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Naturalness-guided gluino mass bound from the minimal mixed mediation of SUSY breaking

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In order to significantly reduce the fine-tuning associated with the electroweak symmetry breaking in the minimal supersymmetric standard model (MSSM), we consider not only the minimal gravity mediation effects but also the minimal gauge mediation ones for a common supersymmetry breaking source at a hidden sector. In this "minimal mixed mediation model," the minimal forms for the Kähler potential and the gauge kinetic function are employed at tree level. The MSSM gaugino masses are radiatively generated through the gauge mediation. Since a "focus point" of the soft Higgs mass parameter, $m_{h_u}^2$ appears around 3–4 TeV energy scale in this case, $m_{h_u}^2$ is quite insensitive to top squark masses. Instead, the naturalness of the small $m_{h_u}^2$ is more closely associated with the gluino mass rather than the top squark mass, unlike the conventional scenario. As a result, even a 3–4 TeV top squark mass, which is known to explain the 125 GeV Higgs mass at three-loop level, can still be compatible with the naturalness of the electroweak scale. On the other hand, the requirements of various fine-tuning measures much smaller than 100 and $|\mu| < 600$ GeV constrain the gluino mass to be 1.6 TeV $\lesssim m_{\tilde{g}} \lesssim$ 2.2 TeV, which is well inside the discovery potential range of LHC Run II.

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I. INTRODUCTION

How to naturally keep the small Higgs boson mass against its quadratically divergent radiative corrections has been one of the most important issues in the particle physics community for the last four decades. Since this question raised in the Standard Model (SM) is associated with stabilization of the EW scale against the grand unified theory (GUT) scale or the Planck scale, many ideas and theories beyond the SM and towards the fundamental theory have been motivated and suggested in order to address this question. The supersymmetric (SUSY) resolution to it is to cancel the quadratic divergences by introducing superpartners with spins different by $1/2$ from those of the SM particles, and their interactions with the same strength as those of the SM. All of them can consistently be controlled within the SUSY framework [1].

Since the top quark and its superpartner the top squark dominantly contribute to the radiative Higgs mass via the large top quark Yukawa coupling, the top squark mass has been regarded as a barometer for naturalness of the minimal SUSY SM (MSSM): a top squark mass lighter than 1 TeV is quite essential for keeping the naturalness of the EW scale and the Higgs boson mass. However, the experimental mass bound on the top squark has already exceeded 700 GeV [2]. Thus, it would be very timely to ask whether the low energy SUSY can still remain natural even with a somewhat heavy top squark mass greater than 1 TeV.

On the other hand, the gluino is not directly involved in this issue, because it does not couple to the Higgs boson at tree level. Instead, the gluino mass dominantly influences the renormalization group (RG) evolution of the top squark mass parameters. In this sense, the gluino affects the Higgs mass parameter $m_{h_u}^2$ just indirectly in the ordinary MSSM. In this paper, however, we attempt to investigate another possibility: the gluino can play a more important role in the naturalness of the small Higgs boson mass. As a consequence, the top squark mass can be much less responsible for it: it can be much heavier than the present experimental bound. Indeed, the gluino can be more easily explored than the top squark at the Large Hadron Collider (LHC). Thus, if a relatively light gluino mass turns out to be needed, this scenario could readily be tested at LHC Run II.

Because of the top quark Yukawa coupling constant y_t of order unity, as mentioned above, the top quark and top squark make the dominant contributions not only to the renormalization of a soft mass parameter of the Higgs h_u ($\equiv \Delta m_{h_u}^2$), but also to the radiative physical Higgs mass ($\equiv \Delta m_H^2$) [1,3]:

$$\Delta m_{h_u}^2|_{\text{1-loop}} \approx \frac{3|y_t|^2}{8\pi^2} \tilde{m}_t^2 \log\left(\frac{\tilde{m}_t^2}{\Lambda^2}\right)\left[1+\frac{1}{2}\frac{A_t^2}{\tilde{m}_t^2}\right], \quad (1)$$

$$\Delta m_H^2|_{\text{1-loop}} \approx \frac{3m_t^4}{4\pi^2 v_h^2}\left[\log\left(\frac{\tilde{m}_t^2}{m_t^2}\right)+\frac{A_t^2}{\tilde{m}_t^2}\left(1-\frac{1}{12}\frac{A_t^2}{\tilde{m}_t^2}\right)\right], \quad (2)$$

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where m_t ($\tilde{m}_t$) denotes the top quark (top squark) mass, and v_h is the vacuum expectation value (VEV) of the Higgs boson, $v_h \equiv \sqrt{\langle h_u \rangle^2 + \langle h_d \rangle^2} \approx 174$ GeV with $\tan\beta \equiv \langle h_u \rangle / \langle h_d \rangle$. For simplicity, here we assumed that the $SU(2)_L$-doublet and $SU(2)_L$-singlet top squarks ("LH and RH top squarks") are degenerate, and the "A-term" coefficient corresponding to the top quark Yukawa coupling, A_t, dominates over $\mu \cdot \cot\beta$, where μ is the "Higgsino" mass. By introducing SUSY, thus, the quadratic dependence on the ultraviolet (UV) cutoff Λ in the SM for $\Delta m_{h_u}^2|_{\text{1-loop}}$ is replaced by a logarithmic one as seen in Eq. (1). For a small enough $\Delta m_{h_u}^2|_{\text{1-loop}}$, however, the top squark mass should necessarily be small enough. Otherwise, the Higgs mass parameters, $m_{h_u}^2$ and $m_{h_d}^2$, should be finely tuned with μ to yield the measured value of the Z boson mass $m_Z \approx 91$ GeV, because they are related to each other via the minimization condition of the Higgs potential [1],

$$\frac{1}{2} m_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2\beta}{\tan^2\beta - 1} - |\mu|^2. \tag{3}$$

As seen in Eq. (2), the radiative correction to the physical Higgs mass depends logarithmically on the top squark mass. Actually the tree level Higgs mass in the MSSM should be lighter even than the Z boson mass ($< m_Z \cdot \cos 2\beta$) [1]. Thus, the radiative Higgs mass Eq. (2) is also quite essential for explaining the observed Higgs boson mass. In view of Eq. (2), however, the recently measured Higgs boson mass, 125 GeV [4] is indeed too heavy as a SUSY Higgs mass, because it would require a too heavy top squark mass ("little hierarchy problem"). Many SUSY models have been proposed for raising the Higgs boson mass by extending the MSSM, but still assuming a relatively light top squark, $\tilde{m}_t \lesssim 1$ TeV [5]. However, the experimental mass bound on the top squark has already exceeded 700 GeV [2], as mentioned above. Of course, the second term in Eq. (2) could be helpful for raising the Higgs mass, when it is almost maximized, $A_t^2/\tilde{m}_t^2 \approx 6$ [1,3]. But it is not easy to realize at low energies from a UV model via its RG running, unless we suppose a tachyonic top squark at the GUT scale (M_G) [6].

According to the recent analysis based on three-loop calculations in Ref. [7], a 3–4 TeV top squark mass can account for the 125 GeV Higgs boson mass with ignorable A_t terms. Such a heavy top squark mass would give rise to a more serious fine-tuning problem associated with the light Z boson mass as seen in Eqs. (1) and (3), particularly, when the cutoff scale Λ is about GUT scale ($\sim 10^{16}$ GeV): apparently a fine-tuning of order 10^{-4} (or $\Delta_{m_0^2} \sim 10^{+4}$ in terms of the fine-tuning measure defined later) looks unavoidable in the MSSM. To more precisely discuss the UV dependence of $m_{h_u}^2$, addressing the little hierarchy problem, however, one should analyze the full RG equations under a given specific UV model. If a SUSY UV model turns out to be simple enough, addressing the above question, SUSY could still be recognized as an attractive solution to the gauge hierarchy problem.

A potentially promising UV model is the "focus point (FP) scenario" [8]. Since it is based on the minimal gravity mediation (mGrM) of SUSY breaking, all the soft squared masses, including the two Higgs mass parameters $m_{h_u}^2$ and $m_{h_d}^2$, LH and RH top squark's squared masses $m_{q_3}^2$ and $m_{u_3^c}^2$, etc., as well as the MSSM gaugino masses, take the universal forms [1,9]:

$$m_{h_u}^2 = m_{h_d}^2 = m_{q_3}^2 = m_{u_3^c}^2 = \cdots \equiv m_0^2 \quad \text{and} \\ M_3 = M_2 = M_1 \equiv m_{1/2}, \tag{4}$$

where $M_{3,2,1}$ denote the gluino, wino, and bino masses, respectively. In this case, as noticed in Ref. [8], the RG flows of $m_{h_u}^2$ converge about the Z boson mass scale to a small negative value, regardless of its initial values taken at the GUT scale, i.e., various m_0^2 values, only if the A_t and $m_{1/2}$ are sufficiently suppressed. Since $m_{h_u}^2$ is almost independent of m_0^2, a small enough $m_{1/2}$ turns out to be responsible for a small negative $m_{h_u}^2$, naturally explaining the smallness of the EW scale or m_Z compared to the GUT or Planck scale. Such a parameter choice can indeed reduce the fine-tuning considerably. Several different definitions of the fine-tuning report a similar tendency around the "FP region" in the MSSM parameter space [10]. On the other hand, the low energy values of other soft mass parameters such as $m_{q_3}^2$ and $m_{u_3^c}^2$ are very sensitive to m_0^2 values. These features in the mGrM might open a possibility to naturally explain the smallness of $m_{h_u}^2$ in contrast to large top squark mass parameters.

However, the experimental gluino mass bound has already exceeded 1.3 TeV [11], and so the unified gaugino mass $m_{1/2}$ cannot be small any longer. Also the naturalness on a small A-term would be questionable. Most of all, if the top squark masses are around 3–4 TeV, they should decouple below the 3–4 TeV energy scale from the ordinary MSSM RG equations, and so the FP behavior of $m_{h_u}^2$ becomes seriously spoiled below the top squark mass scale [12]. Basically the FP scale in the mGrM is too far below the top squark mass scale desired for explaining the 125 GeV Higgs boson mass. All such problems in the FP scenario arise because heavier masses for the Higgs, top squark, and gluino are experimentally and/or theoretically compelled.

The best resolution to such problems would be to somehow push the FP scale from the Z boson mass scale to the desired top squark mass scale ("shifted FP" [13]) such that the m_0^2 dependence of $m_{h_u}^2$ becomes suppressed before top squarks are decoupled from the RG equation of

$m_{h_u}^2$ [12,13]. Actually, it is indispensable for restoring the naturalness of the low energy SUSY in the framework of the FP scenario. $m_{h_u}^2$ below the top squark mass scale or at the Z boson mass scale can be estimated using the Coleman-Weinberg potential [1,14]:

$$m_{h_u}^2(m_Z) \approx m_{h_u}^2(\Lambda_T) + \frac{3|y_t|^2}{16\pi^2}\left[\sum_{i=q_3,u_3^c} m_i^2\left\{\log\frac{m_i^2}{\Lambda_T^2} - 1\right\} - 2m_t^2\left\{\log\frac{m_t^2}{\Lambda_T^2} - 1\right\}\right]\Bigg|_{\Lambda_T}$$
$$\approx m_{h_u}^2(\Lambda_T) - \frac{3|y_t|^2}{16\pi^2}\left\{m_{q_3}^2 + m_{u_3^c}^2\right\} \times \left[1 - \frac{m_{q_3}^2 - m_{u_3^c}^2}{2(m_{q_3}^2 + m_{u_3^c}^2)}\log\frac{m_{q_3}^2}{m_{u_3^c}^2}\right]\Bigg|_{\Lambda_T}, \quad (5)$$

where the cutoff Λ_T is set to the top squark decoupling scale ($\approx\sqrt{m_{q_3}m_{u_3^c}}$). The last term of the second line in Eq. (5) is relatively suppressed. Since the m_0^2 dependence of top squark masses would be loop suppressed, $m_{h_u}^2$ needs to be well focused around Λ_T. Due to the additional negative contribution to $m_{h_u}^2(m_Z)$ below Λ_T, a small positive $m_{h_u}^2(\Lambda_T)$ would be more desirable.

In order to push the FP scale up to the desired top squark mass scale, 3–4 TeV, we will consider the gauge mediation effects as well as the mGrM effects for a common SUSY breaking source at the hidden sector, introducing some messenger fields: we will attempt to combine the two representative SUSY breaking mediation scenarios, the mGrM and the minimal gauge mediation (mGgM) at the GUT scale in a single supergravity (SUGRA) framework [13]. We call it the "minimal mixed mediation" of SUSY breaking. For a qualitative understanding on the FP behaviors, in this paper we will present the semianalytic solutions to the relevant RG equations for small $\tan\beta$ cases. Also we will perform their full numerical analyses for large $\tan\beta$ cases. Based on these results, we will explore the parameter space that can naturally explain the small Higgs mass parameter, and then derive the gluino mass bound consistent with it.

This paper is organized as follows: in Sec. II we will present semianalytic RG solutions for $m_{h_u}^2$ and the top squark masses in the MSSM with a small $\tan\beta$. They will be utilized in the subsequent sections. We will leave the details of their derivations to the Appendix. In Sec. III, we will discuss why the fine-tunings become more serious in the mGrM with relatively heavy top squark masses. In Sec. IV, we will introduce the minimal mixed mediation of SUSY breaking and show that it significantly reduces the fine-tunings of the MSSM. In this section, we will derive a proper gluino mass bound consistent with the naturalness of the EW scale and the Higgs boson mass. Section V will be devoted to the Conclusion.

II. SEMIANALYTIC RG SOLUTIONS

In this section, we will first present our semianalytic solutions to the RG equations of some soft SUSY breaking mass parameters in small $\tan\beta$ cases. When $\tan\beta$ is large, the expressions on them are not simple enough, and so one should perform a full numerical analysis. As will be seen later, however, large $\tan\beta$ cases turn out to be much more useful for reducing the fine-tuning of the EW scale. Nonetheless, discussions on the small $\tan\beta$ case would be helpful for a qualitative understanding on the structure of the FP of $m_{h_u}^2$ and for getting an intuition on how to resolve the problem.

When $\tan\beta$ is small enough and the RH neutrinos are decoupled (by assuming their small Yukawa couplings), the RG evolutions of the soft mass parameters, $m_{q_3}^2$, $m_{u_3^c}^2$, $m_{h_u}^2$, and A_t are described with the following simple equations [1]:

$$16\pi^2\frac{dm_{q_3}^2}{dt} = 2y_t^2(X_t + A_t^2) - \frac{32}{3}g_3^2M_3^2 - 6g_2^2M_2^2 - \frac{2}{15}g_1^2M_1^2, \quad (6)$$

$$16\pi^2\frac{dm_{u_3^c}^2}{dt} = 4y_t^2(X_t + A_t^2) - \frac{32}{3}g_3^2M_3^2 - \frac{32}{15}g_1^2M_1^2, \quad (7)$$

$$16\pi^2\frac{dm_{h_u}^2}{dt} = 6y_t^2(X_t + A_t^2) - 6g_2^2M_2^2 - \frac{6}{5}g_1^2M_1^2, \quad (8)$$

$$8\pi^2\frac{dA_t}{dt} = 6y_t^2A_t - \frac{16}{3}g_3^2M_3 - 3g_2^2M_2 - \frac{13}{15}g_1^2M_1, \quad (9)$$

where t parametrizes the renormalization scale Q, $t - t_0 = \log\frac{Q}{M_G}$, and X_t is defined as $m_{q_3}^2 + m_{u_3^c}^2 + m_{h_u}^2$. Here we neglected the bottom quark Yukawa coupling y_b, the sbottom quark's squared mass $m_{d_3^c}^2$, and also the leptonic contributions due to the smallness of $\tan\beta$. In the above equations, the RG evolutions for the MSSM gauge couplings $g_{3,2,1}$ and the gaugino masses $M_{3,2,1}$ are already well known [1]:

$$g_a^2(t) = \frac{g_0^2}{1 - \frac{g_0^2}{8\pi^2}b_a(t - t_0)}, \quad \text{and} \quad \frac{M_a(t)}{g_a^2(t)} = \frac{m_{1/2}}{g_0^2}, \quad (10)$$

where g_0 and $m_{1/2}$ denote the unified gauge coupling constant and the unified gaugino mass, respectively, and b_a ($a = 3, 2, 1$) means the beta function coefficients for the MSSM field contents, $(b_3, b_2, b_1) = (-3, 1, \frac{33}{5})$. For the full RG equations valid when $\tan\beta$ is large, refer to the Appendix of Ref. [12]. The semianalytic solutions for $m_{q_3}^2$, $m_{u_3^c}^2$, and $m_{h_u}^2$ turn out to take the following forms:

$$m_{q_3}^2(t) = m_{q_30}^2 + \frac{X_0}{6}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2} - 1\right] + \frac{F(t)}{6} + \left(\frac{m_{1/2}}{g_0^2}\right)^2\left[\frac{8}{9}\{g_3^4(t)-g_0^4\} - \frac{3}{2}\{g_2^4(t)-g_0^4\} - \frac{1}{198}\{g_1^4(t)-g_0^4\}\right], \quad (11)$$

$$m_{u_3^c}^2(t) = m_{u_3^c0}^2 + \frac{X_0}{3}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2} - 1\right] + \frac{F(t)}{3} + \left(\frac{m_{1/2}}{g_0^2}\right)^2\left[\frac{8}{9}\{g_3^4(t)-g_0^4\} - \frac{8}{99}\{g_1^4(t)-g_0^4\}\right], \quad (12)$$

$$m_{h_u}^2(t) = m_{h_u0}^2 + \frac{X_0}{2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2} - 1\right] + \frac{F(t)}{2} - \left(\frac{m_{1/2}}{g_0^2}\right)^2\left[\frac{3}{2}\{g_2^4(t)-g_0^4\} + \frac{1}{22}\{g_1^4(t)-g_0^4\}\right], \quad (13)$$

where the subscript 0 in $m_{q_30}^2$, $m_{u_3^c0}^2$, $m_{h_u0}^2$, and X_0 ($\equiv m_{q_30}^2 + m_{u_3^c0}^2 + m_{h_u0}^2$) means the values of the corresponding mass parameters at the GUT scale, or $t = t_0 \equiv \log(M_G/\mathrm{GeV})$. In these solutions, $F(t)$ is given by

$$\begin{aligned} F(t) \equiv{}& \frac{1}{64\pi^4}\left(\frac{m_{1/2}}{g_0^2}\right)^2\left[\left(e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\int_{t_0}^t dt' G_A e^{\frac{-3}{4\pi^2}\int_{t_0}^{t'} dt'' y_t^2}\right)^2 - 2e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\int_{t_0}^t dt' G_A \int_{t_0}^{t'} dt'' G_A e^{\frac{-3}{4\pi^2}\int_{t_0}^{t''} dt''' y_t^2}\right] \\ &- \frac{1}{4\pi^2}\left(\frac{m_{1/2}}{g_0^2}\right)^2\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\int_{t_0}^t dt' G_X^2 e^{\frac{-3}{4\pi^2}\int_{t_0}^{t'} dt'' y_t^2} - \int_{t_0}^t dt' G_X^2\right] \\ &+ \frac{A_0}{4\pi^2}\left(\frac{m_{1/2}}{g_0^2}\right)e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\left[\int_{t_0}^t dt' G_A - e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\int_{t_0}^t dt' G_A e^{\frac{-3}{4\pi^2}\int_{t_0}^{t'} dt'' y_t^2}\right] \\ &+ A_0^2 e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2} - 1\right], \end{aligned} \quad (14)$$

where $A_0 \equiv A_t(t = t_0)$, and G_A and G_X^2 are defined as

$$G_A(t) \equiv \left[\frac{16}{3}g_3^4(t) + 3g_2^4(t) + \frac{13}{15}g_1^4(t)\right] \quad \text{and}$$
$$G_X^2(t) \equiv \left[\frac{16}{3}g_3^6(t) + 3g_2^6(t) + \frac{13}{15}g_1^6(t)\right], \quad (15)$$

respectively. For details of the above solutions, refer to the Appendix. Numerical calculation shows that the sign of $F(t)$ is negative, and $|F(t)/2|$ is larger than the second line of Eq. (13), which is positive. Consequently larger values of $(m_{1/2}/g_0^2)$ and A_0 lead to large negative values of $m_{h_u}^2$ at low energies [12].

The initial values, $m_{q_30}^2$, $m_{u_3^c0}^2$, and $m_{h_u0}^2$, should be determined by a UV model. They would be associated with a SUSY breaking mechanism. We will discuss it in the following sections.

III. MINIMAL GRAVITY MEDIATION

The FP scenario is based on the mGrM model. In this section, we will first review the mGrM of SUSY breaking, particularly investigating the UV boundary conditions on the relevant soft mass parameters, and then discuss the FP in the mGrM model.

A. Basic setup in the minimal gravity mediation

The $N = 1$ SUGRA Lagrangian is described basically with the Kähler potential K, superpotential W, and gauge kinetic function f_{ab}. In the mGrM scenario or minimal SUGRA (mSUGRA) model, particularly, the minimal form of the Kähler potential is employed, and the superpotentials of the hidden and observable sectors are separated:

$$K = \sum_i |z_i|^2 + \sum_r |\phi_r|^2, \qquad W = W_H(z_i) + W_O(\phi_r), \quad (16)$$

where z_i (ϕ_r) denotes scalar fields in the hidden (observable) sector. The kinetic terms of z_i and ϕ_r, hence, have the canonical form. For the hidden sector scalar fields z_i's and the hidden sector superpotential W_H, nonzero VEVs are assumed [9]:

$$\langle z_i \rangle = b_i M_P, \quad \langle \partial_{z_i} W_H \rangle = a_i^* m M_P, \quad \langle W_H \rangle = m M_P^2, \quad (17)$$

where a_i and b_i are dimensionless numbers and M_P ($\approx 2.4 \times 10^{18}$ GeV) means the reduced Planck mass. Then, $\langle W_H \rangle$ or m yields the gravitino mass, $m_{3/2} = e^{\langle K \rangle/(2M_P)}|\langle W \rangle|/M_P^2 = e^{\sum_i |b_i|^2/2} m$.

The soft SUSY breaking terms can read from the scalar potential in SUGRA:

$$V_F = e^{\frac{K}{M_P^2}}\left[\sum_i |F_{z_i}|^2 + \sum_r |F_{\phi_r}|^2 - \frac{3}{M_P^2}|W|^2\right], \tag{18}$$

where the "F-terms," F_X $[= (D_X W)^* = (\partial_X W + \partial_X K W/M_P^2)^*]$, are, in the minimal SUGRA, given by

$$F_{z_i}^* = \frac{\partial W_H}{\partial z_i} + z_i^* \frac{W}{M_P^2} = M_P\left[(a_i^* + b_i^*)m + b_i^* \frac{W_O}{M_P^2}\right],$$
$$F_{\phi_r}^* = \frac{\partial W_O}{\partial \phi_r} + \phi_r^* \frac{W}{M_P^2} = \frac{\partial W_O}{\partial \phi_r} + \phi_r^*\left(m + \frac{W_O}{M_P^2}\right). \tag{19}$$

Note that VEVs of F_{z_i} are of order $\mathcal{O}(mM_P)$. For the vanishing cosmological constant (C.C.), a fine-tuning between $\langle F_{z_i}\rangle$ and $\langle W_H\rangle$, $\sum_i \langle |F_{z_i}|^2\rangle = 3|\langle W_H\rangle|^2/M_P^2$, or $\sum_i |a_i + b_i|^2 = 3$, is required from Eq. (18). Neglecting the Planck-suppressed nonrenormalizable terms, Eq. (18) is rewritten as [9]

$$V_F \approx |\partial_{\phi_r}\tilde{W}_O|^2 + m_0^2|\phi_r|^2 + m_0[\phi_r \partial_{\phi_r}\tilde{W}_O + (A_\Sigma - 3)\tilde{W}_O + \text{H.c.}], \tag{20}$$

where summations for ϕ_r are assumed. A_Σ is defined as $A_\Sigma \equiv \sum_i b_i^*(a_i + b_i)$ and m_0 is identified with the gravitino mass $m_{3/2}$ $(= e^{\sum_i |b_i|^2/2} m)$. $\tilde{W}_O$ $(\equiv e^{\sum_i |b_i|^2/2} W_O)$ means the rescaled W_O. From now on, we will drop out the "tilde" for simplicity. In Eq. (20), the first term is nothing but the F-term scalar potential in global SUSY. The second and other terms imply that the soft scalar mass terms and soft SUSY breaking A-terms parametrized with m_0 are *universal* at the GUT scale in the mGrM. If there are no quadratic or higher powers of ϕ_r in W_O, one can get negative (positive) A-terms with $A_\Sigma < 2$ $(A_\Sigma > 2)$. Here the *universal* A-parameter $(\equiv A_0 = A_t)$ does not include Yukawa coupling constants, but it is proportional to m_0. We will set the universal A-term to

$$A_0 \equiv a_Y m_0, \tag{21}$$

where a_Y is a dimensionless number. Using the vanishing C.C. condition, the universal soft mass parameter, m_0 $(= e^{\langle K\rangle/(2M_P^2)}\langle W_H\rangle/M_P^2)$ can be expressed as $e^{\langle K\rangle/(2M_P^2)}(\sum_i |\langle F_{z_i}\rangle|^2)^{1/2}/\sqrt{3}M_P$. It is the conventional form of m_0 in the mGrM scenario.

In $N = 1$ SUGRA, the gauge kinetic function f_{ab}, which is a holomorphic function of scalar fields, not only determines the form of the gauge fields' kinetic terms $[= -\frac{1}{4}(\text{Re} f_{ab})F^{a\mu\nu}F^b_{\mu\nu}]$, but also contributes to the gaugino mass term [9]:

$$\frac{M_P}{4} e^{G/(2M_P^2)} \frac{\partial f_{ab}^*}{\partial z_i^*}\frac{\partial G}{\partial z_i}\lambda^a\lambda^b = \frac{1}{4} e^{\sum_i |b_i|^2/2} \frac{\partial f_{ab}^*}{\partial z_i^*} F_{z_i}^* \lambda^a\lambda^b, \tag{22}$$

where G is defined as $G \equiv K + M_P^2 \log(W/M_P^3)$, and $\lambda^{a,b}$ stand for the gaugino fields. If SUSY is broken $(F_{z_i} \neq 0)$ and the gauge kinetic function is nontrivial $(\partial f_{ab}/\partial z_i \neq 0)$, the gaugino masses can be generated. In the mGrM scenario, the unified gaugino mass $m_{1/2}$ is regarded as an independent parameter, assuming the canonical kinetic terms for the gauge fields. In our model that will be discussed in Sec. IV, however, we will employ the minimal form of the gauge kinetic function $(= \delta_{ab})$ at tree level: the gaugino masses can be generated radiatively.

B. Focus point in the minimal gravity mediation

As seen in Eq. (20), the soft SUSY breaking masses squared for the superpartners of chiral fermions are universal at the GUT scale in the mGrM. Accordingly, the $m_{q_3 0}^2$, $m_{u_3^c 0}^2$, and $m_{h_u 0}^2$ in Eqs. (11), (12), and (13) should be set to be the same as m_0^2 in the mGrM:

$$m_{q_3 0}^2 = m_{u_3^c 0}^2 = m_{h_u 0}^2 = m_0^2, \quad \text{and so} \quad X_0 = 3m_0^2. \tag{23}$$

Thus, the semianalytic RG solutions take the following form:

$$m_{h_u}^2(t) = \frac{3m_0^2}{2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2} - \frac{1}{3}\right] + \frac{F(t)}{2} - \left(\frac{m_{1/2}}{g_0^2}\right)^2 \times \left[\frac{3}{2}\{g_2^4(t) - g_0^4\} + \frac{1}{22}\{g_1^4(t) - g_0^4\}\right] \tag{24}$$

and

$$\{m_{q_3}^2(t) + m_{u_3^c}^2(t)\} = \frac{3m_0^2}{2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2} + \frac{1}{3}\right] + \frac{F(t)}{2} + \left(\frac{m_{1/2}}{g_0^2}\right)^2\left[\frac{16}{9}\{g_3^4(t) - g_0^4\} - \frac{3}{2}\{g_2^4(t) - g_0^4\} - \frac{17}{198}\{g_1^4(t) - g_0^4\}\right], \tag{25}$$

where $F(t)$ has been presented in Eq. (14). The A-term contributions to the above solutions are all included in $F(t)$. The independent parameters in Eqs. (24) and (25) are, thus, m_0^2, $(m_{1/2}/g_0^2)$, and a_Y: we regard t_0 (or M_G) as a given parameter, whose value is determined with the MSSM field contents and their interactions. Note that the above semianalytic solutions are valid only for small $\tan\beta$ cases. For the solutions in larger $\tan\beta$ cases, numerical analyses on the full RG equations should be implemented. Most of all, the above solutions are not valid any longer below the top squark mass scale, since the top squarks should decouple from the RG equations: the RG equations should be modified below that scale.

In the original FP scenario [8], it was pointed out that $e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}$ in Eq. (24) *happens to be almost* $\frac{1}{3}$ for $t \sim t_Z$

$[\equiv \log(M_Z/\mathrm{GeV})]$, if the top squarks were not decoupled and Eq. (24) was valid down to the Z boson mass scale. In that case, the coefficient of m_0^2 in Eq. (24) becomes very small, and so $m_{h_u}^2$ can almost be independent of m_0^2 around the Z boson mass scale. It implies that a FP of $m_{h_u}^2(t)$ appears around the Z boson mass scale. Note that the top squark masses squared are quite sensitive to m_0^2 for $e^{\frac{3}{4\pi^2}\int_{t_0}^{t_Z} dt' y_t^2} \approx \frac{1}{3}$, as seen in Eq. (25). The coefficient of $(m_{1/2}/g_0^2)^2$ included in $F(t)/2$, which is generically bigger than those in the second line of Eq. (24), turns out to be negative. Unlike the top squark masses, therefore, $m_{h_u}^2$ can be naturally small at the Z boson mass scale, only if $(m_{1/2}/g_0^2)$ and a_Y are small enough.

As mentioned in the Introduction, however, the top squark mass needs to be about 3–4 TeV for explaining the 126 GeV Higgs mass. It means that Eqs. (24) and (25) are valid just down to 3–4 TeV, and below the top squark mass scale the estimation Eq. (5) should be applied for $m_{h_u}^2$. This process would leave a sizable coefficient of m_0^2 in $m_{h_u}^2(t_Z)$, particularly in large $\tan\beta$ cases. Hence a quite heavy top squark mass would spoil the FP behavior of $m_{h_u}^2(t)$. To get a top squark mass of 3–4 TeV, moreover, m_0^2 needs to be large enough in Eq. (25), which could require a large enough $(m_{1/2}/g_0^2)^2$ for EW symmetry breaking in large $\tan\beta$ cases.

The coefficients of $m_0^2, (m_{1/2}/g_0^2)^2, \ldots$, etc. in Eqs. (24) and (25) can numerically be calculated:

$$m_{h_u}^2(t_T) \approx [0.03 - 0.11 a_Y^2] m_0^2 - 0.25\left(\frac{m_{1/2}}{g_0^2}\right)^2 - 0.16\left(\frac{m_{1/2}}{g_0^2}\right) a_Y m_0,$$
$$\{m_{q_3}^2(t_T) + m_{u_3^c}^2(t_T)\} \approx [1.03 - 0.11 a_Y^2] m_0^2 + 1.20\left(\frac{m_{1/2}}{g_0^2}\right)^2 - 0.16\left(\frac{m_{1/2}}{g_0^2}\right) a_Y m_0, \tag{26}$$

which are the values at the top squark decoupling scale, $t = t_T \approx 8.2$ (i.e. $Q_T = 3.5$ TeV) with $\tan\beta = 5$. From the above expression of $m_{h_u}^2(t_T)$, we can expect that a FP of $m_{h_u}^2$ appears below (above) t_T (or $Q_T = 3.5$ TeV) when $a_Y^2 < 0.03/0.11 \approx 0.27$ ($a_Y^2 \gtrsim 0.27$). As mentioned above, $\{m_{u_3^c}^2(t_T) + m_{q_3}^2(t_T)\}$ should be constrained to be around $2\cdot(3.5\ \mathrm{TeV})^2$ in order to get the 126 GeV Higgs boson mass. While the top squark masses would be frozen, thus, $m_{h_u}^2$ further decreases below the top squark mass scale dominantly through the top quark Yukawa coupling: $m_{h_u}^2$ at the Z boson mass scale can be estimated using Eq. (5). It has the following structure:

$$m_{h_u}^2(t_Z) = C_s m_0^2 - C_g\left(\frac{m_{1/2}}{g_0^2}\right)^2 - C_m a_Y m_0\left(\frac{m_{1/2}}{g_0^2}\right), \tag{27}$$

where the coefficients, C_s, C_g, and C_m are approximately given by

$$C_s \approx 0.03 - 0.11 a_Y^2 - \frac{3|y_t|^2}{16\pi^2} \times (1.03 - 0.11 a_Y^2),$$
$$C_g \approx 0.25 + \frac{3|y_t|^2}{16\pi^2} \times 1.20, \quad \text{and}$$
$$C_m \approx 0.16 - \frac{3|y_t|^2}{16\pi^2} \times 0.16, \tag{28}$$

for $\tan\beta = 5$. Since the $SU(3)_c$ gauge coupling becomes almost unity around the 3.5 TeV energy scale, $(m_{1/2}/g_0^2)$ in the above equations can approximately be regarded as the low energy gluino (running) mass:

$$\frac{m_{1/2}}{g_0^2} = \frac{M_3(t_T)}{g_3^2(t_T)} \approx M_3(t_T). \tag{29}$$

For $m_0^2 \gg M_3^2(t_T)$ and $a_Y^2 \ll 1$, $m_0^2 \sim (4.2\ \mathrm{TeV})^2$–$(5.6\ \mathrm{TeV})^2$ is needed for 3–4 TeV top squark masses in Eq. (26). Although the semianalytic solutions, Eqs. (24) and (25), are not valid any longer for large $\tan\beta$ cases, the basic structure of $m_{h_u}^2(t_Z)$ in those cases would still have the form of Eq. (27), but with different values for C_s, C_g, and C_m from Eq. (27).

Figure 1 displays the full numerical results on the RG behaviors of $m_{h_u}^2(t)$ for $\tan\beta = 50$ (solid lines) and $\tan\beta = 5$ (dotted lines) under various trial m_0^2, based on the full RG equations including $y_{b,\tau}$, $A_{b,\tau}$, m_{b,τ,h_d}^2, etc., when $(m_{1/2}/g_0^2) = 2.3$ TeV and $A_t = a_Y = 0$ at the GUT scale. In fact the RG runnings of $m_{h_u}^2(t)$ had to be modified below the top squark decoupling scale. Nonetheless, we extrapolate $m_{h_u}^2(t)$'s below $t = t_T$, keeping heavy superpartners in the RG evolutions, in order to discuss the FPs of $m_{h_u}^2$. As seen in Fig. 1, the FP appears at a scale relatively close to t_T for $\tan\beta = 5$, when $a_Y = 0$. That is the reason why the coefficient of m_0^2 in $m_{h_u}^2(t_T)$ of Eq. (26) is small. For $\tan\beta = 50$, thus, we can expect that the coefficient of m_0^2 is quite sizable, since the FP is relatively far from t_T.

From Eq. (27), we see that the gluino mass should be heavier than 1.3 TeV for EW symmetry breaking, i.e. $m_{h_u}^2(t_Z) < 0$ with $m_0^2 \sim (4.5\ \mathrm{TeV})^2$ and $a_Y^2 \ll 1$. To meet

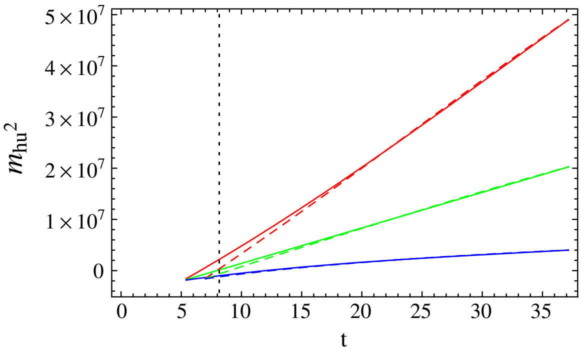

FIG. 1 (color online). RG evolutions of $m_{h_u}^2$ in the mGrM with t [$\equiv \log(Q/\mathrm{GeV})$] for $m_0^2 = (7\ \mathrm{TeV})^2$ (red), $(4.5\ \mathrm{TeV})^2$ (green), and $(2\ \mathrm{TeV})^2$ (blue), when $m_{1/2}/g_0^2 = 2.3$ TeV and $A_t = 0$ at the GUT scale. The tilted solid (dotted) lines correspond to the case of $\tan\beta = 50$ ($\tan\beta = 5$). The vertical dotted line at $t = t_T \approx 8.2$ ($Q_T = 3.5$ TeV) indicates the desired top squark mass scale. Below the top squark decoupling scale, in fact, the RG evolutions should be modified from this figure. The FP of $m_{h_u}^2$ would appear around $t \approx 5.3$ ($Q \approx 200$ GeV) [$t \approx 7.0$ ($Q \approx 1.1$ TeV)], however, if its RG evolutions are extrapolated below $t = t_T$, keeping heavy superpartners.

the experimental bound $M_3(t_T) > 1.3$ TeV, therefore, $\tan\beta$ should be larger than 5, when top squark masses are 3–4 TeV top squark masses and $|a_Y| \ll 1$. For larger $\tan\beta$ cases, heavier low energy gluino masses are necessary for EW symmetry breaking. Since $y_{b,\tau}$, $A_{b,\tau}$, etc. are quite small in small $\tan\beta$ cases, however, the RG evolution of $m_{h_d}^2$ would be negligible and so its low energy values are almost the same as m_0^2. As a result, $|\mu|$ consistent with $m_Z \approx 91$ GeV in Eq. (3) exceeds 900 GeV for $\tan\beta = 5$ and $m_0^2 = (4.5\ \mathrm{TeV})^2$. A larger m_0^2 or a larger $(m_{1/2}/g_0^2)^2$ requires a larger $|\mu|^2$ in general.

In fact, the RG equation of μ is completely separated from those of the soft parameters at one-loop level. Moreover, its generation scale is quite model dependent. Thus, we do not discuss them in this paper. To avoid a potentially problematic fine-tuning issue associated with μ, however, we will consider only the cases of $\frac{1}{2}m_Z^2/|\mu|^2 > 0.01$ or $|\mu| < 600$ GeV. Numerical analyses show that $\tan\beta$ should be larger than 8 for $|\mu| < 600$ GeV, when $m_0^2 = (4.5\ \mathrm{TeV})^2$ and $|a_Y| \ll 1$. In this case, the low energy gluino mass should be heavier than 1.9 TeV for EW symmetry breaking.

Since the coefficients of m_0^2 change slowly under a small variation δm_0^2, the small change of $\delta m_{h_u}^2$ under δm_0^2 at the Z boson mass scale is roughly estimated as

$$\frac{\delta m_{h_u}^2}{\delta m_0^2} \approx C_s - \frac{a_Y C_m}{2m_0}\left(\frac{m_{1/2}}{g_0^2}\right), \qquad (30)$$

which makes contribution to the fine-tuning measure [15],

$$\Delta_{m_0^2} = \frac{\delta \log m_Z^2}{\delta \log m_0^2} = \frac{m_0^2}{m_Z^2}\frac{\delta m_Z^2}{\delta m_0^2} = 2\left(\frac{m_0^2}{m_Z^2}\right)\left[\frac{(\delta m_{h_d}^2/\delta m_0^2) - \tan^2\beta(\delta m_{h_u}^2/\delta m_0^2)}{\tan^2\beta - 1}\right]. \qquad (31)$$

Note that (m_0^2/m_Z^2) is a very large number, because a quite large m_0^2 [$(4.2\ \mathrm{TeV})^2$–$(5.6\ \mathrm{TeV})^2$] is necessary for a 3–4 TeV top squark mass. Hence, the other parts in Eq. (31) should sufficiently be suppressed to get a small enough $\Delta_{m_0^2}$. As clearly seen in Eq. (30), the variation of $m_{h_u}^2$ under δm_0^2, $(\delta m_{h_u}^2/\delta m_0^2)$ cannot be zero at the top squark mass scale, unless a_Y is finely tuned. As mentioned above, moreover, low energy values of $m_{h_d}^2$ are almost the same as m_0^2's in small $\tan\beta$ cases. Accordingly, $(\delta m_{h_d}^2/\delta m_0^2)$ would be about unity in Eq. (31). Therefore, $\Delta_{m_0^2}$ and $|\mu|$ cannot be small enough in small $\tan\beta$ cases, when top squark masses are 3–4 TeV or heavier.

In large $\tan\beta$ cases, $(\delta m_{h_d}^2/\delta m_0^2)$ is relatively suppressed as seen in Eq. (31). In fact, $m_{h_d}^2$ is not focused at all. Hence, a larger $\tan\beta$ would be more desirable in the FP scenario. In the case of $\tan\beta = 50$, for instance, the physical (low energy running) gluino mass should be heavier than 2.6 TeV (2.2 TeV) for EW symmetry breaking, but lighter than 2.8 TeV (2.6 TeV) for $|\mu| < 600$ GeV, when $m_0^2 = (4.5\ \mathrm{TeV})^2$ and $|a_Y| \ll 1$. However, the FP scale is basically too far from the top squark mass scale as shown in Fig. 1. Consequently, $(\delta m_{h_u}^2/\delta m_0^2)$ in Eq. (31) or C_s in Eq. (27) is quite sizable, and so $\Delta_{m_0^2}$ is hard to be small enough also in large $\tan\beta$ cases. We should note here that a sizable C_s in Eq. (27) requires also a sizable $C_g(m_{1/2}/g_0^2)^2$ or $C_m a_Y(m_{1/2}/g_0^2)$ for EW symmetry breaking.

Table I lists soft squared masses of the top squarks and Higgs bosons at $t = t_T \approx 8.2$ ($Q_T = 3.5$ TeV) for various trial m_0^2's and A_0, when $\tan\beta = 50$ and $M_3(t_T) = 2.5$ TeV. They are results generated by SOFTSUSY-3.6.2 [16], analyzing the full RG equations. We can see that $\Delta_{m_0^2}$'s for $m_{h_u}^2$ are of order 10^2 for $|\mu| < 600$ GeV. It is because the FP of $m_{h_u}^2$ appears too far below $t = t_T$ as discussed above.

To summarize, $|\mu|$ and $\Delta_{m_0^2}$ are too large in small $\tan\beta$ cases in the mGrM, even if the FP emerges somewhat close to the top squark mass scale. It is because the m_0^2 needed for the desired top squark mass is quite heavy, and $m_{h_d}^2$ ($\approx m_0^2$) is not focused at all. In large $\tan\beta$ cases, on the other hand, the FP scale of $m_{h_u}^2$ is too low compared with the top squark mass scale.

To keep a small enough μ even with 3–4 TeV top squark masses, thus, we should consider a large $\tan\beta$ case. But we need to somehow push the FP scale up to the desired top squark mass scale in order to reduce $\Delta_{m_0^2}$ in this case. Of course, there still remains a possibility to achieve it by

TABLE I. Soft squared masses of the top squarks and Higgs bosons at $t = t_T \approx 8.2$ ($Q_T = 3.5$ TeV) in the mGrM for various trial m_0^2's when $\tan\beta = 50$. $\Delta_{m_0^2}$ indicates the fine-tuning measure for m_0^2 around $(4.5\ \text{TeV})^2$ for each case.

$A_0/m_0 = 0.3$	$M_3(t_T) = 2.5$ TeV	$\|\mu\| = 903$ GeV	$\Delta_{m_0^2} = 276$
m_0^2	$(5.5\ \text{TeV})^2$	$(4.5\ \text{TeV})^2$	$(3.5\ \text{TeV})^2$
$m_{q_3}^2(t_T)$	$(4437\ \text{GeV})^2$	$(3817\ \text{GeV})^2$	$(3238\ \text{GeV})^2$
$m_{u_3^c}^2(t_T)$	$(3857\ \text{GeV})^2$	$(3329\ \text{GeV})^2$	$(2839\ \text{GeV})^2$
$\mathbf{m_{h_u}^2(t_T)}$	$(\mathbf{461}\ \text{GeV})^2$	$-(\mathbf{694}\ \text{GeV})^2$	$-(\mathbf{1007}\ \text{GeV})^2$
$m_{h_d}^2(t_T)$	$(2585\ \text{GeV})^2$	$(2032\ \text{GeV})^2$	$(1450\ \text{GeV})^2$
$A_0/m_0 = 0$	$M_3(t_T) = 2.5$ TeV	$\|\mu\| = 387$ GeV	$\Delta_{m_0^2} = 378$
m_0^2	$(5.5\ \text{TeV})^2$	$(4.5\ \text{TeV})^2$	$(3.5\ \text{TeV})^2$
$m_{q_3}^2(t_T)$	$(4497\ \text{GeV})^2$	$(3870\ \text{GeV})^2$	$(3285\ \text{GeV})^2$
$m_{u_3^c}^2(t_T)$	$(3933\ \text{GeV})^2$	$(3396\ \text{GeV})^2$	$(2897\ \text{GeV})^2$
$\mathbf{m_{h_u}^2(t_T)}$	$(\mathbf{1044}\ \text{GeV})^2$	$(\mathbf{442}\ \text{GeV})^2$	$-(\mathbf{721}\ \text{GeV})^2$
$m_{h_d}^2(t_T)$	$(2749\ \text{GeV})^2$	$(2189\ \text{GeV})^2$	$(1607\ \text{GeV})^2$
$A_0/m_0 = -1.0$	$M_3(t_T) = 2.5$ TeV	$\|\mu\| = 753$ GeV	$\Delta_{m_0^2} = 83$
m_0^2	$(5.5\ \text{TeV})^2$	$(4.5\ \text{TeV})^2$	$(3.5\ \text{TeV})^2$
$m_{q_3}^2(t_T)$	$(4427\ \text{GeV})^2$	$(3840\ \text{GeV})^2$	$(3289\ \text{GeV})^2$
$m_{u_3^c}^2(t_T)$	$(3840\ \text{GeV})^2$	$(3354\ \text{GeV})^2$	$(2900\ \text{GeV})^2$
$\mathbf{m_{h_u}^2(t_T)}$	$(\mathbf{105}\ \text{GeV})^2$	$-(\mathbf{478}\ \text{GeV})^2$	$-(\mathbf{702}\ \text{GeV})^2$
$m_{h_d}^2(t_T)$	$(2385\ \text{GeV})^2$	$(1952\ \text{GeV})^2$	$(1498\ \text{GeV})^2$

assuming a (fine-tuned) a_Y with a large $\tan\beta$. A fine-tuned Dirac Yukawa coupling of a RH neutrino, y_N, is also helpful for pushing the FP [12,17]. However, it is very hard to contrive a model to naturally explain such a special value of a_Y or y_N, reducing also Δ_{A_0} or Δ_{y_N}. In the next section, we will propose another way to move the FP scale up to the desired top squark mass scale in a large $\tan\beta$ case.

IV. MINIMAL MIXED MEDIATION

In large $\tan\beta$ cases, as mentioned above, C_s is sizable in Eq. (27) because the FP of $m_{h_u}^2$ is far below the top squark decoupling scale, and $C_g(m_{1/2}/g_0^2)^2$ and/or $C_m a_Y(m_{1/2}/g_0^2)$ are also required to be large enough for EW symmetry breaking. While the C_s term makes a positive contribution to $m_{h_u}^2(t_Z)$ for small a_Y's, the other terms make negative contributions to it. In this section, we will attempt to investigate a mechanism in which the two sizable contributions can automatically be canceled to eventually yield a small enough C_s even in a large $\tan\beta$ case.

A. Basic setup in the minimal mixed mediation

On top of the mGrM setup, we consider also the mGgM effects by introducing one pair of messenger fields $\{\mathbf{5}_M, \bar{\mathbf{5}}_M\}$ which are the SU(5) fundamental representations. Through their coupling with an MSSM singlet superfield S,

$$W_m = y_S S \mathbf{5}_M \bar{\mathbf{5}}_M, \tag{32}$$

the soft masses of the MSSM gauginos and scalar superpartners are generated at one- and two-loop levels, respectively, if the scalar and F-term components of S develop nonzero VEVs [1]:

$$M_a|_M = \frac{g_a^2(t_M)\langle F_S\rangle}{16\pi^2\langle S\rangle},$$
$$\delta m_{\phi_r}^2|_M = 2\sum_{a=1}^{3}\left[\frac{g_a^2(t_M)\langle F_S\rangle}{16\pi^2\langle S\rangle}\right]^2 C_a(r), \tag{33}$$

where $C_a(r)$ denotes the quadratic Casimir invariant for a superfield Φ_r; $(T^aT^a)_r^{r'} = C_a(r)\delta_r^{r'}$; and g_a ($a = 3, 2, 1$) is the MSSM gauge couplings. $\langle S\rangle$ and $\langle F_S\rangle$ are VEVs of the scalar and F-term components of the superfield S. Note that M_a and $m_{\Phi_r}^2$ are almost independent of y_S only if $\langle F_S\rangle \lesssim y_S\langle S\rangle^2$ [1]. However, such mGgM effects appear below the messenger mass scale, $y_S\langle S\rangle$. In this paper, we assume the messenger mass scale is lower than the GUT scale. Otherwise, $\delta m_{\phi_r}^2|_M$ and $M_a|_M$ could become relatively universal at the GUT scale (as in the mGrM), respecting the relations required by a given GUT, since non-MSSM gauge sectors contained in a SUSY GUT such as "X" and "Y" in the SU(5) GUT also contribute to $\delta m_{\phi_r}^2|_M$.

Once the hidden sector superpotential W_H develops a VEV, the F-term of S and the F-terms of superfields in the hidden sector can also get VEVs proportional to $\langle W_H\rangle$ ($\equiv mM_P^2$). For instance, let us consider the following Kähler potential in addition to Eq. (16):

$$K \supset f(z)S + \text{H.c.}, \tag{34}$$

where $f(z)$ is a *holomorphic* monomial of hidden sector fields z_i's with VEVs of order M_P in Eq. (17), and so $f(z)$ should be of order $\mathcal{O}(M_P)$. Its specific form can be controlled by introducing hidden local symmetries. Note that the above term leaves intact the kinetic terms of z_i's, and so they still remain as the canonical form. $M_P f(z)S$ in the superpotential can be forbidden by the $U(1)_R$ symmetry. By including the SUGRA corrections with $\langle W_H\rangle = mM_P^2$, then, $\langle F_S\rangle$ can be

$$\langle F_S^*\rangle \approx m[\langle f(z)\rangle + \langle S^*\rangle], \tag{35}$$

if $\langle\partial_S W\rangle$ is relatively suppressed by relevant small (or zero) Yukawa couplings. Thus, the VEV of F_S is of order $\mathcal{O}(mM_P)$ like F_{z_i} in Eq. (19). They should be fine-tuned for the vanishing C.C.: a precise determination of $\langle F_S\rangle$ is indeed associated with the C.C. problem. Here we set $\langle F_S\rangle = m_0 M_P$. F_{ϕ_r} is still given by Eq. (19), which induces the universal soft mass terms at tree level for the observable scalar fields. Consequently, both the gravity and gauge mediation effects are induced from a *single* SUSY breaking source, and they all are parametrized with m_0.

We assume that $\langle S\rangle$ has the same magnitude as the VEV of the SU(5) breaking Higgs ($\equiv v_G$), $\langle \mathbf{24}_H\rangle = v_G \times \mathrm{diag}(2,2,2;-3,-3)/\sqrt{60}$. It can be realized by constructing a proper model, in which a GUT breaking mechanism causes $\langle S\rangle$. For example, let us consider the following Kähler potential and superpotential:

$$K \supset z^c \bar{z}^c S + \mathrm{H.c.},$$
$$W \supset (z\bar{z})^2 S^c S^c + (z^c\bar{z}^c)^2 \mathrm{Tr}[\mathbf{24}\mathbf{24}] + \Sigma_R \mathrm{Tr}[\mathbf{24}'\mathbf{24}'] + \mathrm{Tr}[\mathbf{24}'\{(S+\lambda z\bar{z})\mathbf{24}^c - (z\bar{z})^2 \mathbf{24}^c\mathbf{24}^c\}], \qquad (36)$$

where we drop the $\mathcal{O}(1)$ dimensionless coupling constants and set $M_P = 1$ for simple expressions except for λ ($\sim 10^{-2}$). Here we introduced a $\mathrm{U}(1)_Z$ gauge symmetry and supposed that *some* hidden sector fields $\{z,\bar{z},z^c,\bar{z}^c\}$ [$\subset \{z_i\}$ in Eq. (16)], which are nontrivial representations of a hidden gauge group G_H ($\{R,\bar{R}\}$), carry $\mathrm{U}(1)_Z$ charges as well. We also introduce the global $\mathrm{U}(1)_R$ symmetry and the SU(5) visible gauge symmetry [18], under which $\{z,\bar{z},z^c,\bar{z}^c\}$ remain neutral. The other relevant superfields and their charges are presented in Table II. $\{\mathbf{24}',\mathbf{24},\mathbf{24}^c\}$ are all SU(5) adjoint representations, while $\{S,S^c\}$ are singlets. Σ_R denotes a spurion field, whose VEV breaks the $\mathrm{U}(1)_R$ to the Z_2 symmetry. W_m in Eq. (32) can be reproduced by assigning the unit $\mathrm{U}(1)_R$ charge to $\{\mathbf{5}_M,\bar{\mathbf{5}}_M\}$ from $W_m = z^c\bar{z}^c S\mathbf{5}_M\bar{\mathbf{5}}_M$. Note that the field contents in Table II do not yield any gauge anomaly.

As in $\{z_i\}$ of Eq. (17), $\{z,\bar{z},z^c,\bar{z}^c\}$ in Eq. (36) are assumed to get VEVs of the Planck scale. Note that the combinations of them, $z\bar{z}^c$ ($\equiv u$) and $\bar{z}z^c$ ($\equiv v$) do not carry any quantum numbers. Thus, the Kähler potential and superpotential in the hidden sector would take the forms of $K_H = K_H(u,v)$ and $W_H = W_H(u,v)$, neglecting the asymmetric term $K \supset z^c\bar{z}^c S + \mathrm{H.c.}$ because of its smallness: the consistency of $\langle S\rangle \ll M_P$ will be confirmed. Accordingly, the F-terms of $\{z,\bar{z},z^c,\bar{z}^c\}$ are given by $F_z^* = \partial_z W_H + W_H \partial_z K_H = \bar{z}^c(\partial_u W_H + W_H\partial_u K_H)$, $F_{\bar{z}^c}^* = \partial_{\bar{z}^c} W_H + W_H \partial_{\bar{z}^c} K_H = z(\partial_u W_H + W_H \partial_u K_H)$, etc., which are all assumed to be of order $\mathcal{O}(mM_P)$. Since $|z| = |\bar{z}^c|$ minimizes $|F_z|^2 + |F_{\bar{z}^c}|^2$ [$= (|z|^2 + |\bar{z}^c|^2)|\partial_u W_H + W_H\partial_u K_H|^2$], $\langle z\rangle$ and $\langle \bar{z}^c\rangle$ would be developed along the direction of $|\langle z\rangle| = |\langle \bar{z}^c\rangle|$. Note that the minimization of $|\partial_u W_H + W_H \partial_u K_H|^2$ would determine just u or v. Similarly, $\langle z^c\rangle$ and $\langle \bar{z}^c\rangle$ would be developed along the $|\langle z^c\rangle| = |\langle \bar{z}^c\rangle|$ direction, minimizing $|F_{\bar{z}}|^2 + |F_{z^c}|^2$. Moreover, such directions are the D-flat directions of G_H. Although the full F-term potential could be further minimized, both $|\langle z\rangle| = |\langle \bar{z}^c\rangle|$ and $|\langle z^c\rangle| = |\langle \bar{z}^c\rangle|$ should still be maintained.

TABLE II. Quantum numbers of superfields for a local $\mathrm{U}(1)_Z$, hidden gauge G_H, and the global $\mathrm{U}(1)_R$ symmetries. Only the hidden sector fields $\{z,\bar{z},z^c,\bar{z}^c\}$ carry proper nontrivial quantum numbers $\{R,\bar{R}\}$ under a hidden gauge group G_H.

	S	S^c	$\mathbf{24}'$	$\mathbf{24}$	$\mathbf{24}^c$	z	$\bar{z}$	z^c	$\bar{z}^c$	Σ_R
$\mathrm{U}(1)_Z$	+1	−1	0	+1	−1	$\frac{+1}{2}$	$\frac{+1}{2}$	$\frac{-1}{2}$	$\frac{-1}{2}$	0
G_H	**1**	**1**	**1**	**1**	**1**	R	$\bar{R}$	R	$\bar{R}$	**1**
$\mathrm{U}(1)_R$	0	1	2	1	0	0	0	0	0	−2

Due to the mass terms by the VEVs of $\{z,\bar{z},z^c,\bar{z}^c\}$ and Σ_R in the superpotential of Eq. (36), then, we have $\langle S^c\rangle = \langle \mathbf{24}\rangle = \langle \mathbf{24}'\rangle = 0$ even after including the SUGRA corrections. On the other hand, $\mathbf{24}^c$ can develop a VEV of the order GUT scale in the $\mathrm{U}(1)_Y$ direction from the second line of W in Eq. (34) as in the ordinary minimal SU(5) GUT [19]. It is identified with $\mathbf{24}_H$ discussed above. Both $\langle \mathbf{24}^c\rangle$ and $\langle S\rangle$ are completely determined by the minimum conditions for $F_{\mathbf{24}'}$ and the D-term of $\mathrm{U}(1)_z$ [9],

$$D_z = g_z \sum_j q_j \left[\partial_{\varphi_j} K + M_P^2 \frac{\partial_{\varphi_j} W}{W}\right]\varphi_j = g_z(|S|^2 - \mathrm{Tr}|\mathbf{24}^c|^2 + \cdots), \qquad (37)$$

where g_z and q_j mean the $\mathrm{U}(1)_z$ gauge coupling and charge of a field φ_j. "$\cdots$" contains the contributions by $\{z,\bar{z},z^c,\bar{z}^c\}$ and other scalar fields with zero VEVs. However, the VEVs of z_i are canceled out from Eq. (37) because of $|\langle z\rangle| = |\langle \bar{z}^c\rangle|$ and $|\langle z^c\rangle| = |\langle \bar{z}^c\rangle|$. In the SUSY limit, thus, all the VEVs of the fields in Table II have been determined: $\langle S\rangle = v_G$ and others are vanishing. By including the SUGRA corrections by $\langle W_H\rangle = mM_P^2$, we can read the SUSY breaking effects: $\langle F_S^*\rangle = m(z^c\bar{z}^c + S^*) \gg \langle F_{\mathbf{24}^c}\rangle = mv_G$. Thus, VEVs of F_S and $\{F_z, F_{\bar{z}}, F_{z^c}, F_{\bar{z}^c}\}$ are all $\mathcal{O}(mM_P)$. They should be fine-tuned for the vanishing C.C.: precise determination of $\langle F_S\rangle$ is associated with the C.C. problem as mentioned above.

v_G induces the superheavy masses of X and Y gauge bosons and their superpartners in the SU(5) GUT, M_X and M_Y. Since the GUT gauge interactions would become active above their mass scale, $M_X^2 = M_Y^2 = \frac{5}{24} g_G^2 v_G^2$ [19], it is identified with the MSSM gauge coupling unification scale. Thus, $\langle S\rangle$ ($= v_G$) is *fixed* by the relation with the unification scale. When the superpartners of the SM chiral fermions are heavier than 3–4 TeV, the unification scale is about $(0.9\text{–}1.7)\times 10^{16}$ GeV. In fact, the three MSSM gauge couplings are not exactly unified at a unique scale only with the MSSM field contents, because the superpartners are relatively heavy in this case. However, various threshold effects would arise around that scale. Here we will take the central value of the above range, i.e. 1.3×10^{16} GeV for the unification scale. Then the mGgM SUSY breaking effects in Eq. (33) can be estimated with a parameter f_G:

$$f_G \cdot m_0 \equiv \frac{\langle F_S \rangle}{16\pi^2 \langle S \rangle} = \frac{m_0 M_P}{16\pi^2 M_X}\sqrt{\frac{5}{24}} g_G \approx 0.36 m_0. \tag{38}$$

Note that the m_0 dependence appears because F_S is proportional to m_0 in the minimal mixed mediation as discussed above. f_G is basically a parameter determined by a model. From now on, however, we will leave f_G as an unknown parameter.

From Eq. (33), the soft squared masses for the MSSM Higgs and the superpartners of (the third generation of) chiral fermions at the messenger scale are expressed as follows:

$$\delta m^2_{h_u}|_M = \delta m^2_{h_d}|_M = \delta m^2_{l_3}|_M = f_G^2 m_0^2 \left[\frac{3}{2} g_2^4(t_M) + \frac{3}{10} g_1^4(t_M)\right], \tag{39}$$

$$\delta m^2_{q_3}|_M = f_G^2 m_0^2 \left[\frac{8}{3} g_3^4(t_M) + \frac{3}{2} g_2^4(t_M) + \frac{1}{30} g_1^4(t_M)\right], \tag{40}$$

$$\delta m^2_{u_3^c}|_M = f_G^2 m_0^2 \left[\frac{8}{3} g_3^4(t_M) + \frac{8}{15} g_1^4(t_M)\right], \tag{41}$$

$$\delta m^2_{d_3^c}|_M = f_G^2 m_0^2 \left[\frac{8}{3} g_3^4(t_M) + \frac{2}{15} g_1^4(t_M)\right], \tag{42}$$

$$\delta m^2_{e_3^c}|_M = f_G^2 m_0^2 \left[\frac{6}{5} g_1^4(t_M)\right], \tag{43}$$

where $g_a(t_M)$'s $(a = 3, 2, 1)$ denote the MSSM gauge coupling constants at the messenger scale. Hence, $\delta X|_M$ ($\equiv \delta m^2_{q_3}|_M + \delta m^2_{u_3^c}|_M + \delta m^2_{h_u}|_M$) is given by

$$\delta X|_M = f_G^2 m_0^2 \left[\frac{16}{3} g_3^4(t_M) + 3 g_2^4(t_M) + \frac{13}{15} g_1^4(t_M)\right]. \tag{44}$$

Note that the above soft masses, Eqs. (39)–(43) are not universal even around the GUT scale unlike the mGrM, since only the MSSM gauge sector makes contributions to $\delta m^2_{\phi_r}|_M$ and superheavy gauge sectors contained in a SUSY GUT would decouple at the GUT scale.

In contrast to the soft masses for the superpartners of SM chiral fermions, the gaugino masses are assumed to be generated dominantly only by the mGgM effect, i.e., M_a of Eq. (33). It is possible by employing the constant gauge kinetic function ($= \delta_{ab}$) at tree level, which is the *minimal* gauge kinetic function, yielding the canonical kinetic terms for gauge fields. Above the messenger mass scale, hence, the gaugino mass contributions to the RG equation should be negligible: the gaugino masses via mGrM must be small as seen in Eq. (22). On the contrary, A-terms in the mGgM are generically much suppressed compared to those in the mGrM [1]. So the universal A-terms coming from Eq. (20), which are proportional to m_0, should be dominant ones.

Since the MSSM RG equations are valid below the messenger scale, the boundary conditions at the messenger scale, Eqs. (33) and (38), yield

$$\frac{M_a(t)}{g_a^2(t)} = \frac{M_a(t_M)}{g_a^2(t_M)} = f_G \cdot m_0. \tag{45}$$

Hence, the low energy gaugino (running) masses are determined with the low energy values of the SM gauge couplings and $f_G m_0$:

$$M_a(t_T) = f_G m_0 \times g_a^2(t_T). \tag{46}$$

As discussed before, m_0 is determined such that the low energy top squark masses are around 3–4 TeV for explaining the 126 GeV Higgs mass. We will discuss the valid range of f_G in view of naturalness. Note that the low energy gaugino masses, Eq. (46), are not affected by a messenger scale.

Above the messenger mass scale, however, the RG evolution of the MSSM gauge couplings should be modified by the messenger fields, $\{\mathbf{5}_M, \bar{\mathbf{5}}_M\}$: the mGgM effects enter in the RG equations at the messenger mass scale $y_S\langle S\rangle$. Accordingly, all the RG evolutions of the MSSM Yukawa couplings and soft mass parameters should also be modified above the messenger scale.

Although y_S does not contribute to the soft masses in Eq. (33), it does to the messenger mass scale. Nonetheless, we will show later that the low energy mass spectra are not sensitive to y_S. Since F_S is proportional to m_0, the MSSM gaugino masses are also proportional to m_0. As a result, they could be useful for reducing the size of the m_0^2 coefficient, and so for improving the fine-tuning associated with the EW scale and the Higgs boson mass in the mGrM or mSUGRA. We will discuss this issue in more detail later.

B. Focus point in the minimal mixed mediation

In this subsection we will discuss the focus point of $m^2_{h_u}$ and fine-tunings in the minimal mixed mediation of SUSY breaking.

1. Case for $Q_M \lesssim M_{\rm GUT}$

We first consider the case that the messenger mass scale is of order the GUT scale or slightly lower. It corresponds to the case of $|y_S| \sim \mathcal{O}(1)$, assuming $\langle S\rangle \sim \mathcal{O}(M_G)$. For simplicity, we neglect the contributions from GUT gauge multiplets such as X, Y, and their superpartners to Eq. (33), since they would not much affect the low energy values of $\{m^2_{h_u}, m^2_{q_3}, m^2_{u_3^c}\}$ as in the case of $|y_S\langle S\rangle| \ll \mathcal{O}(M_G)$. The discussion on such a relatively simple case is necessary also for the discussion on the case of $|y_S| \ll \mathcal{O}(1)$, i.e., the case of low messenger scale. As will be seen later, how small the messenger mass scale is compared to the GUT scale is indeed not very important. Since the gaugino masses are assumed to be generated dominantly by mGgM, "$(m_{1/2}/g_0^2)$" in Eqs. (9)–(14) is just replaced by

$$\frac{m_{1/2}}{g_0^2} \approx f_G m_0, \tag{47}$$

because they are generated around the GUT scale, $\frac{M_a(t_0)}{g_a^2(t_0)} = f_G m_0$ $(a = 3, 2, 1)$ in this case. As a result, we can expect that in the minimal mixed mediation, the C_g terms, as well as the C_m terms in Eq. (27), are converted to members of C_s terms. Since they make negative contributions to $m_{h_u}^2(t_T)$, they would be helpful for reducing the size of C_s and eventually $\Delta_{m_0^2}$ [20], particularly in large $\tan\beta$ cases.

On the other hand, the soft squared masses are induced by both the mGrM and mGgM effects at the GUT scale. In Eqs. (11), (12), and (13), hence, $m_{q_3 0}^2$, $m_{u_3^c 0}^2$, $m_{h_u 0}^2$, and X_0 are written down as follows:

$$m_{q_3 0}^2 \approx m_0^2 + f_G^2 m_0^2\left[\frac{8}{3}g_3^4(t_0) + \frac{3}{2}g_2^4(t_0) + \frac{1}{30}g_1^4(t_0)\right] \approx m_0^2\left(1 + \frac{21}{5}f_G^2 g_0^4\right) \tag{48}$$

$$m_{u_3^c 0}^2 \approx m_0^2 + f_G^2 m_0^2\left[\frac{8}{3}g_3^4(t_0) + \frac{8}{15}g_1^4(t_0)\right] \approx m_0^2\left(1 + \frac{16}{5}f_G^2 g_0^4\right) \tag{49}$$

$$m_{h_u 0}^2 \approx m_0^2 + f_G^2 m_0^2\left[\frac{3}{2}g_2^4(t_0) + \frac{3}{10}g_1^4(t_0)\right] \approx m_0^2\left(1 + \frac{9}{5}f_G^2 g_0^4\right) \tag{50}$$

$$X_0 \approx 3m_0^2 + f_G^2 m_0^2\left[\frac{16}{3}g_3^4(t_0) + 3g_2^4(t_0) + \frac{13}{15}g_1^4(t_0)\right] \approx 3m_0^2\left(1 + \frac{46}{15}f_G^2 g_0^4\right). \tag{51}$$

For $t \leq t_0$, therefore, the semianalytic RG solutions Eqs. (11)–(13) are given as the following expressions in the mGgM case:

$$\begin{aligned} m_{h_u}^2(t) \approx{}& \frac{3m_0^2}{2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t}dt' y_t^2} - \frac{1}{3}\right] + f_G^2 m_0^2\left[\frac{3}{2}g_2^4(t_0) + \frac{3}{10}g_1^4(t_0)\right] \\ &+ \frac{f_G^2 m_0^2}{2}\left[\frac{16}{3}g_3^4(t_0) + 3g_2^4(t_0) + \frac{13}{15}g_1^4(t_0)\right]\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t}dt' y_t^2} - 1\right] \\ &+ \frac{F(t)}{2} - f_G^2 m_0^2\left[\frac{3}{2}\{g_2^4(t) - g_2^4(t_0)\} + \frac{1}{22}\{g_1^4(t) - g_1^4(t_0)\}\right] \end{aligned} \tag{52}$$

and

$$\begin{aligned} \{m_{q_3}^2(t) + m_{u_3^c}^2(t)\} \approx{}& \frac{3m_0^2}{2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t}dt' y_t^2} + \frac{1}{3}\right] + f_G^2 m_0^2\left[\frac{16}{3}g_3^4(t_0) + \frac{3}{2}g_2^4(t_0) + \frac{17}{30}g_1^4(t_0)\right] \\ &+ \frac{f_G^2 m_0^2}{2}\left[\frac{16}{3}g_3^4(t_0) + 3g_2^4(t_0) + \frac{13}{15}g_1^4(t_0)\right]\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t}dt' y_t^2} - 1\right] + \frac{F(t)}{2} \\ &+ f_G^2 m_0^2\left[\frac{16}{9}\{g_3^4(t) - g_3^4(t_0)\} - \frac{3}{2}\{g_2^4(t) - g_2^4(t_0)\} - \frac{17}{198}\{g_1^4(t) - g_1^4(t_0)\}\right], \end{aligned} \tag{53}$$

where $F(t)$ is basically given by Eq. (14) except that $m_{1/2}/g_0^2$ should be replaced by $f_G m_0$. In fact, $g_{3,2,1}^4(t_0)$ in the above equations are all the same as the unified gauge coupling constant g_0^4. For future convenience, however, we leave them in the present form. Note that these solutions are valid only when $\tan\beta$ is small enough to neglect $y_{b,\tau}$, $A_{b,\tau}$, $m_{d_3^c, e_3^c, l_3, h_d}^2$, etc. The above semianalytic solutions admit the following numerical estimations:

$$\begin{aligned} m_{h_u}^2(t_T) &\approx m_0^2[0.03 - 0.52 f_G^2 - 0.16 f_G a_Y - 0.11 a_Y^2], \\ \{m_{q_3}^2(t_T) + m_{u_3^c}^2(t_T)\} &\approx m_0^2[1.03 + 2.22 f_G^2 - 0.16 f_G a_Y - 0.11 a_Y^2] \end{aligned} \tag{54}$$

for $\tan\beta = 5$ and $t = t_T \approx 8.2$ $(Q_T = 3.5$ TeV).

TABLE III. Soft squared masses of the top squarks and Higgs bosons at $t = t_T \approx 8.2$ ($Q_T = 3.5$ TeV) for various trial m_0^2's when the messenger scale is $Q_M \approx 1.3 \times 10^{16}$ GeV with $f_G^2 = 0.13$ [13]. $\Delta_{m_0^2}$ indicates the fine-tuning measure for $m_0 = 4.5$ TeV for each case. $m_{h_u}^2$'s further decrease to be negative below $t = t_T$. The above mass spectra are generated using SOFTSUSY.

Case I	$A_0 = 0$	$\tan\beta = 50$	$\Delta_{m_0^2} = 1$
m_0^2	$(5.5\ \text{TeV})^2$	$(4.5\ \text{TeV})^2$	$(3.5\ \text{TeV})^2$
$m_{q_3}^2(t_T)$	$(4363\ \text{GeV})^2$	$(3551\ \text{GeV})^2$	$(2744\ \text{GeV})^2$
$m_{u_3^c}^2(t_T)$	$(3789\ \text{GeV})^2$	$(3098\ \text{GeV})^2$	$(2406\ \text{GeV})^2$
$\mathbf{m_{h_u}^2(t_T)}$	$(\mathbf{431}\ \text{GeV})^2$	$(\mathbf{189}\ \text{GeV})^2$	$-(\mathbf{251}\ \text{GeV})^2$
$m_{h_d}^2(t_T)$	$(2022\ \text{GeV})^2$	$(1512\ \text{GeV})^2$	$(1008\ \text{GeV})^2$
Case II	$A_0 = -0.2m_0$	$\tan\beta = 50$	$\Delta_{m_0^2} = 16$
m_0^2	$(5.5\ \text{TeV})^2$	$(4.5\ \text{TeV})^2$	$(3.5\ \text{TeV})^2$
$m_{q_3}^2(t_T)$	$(4376\ \text{GeV})^2$	$(3563\ \text{GeV})^2$	$(2752\ \text{GeV})^2$
$m_{u_3^c}^2(t_T)$	$(3798\ \text{GeV})^2$	$(3106\ \text{GeV})^2$	$(2413\ \text{GeV})^2$
$\mathbf{m_{h_u}^2(t_T)}$	$(\mathbf{539}\ \text{GeV})^2$	$(\mathbf{361}\ \text{GeV})^2$	$-(\mathbf{44}\ \text{GeV})^2$
$m_{h_d}^2(t_T)$	$(2053\ \text{GeV})^2$	$(1565\ \text{GeV})^2$	$(1046\ \text{GeV})^2$
Case III	$A_0 = -0.5m_0$	$\tan\beta = 50$	$\Delta_{m_0^2} = 9$
m_0^2	$(5.5\ \text{TeV})^2$	$(4.5\ \text{TeV})^2$	$(3.5\ \text{TeV})^2$
$m_{q_3}^2(t_T)$	$(4284\ \text{GeV})^2$	$(3532\ \text{GeV})^2$	$(2630\ \text{GeV})^2$
$m_{u_3^c}^2(t_T)$	$(3755\ \text{GeV})^2$	$(3088\ \text{GeV})^2$	$(2373\ \text{GeV})^2$
$\mathbf{m_{h_u}^2(t_T)}$	$-(\mathbf{363}\ \text{GeV})^2$	$-(\mathbf{41}\ \text{GeV})^2$	$-(\mathbf{546}\ \text{GeV})^2$
$m_{h_d}^2(t_T)$	$(1447\ \text{GeV})^2$	$(1359\ \text{GeV})^2$	$-(950\ \text{GeV})^2$
Case IV	$A_0 = 0$	$\tan\beta = 25$	$\Delta_{m_0^2} = 57$
m_0^2	$(5.5\ \text{TeV})^2$	$(4.5\ \text{TeV})^2$	$(3.5\ \text{TeV})^2$
$m_{q_3}^2(t_T)$	$(4915\ \text{GeV})^2$	$(4025\ \text{GeV})^2$	$(3134\ \text{GeV})^2$
$m_{u_3^c}^2(t_T)$	$(3770\ \text{GeV})^2$	$(3086\ \text{GeV})^2$	$(2400\ \text{GeV})^2$
$\mathbf{m_{h_u}^2(t_T)}$	$(\mathbf{152}\ \text{GeV})^2$	$-(\mathbf{220}\ \text{GeV})^2$	$-(\mathbf{293}\ \text{GeV})^2$
$m_{h_d}^2(t_T)$	$(5057\ \text{GeV})^2$	$(4136\ \text{GeV})^2$	$(3215\ \text{GeV})^2$

For larger $\tan\beta$ cases, refer to Table III: it shows the results obtained by performing numerical analyses for the full RG equations with $\tan\beta = 50$ (cases I, II, and III) and $\tan\beta = 25$ (case IV) [13]. In all the cases, f_G^2 is set to be 0.13 (i.e. $f_G \approx 0.36$). The fine-tuning measure $\Delta_{m_0^2}$ ($\equiv |\frac{\partial \log m_Z^2}{\partial \log m_0^2}| = |\frac{m_0^2}{m_Z^2}\frac{\partial m_Z^2}{\partial m_0^2}|$ [15]) listed for each case is indeed amazing:

$$\Delta_{m_0^2} \approx \{1, 16, 9; 57\} \tag{55}$$

around $m_0^2 = (4.5\ \text{TeV})^2$ for cases I, II, III, and IV, respectively. Case I in Table III actually gives almost the minimum value of it for $\tan\beta = 50$. Δ_{A_0} ($= |\frac{A_0}{m_Z^2}\frac{\partial m_Z^2}{\partial A_0}|$) are

$$\Delta_{A_0} \approx \{0, 10, 118; 0\} \tag{56}$$

for cases I, II, III, and IV, respectively. The $m_{h_u}^2$'s at the top squark mass scale in Table III further decrease to be negative at the Z boson mass scale by Eq. (5). Using Eq. (3), $|\mu|$'s required for the desired value of $m_Z^2 \approx (91\ \text{GeV})^2$ are estimated as

$$|\mu| \approx \{485\ \text{GeV}, 392\ \text{GeV}, 516\ \text{GeV}; 586\ \text{GeV}\} \tag{57}$$

for cases I, II, III, and IV, respectively. When $A_0/m_0 = +0.1$, $\{\Delta_{m_0^2}, \Delta_{A_0}, |\mu|\}$ turn out to be about $\{22, 33, 569\ \text{GeV}\}$. Therefore, we can conclude that the parameter range

$$-0.5 < A_0/m_0 \lesssim +0.1 \quad \text{and} \quad \tan\beta \gtrsim 25 \tag{58}$$

allows $\{\Delta_{m_0^2}, \Delta_{A_0}\}$ and $|\mu|$ to be smaller than 100 and 600 GeV, respectively. Note that $\tan\beta = 50$ is easily achieved, e.g., from the minimal SO(10) [19] or even from the MSSM embedded in a class of the heterotic stringy models [21].

f_G is also a UV parameter in the minimal mixed mediation and so a comment on Δ_{f_G} might be needed. While $\langle S\rangle$ can be fixed to be v_G by a GUT model, $\langle F_S\rangle/m_0$ is associated with the vanishing C.C. as discussed in Sec. III. Once $\langle F_S\rangle/m_0$ is determined through a fine-tuning with other F-term VEVs divided by m_0 and $\langle W_H\rangle/m_0$ such that the C.C. vanishes, its variation yields a nonzero C.C. This problem also arises even in the mGrM or mSUGRA, as discussed below Eq. (19). Also in the mGgM scenario, a variation of $\langle F_S\rangle/\langle S\rangle$ could give a different C.C. Discussions on the vanishing C.C. are beyond the scope of our paper. We will present the valid range of f_G in Sec. IV C.

With $f_G^2 = 0.13$ and $m_0^2 = (4.5\ \text{TeV})^2$, Eq. (46) yields the gluino, wino, and bino masses as follows:

$$M_{3,2,1} \approx \{1.7\ \text{TeV}, 660\ \text{GeV}, 360\ \text{GeV}\} \tag{59}$$

for all the cases considered in Table III. Note that they are all low energy running masses. The physical mass particularly for the gluino would be a bit heavier than it [22]. Since low energy gaugino masses are not affected by a messenger scale, Eq. (59) should be valid even for other choices of y_S.

In the above cases, the sbottom and sleptons turn out to be quite heavier than 3 TeV. The first two generations of SUSY particles must be much heavier than them because of their extremely small relevant Yukawa couplings. Accordingly, the bino is the lightest superparticle (LSP). To avoid overclose of the bino dark matter in the Universe, some entropy production [23] or other lighter dark matter such as the axino and axion is needed [24].

2. Case for $Q_M \ll M_{\rm GUT}$

Since the mass of the messenger fields $\{\mathbf{5}_M, \bar{\mathbf{5}}_M\}$ is given by $y_S\langle S\rangle$, the RG evolutions of the gauge and Yukawa coupling constants and soft mass parameters should be modified by them from those of the MSSM above the messenger mass scale, $Q > y_S\langle S\rangle$. Although $\langle S\rangle$ can be fixed with a proper UV model, y_S still remains as a free parameter. Thus, one might anticipate that low energy values of $m_{h_u}^2$ would be quite sensitive to y_S. In this subsection, we attempt to show that $\{m_{h_u}^2, m_{q_3}^2, m_{u_3^c}^2\}$ at the top squark decoupling scale are very *insensitive* to y_S unlike the naive expectation. Although we first discuss a small $\tan\beta$ case for a qualitative understanding, using semianalytic expressions, the result is quite general: we will display later the numerical result for a large $\tan\beta$ case.

In the energy scale between the GUT and the messenger scales, only the mGrM effects are active: the mGgM effects come in below the messenger scale. Since we neglect the gaugino masses by mGrM in this paper, $m_{q_3}^2$, $m_{u_3^c}^2$, and $m_{h_u}^2$ for $t_M < t < t_0$ are simply

$$m_{q_3 1}^2(t) = m_0^2 + \frac{3m_0^2}{6}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt\bar{y}_t^2} - 1\right] + \frac{F_1(t)}{6}, \quad (60)$$

$$m_{u_3^c 1}^2(t) = m_0^2 + \frac{3m_0^2}{3}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt\bar{y}_t^2} - 1\right] + \frac{F_1(t)}{3}, \quad (61)$$

$$m_{h_u 1}^2(t) = m_0^2 + \frac{3m_0^2}{2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt\bar{y}_t^2} - 1\right] + \frac{F_1(t)}{2}, \quad (62)$$

where $\bar{y}_t$ means the top quark Yukawa coupling constant modified by the messenger fields for $t > t_M$. They can be obtained from Eqs. (11)–(13) and (23). $F_1(t)$ in the above equations is obtained just by neglecting $m_{1/2}/g_0^2$ and setting $A_0 = a_Y m_0$ in Eq. (14):

$$F_1(t) = a_Y^2 m_0^2 e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt\bar{y}_t^2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt'\bar{y}_t^2} - 1\right]. \quad (63)$$

Hence, we have

$$\begin{aligned} X_{t1}(t) &= m_{q_3 1}^2(t) + m_{u_3^c 1}^2(t) + m_{h_u 1}^2(t) \\ &= 3m_0^2 e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt\bar{y}_t^2} + F_1(t). \end{aligned} \quad (64)$$

At the messenger scale $t = t_M$, the mGgM effects become active: the additional soft masses squared, Eqs. (39)–(41), and the gaugino masses by Eq. (33) should be imposed to the RG solutions, Eqs. (11)–(13), at $t = t_M$. For $t_T \le t \le t_M$, therefore, we get

$$\begin{aligned} m_{q_3}^2(t) = {}& m_{q_3}^2(t_M) + \frac{X_t(t_M)}{6}\left[e^{\frac{3}{4\pi^2}\int_{t_M}^{t} dt' y_t^2} - 1\right] + \frac{F_2(t)}{6} \\ &+ f_G^2 m_0^2\left[\frac{8}{9}\{g_3^4(t) - g_3^4(t_M)\} - \frac{3}{2}\{g_2^4(t) - g_2^4(t_M)\} - \frac{1}{198}\{g_1^4(t) - g_1^4(t_M)\}\right], \end{aligned} \quad (65)$$

$$m_{u_3^c}^2(t) = m_{u_3^c}^2(t_M) + \frac{X_t(t_M)}{3}\left[e^{\frac{3}{4\pi^2}\int_{t_M}^{t} dt' y_t^2} - 1\right] + \frac{F_2(t)}{3} + f_G^2 m_0^2\left[\frac{8}{9}\{g_3^4(t) - g_3^4(t_M)\} - \frac{8}{99}\{g_1^4(t) - g_1^4(t_M)\}\right], \quad (66)$$

$$m_{h_u}^2(t) = m_{h_u}^2(t_M) + \frac{X_t(t_M)}{2}\left[e^{\frac{3}{4\pi^2}\int_{t_M}^{t} dt' y_t^2} - 1\right] + \frac{F_2(t)}{2} - f_G^2 m_0^2\left[\frac{3}{2}\{g_2^4(t) - g_2^4(t_M)\} + \frac{1}{22}\{g_1^4(t) - g_1^4(t_M)\}\right], \quad (67)$$

where $m_{q_3}^2(t_M) = m_{q_3 1}^2(t_M) + \delta m_{u_3^c}^2|_M$, $m_{u_3^c}^2(t_M) = m_{u_3^c 1}^2(t_M) + \delta m_{u_3^c}^2|_M$, $m_{h_u}^2(t_M) = m_{h_u 1}^2(t_M) + \delta m_{h_u}^2|_M$, $X_t(t_M) = X_{t1}(t_M) + \delta X_t|_M$, etc., and so

$$m_{q_3}^2(t_M) = \frac{3m_0^2}{2}\left[\frac{1}{3}e^{\frac{3}{4\pi^2}\int_{t_0}^{t_M} dt\bar{y}_t^2} + \frac{1}{3}\right] + \frac{F_1(t_M)}{6} + f_G^2 m_0^2\left[\frac{8}{3}g_3^4(t_M) + \frac{3}{2}g_2^4(t_M) + \frac{g_1^4(t_M)}{30}\right], \quad (68)$$

$$m_{u_3^c}^2(t_M) = \frac{3m_0^2}{2}\left[\frac{2}{3}e^{\frac{3}{4\pi^2}\int_{t_0}^{t_M} dt\bar{y}_t^2} + 0\right] + \frac{F_1(t_M)}{3} + f_G^2 m_0^2\left[\frac{8}{3}g_3^4(t_M) + \frac{8}{15}g_1^4(t_M)\right], \quad (69)$$

$$m_{h_u}^2(t_M) = \frac{3m_0^2}{2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t_M} dt\bar{y}_t^2} - \frac{1}{3}\right] + \frac{F_1(t_M)}{2} + f_G^2 m_0^2\left[\frac{3}{2}g_2^4(t_M) + \frac{3}{10}g_1^4(t_M)\right], \quad (70)$$

$$X_t(t_M) = 3m_0^2 e^{\frac{3}{4\pi^2}\int_{t_0}^{t_M} dt\bar{y}_t^2} + F_1(t_M) + f_G^2 m_0^2\left[\frac{16}{3}g_3^4(t_M) + 3g_2^4(t_M) + \frac{13}{15}g_1^4(t_M)\right]. \quad (71)$$

Here, $g_i^4(t_M)$'s ($i = 3, 2, 1$) are extrapolated from their low energy values, using the ordinary MSSM RG equations without the messenger fields. In the above equations, $F_2(t)$ is basically given by Eq. (14), but t_0 should be replaced by t_M. For its definition, refer to the Appendix.

We should note that the top quark Yukawa coupling in the presence of the messengers $\{\mathbf{5}_M, \bar{\mathbf{5}}_M\}$, $\bar{y}(t)$ is *not* much different from $y_t(t)$, i.e., that in the absence of them above the messenger scale. As a result, we have

$$\frac{e^{\frac{3}{4\pi^2}\int_{t_0}^{t_M} dt\bar{y}_t^2}}{e^{\frac{3}{4\pi^2}\int_{t_0}^{t_M} dt y_t^2}} \approx 1.005\,[1.014, 1.032] \tag{72}$$

even for $t_M \approx 23.0$ ($Q_M \approx 1.0 \times 10^{10}$ GeV) [$t_M \approx 18.4$ ($Q_M = 1.0 \times 10^8$ GeV), $t_M \approx 13.8$ ($Q_M = 1.0 \times 10^6$ GeV)], namely, $y_S \sim \mathcal{O}(10^{-6})$ [$\mathcal{O}(10^{-8})$, $\mathcal{O}(10^{-10})$]. For a higher scale t_M, of course, the ratio must be closer to unity. With much larger $\tan\beta$'s, we get almost the same results. From now on, thus, we will set $e^{\frac{3}{4\pi^2}\int_{t_0}^{t_M} dt\bar{y}_t^2} = e^{\frac{3}{4\pi^2}\int_{t_0}^{t_M} dt y_t^2}$, just when we show the insensitivity of $m_{h_u}^2(t_T)$ to y_S. Then, one can arrive at the following results:

$$\begin{aligned} m_{h_u}^2(t) \approx{}& \frac{3m_0^2}{2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt' y_t^2} - \frac{1}{3}\right] + f_G^2 m_0^2\left[\frac{3}{2}g_2^4(t_M) + \frac{3}{10}g_1^4(t_M)\right] \\ &+ \frac{f_G^2 m_0^2}{2}\left[\frac{16}{3}g_3^4(t_M) + 3g_2^4(t_M) + \frac{13}{15}g_1^4(t_M)\right]\left[e^{\frac{3}{4\pi^2}\int_{t_M}^{t} dt' y_t^2} - 1\right] \\ &+ \frac{a_Y^2 m_0^2}{2}e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt' y_t^2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t_M} dt\bar{y}_t^2} - 1\right] + \frac{F_2(t)}{2} - f_G^2 m_0^2\left[\frac{3}{2}\{g_2^4(t) - g_2^4(t_M)\} + \frac{1}{22}\{g_1^4(t) - g_1^4(t_M)\}\right], \end{aligned} \tag{73}$$

and

$$\begin{aligned} \{m_{q_3}^2(t) + m_{u_3^c}^2(t)\} \approx{}& \frac{3m_0^2}{2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt' y_t^2} + \frac{1}{3}\right] + f_G^2 m_0^2\left[\frac{16}{3}g_3^4(t_M) + \frac{3}{2}g_2^4(t_M) + \frac{17}{30}g_1^4(t_M)\right] \\ &+ \frac{f_G^2 m_0^2}{2}\left[\frac{16}{3}g_3^4(t_M) + 3g_2^4(t_M) + \frac{13}{15}g_1^4(t_M)\right]\left[e^{\frac{3}{4\pi^2}\int_{t_M}^{t} dt' y_t^2} - 1\right] \\ &+ \frac{a_Y^2 m_0^2}{2}e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt' y_t^2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t_M} dt\bar{y}_t^2} - 1\right] + \frac{F_2(t)}{2} \\ &+ f_G^2 m_0^2\left[\frac{16}{9}\{g_3^4(t) - g_3^4(t_M)\} - \frac{3}{2}\{g_2^4(t) - g_2^4(t_M)\} - \frac{17}{198}\{g_1^4(t) - g_1^4(t_M)\}\right], \end{aligned} \tag{74}$$

where $F_2(t)$ is recast to

$$\begin{aligned} F_2(t) \approx{}& \frac{f_G^2 m_0^2}{64\pi^4}\left[\left(e^{\frac{3}{4\pi^2}\int_{t_M}^{t} dt' y_t^2}\int_{t_M}^{t} dt' G_A e^{\frac{-3}{4\pi^2}\int_{t_M}^{t'} dt'' y_t^2}\right)^2 - 2e^{\frac{3}{4\pi^2}\int_{t_M}^{t} dt' y_t^2}\int_{t_M}^{t} dt' G_A\int_{t_M}^{t'} dt'' G_A e^{\frac{-3}{4\pi^2}\int_{t_M}^{t''} dt''' y_t^2}\right] \\ &- \frac{f_G^2 m_0^2}{4\pi^2}\left[e^{\frac{3}{4\pi^2}\int_{t_M}^{t} dt' y_t^2}\int_{t_M}^{t} dt' G_X^2 e^{\frac{-3}{4\pi^2}\int_{t_M}^{t'} dt'' y_t^2} - \int_{t_M}^{t} dt' G_X^2\right] \\ &+ \frac{f_G a_Y m_0^2}{4\pi^2}e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt' y_t^2}\left[\int_{t_M}^{t} dt' G_A - e^{\frac{3}{4\pi^2}\int_{t_M}^{t} dt' y_t^2}\int_{t_M}^{t} dt' G_A e^{\frac{-3}{4\pi^2}\int_{t_M}^{t'} dt'' y_t^2}\right] \\ &+ a_Y^2 m_0^2 e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt' y_t^2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt' y_t^2} - e^{\frac{3}{4\pi^2}\int_{t_0}^{t_M} dt'\bar{y}_t^2}\right]. \end{aligned} \tag{75}$$

The coefficients of a_Y^2 in Eqs. (73) and (74) are determined from the third lines of them and the last line of Eq. (75):

$$\frac{a_Y^2 m_0^2}{2}e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt' y_t^2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t} dt' y_t^2} - 1\right], \tag{76}$$

which is coincident with those of Eqs. (52) and (53). See the last line of Eq. (14).

Now let us compare Eq. (73) with (52). The first two terms of Eq. (73) are the same as those of (52). The largest terms among the other ones would be those proportional to

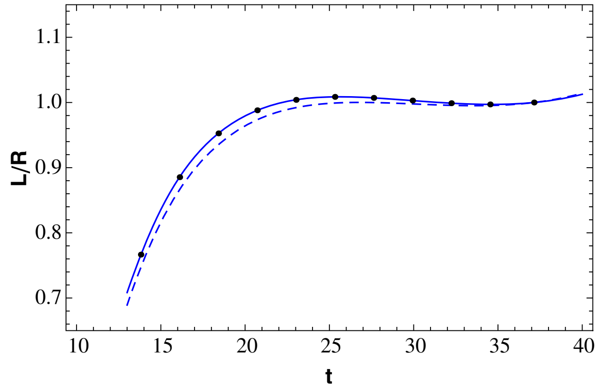

FIG. 2 (color online). Left-hand side/right-hand side of Eq. (77) vs t [$\equiv \log(Q/\mathrm{GeV})$]. The solid (dotted) line corresponds to the case of $\tan\beta = 5$ ($\tan\beta = 50$). In both cases, Eq. (77) becomes approximately valid for $t \gtrsim 18.4$ (or $Q \gtrsim 10^8$ GeV).

$g_3^4(t_M)$. Interestingly enough, the terms in the second line of both equations are almost the same:

$$\left[\frac{16}{3}g_3^4(t_M) + 3g_2^4(t_M) + \frac{13}{15}g_1^4(t_M)\right]\left[e^{\frac{3}{4\pi^2}\int_{t_M}^{t_T}dty_t^2} - 1\right] \approx \left[\frac{16}{3}g_3^4(t_0) + 3g_2^4(t_0) + \frac{13}{15}g_1^4(t_0)\right]\left[e^{\frac{3}{4\pi^2}\int_{t_0}^{t_T}dt'y_t^2} - 1\right] \tag{77}$$

even for $t_M \ll t_0$. Figure 2 shows the ratio between the left-hand side ("L") and the right-hand side ("R") of Eq. (77) with t [$\equiv \log(Q/\mathrm{GeV})$]: Eq. (77) becomes approximately valid for $t \gtrsim 18.4$ or $Q \gtrsim 10^8$ GeV regardless of the size of $\tan\beta$. Note that both $g_i^4(t_M)$'s in Eq. (73) and $g_i^4(t_0)$'s in Eq. (52) are determined from their low energy values with the ordinary MSSM RG equations without the messenger fields.

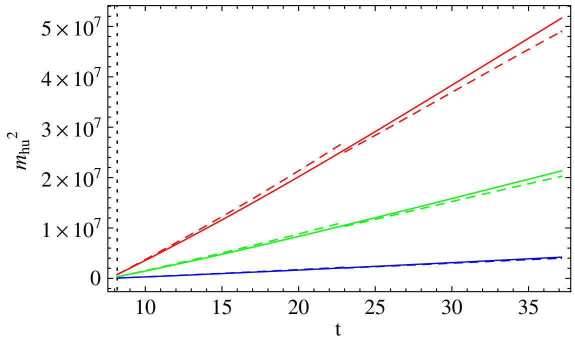

FIG. 3 (color online). RG evolutions of $m_{h_u}^2$ with t [$\equiv \log(Q/\mathrm{GeV})$] for $m_0^2 = (7\ \mathrm{TeV})^2$ (red), $(4.5\ \mathrm{TeV})^2$ (green), and $(2\ \mathrm{TeV})^2$ (blue) when $f_G^2 = 0.13$, $A_0 = -0.2m_0$, and $\tan\beta = 50$ [13]. The tilted solid (dotted) lines correspond to the case of $t_M \approx 37$ (or $Q_M \approx 1.3\times10^{16}$ GeV, "Case A") [$t_M \approx 23$ (or $Q_M = 1.0\times10^{10}$ GeV, "Case B")]. The vertical dotted line at $t = t_T \approx 8.2$ ($Q_T = 3.5$ TeV) indicates the desired top squark decoupling scale. The discontinuities of $m_{h_u}^2(t)$ should appear at the messenger scales.

Both $g_2^4(t_M)$ and $g_1^4(t_M)$ are quite small for $t_M \ll t_0$. Since the beta function coefficient of $g_2^2(t)$ is still small enough ($=1$), $g_2^4(t_M)$ of Eq. (73) is similar to $g_2^4(t_0)$ of Eq. (52): $g_2^4(t_M)/g_2^4(t_0)$ is about 0.943, 0.848, and 0.767 for $t_M \approx 32.2$ ($Q_M = 10^{14}$ GeV), $t_M \approx 23.0$ ($Q_M = 10^{10}$ GeV), and $t_M \approx 13.8$ ($Q_M = 10^6$ GeV), respectively. $g_1^4(t)$ is more suppressed than $g_2^4(t)$. $F(t)$ and $F_2(t)$ cannot make a big difference between Eqs. (52) and (73): although they contain g_3^4, g_3^6, etc., they are suppressed with large numbers (like $64\pi^4$) and/or effectively cancel each other. As shown before, moreover, the coefficients of a_Y^2 must be the same.

The numerical results for the semianalytic solutions, Eqs. (73) and (74) are given by

$$m_{h_u}^2(t_T) \approx m_0^2[0.03 - 0.64f_G^2 - 0.07f_Ga_Y - 0.11a_Y^2],$$
$$\{m_{q_3}^2(t_T) + m_{u_3^c}^2(t_T)\} \approx m_0^2[1.03 + 2.73f_G^2 - 0.07f_Ga_Y - 0.11a_Y^2] \tag{78}$$

for $\tan\beta = 5$ and $t_M \approx 23.0$ ($Q_M = 10^{10}$ GeV). The main difference in $m_{h_u}^2(t_T)$'s of Eqs. (52) and (73) arises from the difference between $g_2^4(t_0)$ and $g_2^4(t_M)$,

$$\Delta m_{h_u}^2(t_T) \approx f_G^2m_0^2 \times 3[g_2^4(t_0) - g_2^4(t_M)] \approx f_G^2m_0^2 \times 0.10, \tag{79}$$

which is approximately the difference between Eqs. (54) and (78). Similarly, the main difference in $\{m_{q_3}^2(t_T) + m_{u_3^c}^2(t_T)\}$ comes from the $f_G^2m_0^2$ parts in the first and last lines of Eqs. (53) and (74). Considering the extremely large energy scale difference between the GUT and 10^{10} GeV, the differences in Eqs. (54) and (78) are quite small. Moreover, such differences become more negligible for a small enough f_G^2 [$\sim\mathcal{O}(0.1)$]. Actually, we need such a small f_G^2 also to suppress the m_0^2 dependence of $m_{h_u}^2(t_T)$.

Figure 3 exhibits some RG evolutions of $m_{h_u}^2$ under various trial m_0^2 when $f_G^2 = 0.13$, $A_0 = -0.2m_0$, and $\tan\beta = 50$ [13]. The solid (dotted) lines correspond to the case of $t_M \approx 37$ (or $Q_M \approx 1.3\times10^{16}$ GeV, "Case A")

[$t_M \approx 23$ (or $Q_M = 1.0 \times 10^{10}$ GeV, "Case B")]. Since the soft masses induced by the mGgM effect are added at the messenger scale, the discontinuities of $m_{h_u}^2(t)$ should arise there. As seen in Fig. 3, in the case of the minimal mixed mediation, the FP of $m_{h_u}^2$ always appears at the desired top squark mass scale ($t = t_T \approx 8.2$) regardless of the messenger scales: the FP scale is not affected by messenger scales or the size of y_S. As defined in Sec. III, in fact, m_0 is originally a parameter associated with the VEV of the hidden sector superpotential, $\langle W_H \rangle$, which triggers SUSY breaking in the observable sector, via both the gravity and gauge mediations, determining the soft mass spectrum. Hence, the low energy value of $m_{h_u}^2$ can remain insensitive to the scale of $\langle W_H \rangle$ and the coupling strength to the hidden sector: the wide ranges of UV parameters can allow almost the same $m_{h_u}^2$'s at low energy. Under this situation, one can guess that $m_0^2 \approx (4.5\ \text{TeV})^2$ happens to be selected by nature, yielding the 3–4 TeV top squark mass and eventually also the 126 GeV Higgs mass. As mentioned above, the gaugino masses are also not affected by a messenger scale. In both cases of Fig. 3, thus, the gaugino masses are given by Eq. (59).

C. Gluino mass bound

Figures 4 and 5 show various scatter plots for given ranges of $\{f_G, a_Y\}$ with $\tan\beta = 50$. m_0^2 in Figs. 4 and 5 are

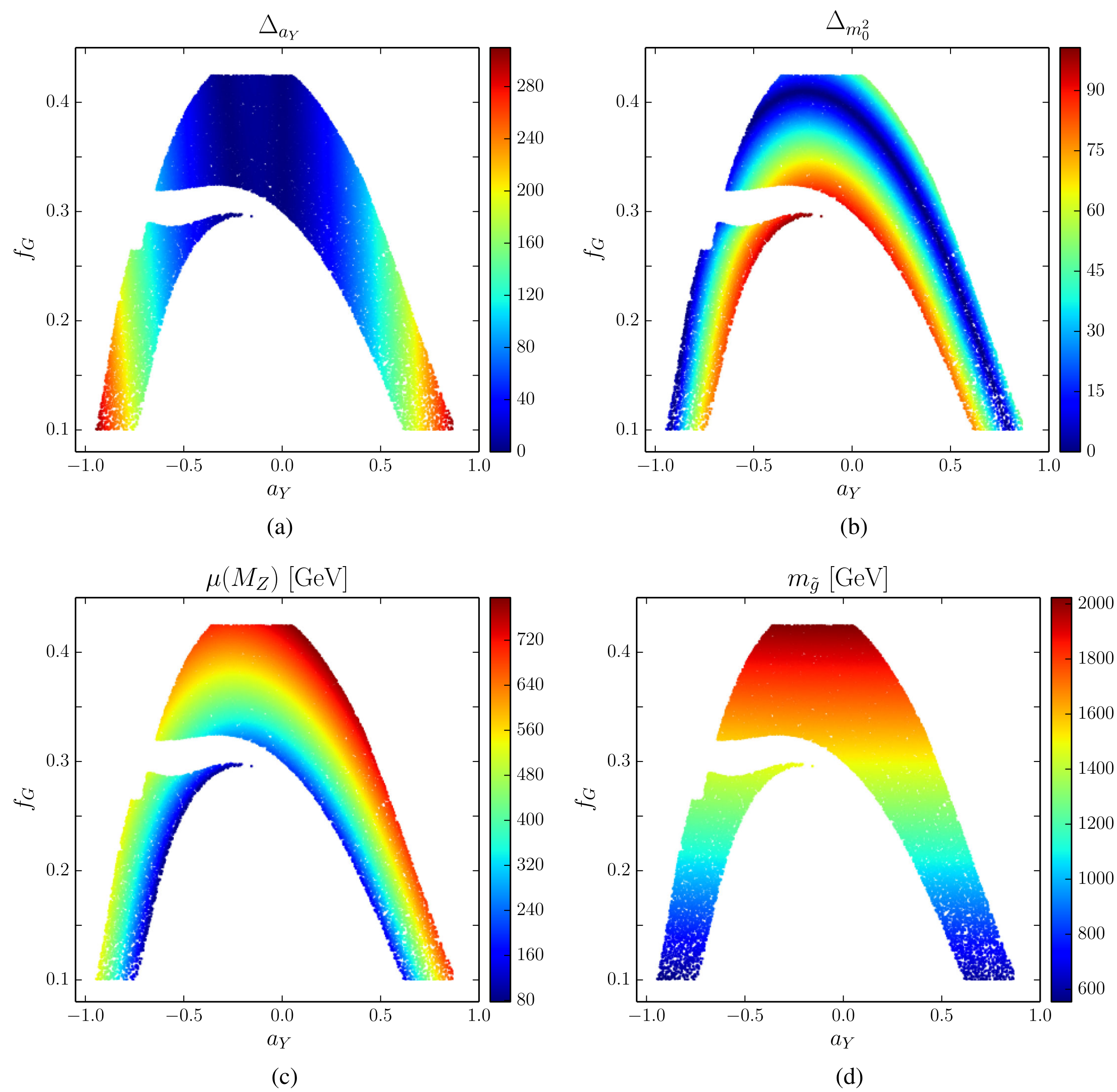

FIG. 4 (color online). Scatter plots for (a) Δ_{a_Y}, (b) $\Delta_{m_0^2}$, and (c) $|\mu|$ at the M_Z scale, and (d) physical gluino mass when $m_0^2 = (4\ \text{TeV})^2$ and $\tan\beta = 50$. The top squark mass scale is about 3.0 TeV.

taken, respectively, to be $(4\ \mathrm{TeV})^2$ and $(5\ \mathrm{TeV})^2$. As a result, the top squark mass scales are about 3.0 and 3.7 TeV, respectively. Here we set M_G as the scale where the EW gauge couplings, g_2 and g_1, meet. It is approximately 1.7×10^{16} GeV in these cases. They all are drawn using SOFTSUSY-3.6.2. As expected from Eqs. (54) and (78), they have "rainbow" shapes. The two "legs" of the "rainbow" in those figures, which are located in the left and right sides of the figures, are relatively narrow. Note that the origin of disconnected points on the left legs is the convergence problem of the iterations of the SOFTSUSY calculation. Their colors are, therefore, supposed to be interpolated continuously since they are not physically forbidden.

As a_Y (or A_0/m_0) is deviated from zero, $m^2_{h_u}$ is expected to rapidly change from Eqs. (54) and (78). Accordingly, m_Z^2 would also rapidly change. It implies that Δ_{a_Y} would rapidly increase as shown in Figs. 4(a) and 5(a), which was seen also in Eq. (56). For a small enough Δ_{a_Y}, thus, we are more interested in the thick central parts around $a_Y = 0$ in the figures,

$$-0.7 \lesssim a_Y \lesssim 0.5, \tag{80}$$

which satisfies $\Delta_{a_Y} < 100$. As discussed before, in addition, we confine our discussion to cases of $|\mu| < 600$ GeV. In fact, the constraint associated with μ or heavy gluino effects could be relaxed by assuming very heavy masses for

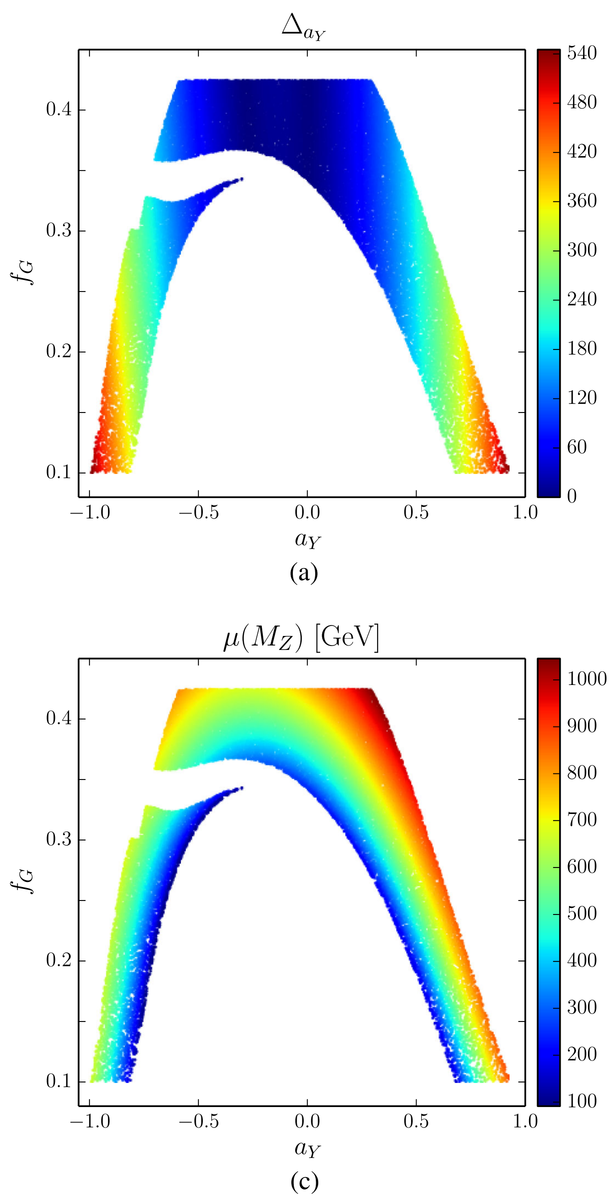

FIG. 5 (color online). Scatter plots for (a) Δ_{a_Y}, (b) $\Delta_{m_0^2}$, and (c) $|\mu|$ at the M_Z scale, and (d) physical gluino mass when $m_0^2 = (5\ \mathrm{TeV})^2$ and $\tan\beta = 50$. The top squark mass scale is about 3.7 TeV.

the superpartners of the first and second generations of the SM chiral fermions [12]. For simplicity, however, we do not consider such a possibility in this paper. Below $f_G \approx 0.3$, the EW symmetry breaking does not occur. From Figs. 4(c) and 5(c), thus, f_G is constrained to

$$0.3 \lesssim f_G \lesssim 0.4, \tag{81}$$

which is consistent with $\Delta_{m_0^2} < 100$ as seen in Figs. 4(b) and 5(b). From Figs. 4(d) and 5(d), we see that the above ranges confine the physical gluino mass to

$$1.6 \text{ TeV} \lesssim m_{\tilde{g}} \lesssim 2.2 \text{ TeV}. \tag{82}$$

Note that this gluino mass bound is a theoretical constraint obtained by considering the naturalness of the EW scale in the minimal mixed mediation scenario. It is well inside the discovery potential range of LHC Run II. Actually the relevant energy scale for the naturalness of the low energy SUSY in the minimal mixed mediation scenario was outside the range of LHC Run I, but it can be covered by LHC Run II. Accordingly, the future exploration for the SUSY particle, particularly the gluino at the LHC, would be more important.

V. CONCLUSION

In this paper, we have studied the SUSY breaking effects by the mGrM parametrized with m_0, combined with the mGgM parametrized with $f_G \cdot m_0$ for a common SUSY breaking source at a hidden sector, $\langle W_H \rangle$ ($\sim m_0 M_P^2$) in a SUGRA framework. When the minimal Kähler potential and the minimal gauge kinetic function ($= \delta_{ab}$) are employed at tree level, a FP of $m_{h_u}^2$ appears at a bit higher energy scale than m_Z (shifted FP), depending on f_G. Basically f_G is a parameter determined by a model. For $0.3 \lesssim f_G \lesssim 0.4$, the FP of $m_{h_u}^2$ emerges at the 3–4 TeV scale, which is the top squark mass scale desired for explaining the 125 GeV Higgs mass, and so $m_{h_u}^2$ becomes quite insensitive to top squark masses or m_0^2. Thus, this range of f_G and $-0.7 \lesssim a_Y \lesssim 0.3$ can admit the fine-tuning measures and μ to be much smaller than 100 and 600 GeV, respectively. The range $0.3 \lesssim f_G \lesssim 0.4$ is directly translated into, e.g., the gluino mass bound, $1.6 \text{ TeV} \lesssim m_{\tilde{g}} \lesssim 2.2$ TeV, which could readily be tested at LHC Run II in the near future.

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APPENDIX: RG EQUATIONS AND SOLUTIONS

We present our semianalytic solutions to the RG equations. When $\tan\beta$ is small enough and the RH neutrinos are decoupled, the RG evolutions of the soft mass parameters, $m_{q_3}^2$, $m_{u_3^c}^2$, $m_{h_u}^2$, and A_t are simplified approximately as

$$16\pi^2 \frac{dm_{q_3}^2}{dt} = 2y_t^2(X_t + A_t^2) - \frac{32}{3} g_3^2 M_3^2 - 6g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2, \tag{A1}$$

$$16\pi^2 \frac{dm_{u_3^c}^2}{dt} = 4y_t^2(X_t + A_t^2) - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2, \tag{A2}$$

$$16\pi^2 \frac{dm_{h_u}^2}{dt} = 6y_t^2(X_t + A_t^2) - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2, \tag{A3}$$

$$8\pi^2 \frac{dA_t}{dt} = 6y_t^2 A_t - \frac{16}{3} g_3^2 M_3 - 3g_2^2 M_2 - \frac{13}{15} g_1^2 M_1 \equiv 6y_t^2 A_t - \left(\frac{m_{1/2}}{g_0^2}\right) G_A, \tag{A4}$$

assuming $\frac{M_a(t)}{g_a^2(t)} = \frac{m_{1/2}}{g_0^2}$ ($a = 3, 2, 1$). Summation of Eqs. (A1), (A2), and (A3) yields the RG equation for X_t ($\equiv m_{q_3}^2 + m_{u_3^c}^2 + m_{h_u}^2$):

$$\frac{dX_t}{dt} = \frac{3y_t^2}{4\pi^2}(X_t + A_t^2) - \frac{1}{4\pi^2}\left(\frac{m_{1/2}}{g_0^2}\right)^2 G_X^2. \tag{A5}$$

In Eqs. (A4) and (A5), G_A and G_X^2 are defined in Eq. (15). The solutions of A_t and X_t are given by

$$A_t(t) = e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\left[A_0 - \frac{1}{8\pi^2}\left(\frac{m_{1/2}}{g_0^2}\right)\int_{t_0}^t dt' G_A e^{\frac{-3}{4\pi^2}\int_{t_0}^{t'} dt'' y_t^2}\right], \tag{A6}$$

$$X_t(t) = e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\left[X_0 + \int_{t_0}^t dt'\left\{\frac{3}{4\pi^2}y_t^2 A_t^2 - \frac{1}{4\pi^2}\left(\frac{m_{1/2}}{g_0^2}\right)^2 G_X^2\right\} e^{\frac{-3}{4\pi^2}\int_{t_0}^{t'} dt'' y_t^2}\right], \tag{A7}$$

where A_0 and X_0 denote the GUT scale values of A_t and X_t, $A_0 \equiv A_t(t=t_0)$, and $X_0 \equiv X_t(t=t_0) = m_{q_30}^2 + m_{u_3^c0}^2 + m_{h_u0}^2$.

With Eqs. (A5) and (A7), one can solve Eqs. (A1), (A2), and (A3):

$$m_{q_3}^2(t) = m_{q_30}^2 + \frac{X_0}{6}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2} - 1\right] + \frac{F(t)}{6} + \left(\frac{m_{1/2}}{g_0^2}\right)^2\left[\frac{8}{9}\{g_3^4(t) - g_0^4\} - \frac{3}{2}\{g_2^4(t) - g_0^4\} - \frac{1}{198}\{g_1^4(t) - g_0^4\}\right], \tag{A8}$$

$$m_{u_3^c}^2(t) = m_{u_3^c0}^2 + \frac{X_0}{3}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2} - 1\right] + \frac{F(t)}{3} + \left(\frac{m_{1/2}}{g_0^2}\right)^2\left[\frac{8}{9}\{g_3^4(t) - g_0^4\} - \frac{8}{99}\{g_1^4(t) - g_0^4\}\right], \tag{A9}$$

$$m_{h_u}^2(t) = m_{h_u0}^2 + \frac{X_0}{2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2} - 1\right] + \frac{F(t)}{2} - \left(\frac{m_{1/2}}{g_0^2}\right)^2\left[\frac{3}{2}\{g_2^4(t) - g_0^4\} + \frac{1}{22}\{g_1^4(t) - g_0^4\}\right], \tag{A10}$$

where $F(t)$ is defined as

$$F(t) \equiv e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\int_{t_0}^t dt' \frac{3}{4\pi^2} y_t^2 A_t^2 e^{\frac{-3}{4\pi^2}\int_{t_0}^{t'} dt'' y_t^2} - \frac{1}{4\pi^2}\left(\frac{m_{1/2}}{g_0^2}\right)^2\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\int_{t_0}^t dt' G_X^2 e^{\frac{-3}{4\pi^2}\int_{t_0}^{t'} dt'' y_t^2} - \int_{t_0}^t dt' G_X^2\right]. \tag{A11}$$

Using Eq. (10), one can obtain the following results:

$$\int_{t_0}^t dt' g_i^2 M_i^2 = \frac{4\pi^2}{b_i}\left(\frac{m_{1/2}}{g_0^2}\right)^2\{g_i^4(t) - g_0^4\}, \tag{A12}$$

$$\int_{t_0}^t dt' g_i^2 M_i = \frac{8\pi^2}{b_i}\left(\frac{m_{1/2}}{g_0^2}\right)\{g_i^2(t) - g_0^2\}, \tag{A13}$$

$$\int_{t_0}^t dt' g_i^4 = \frac{8\pi^2}{b_i}\{g_i^2(t) - g_0^2\}, \tag{A14}$$

which are useful to get the solutions, Eqs. (A8), (A9), and (A10).

With Eq. (A6), the first line of Eq. (A11) is recast to

$$\begin{aligned}
&A_t^2(t) - e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\left\{A_0^2 - \frac{1}{4\pi^2}\left(\frac{m_{1/2}}{g_0^2}\right)\int_{t_0}^t dt' G_A(t') A_t(t') e^{\frac{-3}{4\pi^2}\int_{t_0}^{t'} dt'' y_t^2}\right\} \\
&= \frac{1}{64\pi^4}\left(\frac{m_{1/2}}{g_0^2}\right)^2\left[\left(e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\int_{t_0}^t dt' G_A e^{\frac{-3}{4\pi^2}\int_{t_0}^{t'} dt'' y_t^2}\right)^2\right. \\
&\quad \left. - 2e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\int_{t_0}^t dt' G_A \int_{t_0}^{t'} dt'' G_A e^{\frac{-3}{4\pi^2}\int_{t_0}^{t''} dt''' y_t^2}\right] \\
&\quad + \frac{A_0}{4\pi^2}\left(\frac{m_{1/2}}{g_0^2}\right) e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\left[\int_{t_0}^t dt' G_A - e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\int_{t_0}^t dt' G_A e^{\frac{-3}{4\pi^2}\int_{t_0}^{t'} dt'' y_t^2}\right] \\
&\quad + A_0^2 e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2}\left[e^{\frac{3}{4\pi^2}\int_{t_0}^t dt' y_t^2} - 1\right].
\end{aligned} \tag{A15}$$

When the gaugino masses are generated below t_M with $\frac{M_a(t_M)}{g_a^2(t_M)} = f_G m_0$ ($a = 3, 2, 1$), the solutions for $t_f < t_i < t_M$ are

$$m_{q_3}^2(t_f) = m_{q_3}^2(t_i) + \frac{1}{6}\{X_t(t_f) - X_t(t_i)\} + \frac{f_G^2 m_0^2}{24\pi^2}\int_{t_i}^{t_f} dt G_X^2 + f_G^2 m_0^2\left[\frac{8}{9}\{g_3^4(t_f) - g_3^4(t_i)\} - \frac{3}{2}\{g_2^4(t_f) - g_2^4(t_i)\} - \frac{1}{198}\{g_1^4(t_f) - g_1^4(t_i)\}\right], \tag{A16}$$

$$m_{u_3^c}^2(t_f) = m_{u_3^c}^2(t_i) + \frac{1}{3}\{X_t(t_f) - X_t(t_i)\} + \frac{f_G^2 m_0^2}{12\pi^2}\int_{t_i}^{t_f} dt G_X^2 + f_G^2 m_0^2\left[\frac{8}{9}\{g_3^4(t_f) - g_3^4(t_i)\} - \frac{8}{99}\{g_1^4(t_f) - g_1^4(t_i)\}\right], \tag{A17}$$

$$m_{h_u}^2(t_f) = m_{h_u}^2(t_i) + \frac{1}{2}\{X_t(t_f) - X_t(t_i)\} + \frac{f_G^2 m_0^2}{8\pi^2}\int_{t_i}^{t_f} dt G_X^2 - f_G^2 m_0^2\left[\frac{3}{2}\{g_2^4(t_f) - g_2^4(t_i)\} + \frac{1}{22}\{g_1^4(t_f) - g_1^4(t_i)\}\right], \tag{A18}$$

where

$$X_t(t_f) - X_t(t_i) = X_t(t_i)\left[e^{\frac{3}{4\pi^2}\int_{t_i}^{t_f} dt y_t^2} - 1\right] + e^{\frac{3}{4\pi^2}\int_{t_i}^{t_f} dt y_t^2}\int_{t_i}^{t_f} dt'\left(\frac{3}{4\pi^2}y_t^2 A_t^2 - \frac{f_G^2 m_0^2}{4\pi^2}G_X^2\right)e^{\frac{-3}{4\pi^2}\int_{t_i}^{t'} dt'' y_t^2} \tag{A19}$$

and

$$A_t(t_f) = e^{\frac{3}{4\pi^2}\int_{t_i}^{t_f} dt y_t^2}\left[A_t(t_i) - \frac{f_G m_0}{8\pi^2}\int_{t_i}^{t_f} dt' G_A e^{\frac{-3}{4\pi^2}\int_{t_i}^{t'} dt'' y_t^2}\right]. \tag{A20}$$

In the main text, we set $t_i = t_M$, $t_f = t$, and define $F_2(t)$ as

$$F_2(t) \equiv e^{\frac{3}{4\pi^2}\int_{t_M}^{t} dt' y_t^2}\int_{t_M}^{t} dt' \frac{3}{4\pi^2}y_t^2 A_t^2 e^{\frac{-3}{4\pi^2}\int_{t_M}^{t'} dt'' y_t^2} - \frac{f_G^2 m_0^2}{4\pi^2}\left[e^{\frac{3}{4\pi^2}\int_{t_M}^{t} dt' y_t^2}\int_{t_M}^{t} dt' G_X^2 e^{\frac{-3}{4\pi^2}\int_{t_M}^{t'} dt'' y_t^2} - \int_{t_M}^{t} dt' G_X^2\right]. \tag{A21}$$

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