 \end{pmatrix} \\ &+ (u \leftrightarrow d). \end{aligned} \tag{45} $$

In the custodial $SU(2)$ limit, we can drop the u and d labels since the Higgs coupling matrix is identical for both sets of fields. If we further take the limit of an approximately symmetric mass matrix, Eq. (28), the Higgs couplings simplify to

$$ \begin{aligned} \mathcal{L} \supset &\frac{yh}{M_2 - M_1} (\, \bar{\Psi}_1 \quad \bar{\Psi}_2 \,) \left[\begin{pmatrix} yv & -\sqrt{2}\Delta \\ -\sqrt{2}\Delta & -yv \end{pmatrix} + O(\epsilon_y) \right] \\ &\times \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}. \end{aligned} \tag{46} $$

We observe both diagonal and off-diagonal Higgs couplings to the fermions. The off-diagonal dark flavor-changing interactions vanish in the limit $\Delta \to 0$ and $\epsilon_y \to 0$. In this limit an enhanced flavor symmetry among the fermions is restored, and the analogue of the GIM mechanism forbids such interactions at tree level. The off-diagonal Higgs couplings lead to an inelastic scattering cross section when a single Higgs is exchanged. This is highly suppressed unless the mass difference $M_2 - M_1$ is near the (nonrelativistic) kinetic energy of the dark matter in galaxy. Two off-diagonal Higgs couplings can be combined in a loop involving one heavier dark fermion and double Higgs exchange, but this is suppressed by the square of the Higgs couplings times a loop factor, as well as by the mass of the heavier fermions.

The single Higgs coupling to the lightest fermions is finally

$$ \mathcal{L} \supset y_\Psi h \bar{\Psi}_1 \Psi_1 \tag{47} $$

where

$$y_\Psi = \frac{y^2 v}{M_2 - M_1} + O(\epsilon_y) \quad \simeq \begin{cases} \frac{y}{\sqrt{2}} & \text{linear case} \\ \frac{y^2 v}{2\Delta} & \text{quadratic case.} \end{cases} \tag{48}$$

(Note also that the single Higgs coupling to the heaviest fermions Ψ_2 is identical up to an overall sign.) Depending on the relative size of yv and Δ, the Higgs boson couples linearly or quadratically proportional to the Yukawa coupling y. The additional suppression of yv/Δ in the quadratic case will imply that spin independent scattering through single Higgs exchange can be significantly weaker when the mass difference between the lightest and heaviest fermions is dominated by the electroweak preserving mass Δ.

VIII. DIRECT DETECTION BOUNDS FROM HIGGS EXCHANGE

In a previous paper [33], we set up a framework for the study of direct-detection bounds on scalar baryonic dark matter candidates through Higgs exchange, and we presented detailed numerical results for an $SU(4)$ gauge group. Our notation in this section closely follows [33]. The model-independent result was expressed in terms of the effective Higgs coupling to the baryon

$$g_B = \frac{m_B}{v} \alpha f_f^{(B)}. \tag{49}$$

The first factor, the baryon mass m_B (divided by the electroweak VEV), as well as the third factor,

$$f_f^{(B)} = \frac{\langle B | M_1 \bar{\Psi}_1 \Psi_1 | B \rangle}{m_B} = \frac{M_1}{m_B} \frac{\partial m_B}{\partial M_1},$$

are extracted from our lattice results [33]. The second factor,

$$\alpha \equiv \frac{v}{M_1} \frac{\partial M_1(h)}{\partial h}\bigg|_{h=v} \simeq \begin{cases} \frac{yv}{\sqrt{2} M_1} & \text{linear case} \\ \frac{(yv)^2}{2 M_1 \Delta} & \text{quadratic case,} \end{cases} \tag{50}$$

provides the effective coupling of the Higgs boson to the fermions (multiplied by v/M_1), and we have evaluated the derivative for the two cases in our model.

Given the dark fermion mass parameters M_1 and Δ, combined with the dark baryon mass and the coupling $f_f^{(B)}$ as determined in [33], we can in principle calculate a bound on the Higgs couplings to stealth dark matter, which could provide useful input into more precise calculations for electroweak precision tests and dark matter abundance. However, we currently only know the "bare" fermion mass parameters in units of the lattice spacing. We, therefore, characterize the fermion mass using the ratio of pseudoscalar to vector meson masses m_Π / m_V as a proxy. We can construct a regularization-independent parameter, the effective Yukawa coupling $y_{\rm eff}$, that is closely related to the model parameters:

$$y_{\rm eff} \equiv \begin{cases} y \frac{m_B}{\sqrt{2} M_1} & \text{linear case} \\ y \frac{m_B}{\sqrt{2 \Delta M_1}} & \text{quadratic case.} \end{cases} \tag{51}$$

The α parameter is, therefore,

$$\alpha \simeq \begin{cases} y_{\rm eff} \frac{v}{m_B} & \text{linear case} \\ y_{\rm eff}^2 \frac{v^2}{m_B^2} & \text{quadratic case.} \end{cases} \tag{52}$$

Recasting our previous constraints in α-m_B space into $y_{\rm eff}$-m_B space, we can identify the region of parameter space that remains viable. The constraints for the linear case are shown in Fig. 2 and the quadratic case in Fig. 3. In the top two plots for the respective figures, the region above the LUX bounds represents the excluded parameter space for the model at a given dark matter mass (m_B) and effective Yukawa coupling ($y_{\rm eff}$). The figures show a clear qualitative trend in how the predictions change as a function of dark matter mass. In particular, the cross section is independent of m_B for the linear case and inversely proportional to m_B in the quadratic case. The bottom plots in Figs. 2 and 3 show the maximum $y_{\rm eff}$ allowed for a given dark matter mass. By increasing the splitting Δ between the vectorlike mass terms, significantly more $y_{\rm eff}$ parameter space becomes available.

IX. ABUNDANCE

We now provide a brief discussion of the relic abundance of stealth dark matter. In the regime where the dark fermions have masses comparable to the confinement scale of the dark force, calculating the relic abundance is an intrinsically strongly coupled calculation. Unfortunately, this calculational difficulty is not easily overcome with lattice simulations, due to the different initial and final states. Nevertheless, it is straightforward to see that the relic abundance *can* match the cosmological abundance through at least two distinct mechanisms that lead to two different mass scales for stealth dark matter. In this section we discuss obtaining the abundance of stealth dark matter through thermal freezeout, leading to a symmetric abundance of dark baryons and antibaryons. Separately, we consider the possibility of an asymmetric abundance generated through electroweak sphalerons.

A. Symmetric abundance

In the early universe at temperatures well above the confinement scale of the $SU(4)$ dark gauge force, the dark fermions are in thermal equilibrium with the thermal bath through their electroweak interactions. As the universe cools to temperatures below the confinement scale, the degrees of freedom change from dark fermions and gluons into the dark baryons and mesons of the low-energy

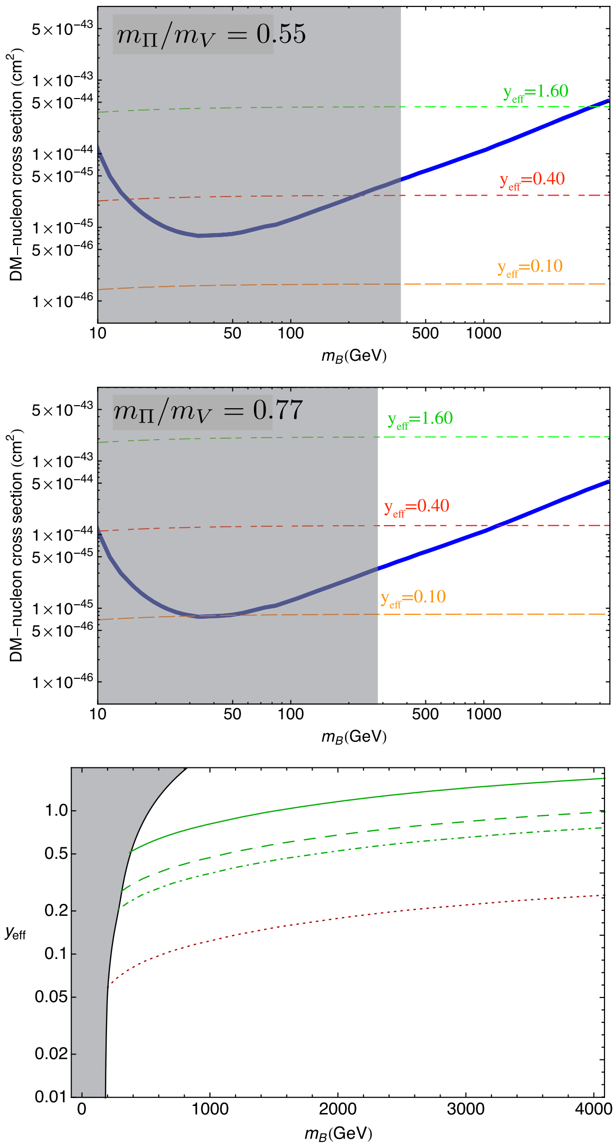

FIG. 2 (color online). Constraints on the stealth dark matter model in the linear case of the model. The top and middle figures show the predicted values for the smallest and largest fermion mass explored in our simulations (corresponding to the pseudoscalar to vector mass ratio $m_\Pi/m_V = 0.55, 0.77$) as well as LUX bounds. Various $y_{\rm eff}$ values are plotted on the figure, where $y_{\rm eff} \approx y m_B/M_1$ in this case. The dark grey region is excluded by the LEP constraints on charged dark mesons. The bottom figure displays the maximum $y_{\rm eff}$ allowed for a given dark matter mass. Each of the green curves represents a different fermion mass in the lattice calculation, $m_\Pi/m_V = 0.55, 0.7, 0.77$ from top to bottom, and the bottom red curve is the result in the heavy fermion limit.

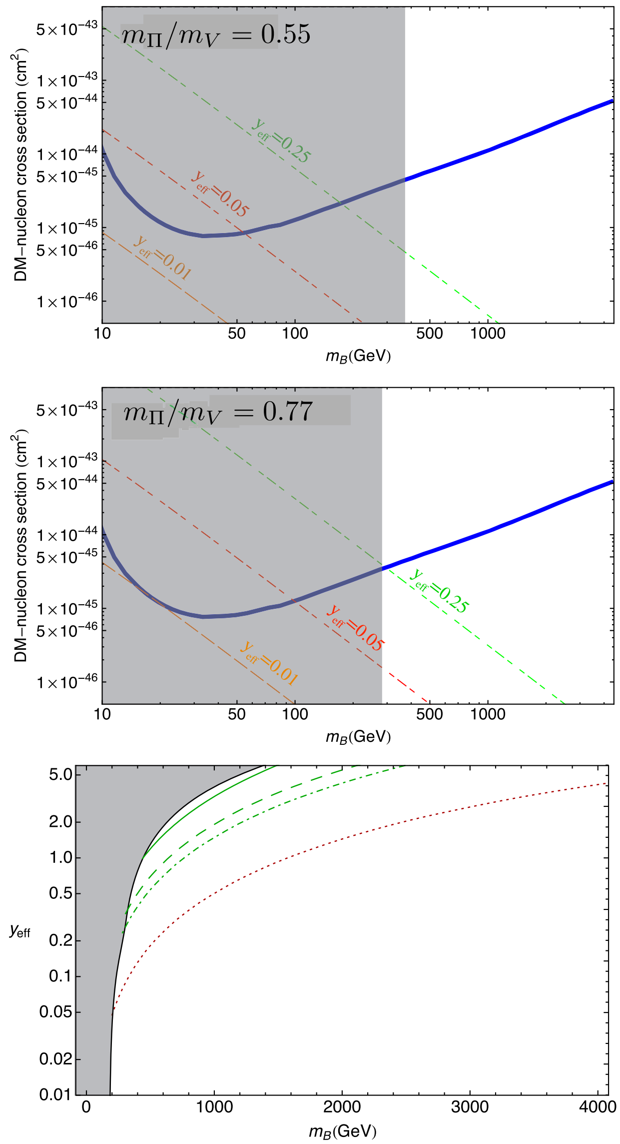

FIG. 3 (color online). Same as Fig. 2 but for the quadratic case of the model. In this case, $y_{\rm eff} \approx y m_B/\sqrt{M_1 \Delta}$.

description. Some of the dark mesons carry electric charge, and so the dark mesons remain in thermal equilibrium with the Standard Model quarks, leptons, and gauge fields. Since the dark baryons are strongly coupled to the dark mesons, they also are kept in thermal equilibrium. As the temperature of the universe falls well below the mass of the dark baryons, they annihilate into dark mesons that subsequently thermalize and decay (or decay then thermalize) into Standard Model particles. The symmetric abundance of dark baryons is, therefore, determined by the annihilation rate of dark baryons into dark mesons.

The annihilation of dark baryons to dark mesons is a strongly coupled process. We expect $B^*B \to \Pi\Pi$, $B^*B \to 3\Pi$, and $B^*B \to 4\Pi$, (and to possibly more mesons if kinematically allowed) to occur, but we do not know the dominant annihilation channel. If the 2-to-2 process $B^*B \to \Pi\Pi$ dominates, one approach is to use partial wave unitarity to estimate the thermally averaged annihilation rate [53,69],

$$\langle \sigma v \rangle \sim \frac{4\pi \langle v^{-1} \rangle}{m_B^2}, \tag{53}$$

where $\langle v^{-1}\rangle \simeq 2.5$ at freeze-out [69]. Matching this cross section to the required thermal relic abundance yields $m_B \sim 100$ TeV. An alternative approach is to use naive dimensional analysis [70–72], which appears to lead to a larger dark matter mass.

If the 2-to-3 or 2-to-4 processes dominate instead, the additional phase space and kinematic suppression lowers the annihilation rate and, therefore, lowers the scalar baryon mass needed to obtain the cosmological abundance. For recent work that has considered the thermal relic abundance in multibody processes, see [25,28]. Suffice it to say a symmetric thermal abundance of dark baryons will match the cosmological abundance for a relatively large baryon mass that is of order tens to hundreds of TeV.

B. Asymmetric abundance

Early work on technibaryons demonstrated that strongly coupled dark matter could arise from an asymmetric abundance [4–8]. The main ingredient to obtain the correct cosmological abundance is the electroweak sphaleron (the non-perturbative solution at finite temperature that allows for transitions between vacua with different[5] $B+L$ numbers).[6] In the early Universe, at temperatures much larger than the electroweak scale, electroweak sphalerons are expected to violate one accidental global symmetry, $B+L+D$ number, leaving $B-L$ and $B-D$ numbers unaffected [7,8,16]. Here D number is proportional to the dark baryon number, with some appropriate normalization (for examples; see [7,16]).

Given a baryogenesis mechanism, the electroweak sphalerons redistribute baryon number into lepton number and dark baryon number. As the Universe cools, the mass of the technibaryon becomes larger than the temperature of the Universe. Eventually, the Universe cools to the point where electroweak sphalerons "freeze out" and can no longer continue exchanging B, D, and L numbers. The residual abundance of dark baryons is $\rho \sim m_B n_B$ where the number density is proportional to $\exp[-m_{\mathrm{B}}/T_{\mathrm{sph}}]$, where T_{sph} is the temperature at which sphaleron interactions shut off.

If the baryon and dark baryon number densities are comparable, the would-be overabundance of dark matter (from $m_B \gg m_{\mathrm{nucleon}}$) is compensated by the Boltzmann suppression. Very roughly, $m_B \sim 1$–2 TeV is the natural mass scale that matches the cosmological abundance of dark matter [6]. A crucial component of the early technibaryon papers [4–6] is that the technifermions were in a purely chiral representation of the electroweak group, like the fermions of the Standard Model.

[5]In this section, B refers to baryon number and is to not be confused with the field defined earlier.

[6]In addition, an asymmetric abundance could be generated through other mechanisms, see Ref. [73], in which case the mass scales and parameters depend on the details of the particular mechanism.

In stealth dark matter, given an early baryogenesis mechanism (or other analogous mechanism to generate an asymmetry in a globally conserved quantity [8,74–80]), it is possible that electroweak sphalerons could also lead to the correct relic abundance of dark baryons consistent with cosmology.

There is one critical difference from the early technicolor models (as well as the quirky dark matter model): The dark fermions in stealth dark matter have both vectorlike and electroweak symmetry breaking masses. This leads to a suppression of the effectiveness of the electroweak sphalerons by a factor of α, cf. Eq. (50), leading to a somewhat *smaller* stealth baryon mass to obtain the correct relic abundance compared with a technicolor model (all other parameters equal). A more quantitative estimate is complicated by several factors:

(i) Determining how the electroweak sphaleron redistributes the conserved global charges in the presence of fermions that acquire both electroweak preserving and electroweak breaking masses. To the best of our knowledge, this calculation has never been done.

(ii) Determining the precise temperature at which electroweak sphalerons shut off, in the presence of both the Standard Model and stealth dark matter degrees of freedom contributing to the thermal bath.

(iii) The baryogenesis mechanism itself, that determines the initial $B-L$ and $B-D$ numbers.

Given the exponential suppression of the asymmetric abundance as the dark baryon mass is increased, it is clear that the upper bound on the dark baryon mass is nearly the same as the technibaryon calculation (updated to the current cosmological parameters), when stealth dark fermions have vectorlike masses comparable to electroweak symmetry breaking masses. (This case is, however, constrained by the S parameter; see Sec. VI). We can, therefore, anticipate that a range of stealth dark matter masses will be viable, up to about a TeV. More precise predictions require further detailed investigation that is beyond the scope of this paper.

X. DISCUSSION

We have presented a concrete model, "stealth dark matter," that is a composite baryonic scalar of a new $SU(N_D)$ strongly coupled confining gauge theory with dark fermions transforming under the electroweak group. Though the stealth dark matter model has a wide parameter space, we focused on dark fermion masses that respect an exact custodial $SU(2)$. Custodial $SU(2)$ implies the lightest bosonic baryonic composite is an electrically neutral scalar (and not a vector or spin-2) of the $SU(N_D)$ dark spectrum, and in addition does not have a charge radius. This yields an exceptionally "stealthy" dark matter candidate, with spin-independent direct detection scattering proceeding only through Higgs exchange (studied in this paper) and the polarizability interaction (studied in our companion paper [81]). Custodial $SU(2)$ also allows for stealth dark matter to completely avoid the constraints from the T parameter. While contributions to the S parameter are present, they are suppressed by the ratio of the electroweak symmetry breaking mass-squared divided by a vectorlike mass squared of the dark fermions. We also verified the lightest non-singlet mesons decay rapidly (so long as $\epsilon_y \neq 0$), avoiding any cosmological issues with stable electrically charged dark mesons.

Specializing to the case of $N_D = 4$, we then applied our earlier model-independent lattice results [33] to the parameters of stealth dark matter, and obtained constraints on the effective Higgs interaction. We find that the present LUX bound is able only to mildly constrain the Higgs coupling to stealth dark matter for relatively light dark baryons. Even weaker constraints arise when the effective Higgs interaction is quadratic in the Yukawa coupling, which is a natural possibility when the two pairs of dark fermions are split dominantly by vectorlike masses, i.e., $yv \ll \Delta$.

While we have considered many aspects of stealth dark matter, several avenues warrant further investigation:

(i) Chiral symmetry forbids additive renormalization of the fermion masses; we have focused on the regime where the constituent fermion mass is comparable to the confinement scale $M_f \sim \Lambda_D$, since this is best suited for lattice simulations, exactly where analytic estimates are least useful. It would be interesting to consider a broader range of fermion masses relative the confinement scale, to understand the relative scaling of the Higgs interactions.

(ii) A more precise calculation of the S parameter is possible using lattice simulations for the relevant correlators. This would allow us to place numerical bounds on the parameters of the theory, that could be stronger than the bounds from the non-observation through direct detection.

(iii) We would like to unpack y_{eff} [cf. Eq. (51)] and obtain constraints on the Yukawa couplings of the model. However, this requires translating the fermion masses from the lattice regularization into a continuum regularization.

(iv) Dark meson production and decay at the LHC is ripe for exploration. Dark meson pair production would proceed through off-shell EW gauge bosons, $q\bar{q} \to \Pi^+\Pi^-$, $q\bar{q} \to \Pi^0\Pi^0$, and $q\bar{q}' \to \Pi^\pm\Pi^0$. These could have spectacular signals at the LHC. Neutral mesons decay into fermion pairs and dibosons (explored in other related models in [20,42,82,83]). For charged dark mesons, with masses in the range $m_{\Pi^\pm} \sim 90$–180 GeV, the decay $\Pi^+ \to \tau^+\nu_\tau$ dominates, while for masses above this, $\Pi^+ \to t\bar{b}$ is dominant. Charged pion pair production could, therefore, lead to $t\bar{b}b\bar{t}$ signals with the $t\bar{b}$ and $b\bar{t}$ pairs reconstructing to the same mass. To the best of our knowledge, this type of resonance search is not being performed at the LHC.

(v) More insight into the thermal abundance of stealth dark matter, perhaps using lattice simulations, would help narrow the interesting mass range that matches cosmological data.

(vi) Asymmetric production of stealth dark matter seems very promising, but has several calculational obstacles to overcome to arrive at a quantitative relationship between the abundance and the other parameters of the theory.

(vii) If stealth dark matter has an asymmetric abundance, there are potential limits from neutron star lifetimes [84–86] though the precise bounds depend sensitively on the equation of state of the neutron stars.

(viii) There are tantalizing signals of a γ-ray excess between about 1–10 GeV in the Galactic center (see for example [87–91]). A recent analysis [92] suggests that this could arise from dark matter up to 300 GeV. It is intriguing to consider the γ-ray signal spectrum that could arise from a symmetric abundance of stealth dark matter with annihilation into a multibody final state [93] with mixtures of four or more heavy fermions and multi-gauge bosons (from $BB^* \to \Pi\Pi\ldots \to$ SM states).

Clearly there are several characteristic signals of stealth composite dark matter. If the Higgs couplings to stealth dark matter are significant, this could also lead to modifications of Higgs properties, and provide a channel for direct production of the dark baryons at colliders. On the other hand, if the Higgs couplings are suppressed, then we find that direct detection proceeds through the electromagnetic polarizability, which is discussed in our companion paper [81]. The polarizability channel is particularly interesting, with a double-photon exchange interaction which gives a per-nucleon cross section expected to scale as Z^4/A^2 (where Z and A are the atomic and mass numbers of the target nucleus, respectively), favoring larger nuclei over smaller ones.

An unambigious prediction of stealth dark matter is the rich spectrum of other composite states made from the constituent dark fermions. The states most likely to be accessible to collider energies are the dark mesons, and their production and detection may provide the best way to investigate the presence of a new strongly coupled sector. The excited states of dark matter itself—the heavier dark baryons—may also provide complementary evidence of the compositeness of the dark sector, for example through emission lines detectable in gamma-ray telescope experiments. We leave detailed investigations to future work.

Finally, there are broader model-building questions to consider. One is the choice of scales $M_f \sim \Lambda_D$ that has been the focus of this work. This could arise dynamically. For example, if there are sufficient flavors in the $SU(N_D)$ gauge theory such that it is approximately conformal at high energies, then as the theory is run down through the dark fermion mass scale M_f, the dark fermions integrate out, and confinement sets in at $\Lambda_D \sim M_f$. This is well known to occur for supersymmetric $SU(N)$ theories in the conformal window that flow to confining theories once the number of flavors drops below $N_f < 3N/2$ [94]. The origin of the vectorlike masses of the fermions is also an interesting model-building puzzle. However, just as SM fermion masses are vectorlike below the electroweak breaking scale, we can imagine dark fermion vectorlike masses could be revealed as arising from dynamics that breaks the flavor symmetries of our dark fermions at some higher scale.

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APPENDIX: WEAK CURRENTS

We examine the dark fermion contributions to the electroweak currents. In the gauge eigenstate basis, the currents are

$$j^\mu_+ = -\frac{1}{\sqrt{2}}(F_1^{u\dagger}\bar{\sigma}^\mu F_1^d + F_2^{u\dagger}\bar{\sigma}^\mu F_2^d) \tag{A1}$$

$$j^\mu_- = -\frac{1}{\sqrt{2}}(F_1^{d\dagger}\bar{\sigma}^\mu F_1^u + F_2^{d\dagger}\bar{\sigma}^\mu F_2^u) \tag{A2}$$

$$j^\mu_3 = -\frac{i}{2}\sum_{i=1,2}(F_i^{u\dagger}\bar{\sigma}^\mu F_i^u - F_i^{d\dagger}\bar{\sigma}^\mu F_i^d) \tag{A3}$$

$$j^\mu_Y = -\frac{i}{2}\sum_{i=3,4}(F_i^{u\dagger}\bar{\sigma}^\mu F_i^u - F_i^{d\dagger}\bar{\sigma}^\mu F_i^d). \tag{A4}$$

In the mass eigenstate basis given by Eqs. (14)–(17), the currents can be rewritten in terms of the 4-component Dirac fermions defined by Eqs. (19) and (20). After some algebra, one obtains

$$\begin{aligned} j^\mu_+ = -\frac{1}{\sqrt{2}}[&\overline{\Psi_1^u}\gamma^\mu(c_1^u c_1^d P_L + c_2^u c_2^d P_R)\Psi_1^d + \overline{\Psi_2^u}\gamma^\mu(s_1^u s_1^d P_L + s_2^u s_2^d P_R)\Psi_2^d \\ &+ \overline{\Psi_1^u}\gamma^\mu(c_1^u s_1^d P_L + c_2^u s_2^d P_R)\Psi_2^d + \overline{\Psi_2^u}\gamma^\mu(s_1^u c_1^d P_L + s_2^u c_2^d P_R)\Psi_1^d] \end{aligned} \tag{A5}$$

$$\begin{aligned} j^\mu_- = -\frac{1}{\sqrt{2}}[&\overline{\Psi_1^d}\gamma^\mu(c_1^d c_1^u P_L + c_2^d c_2^u P_R)\Psi_1^u + \overline{\Psi_2^d}\gamma^\mu(s_1^d s_1^u P_L + s_2^d s_2^u P_R)\Psi_2^u \\ &+ \overline{\Psi_1^d}\gamma^\mu(c_1^d s_1^u P_L + c_2^d s_2^u P_R)\Psi_2^u + \overline{\Psi_2^d}\gamma^\mu(s_1^d c_1^u P_L + s_2^d c_2^u P_R)\Psi_1^u] \end{aligned} \tag{A6}$$

$$\begin{aligned} j^\mu_3 = \frac{1}{2}[&\overline{\Psi_1^u}\gamma^\mu((c_1^u)^2 P_L + (c_2^u)^2 P_R)\Psi_1^u + \overline{\Psi_2^u}\gamma^\mu((s_1^u)^2 P_L + (s_2^u)^2 P_R)\Psi_2^u \\ &- \overline{\Psi_1^d}\gamma^\mu((c_1^d)^2 P_L + (c_2^d)^2 P_R)\Psi_1^d - \overline{\Psi_2^d}\gamma^\mu((s_1^d)^2 P_L + (s_2^d)^2 P_R)\Psi_2^d \\ &+ \overline{\Psi_1^u}\gamma^\mu(c_1^u s_1^u P_L + c_2^u s_2^u P_R)\Psi_2^u + \overline{\Psi_2^u}\gamma^\mu(s_1^u c_1^u P_L + s_2^u c_2^u P_R)\Psi_1^u \\ &- \overline{\Psi_1^d}\gamma^\mu(c_1^d s_1^d P_L + c_2^d s_2^d P_R)\Psi_2^d - \overline{\Psi_2^d}\gamma^\mu(s_1^d c_1^d P_L + s_2^d c_2^d P_R)\Psi_1^d] \end{aligned} \tag{A7}$$

$$\begin{aligned} j^\mu_Y = \frac{1}{2}[&\overline{\Psi_1^u}\gamma^\mu((s_1^u)^2 P_L + (s_2^u)^2 P_R)\Psi_1^u + \overline{\Psi_2^u}\gamma^\mu((c_1^u)^2 P_L + (c_2^u)^2 P_R)\Psi_2^u \\ &- \overline{\Psi_1^d}\gamma^\mu((s_1^d)^2 P_L + (s_2^d)^2 P_R)\Psi_1^d - \overline{\Psi_2^d}\gamma^\mu((c_1^d)^2 P_L + (c_2^d)^2 P_R)\Psi_2^d \\ &- \overline{\Psi_1^u}\gamma^\mu(c_1^u s_1^u P_L + c_2^u s_2^u P_R)\Psi_2^u - \overline{\Psi_2^u}\gamma^\mu(s_1^u c_1^u P_L + s_2^u c_2^u P_R)\Psi_1^u \\ &+ \overline{\Psi_1^d}\gamma^\mu(c_1^d s_1^d P_L + c_2^d s_2^d P_R)\Psi_2^d + \overline{\Psi_2^d}\gamma^\mu(s_1^d c_1^d P_L + s_2^d c_2^d P_R)\Psi_1^d], \end{aligned} \tag{A8}$$

where $c_i^j \equiv \cos\theta_i^j$, $s_i^j \equiv \sin\theta_i^j$ and $P_{L,R} = (1\mp\gamma_5)/2$ are the left- and right-handed projectors. In general, the dark fermions contribute to both the vector and axial currents with strengths given by the mixing angles. It is easy to verify that the electromagnetic current,

$$j^\mu_{\rm em} = j^\mu_3 + j^\mu_Y = \sum_{i=1,2} [Q_u \overline{\Psi^u_i}\gamma^\mu\Psi^u_i + Q_d\overline{\Psi^d_i}\gamma^\mu\Psi^d_i], \tag{A9}$$

with $Q_{u,d} = \pm 1/2$, is consistent with a pure vector coupling of the dark fermions to the photon independent of mass mixing angles.

Interestingly, if the mass matrices Eqs. (9) and (10) are symmetric, i.e., $y^u_{14} = y^u_{23}$ and $y^d_{14} = y^d_{23}$, then just two mixing angles are required, i.e., $\theta^u_1 = \theta^u_2$ and $\theta^d_1 = \theta^d_2$. In this case, the mixing angles factor out of the left-right gamma matrix structure, leaving all of the electroweak currents to be purely vector (with vanishing axial current). This is unlike the Standard Model, where the $SU(2)_L$ currents are purely $V-A$. The difference between this model and the Standard Model is the structure of the dark fermion mass matrices that include both vectorlike and electroweak symmetry breaking masses.

It is also interesting to calculate the neutral current,

$$j^\mu_Z = j^\mu_3 - \sin^2\theta_W j^\mu_{\rm em}. \tag{A10}$$

For the neutral baryon state,

$$\begin{aligned}\langle B|j^\mu_Z|B\rangle \simeq &+\frac{1}{4}((c^u_1)^2+(c^u_2)^2-(c^d_1)^2-(c^d_2)^2)\langle B|\bar\Psi_1\gamma^\mu\Psi_1|B\rangle \\ &+\frac{1}{4}(-(c^u_1)^2+(c^u_2)^2+(c^d_1)^2 \\ &-(c^d_2)^2)\langle B|\bar\Psi_1\gamma^\mu\gamma^5\Psi_1|B\rangle.\end{aligned} \tag{A11}$$

In the limit of zero momentum exchange ($Q^2 = 0$), the vector form factor $\langle B|\bar\Psi_1\gamma^\mu\Psi_1|B\rangle$ evaluates to 1, while the axial-vector form factor $\langle B|\bar\Psi_1\gamma^\mu\gamma^5\Psi_1|B\rangle$ for a scalar baryon vanishes. In the presence of an exact custodial SU(2) symmetry, which is the focus of this paper, we have $c^u_i = c^d_i$ and the Z coupling vanishes identically at any momentum exchange.

On the other hand, if custodial symmetry is broken, then the lightest neutral baryon acquires tree-level couplings to the Z boson. To illustrate the size of these couplings, consider taking the dark fermion mass matrices to be exactly symmetric ($y_{23} = y_{14}$) but allowing for a small, custodial symmetry-violating difference in the Yukawas, $y_u = y + \xi$ and $y_d = y - \xi$ where $\xi/y \ll 1$. The coefficient of the weak neutral vector current becomes

$$\begin{aligned}&(c^u_1)^2 + (c^u_2)^2 - (c^d_1)^2 - (c^d_2)^2 \\ &\simeq \begin{cases} 2\sqrt{2}\frac{\xi}{y}\frac{\Delta}{yv} & \text{linear case} \\ \frac{\xi}{y}\frac{(yv)^2}{\Delta^2} & \text{quadratic case.}\end{cases}\end{aligned} \tag{A12}$$

Custodial symmetry violation is, therefore, restricted by requiring the coupling of the lightest neutral baryon to the Z boson be small enough to have evaded direct detection. There are several limits in which this can occur: $\xi/y \ll 1$ (any scenario), $\Delta/(yv) \ll 1$ (linear case), or $(yv)/\Delta \ll 1$ (quadratic case). This suggests that modest custodial symmetry violation is possible but rather constrained.

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