ELKO fermions as dark matter candidates

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We study the implications of the ELKO fermions as a cold dark matter candidate. Such fermions arise in theories that are not symmetric under the full Lorentz group. Although they do not carry electric charge, ELKOs can still couple to photons through a nonstandard interaction. They also couple to the Higgs but do not couple to other standard model particles. We impose limits on their coupling strength and the ELKO mass assuming that these particles give dominant contribution to the cosmological cold dark matter. We also determine limits imposed by the direct dark matter search experiments on the ELKO-photon and the ELKO-Higgs coupling. Furthermore we determine the limit imposed by the gamma ray bursts time delay observations on the ELKO-Higgs coupling. We find that astrophysical and cosmological considerations rule out the possibility that ELKO may contribute significantly as a cold dark matter candidate. The only allowed scenario in which it can contribute significantly as a dark matter candidate is that it was never in equilibrium with the cosmic plasma. We also obtain a relationship between the ELKO self-coupling and its mass by demanding it to be consistent with observations of dense cores in the galactic centers.

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I. INTRODUCTION

Current cosmological observations indicate that cold dark matter (CDM) contributes 23% of the energy density of the Universe. The nature of this matter is so far unknown but there are many proposals for dark matter [1–5]. In 2005, Ahluwalia and Grumiller proposed a spin half fermion with mass dimension one [6,7]. The field, called ELKO, is an eigenspinor of the charge conjugation operator and hence carries no electric charge. Moreover, the mismatch between the mass dimension of ELKO and the standard model (SM) fermions restricts its interactions with the SM particles [8] making ELKO a suitable candidate for dark matter.

ELKO arises in theories that are not symmetric under the full Lorentz group [8,9] but only a subgroup, such as SIM(2) [10]. Had the SM respected either P, T, CP or CT, then the subgroup SIM(2) would necessarily be enhanced to the full Lorentz group but it breaks these discrete symmetries and allows the possibility of a small violation of the Lorentz invariance. As Cohen and Glashow [10] argued in 2006, "Many empirical successes of special relativity need not demand Lorentz invariance of the underlying framework." These theories have a preferred axis [8,10,11], that breaks Lorentz invariance by breaking rotational symmetry. Along such a preferred axis, the ELKO field enjoys locality [12]. It is intriguing that

cosmological observations also show some evidence for a preferred axis in the Universe [13,14].

ELKO interacts dominantly with the Higgs field and thus acts as a dark matter candidate somewhat analogous to the Higgs portal models, see for example [15–17]. It also has a quadratic self-coupling as well as a coupling to the electromagnetic field tensor $F^{\mu\nu}$. We find that the electromagnetic coupling is severely restricted by direct dark matter searches. At the Large Hadron Collider (LHC) its discovery prospects through its Higgs interaction [18–20] as well as possible indirect detection [21] have been studied. The ELKO spinor driven inflation [22–29] and its application to gravity [30,31] and higher dimensional brane world model [32–34] have been proposed. The causality [35] structure as well as a dynamical mass generation mechanism of the ELKO field [36,37] has been discussed in the literature. The ELKO spinor has been shown as a building block of Duffin-Kemmer-Petiau algebra in Ref. [38].

In the present paper we systematically investigate its implications as a dark matter candidate. In particular we determine the range of parameters over which it can act as a CDM candidate. Furthermore we investigate whether this range is consistent with the known limits on dark matter couplings. This issue has not been addressed so far in the literature.

This paper is organized as follows: In Sec. II, we briefly review the ELKO field and its interactions. In Sec. III we determine the range of values of the ELKO mass and its coupling with the Higgs for which it may be considered as a CDM candidate. In Sec. IV we determine the constraints on the ELKO-photon and the ELKO-Higgs couplings arising

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from the CDMS II limit on the scattering of dark matter with protons. In Sec. V we obtain the constraints on the ELKO-Higgs coupling arising from gamma ray bursts. In Sec. VI we determine the implications of the galactic dark matter cores for the ELKO self-coupling. Finally we conclude in Sec. VII.

II. A BRIEF REVIEW OF ELKO FERMION AND ITS INTERACTIONS

The Fourier decomposition of the ELKO field may be written as [39]

$$
\mathfrak{f}(x) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2mE(\boldsymbol{p})}} \sum_\alpha [a_\alpha(\boldsymbol{p})\lambda_\alpha^S(\boldsymbol{p}) \exp(-ip_\mu x^\mu)
$$
$$
+ b_\alpha^\dagger(\boldsymbol{p})\lambda_\alpha^A(\boldsymbol{p}) \exp(ip_\mu x^\mu)] \tag{1a}
$$

and its dual as

$$
\bar{\mathfrak{f}}(x) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2mE(\boldsymbol{p})}} \sum_\alpha [a_\alpha^\dagger(\boldsymbol{p})\bar{\lambda}_\alpha^S(\boldsymbol{p}) \exp(ip_\mu x^\mu)
$$
$$
+ b_\alpha(\boldsymbol{p})\bar{\lambda}_\alpha^A(\boldsymbol{p}) \exp(-ip_\mu x^\mu)] \tag{1b}
$$

where m is the mass of the ELKO field. The creation and annihilation operators satisfy the following commutation relations

$$
\{a_\alpha(\boldsymbol{p}), a_{\alpha'}^\dagger(\boldsymbol{p}')\} = (2\pi)^3\delta^3(\boldsymbol{p} - \boldsymbol{p}')\delta_{\alpha\alpha'} \tag{2a}
$$

$$
\{a_\alpha(\boldsymbol{p}), a_{\alpha'}(\boldsymbol{p}')\} = 0, \qquad \{a_\alpha^\dagger(\boldsymbol{p}), a_{\alpha'}^\dagger(\boldsymbol{p}')\} = 0 \tag{2b}
$$

with similar relations for b's. The spinors, λ_α^S and λ_α^A are eigenstates of the charge conjugation operator, C, such that

$$
C\lambda_\alpha^S = +\lambda_\alpha^S \qquad C\lambda_\alpha^A = -\lambda_\alpha^A. \tag{3}
$$

Here α is the helicity index. The dual spinors are defined as, for example,

$$
\bar{\lambda}_+^S(p^\mu) = -i[\lambda_-^S]^\dagger\eta
$$
$$
\bar{\lambda}_-^S(p^\mu) = i[\lambda_+^S]^\dagger\eta \tag{4}
$$

with similar relationships for the remaining spinors. The matrix η is given by,

$$
\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{5}
$$

The spinors satisfy the following spin sums,

$$
\sum_\alpha \lambda_\alpha^S\bar{\lambda}_\alpha^S = m(\mathbb{G}(\phi) + I)
$$
$$
\sum_\alpha \lambda_\alpha^A\bar{\lambda}_\alpha^A = m(\mathbb{G}(\phi) - I), \tag{6}
$$

where

$$
\mathbb{G}(\phi) = \begin{pmatrix} 0 & 0 & 0 & -ie^{-i\phi} \\ 0 & 0 & ie^{i\phi} & 0 \\ 0 & -ie^{-i\phi} & 0 & 0 \\ ie^{i\phi} & 0 & 0 & 0 \end{pmatrix}. \tag{7}
$$

The Lagrangian density for the ELKO field can be written as,

$$
\mathcal{L} = \partial^\mu\bar{\mathfrak{f}}\partial_\mu\mathfrak{f}(x) - m^2\bar{\mathfrak{f}}(x)\mathfrak{f}(x) + \mathcal{L}_{\text{int}}, \tag{8}
$$

where the interaction Lagrangian density is given by [39]

$$
\mathcal{L}_{\text{int}} = -g_{\mathfrak{ff}}(\bar{\mathfrak{f}}(x)\mathfrak{f}(x))^2 - g_{\mathfrak{f}\phi}\bar{\mathfrak{f}}(x)\mathfrak{f}(x)\phi^\dagger(x)\phi(x)
$$
$$
- g_{\mathfrak{f}}\bar{\mathfrak{f}}(x)[\gamma_\mu, \gamma_\nu]\mathfrak{f}(x)F^{\mu\nu}(x) \tag{9}
$$

and $g_{\mathfrak{ff}}$, $g_{\mathfrak{f}\phi}$ and $g_{\mathfrak{f}}$ are dimensionless coupling constants. The first term on the right-hand side of Eq. (9) represents the self-interaction of the ELKO field, the second is the interaction with the Higgs field, ϕ and the third its interaction with the electromagnetic field [39].

III. ELKO AS A COLD DARK MATTER CANDIDATE

If the ELKO-Higgs coupling, $g_{\mathfrak{f}\phi}$, is significant then it could maintain these fermions in thermal equilibrium with the cosmic plasma in the early Universe. The processes relevant for this purpose are shown in Figs. 1 and 2. These correspond, respectively, to the ELKO-Higgs scattering and the ELKO-ELKO annihilation into two Higgses.

The amplitude of the ELKO-Higgs scattering process (Fig. 1) is given by,

$$
i\mathcal{M} = \frac{g_{\mathfrak{f}\phi}}{m}\bar{\lambda}_{\alpha'}^S(k')\lambda_{\beta'}^S(k). \tag{10}
$$
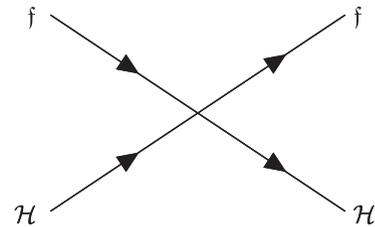
FIG. 1. ELKO-Higgs Scattering.
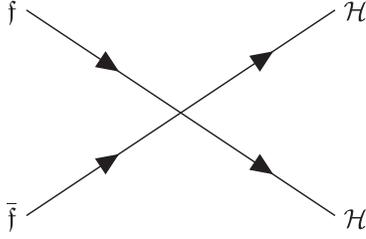
FIG. 2.   Annihilation of ELKOs into a pair of Higgses.

This leads to,

$$|\mathcal{M}|^2 = \frac{g_{\mathfrak{f}\phi}^2}{m^2} 4(EE' - kk'\cos(\theta - \theta'))(1 + \cos(\phi - \phi')).$$

(11)

The thermal averaged cross section for this scattering process is

$$\langle \sigma_s v \rangle = \frac{g_{\mathfrak{f}\phi}^2}{32\pi^2 m^2 s} \frac{1}{2} 4\pi (4EE' - \pi kk' \sin\theta),$$

(12)

where E, E' are initial and final energy of ELKO, respectively. We assume an isotropic distribution of the ELKO momenta. Integrating over θ, the thermally averaged cross section (σ_s) in the nonrelativistic limit is found to be,

$$\langle \sigma_s v \rangle = \frac{g_{\mathfrak{f}\phi}^2}{2\pi(m_H + m)^2},$$

(13)

where $m_H = 125$ GeV is the mass of the Higgs.

The amplitude of the pair annihilation process (Fig. 2) is given by

$$i\mathcal{M} = \frac{g_{\mathfrak{f}\phi}}{m} \neg_{\alpha'}^A(k') \lambda_{\beta'}^S(k).$$

(14)

The square of the invariant amplitude is given by,

$$|\mathcal{M}|^2 = \frac{8g_{\mathfrak{f}\phi}^2}{m^2}(m^2 + 2\mathbf{p}^2).$$

(15)

In the nonrelativistic limit, this gives the following thermal averaged annihilation cross section (σ_a),

$$\langle \sigma_a v \rangle = \frac{g_{\mathfrak{f}\phi}^2}{16\pi m^2}.$$

(16)

If ELKOs act as a CDM candidate, they will decouple from the cosmic plasma when they are nonrelativistic. Let T_f denote their freeze-out temperature [40]. Now, we use the fact that at the time of freeze-out, the interaction rate (Γ) becomes equal to the expansion rate (H), i.e. $\Gamma = H$. Since, both the ELKO-Higgs scattering and the pair annihilation of ELKOs to Higgses would contribute to the total

thermally averaged cross section at the time of decoupling of ELKO from cosmic plasma, the interaction rate is $\Gamma = n\langle \sigma v \rangle = n(\langle \sigma_s v \rangle + n\langle \sigma_a v \rangle)$, where the number density n, in the nonrelativistic limit, is given by,

$$n = g_A \left(\frac{mT_f}{2\pi}\right)^{\frac{3}{2}} e^{-m/T_f}.$$

Here g_A is the degeneracy factor which is equal to 2 for ELKO. Now, the expansion rate or the Hubble constant can be expressed as,

$$H(T_f) = 5.44 \frac{T_f^2}{M_{\text{pl}}},$$

where M_{pl} denotes the Planck mass. Hence $\Gamma = H$ implies,

$$g_A \left(\frac{mT_f}{2\pi}\right)^{\frac{3}{2}} \exp[-m/T_f] \left(\frac{g_{\mathfrak{f}\phi}^2}{2\pi(m_H + m)^2} + \frac{g_{\mathfrak{f}\phi}^2}{16\pi m^2}\right)$$
$$= 5.44 \frac{T_f^2}{M_{\text{pl}}}.$$

(17)

In Fig. 3, we plot the ELKO-Higgs coupling $g_{\mathfrak{f}\phi}$ as a function of its mass, m for a range of values of the decoupling temperature, T_f. We restrict the value of the coupling constant to be less than one so that perturbation theory is applicable. The higher order corrections are suppressed by powers of $\alpha = g_{\mathfrak{f}\phi}^2/4\pi$ and hence are small, less than 10%, as long as $g_{\mathfrak{f}\phi} < 1$. We display the plots for $m \gtrsim 100$ GeV because the Higgs decouples from the cosmic plasma at a temperature of around 80 GeV. Hence, below this temperature ELKOs cannot maintain equilibrium with the cosmic plasma due to their interaction with the Higgs. Only for mass much larger than 100 GeV, ELKOs decouple as nonrelativistic particles and hence act as CDM.

We set the relic density of ELKO fermions equal to the dark matter density $\Omega_s \approx 0.3$, given by [41],

$$\Omega_s = \frac{74.7 S_0 m}{2\pi^2 M_{\text{pl}} \sqrt{g_*} T_f \rho_c \langle \sigma v \rangle_f}$$

(18)

where $S_0 = 2.97 \times 10^3$ cm^{-3} is the present value of entropy density, $\rho_c = 1.05 \times 10^4 h^2$ eV/cm^3 is the critical density of the universe and we have assumed $g_* = 106.75$, corresponding to the relativistic degrees of freedom at the time of decoupling. This leads to,

$$\frac{m^3(m_H + m)^2}{(8m^2 + (m_H + m)^2)T_f} = 3.37 \times 10^8 g_{\mathfrak{f}\phi}^2$$

(19)

in units of GeV^2. This relationship between m and $g_{\mathfrak{f}\phi}$ is also plotted in Fig. 3 as slanted straight lines. For a given
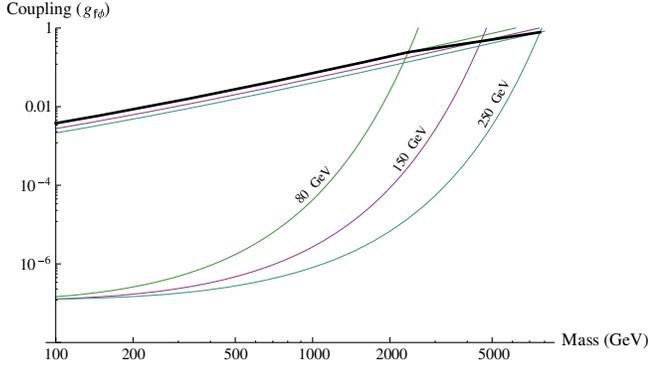
FIG. 3 (color online). The curved lines show the decoupling at different freeze-out temperatures, 250, 150 and 80 GeV. The lowest freeze out temperature is 80 GeV since at this temperature the Higgs decouples from the plasma. The slanted, almost straight lines are obtained by imposing the condition $\Omega_s = 0.3$ for different freeze out temperatures. From bottom to top the decoupling temperatures are 250, 150 and 80 GeV. The dark line corresponds to the values of parameters for which ELKOs dominate the dark matter density. The region above the dark line corresponds to the allowed range of parameters.

temperature, T_f, all parameter values below a particular line will overclose the Universe and hence are ruled out.

The intersections of the two sets of curves give the preferred range of the ELKO mass and its coupling with the Higgs. This is shown in Fig. 3 as a thick dark line. For these parameter values ELKO will give dominant contribution to the cosmological dark matter density. For a given decoupling temperature, larger values of $g_{\mathfrak{f}\phi}$ are also allowed but in this case we also require other dark matter particles in order to fit the observed energy density. Hence we find that ELKO acts as a CDM candidate if $100\text{ GeV} \lesssim m \lesssim 10{,}000\text{ GeV}$ and $0.005 \lesssim g_{\mathfrak{f}\phi} \lesssim 1.0$. As explained earlier, the upper limit on the coupling comes from the perturbative limit. Setting $g_{\mathfrak{f}\phi} = 1$, we find that the corresponding value of ELKO mass is approximately 10,000 GeV.

So far in this section, we have only considered the processes involving ELKO and the Higgs, but there is another type of process that, *a priori*, might also be relevant for maintaining ELKO fermions in equilibrium with the cosmic plasma. An example of these is shown in Fig. 4. These involve the coupling of ELKO with the electromagnetic field tensor. However, as we shall see, direct dark matter searches impose severe restriction on this coupling. Hence these processes do not give any significant contribution.
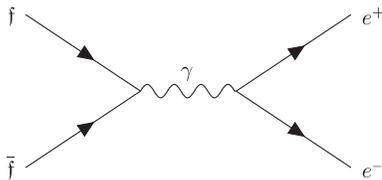
FIG. 4. Annihilation process for ELKOs.

IV. LIMITS ON ELKO FROM DIRECT DARK MATTER SEARCHES

We next consider the limits on ELKO couplings imposed by the direct dark matter searches using the CDMS II [42] results. For this purpose, we consider the scattering of ELKO with proton in nonrelativistic limit. We first determine the constraint on the ELKO-photon coupling and next on the ELKO-Higgs coupling.

A. Constraints on the ELKO-photon coupling

The dominant contribution to the ELKO-proton scattering due to the ELKO electromagnetic coupling is given by the t-channel process, $\mathfrak{f}(k)p(p) \to \mathfrak{f}(k')p(p')$, shown in Fig. 5.

The invariant amplitude for this process is given by

$$i\mathcal{M} = 4ig_{\mathfrak{f}}(\overset{\neg S}{\lambda}_{\alpha'}(k')\sigma^{\mu\nu}\lambda^S_{\beta'}(k))\frac{q_\mu g_{\nu\sigma}}{mq^2}$$
$$\times ie\bar{u}_{s'}(p')\left[F_1\gamma^\sigma + \frac{\kappa}{2m_p}F_2 i\sigma^{\sigma\alpha}q_\alpha\right]u_s(p) \quad (20)$$

where $F_1(q^2)$, $F_2(q^2)$ are the proton form factors, m_p is the mass of proton and κ is the anomalous magnetic moment of proton. The momentum transfer in the process is $q = p' - p$. The amplitude squared becomes

$$|\mathcal{M}|^2 = \frac{16g_{\mathfrak{f}}^2 e^2 q_\mu q_\kappa}{m^2 q^4}(\overset{\neg S}{\lambda}_{\alpha'}\sigma^{\mu\nu}\lambda^S_{\beta'})(\lambda^{S\dagger}_{\beta'}\sigma^{\kappa\tau\dagger}\overset{\neg S\dagger}{\lambda}_{\alpha'})$$
$$\times \text{Tr}\left[(\not{p}' + m_p)\left(F_1\gamma_\nu + \frac{\kappa}{2m_p}F_2 i\sigma_\nu^\alpha q_\alpha\right)\right.$$
$$\left.\times (\not{p} + m_p)\left(F_1\gamma_\tau - \frac{\kappa}{2m_p}F_2 i\sigma_\tau^\rho q_\rho\right)\right]. \quad (21)$$

Since ELKOs are dark matter candidates, we assume that they are moving in random directions with respect to the Milky Way center. We consider an incoming proton, coming from the z-direction i.e. $p_\mu = (E_p, 0, 0, -p_3)$, with velocity $v = 232$ km/s, which is equal to the speed of Sun around the galactic center. We consider its scattering with an ELKO at rest. The proton recoil energy turns out to be of
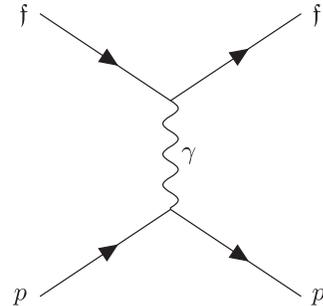
FIG. 5. ELKO-Proton Scattering by exchange of a photon.
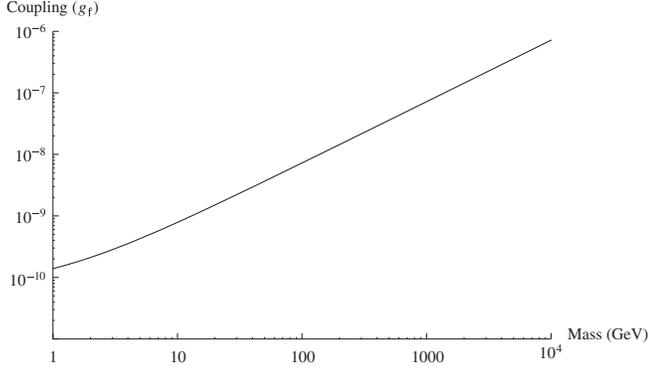
Coupling ($g_{\mathfrak{f}}$)

FIG. 6. The constraint imposed by CDMS II on the ELKO electromagnetic coupling $g_{\mathfrak{f}}$ as a function of the ELKO mass. Only the region below the line is allowed.

order 10 KeV. In the nonrelativistic limit, $F_1(q^2 \approx 0) = 1$, $F_2(q^2 \approx 0) = 1$. The scattering cross section in this limit is found to be,

$$\sigma = \frac{(1.507 \times 10^6 + 97382 \cos(\phi - \phi'))g_{\mathfrak{f}}^2}{m_p^2 + m^2 + 2mE_p} \qquad (22)$$

where ϕ and ϕ' are the azimuthal angles of the momenta of the initial and final state ELKOs. We point out that for the initial state ELKO which is at rest, we first assume a nonzero momentum that is later set to zero. Integrating over ϕ', the cross section becomes

$$\sigma = \frac{9.47 \times 10^6 g_{\mathfrak{f}}^2}{m_p^2 + m^2 + 2mE_p}. \qquad (23)$$

By using their silicon detectors, CDMS II [42] imposed an upper-bound on the WIMP-nucleon scattering cross section σ at 1.9×10^{-41} cm^2 (0.019 fb). The limits on the coupling, $g_{\mathfrak{f}}$, for different ELKO masses, shown in Fig. 6, are obtained by using the CDMS II limit in Eq. (23). Only the region below the line is allowed. For this range of parameters, we find that the coupling $g_{\mathfrak{f}}$ gives negligible contribution for cosmic evolution of ELKOs. In order for this coupling to give a significant contribution to the scattering cross section of ELKOs with cosmic plasma, its value would have to be larger than 0.001 which is far above the limit allowed by CDMS II. Hence ELKO acts as a dark matter candidate predominantly through its interaction with the Higgs.

B. Constraints on the ELKO-Higgs coupling

We next determine the constraint on the ELKO-Higgs coupling, $g_{\mathfrak{f}\phi}$, imposed by CDMS II dark matter search. Expanding scalar field ϕ around the classical ground state [19]

$$\phi = \frac{1}{\sqrt{2}}(H + v), \qquad v = 246 \text{ GeV}, \qquad (24)$$

we obtain

$$\mathcal{L} = -\frac{1}{2} g_{\mathfrak{f}\phi} \vec{\mathfrak{f}}(x)\mathfrak{f}(x)H^2(x) - g_{\mathfrak{f}\phi} v \vec{\mathfrak{f}}(x)\mathfrak{f}(x)H$$
$$\qquad - \frac{1}{2} g_{\mathfrak{f}\phi} \vec{\mathfrak{f}}(x)\mathfrak{f}(x)v^2. \qquad (25)$$

The 2nd term in Eq. (25) gives the 3-point Higgs-ELKO-ELKO vertex. Using this vertex we study the scattering of ELKO off proton in nonrelativistic limits. The Feynman diagram for the ELKO-proton scattering with Higgs as intermediate particle is shown in Fig. 7.

The amplitude for this process is given by

$$i\mathcal{M} = \left(\frac{g_{\mathfrak{f}\phi} v}{m}\right)(\vec{\lambda}_\alpha^S(k')\lambda_\beta^S(k)) \frac{i}{q^2 - m_H^2}$$
$$\qquad \times \left(\frac{m_p F_H}{v}\right) \bar{u}^{s'}(p')u^s(p) \qquad (26)$$

where, as before, $q = p' - p$ is the momentum transferred. The factor $m_p F_H/v$ is the low-energy effective coupling of the Higgs with proton. Here F_H is the Higgs-proton form factor whose value has been estimated to be approximately 0.35 [43–47] in the limit $q^2 \approx 0$. In the approximation $q^2 \ll m_H^2$, we have

$$|\mathcal{M}|^2 = \frac{g_{\mathfrak{f}\phi}^2 m_p^2 F_H^2}{m_H^4 m^2}|(\vec{\lambda}_\alpha^S(k')\lambda_\beta^S(k))(\bar{u}^{s'}(p')u^s(p))|^2$$
$$\qquad = \frac{g_{\mathfrak{f}\phi}^2 m_p^2 F_H^2}{m_H^4 m^2} 4(EE' - kk'\cos(\theta - \theta'))$$
$$\qquad \times (1 + \cos(\phi - \phi')) \times 4(p.p' + m_p^2). \qquad (27)$$

In the nonrelativistic limit the cross-section is given by

$$\sigma = \frac{g_{\mathfrak{f}\phi}^2 m_p^2 F_H^2}{(64\pi^2 s)(4m_H^4 m^2)}$$
$$\qquad \times [16\pi^2((4EE' - \pi kk'\sin\theta)(8m_p^2))]. \qquad (28)$$

FIG. 7. Proton scattering with ELKO.

Coupling [$g_{\bar{f}\phi}$]

FIG. 8.   The constraint imposed by CDMS II on the ELKO-Higgs coupling, $g_{\bar{f}\phi}$. For larger values of the ELKO mass, the lower limit on $g_{\bar{f}\phi}$ is larger than 1.

Assuming an isotropic incident ELKO flux, we obtain, after integrating over θ, in the limit k, $k' \rightarrow 0$,

$$\sigma = \frac{4g_{\bar{f}\phi}^2 m_p^4 F_H^2}{m_H^4 (m_p + m)^2}. \tag{29}$$

Using the CDMS II constraint on this cross section, the limit imposed on the coupling, $g_{\bar{f}\phi}$, as a function of ELKO mass is shown in Fig. 8. The region below the line is the allowed range for the ELKO mass and the coupling, $g_{\bar{f}\phi}$. We point out that here also we have imposed an upper limit on the coupling such that, $g_{\bar{f}\phi} < 1$. The lower limit on $g_{\bar{f}\phi}$ turns out to be greater than unity for larger values of the ELKO mass. As discussed earlier, with this constraint the higher order effects are expected to be smaller than 10%. We find that the CDMS II result does not produce any constraint on the parameter range, shown in Fig. 3, for which ELKO acts as a cold dark matter candidate.

V. CONSTRAINT ON THE ELKO–HIGGS COUPLING FROM GAMMA-RAY BURSTS

The ELKO fermions break Lorentz invariance due to the existence of a preferred axis. Hence they may induce Lorentz violating corrections in the photon dispersion relation through loop effects. There exist stringent constraints [48] on such effects due to data from gamma-ray bursts (GRBs) observed by Fermi–LAT [49]. In this section we determine the constraints imposed by this data on the coupling of ELKO fermions with the Higgs particle.

The modified photon dispersion relation in vacuum in a Lorentz violating theory can be parametrized as [48,50]

$$E^2 = |\vec{p}|^2 c^2 [1 - s_{\pm} \beta E^n] \tag{30}$$

where E is the energy, $\vec{p}$ the three momentum, β and n parametrize the Lorentz violating effects and $s_{\pm} = \pm 1$ is the sign of Lorentz violation. If the Lorentz violation is attributed to quantum gravity effects, then we have $\beta = \frac{1}{(E_{QG})^n}$, where E_{QG} is the scale of quantum gravity. In the present case, however, β is just a parameter which

characterizes the Lorentz violation contribution due to ELKOs and has no apparent relationship to the scale of quantum gravity. The relationship in Eq. (30), implies that the photon group velocity depends upon the photon energy. Here the sign $s_{\pm} = -1(+1)$ corresponds to an increase (decrease) in photon velocity with an increasing photon energy. Hence, two photons of different energies, E_h and E_l ($E_h > E_l$) emitted by a distant point source at the same instant will reach Earth with a time difference Δt. This time difference is related to the Lorentz invariance violation parameter τ_n, defined as [50],

$$\tau_n \equiv \frac{\Delta t}{E_h^n - E_l^n} \approx s_{\pm} \frac{\beta(1 + n)}{2H_0} \times \int_0^z \frac{(1 + x)^n dx}{\sqrt{\Omega_\Lambda + \Omega_M (1 + x)^3}}. \tag{31}$$

Here H_0 is Hubble constant, z is redshift, Ω_M and Ω_Λ are matter and energy density, respectively. As explained in [48] the data analysis can also be generalized to the case of a real GRB in which the photons are not emitted at the same time.

We next determine the change in the photon propagator due to an exchange of the ELKO particles. The leading order diagram which contributes due to the Higgs-ELKO coupling is shown in Fig. 9. The correction to the propagator leads to a modified dispersion relationship for the photons, from which we can extract the parameters β and n of Eq. (30). These can be used to calculate τ_n using the relation Eq. (31) for the redshifts corresponding to different GRBs.

In the Higgs effective field theory *(heft)* [51–53], the coupling of the Higgs with photons is mediated by top quark and W boson loops. The effective loop induced interaction Lagrangian can be written as,

$$L_{\text{heft}} = -\frac{1}{4} g F_{\mu\nu} F^{\mu\nu} H, \tag{32}$$

where the coupling constant g is given by

$$\begin{aligned}
g = &-\frac{\alpha}{\pi v} \frac{47}{18} \left(1 + \frac{66}{235} \tau_w + \frac{228}{1645} \tau_w^2 + \frac{696}{8225} \tau_w^3 \right. \\
&+ \frac{5248}{90475} \tau_w^4 + \frac{1280}{29939} \tau_w^5 + \frac{54528}{1646645} \tau_w^6 - \frac{56}{705} \tau_t \\
&\left. - \frac{32}{987} \tau_t^2 \right).
\end{aligned} \tag{33}$$

FIG. 9.   Loop correction to the photon propagator.

Here $\tau_t = \frac{m_h^2}{4m_t^2}$ and $\tau_w = \frac{m_h^2}{4m_W^2}$. Hence the amplitude for the diagram shown in Fig. 9 can be written as

$$i\Pi^{\mu\nu} = \frac{g^2 g_{\bar{f}\phi}^2 v^2}{4} \int \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} \frac{I^{\mu\nu}}{(k^2 - m_h^2 + i\epsilon)^2}$$
$$\times \frac{\text{Tr}((\mathbb{G}(\phi) + \mathbb{I})(\mathbb{G}(\phi'') + \mathbb{I}))}{(l^2 - m^2 + i\epsilon)((k+l)^2 - m^2 + i\epsilon)((p-k)^2 + i\epsilon)}.$$
(34)

Here $p' = p - k$, ϕ and ϕ'' are the azimuthal angles of $\vec{l}$ and $\vec{k} + \vec{l}$, respectively and

$$I^{\mu\nu} = (p.p')^2 \left(g^{\mu\nu} - \frac{p'^\mu p'^\nu}{p'^2} \right) - (p.p') \left(g^{\nu\lambda} - \frac{p'^\nu p'^\lambda}{p'^2} \right) (p'^\mu p_\lambda)$$
$$- (p_\sigma p'^\nu) \left(g^{\sigma\mu} - \frac{p'^\sigma p'^\mu}{p'^2} \right) (p'.p)$$
$$+ (p_\sigma p'^\nu) \left(g^{\sigma\lambda} - \frac{p'^\sigma p'^\lambda}{p'^2} \right) (p'^\mu p_\lambda).$$
(35)

The vacuum polarization $\Pi^{\mu\nu}$ satisfies the Ward identity, i.e. $p_\mu \Pi^{\mu\nu}(p) = 0$. By gauge invariance, $\Pi^{\mu\nu}(p)$ is proportional to the $(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2})$, i.e.

$$\Pi^{\mu\nu}(p) = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \Pi(p^2, E).$$
(36)

Using Eqs. (34) and (36), we find that the leading order correction to the propagator is given by,

$$\Pi(p^2, E)$$
$$= \frac{g^2 g_{\bar{f}\phi}^2 v^2}{6} \int \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} \frac{(p.p')^2}{(k^2 - m_h^2 + i\epsilon)^2}$$
$$\times \frac{\text{Tr}((\mathbb{G}(\phi) + \mathbb{I})(\mathbb{G}(\phi'') + \mathbb{I}))}{(l^2 - m^2 + i\epsilon)((k+l)^2 - m^2 + i\epsilon)((p-k)^2 + i\epsilon)}.$$
(37)

It is not practical to use the standard Feynman parametrization for evaluating this integral because of the azimuthal angle dependent factors $\mathbb{G}(\phi)$ and $\mathbb{G}(\phi'')$ in the numerator. Instead, we use a different approach to estimate it. We evaluate the dl_0 and dk_0 integral analytically and then, for different photon energies, integrate over $d^3\vec{k}$, $d^3\vec{l}$ numerically using Monte-Carlo integration routine.

The correction to the dispersion relation, $-s_\pm \beta E^n$ is equal to $\Pi(p^2, E)/|\vec{p}|^2$. Since this term is expected to be a small correction, we can consistently set $p^2 = E^2 - |\vec{p}|^2 = 0$ in its evaluation. We are primarily interested in the energy regime $0.1 \text{ GeV} < E < 10 \text{ GeV}$ which overlaps closely with the range of energy of the events observed in GRBs

080916C, 090510, 090902B, 090926A studied in [48]. The result depends on the direction of photon propagation since the basic framework violates Lorentz invariance through the appearance of factors, such as, $\mathbb{G}(\phi)$, in the spin sums. However we find that the result does not depend qualitatively on the direction of propagation and fix the direction such that in our chosen frame the spherical polar coordinates of the photon momentum are $\theta = \phi = \pi/4$. We have verified that the order of magnitude of the final answer does not change with choice of propagation direction. We find that for $E \ll 0.1 \text{ GeV}$ and for $E \gg 10 \text{ GeV}$, the correction factor is almost independent of energy, i.e. corresponds to $n = 0$. However in the range $0.1 \text{ GeV} < E < 10 \text{ GeV}$ we find a small decrease in the correction factor. We restrict ourselves to this energy range while determining the effective value of n. We define a parameter β' such that $\beta = \beta' g_{\bar{f}\phi}^2$. The resulting extracted values of β' and n for different choices of ELKO masses are given in Table I.

The fact that our Lorentz violating correction to the dispersion relations is not proportional to either E or E^2, as is often assumed within the framework of quantum gravity [48,54], is not surprising. The current framework is closest to the Very Special Relativity (VSR) invariant theories which tend to show dominant deviation from Lorentz violation at low energies due to the presence of nonlocal contributions [10,55]. The ELKO framework is somewhat unique since the Lorentz violating terms appear explicitly only in the spin sum and not the action. Hence it is expected to deviate both from the VSR invariant theories, as proposed in [10], and the expectation that Lorentz violating effects might increase with energy as E or E^2 due to quantum gravity effects.

We next use the GRB data to impose a limit on the parameter β and hence on the ELKO coupling, $g_{\bar{f}\phi}$. A detailed data analysis for this purpose is rather complicated and beyond the scope of the present paper. Here we restrict ourselves to extracting an order of magnitude estimate of the limit. In Ref. [48], GRB data was used in order to impose a limit on the quantum gravity scale E_{QG} for $n = 1$, 2. They used the GRBs 080916C, 090510, 090902B, 090926A for this calculation. The data for these bursts is mostly confined to energies less than 10 GeV. In fact most of the data lies in the range $E < 1 \text{ GeV}$ and in Ref. [48] the authors impose a lower limit of 30 MeV. Here

TABLE I. Parameter β' and n for different ELKO masses.

Mass(m) in GeV	β'	n
100	1.33×10^{-6}	-0.18
500	1.30×10^{-6}	-0.12
1000	1.22×10^{-6}	-0.12
2000	9.41×10^{-7}	-0.12
5000	7.05×10^{-7}	-0.12
9500	5.39×10^{-7}	-0.12

we directly use their extracted value of τ_n with $n = 1$ and make an estimate of Δt setting $E_h = 1$ GeV and $E_l = 0.1$ GeV. For all the GRBs it is found that $|\tau_n| \lesssim 1$ s/GeV. Hence we set $|\tau_n| \approx 1$ s/GeV which leads to $\Delta t \approx 1$ sec. We find that the extracted value of Δt does not show a strong dependence on E_l or the chosen value of n, i.e. the value obtained with $n = 2$ is not too different from that corresponding to $n = 1$. Using this value of Δt in Eq. (31), and the β' and n values given in Table I we obtain an order of magnitude estimate of the limit on $g_{\mathfrak{f}\phi}$.

We find that for all the GRBs, 080916C, 090510, 090902B, 090926A and for the entire range of ELKO mass values given in Table I, the limiting values of $g_{f\phi}$ lie in the range 10^{-5} to 10^{-6}. Hence we obtain a conservative upper limit $g_{\mathfrak{f}\phi} < 10^{-5}$. This implies that the Fermi-LAT data actually rules out the parameter space we obtained by demanding that ELKO acts as a CDM candidate subject to the limits imposed by direct detection experiments. Hence we conclude that ELKO fermion cannot be a dominant CDM candidate. It can of course still contribute as a subdominant cold dark matter candidate. Alternatively ELKOs might never have been in equilibrium with the cosmic plasma. In this case they may still contribute significantly to the energy density of the dark matter despite the limit due to GRBs. However we have not investigated this in this paper.

VI. LIMIT ON THE SELF-COUPLING FROM ASTROPHYSICAL DATA

In the earlier sections we have shown that cosmological and astrophysical observations lead to severe constraints on the coupling of ELKO fermions with Higgs and photons. Indeed the entire parameter regime for which it can contribute significantly as a cold dark matter candidate is ruled out. As already mentioned, the only allowed possibility is that ELKO fermions were never in equilibrium with the cosmic plasma. In this case the constraints imposed by cosmological considerations (see Fig. 3) are not applicable and such fermions can contribute significantly to nonrelativistic dark matter density. In this section we assume such a scenario and determine implications of the ELKO self-interaction term of Eq. (9), $g_{\mathfrak{f}\mathfrak{f}}(\overset{\neg}{\mathfrak{f}}(x)\mathfrak{f}(x))^2$, for the nonrelativistic dark matter cores in galactic halos.

The self-interacting nonrelativistic dark matter has been proposed to solve the problem with the small scale structure formation of the Universe. The density of dark matter cores in the galactic centers is observed to be lower than the value predicted by weakly interacting nonrelativistic dark matter. The lower density can be explained by invoking collisional (self-interacting) dark matter. In this model, the dark matter has large scattering cross section and negligible annihilation rate. Assuming that the ELKO particles are non-relativistic, the scattering cross section $\overset{\neg}{\mathfrak{f}}\mathfrak{f} \to \overset{\neg}{\mathfrak{f}}\mathfrak{f}$ is

$$\sigma_{\overset{\neg}{\mathfrak{f}}\mathfrak{f}} = \frac{g_{\mathfrak{f}\mathfrak{f}}^2}{4\pi m^2}. \tag{38}$$

For this scenario to work, the mean free path (λ) of the collisional dark matter should be in the range of 1 Kpc to 1 Mpc at the location of the Sun within the Milky way. Here the mean density of dark matter is 0.4 GeV/cm^3 [56,57]. Using the result for the elastic scattering cross section for such a dark matter [56] and applying this for the ELKO-ELKO scattering we obtain

$$\sigma_{\overset{\neg}{\mathfrak{f}}\mathfrak{f}} = 8.1 \times 10^{-25} \text{ cm}^2 \left(\frac{m}{\text{GeV}}\right) \left(\frac{\lambda}{1 \text{ Mpc}}\right)^{-1}. \tag{39}$$

From Eqs. (38) and (39), we get

$$g_{\mathfrak{f}\mathfrak{f}} = 161.71 \times \left(\frac{m}{\text{GeV}}\right)^{3/2} \left(\frac{\lambda}{1 \text{ Mpc}}\right)^{-1/2}. \tag{40}$$

Typical range of self-interacting dark matter mass is 1 MeV to 10 GeV [56], so depending upon the mean free path, the ELKO self-coupling is constrained by the above relation. In particular as we vary λ from 1 Mpc to 1 Kpc the minimum value of coupling $g_{\mathfrak{f}\mathfrak{f}}$ is found to vary from 0.005 to 0.16. These values are obtained by setting the ELKO mass $m = 1$ MeV. For the range of λ and m values quoted above, the upper limit on the coupling exceeds unity.

VII. CONCLUSION

The ELKO fermion is an interesting and natural dark matter candidate. By its very existence it violates Lorentz invariance and respects only a subgroup. By its intrinsic nature, its interactions with most of the standard model fields are severely restricted. It couples dominantly with the Higgs particle. Hence, in the ELKO proposal we find an interesting prediction that the dark matter sector as well as its coupling to Higgs must violate Lorentz invariance. In the present paper we have made a detailed analysis of the implications of ELKO fermions as a cold dark matter candidate. We find that ELKO acts as a cold dark matter candidate if its mass lies in the range 100 to 10,000 GeV. The upper bound on ELKO mass is obtained by demanding that the Higgs-ELKO coupling $g_{\mathfrak{f}\phi} < 1$, that is, it stays within the perturbative regime. Below the lower limit it will not decouple from cosmic plasma as a nonrelativistic particle. The lower limit on the coupling $g_{\mathfrak{f}\phi}$ is found to be 0.005. However this entire range of coupling is eliminated by the constraint imposed by time delay observations of photons of different energies emitted in gamma ray bursts. This constraint arises since the ELKO fermion induces a Lorentz violating term in the photon dispersion relations. Such a term leads to a delay in arrival times of photons of different energies emitted by gamma ray bursts and hence is constrained by the observed time delay. Hence we conclude that ELKO does not contribute

significantly as a cold dark matter candidate. However it may still contribute significantly to dark matter if it were never in equilibrium with the cosmic plasma.

ELKOs also couple to photon via nonstandard $g_f \bar{\mathfrak{f}}(x)[\gamma_\mu, \gamma_\nu]\mathfrak{f}(x)F^{\mu\nu}(x)$ interaction. We find that this coupling is severely constrained by direct dark matter search experiments, such as CDMS II. However we find that CDMS II does not impose a significant constraint on the ELKO-Higgs coupling.

Finally we have obtained the range of values for the ELKO mass and self-coupling for which it may be consistent with the density of dark matter core in the galactic center. This requires the dark matter to have significant cross section for scattering with other dark matter particles. Hence it provides us with a handle on the self-coupling.

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