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E_6 inspired composite Higgs model

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We consider a composite Higgs model embedded into a grand unified theory (GUT) based on the E_6 gauge group. The phenomenological viability of this E_6inspired composite Higgs model (E_6CHM) implies that standard model (SM) elementary fermions with different baryon or lepton number should stem from 27 different representations of E_6. We present a six-dimensional orbifold GUT model in which the E_6 gauge symmetry is broken to the SM gauge group so that the appropriate splitting of the bulk 27-plets takes place. In this model the strongly coupled sector is localized on one of the branes and possesses an $SU(6)$ global symmetry that contains the $SU(3)_C \times SU(2)_W \times U(1)_Y$ subgroup. In this case the approximate gauge coupling unification can be attained if the right-handed top quark is a composite state and the elementary sector involves extra exotic matter beyond the SM which ensures anomaly cancellation. The breakdown of the approximate $SU(6)$ symmetry at low energies in this model results in a set of the pseudo-Nambu-Goldstone states which include a Higgs doublet and scalar color triplet. We discuss the generation of the masses of the SM fermions in the E_6CHM. The presence of the TeV scale vectorlike exotic quarks and scalar color triplet may provide spectacular new physics signals that can be observed at the LHC.

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I. INTRODUCTION

The properties of a new scalar particle, discovered by the ATLAS [1] and CMS [2] collaborations at CERN, strongly suggest that it is the Higgs boson, the particle related to the mechanism of the electroweak (EW) symmetry breaking (EWSB) in the standard model (SM). Current data do not allow one to distinguish whether this new scalar state is an elementary particle (up to very high energies) or a composite state composed of more fundamental degrees of freedom. The idea of a composite Higgs boson, which was proposed in the 1970s [3] and 1980s [4], implies that there exists a new, strongly coupled sector. This sector generates the EW scale dynamically, in analogy with the origin of the QCD scale. In such models the composite Higgs state has generically a large quartic coupling and tends to be quite heavy. On the other hand, the recently observed Higgs boson is sufficiently light, with mass around $m_h \simeq 125$–126 GeV, that it corresponds to a rather small value of the Higgs quartic coupling, $\lambda \simeq 0.13$. The relatively low values of m_h and λ indicate that the Higgs field can emerge as a pseudo-Nambu-Goldstone boson (pNGB) from the spontaneous breaking of an approximate global symmetry of the strongly coupled sector. This idea was used before in the little-Higgs models [5].

The pNGB Higgs idea is also realized in Randall-Sundrum (RS) extradimensional scenarios, with the SM fields in the bulk [6,7]. Via the AdS/CFT correspondence, these scenarios are dual to the 4D composite Higgs scenarios in which Kaluza-Klein excitations are associated with the low-lying bound states at the compositeness scale, f [6–9]. Thus, these models contain a sector of weakly coupled elementary particles, including the SM gauge bosons and SM fermions, as well as a second strongly interacting sector resulting in a set of composite bound states that involves a Higgs doublet, massive excitations of the elementary fields and so on. The elementary states couple weakly to the composite operators of the strong sector. Because of this, at low energies those states identified with SM fermions (bosons) are a mixture of the corresponding elementary fermionic (bosonic) states and their vectorlike fermionic (bosonic) composite partners. In this framework, which is known as partial compositeness [9,10], the SM states couple to the composite Higgs with a strength which is determined by the fraction of the compositeness of this state. That is, for the effective up- and down-quark Yukawa couplings (y^u_{ij} and y^d_{ij} respectively) one gets

$$y^u_{ij} = s^i_q Y^u_{ij} s^j_u, \qquad y^d_{ij} = s^i_q Y^d_{ij} s^j_d, \tag{1}$$

where $i, j = 1, 2, 3$ run over three generations, Y^u_{ij} and Y^d_{ij} are the effective Yukawa couplings of the composite Higgs field to the composite partners of the up and down quarks, while s^j_u and s^j_d are the fractions of compositeness of the right-handed SM quarks of up and down type, respectively, and s^i_q are the fractions of compositeness of the left-handed SM quarks. The couplings of the elementary states to the strongly interacting sector explicitly break the global symmetry of the latter. As a consequence, the pNGB

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Higgs potential arises from loops containing elementary states and that in turn leads to the suppression of the effective quartic Higgs coupling.

The observed mass hierarchy in the quark and lepton sectors can be accommodated through partial compositeness if the fractions of compositeness of the first- and second-generation fermions are quite small; that is, the couplings of the corresponding elementary states to their composite partners are very weak. Such weak couplings also substantially suppress the flavor-changing effects and the modifications of the W and Z couplings associated with the light SM fermions [9,11], serving as a generalization of Glashow-Iliopoulos-Maiani (GIM) mechanism of the SM [12]. At the same time, the top quark is so heavy that right-handed and left-handed top quarks (t^c and t) should have sizeable fractions of compositeness. Since precision data, such as $Z \to b\bar{b}$ measurements, imply that the left-handed b quark and, hence, t should have a reasonably small admixture of composite partners, t^c is expected to be almost completely composite.

If t^c is entirely composite then the approximate unification of the SM gauge couplings, α_i, can be achieved very naturally [13]. This happens, for example, when all composite objects fill in complete $SU(5)$ representations and the sector of weakly coupled elementary states involves

$$(q_i, d_i^c, \ell_i, e_i^c) + u_\alpha^c + \bar{q} + \bar{d}^c + \bar{\ell} + \bar{e}^c + \eta, \qquad (2)$$

where $\alpha = 1, 2$ runs over the first two generations and $i = 1, 2, 3$ runs over all three. We have denoted here the left-handed quark and lepton doublets by q_i and ℓ_i, the right-handed up- and down-type quarks and charged leptons by u_α^c, d_i^c and e_i^c, while the extra exotic states in Eq. (2), $\bar{q}, \bar{d}^c, \bar{\ell}$ and $\bar{e}^c$, have exactly opposite $SU(3)_C \times SU(2)_W \times U(1)_Y$ quantum numbers to left-handed quark doublets, right-handed down-type quarks, left-handed lepton doublets and right-handed charged leptons, respectively. An extra exotic elementary state η with spin $1/2$, that does not participate in the $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge interactions, is included to ensure the phenomenological viability of such a scenario. This scenario also implies that the dynamics of the strongly interacting sector leads to the composite $\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}$ multiplets of $SU(5)$ which, in turn get combined with $\bar{q}, \bar{d}^c, \bar{\ell}, \bar{e}^c$ and η forming a set of vectorlike states. The only exceptions are the components of the 10–plet associated with the composite t^c, which survive down to the EW scale.

Using the one-loop renormalization group equations (RGEs) it is rather easy to find the value of $\alpha_3(M_Z)$ for which exact gauge coupling unification takes place in this model,

$$\frac{1}{\alpha_3(M_Z)} = \frac{1}{b_1 - b_2}\left[\frac{b_1 - b_3}{\alpha_2(M_Z)} - \frac{b_2 - b_3}{\alpha_1(M_Z)}\right], \qquad (3)$$

where b_i are one-loop beta functions, with the indices 1,2,3 corresponding to the $U(1)_Y$, $SU(2)_W$ and $SU(3)_C$ interactions. Since all composite states come in complete $SU(5)$ multiplets, the strong sector does not contribute to the differential running, which is determined by $(b_i - b_j)$ in the one-loop approximation. Then, for $\alpha(M_Z) = 1/127.9$, $\sin^2\theta_W = 0.231$ and the elementary particle spectrum given by Eq. (2), one finds that exact gauge coupling unification can be obtained for $\alpha_3(M_Z) \simeq 0.109$. Although this value of $\alpha_3(M_Z)$ is substantially lower than the central measured low energy value, this result does indicate that at high energies an approximate gauge coupling unification can be attained within the composite Higgs model with a composite t^c. It was argued that the inclusion of higher order effects coming from the strongly coupled sector, as well as the weakly coupled elementary sector, may improve the gauge coupling unification, which takes place around the scale $M_X \sim 10^{15}$–10^{16} GeV in these models [13–15].

In this context it is especially interesting to consider the embedding of the composite Higgs models into well-known grand unified theories (GUTs) in which all elementary quark and lepton states are the components of some irreducible representation of the GUT gauge group. Here we focus on the E_6 gauge theory. In this GUT all elementary SM fermions can originate from the fundamental 27-dimensional representation of E_6. To suppress baryon and lepton number violating operators, that lead to rapid proton decay and too large masses of the left-handed neutrinos, the low-energy effective Lagrangian of this E_6 inspired composite Higgs model (E$_6$CHM) has to be invariant with respect to the global $U(1)_B$ and $U(1)_L$ symmetries associated with the conservation of baryon and lepton numbers, to a very good approximation. In the simplest case this implies that elementary quark and lepton fields with different baryon or lepton number should come from different 27-plets, whereas all other components of these 27-plets acquire masses somewhat close to the scale M_X where the E_6 gauge symmetry is broken down to the SM gauge group. The corresponding splitting of the 27-plets can take place in the orbifold GUTs.[1]

The layout of this paper is as follows. In Sec. II we present a six-dimensional (6D) orbifold GUT model based on the E_6 gauge group in which E_6 is broken down to $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge symmetry, so that all SM fermions with different baryon or lepton number stem from different fundamental representations of E_6. At low energies the weakly coupled elementary sector of this model involves a set of states given by Eq. (2). In this SUSY GUT model all fields of the strongly interacting sector reside on the brane where E_6 gauge symmetry is broken down to $SU(6)$. This $SU(6)$ symmetry includes

[1]It is worth noting that in this orbifold GUT model, extra-dimensional components of bulk fields cannot be associated with composite states.

$SU(3)_C \times SU(2)_W \times U(1)_Y$ subgroup. Here we assume that $SU(6)$ remains an approximate global symmetry of the strongly coupled sector even at low energies. This can happen when the gauge couplings of the strongly interacting sector are considerably larger than the SM gauge couplings below the scale M_X. Assuming that the breakdown of the approximate global $SU(6)$ symmetry down to its $SU(5)$ subgroup takes place at low energies, the spectrum of the E$_6$CHM involves a set of the pseudo-Nambu-Goldstone states, including a composite Higgs doublet and scalar color triplet. In Sec. III we discuss the generation of masses of the SM fermions and other phenomenological implications of the E$_6$CHM. Our results are summarized in Sec. IV.

II. E_6 ORBIFOLD GUT MODEL IN SIX DIMENSIONS

Higher-dimensional theories offer new possibilities for gauge symmetry breaking. A simple and elegant scheme is provided by orbifold compactifications which have been considered for SUSY GUT models in five dimensions [16–21] and six dimensions [20–24]. These models apply ideas that first appeared in string–motivated work [25], where it was pointed out that the gauge symmetry could be broken by identifications imposed on the gauge fields under the spacetime symmetries of an orbifold. More recently, orbifold compactifications of the heterotic string have been constructed which can account for the SM in four dimensions and which have five-dimensional or six-dimensional GUT structures as intermediate steps, very similar to orbifold GUT models [26]. In the context of Sherk-Schwarz compactification the models of composite quarks and leptons were discussed in [27].

In this section we study an $N=1$ supersymmetric (SUSY) GUT in 6D that can lead at low energies to the field content of the weakly coupled elementary sector given by Eq. (2). In particular, we focus on the SUSY GUT based on the $E_6 \times G_0$ gauge group. This SUSY GUT implies that at high energies E_6 and G_0 are broken down to their subgroups, i.e. $SU(3)_C \times SU(2)_W \times U(1)_Y$ and G, respectively, which are associated with the elementary and strongly coupled sectors. Fields from the strongly coupled sector can be charged under both the E_6 and G_0 gauge groups, while the elementary states participate in the E_6 interactions only.

We further assume that all elementary quark and lepton fields are components of the bulk 27 supermultiplets of E_6. In the four-dimensional $N=1$ SUSY models based on the E_6 gauge group, the fundamental 27-dimensional representation involves components Φ_i that correspond to the left-handed quark and lepton supermultiplets (q_i and ℓ_i), right-handed up- and down-type quark supermultiplets (u_i^c and d_i^c), right-handed charged and neutral lepton superfields (e_i^c and ν_i^c), a SM singlet superfield s_i, charged $\pm 1/3$ exotic quark supermultiplets (h_i^c and h_i) as well as two $SU(2)_W$ doublet superfields (h_i^u and h_i^d) that do not carry baryon or lepton number. The minimal $N=1$ supersymmetry in 6D corresponds to $N=2$ in 4D. Indeed, because a 6D fermion state is composed of two 4D Weyl fermions, ψ_i and ψ_i^c, SUSY implies that each 6D superfield includes two complex scalars, ϕ_i and ϕ_i^c, as well. The fields ψ_i, ψ_i^c, ϕ_i and ϕ_i^c form a 4D $N=2$ hypermultiplet which involves two 4D $N=1$ chiral superfields: $\Phi_i = (\phi_i, \psi_i)$ and its conjugate $\bar{\Phi}_i = (\phi_i^c, \psi_i^c)$, with opposite quantum numbers. Thus, each bulk 27 supermultiplet $\hat{\Phi}_i$ contains two 4D $N=1$ supermultiplets, 27 and $\overline{27}$.

The E_6 gauge supermultiplet that exists in the bulk should contain vector bosons A_M ($M = 0,1,2,3,5,6$) and 6D Weyl fermions (gauginos). The 6D gauginos are composed of two 4D Weyl fermions, λ and λ'. These fields can be grouped into vector and chiral multiplets of the $N=1$ supersymmetry in 4D, i.e.

$$V = (A_\mu, \lambda), \qquad \Sigma = \Big((A_5 + iA_6)/\sqrt{2}, \lambda'\Big), \tag{4}$$

where V, A_M, λ and λ' are matrices in the adjoint representation of E_6 and $\mu = 0,1,2,3$. These two $N=1$ supermultiplets also form an $N=2$ vector supermultiplet in 4D.

We consider the compactification of two extra dimensions on a torus T^2 with two fixed radii, R_5 and R_6. Thus, two extra dimensions $y(=x_5)$ and $z(=x_6)$ are compact, i.e. $y \in (-\pi R_5, \pi R_5]$ and $z \in (-\pi R_6, \pi R_6]$. The sizes of the radii, R_5 and R_6, are determined by the GUT scale, M_X. The orbifold T^2/Z_2 is obtained by dividing the torus T^2 with a Z_2 transformation which acts on T^2 according to $y \to -y$ and $z \to -z$. The components of the bulk supermultiplets transform under the Z_2 action as well. The Lagrangian is invariant under the Z_2 transformation. The orbifold T^2/Z_2 has the following set of fixpoints: (0,0), $(\pi R_5, 0)$, $(0, \pi R_6)$ and $(\pi R_5, \pi R_6)$. The Z_2 transformation can be regarded as an equivalence relation that allows one to reduce the physical region associated with the compactification on the orbifold T^2/Z_2 to a pillow with the four fixed points of the Z_2 transformation as corners.

A. The breakdown of E_6 to $SU(4)' \times SU(2)_W \times SU(2)_N \times U(1)'$

Here we examine 6D SUSY GUT compactified on the orbifold $T^2/(Z_2 \times Z_2^I \times Z_2^{II})$. The Z_2, Z_2^I and Z_2^{II} symmetries are reflections. The Z_2 symmetry transformation is defined as before, i.e. $y \to -y$, $z \to -z$. The transformation associated with the reflection Z_2^I is given by $y' \to -y'$, $z \to -z$, where $y' = y - \pi R_5/2$. The reflection Z_2^{II} acts as $y \to -y$, $z' \to -z'$, with $z' = z - \pi R_6/2$. The additional Z_2^I and Z_2^{II} reflection symmetries introduce extra fixed points that lead to the further reduction of the physical region which is limited by these fixed points. In this case the physically irreducible space is a pillow, in which $y \in [0, \pi R_5/2]$ and $z \in [0, \pi R_6/2]$, with the four 4D walls (branes) located at its corners.

The consistency of the construction requires that the Lagrangian of the orbifold SUSY GUT model under consideration is invariant under Z_2, Z_2^I and Z_2^{II} reflections. Each reflection symmetry, Z_2, Z_2^I and Z_2^{II}, has its own orbifold parity, P, P_I and P_{II}. To ensure the invariance of the Lagrangian, the components Φ_i and $\bar{\Phi}_i$ of the bulk 27 supermultiplet should transform under Z_2, Z_2^I and Z_2^{II} as follows,

$$\begin{aligned}
\Phi_i(x,-y,-z) &= P_{ii}\Phi_i(x,y,z),\\
\bar{\Phi}_i(x,-y,-z) &= -P_{ii}\bar{\Phi}_i(x,y,z),\\
\Phi_i(x,-y',-z) &= P^I_{ii}\hat{\Phi}_i(x,y',z),\\
\bar{\Phi}_i(x,-y',-z) &= -P^I_{ii}\bar{\Phi}_i(x,y',z),\\
\Phi_i(x,-y,-z') &= P^{II}_{ii}\hat{\Phi}_i(x,y,z'),\\
\bar{\Phi}_i(x,-y,-z') &= -P^{II}_{ii}\bar{\Phi}_i(x,y,z'),
\end{aligned} \tag{5}$$

where P, P_I and P_{II} are diagonal matrices with eigenvalues ± 1 that act on each component of the fundamental representation of E_6, making some components positive and some components negative.

One can specify the matrix representation of the orbifold parity assignments in terms of the E_6 weights α_i and gauge shifts, Δ, Δ^I and Δ^{II} corresponding to Z_2, Z_2^I and Z_2^{II}. Then the diagonal elements of the matrices P, P^I and P^{II} can be written in the following form [21],

$$(P)_{ii} = \sigma \exp\{2\pi i \Delta\alpha_i\}, \qquad (P^I)_{ii} = \sigma_I \exp\{2\pi i \Delta^I \alpha_i\},$$
$$(P^{II})_{ii} = \sigma_{II} \exp\{2\pi i \Delta^{II}\alpha_i\}, \tag{6}$$

where σ, σ_I and σ_{II} are parities of the bulk 27 supermultiplet, i.e. $\sigma, \sigma_I, \sigma_{II} \in \{+,-\}$. In the case of the fundamental representation of E_6 the particle assignments of the weights are well known (see, for example [21]). Here we choose the following gauge shifts,

$$\Delta = \left(0,0,0,\frac{1}{2},0,0\right), \qquad \Delta^I = \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0\right),$$
$$\Delta^{II} = \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},0,\frac{1}{2},0\right), \tag{7}$$

that correspond to the orbifold parity assignments shown in Table I.

The supermultiplets V and Σ, which are components of the E_6 gauge supermultiplet, transform under Z_2, Z_2^I and Z_2^{II} as follows:

$$\begin{aligned}
V(x,-y,-z) &= PV(x,y,z)P^{-1},\\
\Sigma(x,-y,-z) &= -P\Sigma(x,y,z)P^{-1},\\
V(x,-y',-z) &= P^I V(x,y',z)(P^I)^{-1},\\
\Sigma(x,-y',-z) &= -P^I\Sigma(x,y',z)(P^I)^{-1},\\
V(x,-y,-z') &= P^{II}V(x,y,z')(P^{II})^{-1},\\
\Sigma(x,-y,-z') &= -P^{II}\Sigma(x,y,z')(P^{II})^{-1}.
\end{aligned} \tag{8}$$

In Eq. (8) $V(x,y,z) = V^A(x,y,z)T^A$ and $\Sigma(x,y,z) = \Sigma^A(x,y,z)T^A$ where T^A is the set of generators of the E_6 group. Since different components of the bulk supermultiplets transform differently under Z_2, Z_2^I and Z_2^{II} reflections, the 4D $N=2$ supersymmetry is broken down to 4D $N=1$ SUSY. Moreover, E_6 gauge symmetry is also broken by the parity assignments specified in Table I, because P, P^I and P^{II} are not unit matrices and do not commute with all E_6 generators.

On the brane O, situated near the fixed point $y=z=0$ which is associated with the Z_2 reflection symmetry, the E_6 gauge symmetry is broken down to $SU(6)\times SU(2)_N$. This follows from the P parity assignment in the bulk 27 supermultiplet. The 27-plet of E_6 decomposes under the $SU(6)\times SU(2)_N$ subgroup as follows,

$$27 \to (15,1) + (\bar{6},2),$$

where the first and second quantities in brackets are the $SU(6)$ and $SU(2)_N$ representations. The multiplet $(\bar{6},2)$ involves two $SU(3)_C$ triplets d^c and h^c, two $SU(2)_W$ doublets ℓ and h^d as well as two SM singlets ν^c and s, which are contained in the 27-plet. In this case the SM gauge group is a subgroup of $SU(6)$, whereas none of the $SU(2)_N$ gauge bosons participate in the $SU(3)_C\times SU(2)_W\times U(1)_Y$ gauge interactions. From Table I one can see that the components of the 27 supermultiplet, that correspond to the multiplet $(\bar{6},2)$, transform differently under the Z_2 symmetry as compared with the other components of the 27-plet which form the (15,1)

TABLE I. Orbifold parity assignments in the bulk 27 supermultiplet with $\sigma = \sigma_I = \sigma_{II} = \sigma_{III} = +1$.

	q	d^c	u^c	ℓ	e^c	ν^c	h^u	h^d	h	h^c	s
Z_2	+	−	+	−	+	−	+	−	+	−	−
Z_2^I	−	+	+	−	+	+	−	−	+	+	+
Z_2^{II}	−	−	+	+	+	−	−	+	+	−	−
Z_2^{III}	+	+	+	+	+	+	+	+	+	+	+

representation of $SU(6)$. We further assume that all fields from the strongly coupled sector reside on the O brane.

The P^I parity assignment associated with the Z_2^I symmetry leads to the breakdown of the E_6 gauge group to the $SU(6)'\times SU(2)_W$ subgroup on the brane O_I located at the fixed point $y=\pi R_5/2$, $z=0$. Indeed, according to Table I all $SU(2)_W$ doublet components of the bulk 27 supermultiplet, which form the (6,2) representation, transform differently under the Z_2^I reflection, as compared with all other components of this supermultiplet which compose $(\overline{15},1)$ of $SU(6)'$. In this case the $SU(3)_C$ symmetry is a subgroup of $SU(6)'$. To ensure the breakdown of the E_6 gauge symmetry to the SM gauge group, we assume that two pairs of the supermultiplets (15,1) and $(\overline{15},1)$ of $SU(6)'$ are confined on the brane O_I.

The E_6 symmetry is also broken on the brane O_{II} placed at the fixed point $y=0$, $z=\pi R_6/2$ of the Z_2^{II} reflection symmetry. The P_{II} parity assignment is such that the 16 components q, d^c, ν^c, h^u, h^c and s of the bulk 27-plet are odd, while all other components are even. Because the symmetry breaking mechanism in the orbifold GUT models preserves the rank of the group, the unbroken subgroup at the fixed point O_{II} should be $SO(10)'\times U(1)'$. Indeed, the 16 components of the bulk 27-plet mentioned above constitute a 16-dimensional spinor representation of $SO(10)'$. The other ten components u^c, ℓ, h^d and h of the bulk 27-plet form a ten-dimensional vector representation of $SO(10)'$, whereas the e^c component represents an $SO(10)'$ singlet. The $SU(3)_C$ and $SU(2)_W$ groups are subgroups of $SO(10)'$. It is worth noting that ordinary $SO(10)$ and $SO(10)'$ are not the same subgroups of E_6. In particular, the 16-plets of $SO(10)$ and $SO(10)'$ are formed by different components of the fundamental representation of E_6. The $U(1)'$ charges of the different components of the 27-plet are given in Table II. The consistency of the orbifold GUT model under consideration requires that three pairs of e_i^c and $\overline{e_i^c}$ superfields as well as 45-dimensional representations of $SO(10)'$ reside on the brane O_{II}.

In addition to the three branes mentioned above, there is a fourth brane O_{III} which is situated at the corner $y=\pi R_5/2$, $z=\pi R_6/2$ of the physically irreducible space. The Z_2^{III} reflection corresponding to this brane is obtained by combining the three symmetries Z_2, Z_2^I and Z_2^{II}. The corresponding parity assignment $P_{III}=PP_IP_{II}$. Combining three parity assignments P, P^I and P^{II} one can see that P^{III} is just an identity matrix. This implies that on the brane O_{III} the E_6 gauge symmetry remains intact, while $N=2$ supersymmetry is broken to $N=1$ SUSY. The consistency of this orbifold GUT model requires that two 27-plets are confined on this brane.

The unbroken gauge group of the low-energy effective 4D theory is given by the intersection of the E_6 subgroups at the fixed points O, O_I, O_{II} and O_{III}. The intersection of $SU(6)\times SU(2)_N$, $SU(6)'\times SU(2)_W$ and $SO(10)'\times U(1)'$ yields the group $SU(4)'\times SU(2)_W\times SU(2)_N\times U(1)'$. The $SU(3)_C$ group is a subgroup of $SU(4)'$, which is in turn a subgroup of $SO(10)'$.

B. The breakdown of $SU(4)'\times SU(2)_W\times SU(2)_N\times U(1)'$ to the SM gauge group

As follows from Table I, in general the bulk 27-plets include components with even and odd parities, P, P^I, P^{II} and P^{III}. However, only components for which all parities are positive are allowed to have zero modes. This means that the corresponding fields can survive below the GUT scale M_X. None of the other components of the bulk 27-plets possess massless modes. The elementary states, u_α^c, e_i^c and $\bar{e^c}$, can originate from the bulk 27 supermultiplets $\hat{\Phi}_i^u$, $\hat{\Phi}_i^{\bar{u}}$, $\hat{\Phi}_i^e$ and $\hat{\Phi}_i^{\bar{e}}$ that decompose as follows,

$$\hat{\Phi}_i^u=(27,+,+,+,+),\qquad \hat{\Phi}_i^{\bar{u}}=(27,-,-,-,-),$$
$$\hat{\Phi}_i^e=(27,+,+,+,+),\qquad \hat{\Phi}_i^{\bar{e}}=(27,-,-,-,-),\tag{9}$$

where the quantities in brackets are the E_6 representation as well as the values of σ, σ_I, σ_{II} and σ_{III} associated with this representation. In Eq. (9) $i=1,2,3$ as before. The parities of $\hat{\Phi}_i^u$ are chosen so that their u_i^c, e_i^c and h_i components of the $N=1$ chiral supermultiplet, Φ_i^u, have positive parities with respect to all reflection symmetries (see Table I). Because the invariance of the 6D action requires that the parities of the 4D chiral supermultiplets Φ_i^u and $\bar{\Phi}_i^u$ are opposite, the $N=1$ chiral supermultiplet $\bar{\Phi}_i^u$ does not contain any even components. On the other hand, in the case of $\hat{\Phi}_i^{\bar{u}}$ only components $\bar{u^c}_i$, $\bar{e^c}_i$ and $\bar{h}_i$ of the $N=1$ chiral supermultiplet $\bar{\Phi}_i^{\bar{u}}$ are even. Thus, the Kaluza-Klein (KK) expansion of the bulk supermultiplets $\hat{\Phi}_i^u$ and $\hat{\Phi}_i^{\bar{u}}$ contains zero modes that form $N=1$ chiral supermultiplets

TABLE II. The $U(1)'$, $\tilde{U}(1)$, $U(1)_N$ and $U(1)_Y$ charges (Q_i', $\tilde{Q}_i$, Q_i^N and Q_i^Y, respectively) of the different components of the 27-plet.

	q	d^c	u^c	ℓ	e^c	ν^c	h^u	h^d	h	h^c	s
$\sqrt{24}Q_i'$	1	1	−2	−2	4	1	1	−2	−2	1	1
$\sqrt{24}\tilde{Q}_i$	1	−1	2	0	0	3	−3	0	−2	−1	3
$\sqrt{40}Q_i^N$	1	2	1	2	1	0	−2	−3	−2	−3	5
$\sqrt{\frac{5}{3}}Q_i^Y$	$\frac{1}{6}$	$\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	0

TABLE III. The components of the bulk 27-plets that survive below the scales M_X, φ_0 and ϕ_0. The index $i = 1, 2, 3$ runs over all three generations.

	$\hat{\Phi}^u_i$	$\hat{\Phi}^e_i$	$\hat{\Phi}^q_i$	$\hat{\Phi}^d_i$	$\hat{\Phi}^\ell_i$	$\hat{\Phi}^{\bar{u}}_i$	$\hat{\Phi}^{\bar{e}}_i$	$\hat{\Phi}^{\bar{q}}_i$	$\hat{\Phi}^{\bar{d}}_i$	$\hat{\Phi}^{\bar{\ell}}_i$
$E \lesssim M_X$	u^c_i, e^c_i, h_i	u^c_i, e^c_i, h_i	q_i, h^u_i	d^c_i, ν^c_i, h^c_i, s_i	ℓ_i, h^d_i	$\bar{u^c}_i$, $\bar{e^c}_i$, $\bar{h}_i$	$\bar{u^c}_i$, $\bar{e^c}_i$, $\bar{h}_i$	$\bar{q}_i$, $\bar{h^u_i}$	$\bar{d^c_i}$, $\bar{\nu^c_i}$, $\bar{h^c_i}$, $\bar{s}_i$	$\bar{\ell}_i$, $\bar{h^d_i}$
$E \lesssim \varphi_0$	u^c_i, h_i	e^c_i,	q_i, h^u_i	d^c_i, ν^c_i, h^c_i, s_i	ℓ_i, h^d_i	$\bar{u^c}_i$, $\bar{h}_i$	$\bar{e^c}_i$,	$\bar{q}_i$, $\bar{h^u_i}$	$\bar{d^c_i}$, $\bar{\nu^c_i}$, $\bar{h^c_i}$, $\bar{s}_i$	$\bar{\ell}_i$, $\bar{h^d_i}$
$E \lesssim \phi_0$	u^c_i	e^c_i	q_i	d^c_i	ℓ_i	$\bar{u^c}$	$\bar{e^c}$	$\bar{q}$	$\bar{d^c}$	$\bar{\ell}$

with the quantum numbers of u^c_i, e^c_i, h_i, $\bar{u^c}_i$, $\bar{e^c}_i$ and $\bar{h}_i$. Similar zero modes come from the KK expansion of $\hat{\Phi}^e_i$ and $\hat{\Phi}^{\bar{e}}_i$.

Here we assume that one component, φ, of the 45-dimensional representations of $SO(10)'$ localized on the brane O_{II}, which is associated with the Cartan algebra generator of the $\tilde{U}(1)$ subgroup of $SO(10)'$ (see Table II), acquires a nonzero vacuum expectation value (VEV), φ_0, which is somewhat smaller than the GUT scale, breaking the $SU(4)' \times SU(2)_W \times SU(2)_N \times U(1)'$ gauge group down to $SU(3)_C \times SU(2)_W \times SU(2)_N \times \tilde{U}(1) \times U(1)'$. If this superfield couples to $\hat{\Phi}^e_i$ and $\hat{\Phi}^{\bar{e}}_i$ then the zero modes u^c_i and $\bar{u^c}_i$ as well as h_i and $\bar{h}_i$ gain large masses ($\sim\varphi_0$), forming vectorlike states. Therefore, only $N = 1$ chiral superfields with the quantum numbers of e^c_i and $\bar{e^c}_i$ remain massless in this case. To forbid any couplings of φ to other bulk supermultiplets one can impose a discrete Z^u_2 symmetry under which φ and $\hat{\Phi}^e_i$ are odd whereas all other supermultiplets are even. Since the superfield φ resides on the brane O_{II}, it does not interact with the superfields localized on the brane O. Therefore, the $SU(6) \times SU(2)_N$ global symmetry of the strongly coupled sector remains intact.

Another discrete Z^e_2 symmetry, under which only three superfields e^c_i, which are confined on brane O_{II}, and $\hat{\Phi}^{\bar{u}}_i$ are odd while all other supermultiplets are even, allows us to suppress the interaction between e^c_i and the corresponding components of all bulk supermultiplets except $\hat{\Phi}^{\bar{u}}_i$. In this case the superfield e^c_i gets combined with the appropriate zero modes of $\hat{\Phi}^{\bar{u}}_i$ so that the resulting vectorlike states gain masses of order of M_X. As a consequence, only zero modes associated with the $\bar{u^c}_i$ and $\bar{h}_i$ components of $\hat{\Phi}^{\bar{u}}_i$ remain massless. In addition, we impose a $Z^{\bar{e}}_2$ symmetry which implies that $\bar{e^c_i}$ and $\hat{\Phi}^u_i$ are odd while all other supermultiplets are even. This symmetry allows for the formation of vectorlike states which are formed by the components of the superfield $\bar{e^c_i}$ and the appropriate zero modes of $\hat{\Phi}^u_i$. Again in general these states gain masses of order of M_X. Because of this, the set of the zero modes involves only the u^c_i and h_i components of $\hat{\Phi}^u_i$.

The 4D supermultiplets q_i, d^c_i, $\bar{q}$ and $\bar{d^c}$ can stem from another six pairs of the bulk 27-plets:

$$\hat{\Phi}^q_i = (27, +, -, -, +), \qquad \hat{\Phi}^{\bar{q}}_i = (27, -, +, +, -),$$
$$\hat{\Phi}^d_i = (27, -, +, -, +), \qquad \hat{\Phi}^{\bar{d}}_i = (27, +, -, +, -). \tag{10}$$

Using the orbifold parity assignments presented in Table I, one can check that all parities of q_i and h^u_i components of $\hat{\Phi}^q_i$, $\overline{q_i}$ and $\bar{h^u_i}$ components of $\hat{\Phi}^{\bar{q}}_i$ as well as d^c_i, ν^c_i, h^c_i and s_i components of $\hat{\Phi}^d_i$ and $\overline{d^c_i}$, $\bar{\nu^c_i}$, $\bar{h^c_i}$ and $\bar{s}_i$ components of $\hat{\Phi}^{\bar{d}}_i$ are positive so that the KK expansions of the corresponding 6D superfields should contain the appropriate zero modes. Finally, in order to get 4D supermultiplets ℓ_i and $\bar{\ell}$ the set of the bulk 27-plets should be supplemented by

$$\hat{\Phi}^\ell_i = (27, -, -, +, +), \qquad \hat{\Phi}^{\bar{\ell}}_i = (27, +, +, -, -). \tag{11}$$

The set of zero modes of $\hat{\Phi}^\ell_i$ and $\hat{\Phi}^{\bar{\ell}}_i$ involves $N = 1$ chiral supemultiplets with the quantum numbers of ℓ_i, h^d_i, $\bar{\ell}_i$ and $\bar{h^d_i}$. The complete set of the bulk 27-plets and their zero modes, which survive below $\langle\varphi\rangle = \varphi_0$, are specified in Table III. It is assumed that the mass terms involving zero modes of the 6D supermultiplets with exactly opposite $SU(3)_C \times SU(2)_W \times U(1)_Y$ quantum numbers are not allowed. Such mass terms can be forbidden by the Z^b_2 symmetry, under which $\hat{\Phi}^u_i$, $\hat{\Phi}^e_i$, $\hat{\Phi}^q_i$, $\hat{\Phi}^d_i$, $\hat{\Phi}^\ell_i$ are even, whereas $\hat{\Phi}^{\bar{u}}_i$, $\hat{\Phi}^{\bar{e}}_i$, $\hat{\Phi}^{\bar{q}}_i$, $\hat{\Phi}^{\bar{d}}_i$, $\hat{\Phi}^{\bar{\ell}}_i$ are odd.

The 6D supermultiplets mentioned above lead to the set of zero modes which compose three pairs of complete $N = 1$ chiral 27 and $\overline{27}$-plets. Two $\overline{27}$ supermultiplets (say, associated with $i = 1, 2$) can get combined with two 27-plets which are located on the O_{III} brane resulting in the set of vectorlike states. The mass scale M_0 associated with the masses of these states can be chosen slightly lower than φ_0. We also assume that the ν^c and $\bar{\nu^c}$ components of one pair of $\overline{15}$ and 15 of $SU(6)'$, as well as the s and $\bar{s}$ components of another pair of $\overline{15}$ and 15 of $SU(6)'$ localized on the brane O_I, acquire nonzero VEVs of order of ϕ_0 but somewhat below M_0 and φ_0. The VEVs of ν^c and $\bar{\nu^c}$ break $SU(3)_C \times SU(2)_W \times SU(2)_N \times \tilde{U}(1) \times U(1)'$ gauge symmetry down to the $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_N$ subgroup.[2] The VEVs of s and $\bar{s}$ break $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_N$

[2]Different phenomenological aspects of SUSY models with extra $U(1)_N$ gauge symmetry were considered in [28].

to the SM gauge group. These VEVs also generate the following set of the mass terms in the superpotential of the model under consideration,

$$\delta W_{\rm mass} = M^{\eta}_{ij} h_i h^c_j + M^{\zeta}_{ij} h^u_i h^d_j + M^{\xi}_{ij} s_i s_j + M^{\nu}_{ij} \nu^c_i \nu^c_j + \bar{M}^{\eta} \bar{h}\, \bar{h}^c + \bar{M}^{\zeta} \bar{h}^u\, \bar{h}^d + \bar{M}^{\xi} \bar{s}^2 + \bar{M}^{\nu} \bar{\nu}^{c\,2}, \tag{12}$$

where $M^{\eta}_{ij} \sim M^{\zeta}_{ij} \sim M^{\xi}_{ij} \sim M^{\nu}_{ij} \sim \bar{M}^{\eta} \sim \bar{M}^{\zeta} \sim \bar{M}^{\xi} \sim \bar{M}^{\nu} \sim \phi_0$ and $i, j = 1, 2, 3$. Two pairs of $\overline{15}$ and 15 of $SU(6)'$ are expected to form a set of vectorlike states with masses close to ϕ_0. Since these supermultiplets are confined on the brane O_I, they do not interact with the superfields which reside on the brane O, so that the $SU(6) \times SU(2)_N$ global symmetry of the strongly coupled sector remains unbroken.[3]

Finally, at the scale M_S, which is one or two orders of magnitude lower than M_X, SUSY gets broken and scalar components of all superfields including the pseudo-Goldstone bosons gain masses of order M_S. We assume that near the supersymmetry breaking scale the SM singlet superfield S, which interacts only with the components of the 6D supermultiplets $\hat{\Phi}^u_3$ and $\hat{\Phi}^{\bar{u}}_3$, acquires a VEV giving rise to the masses of the zero modes with the quantum numbers of u_3 and $\bar{u_3}$. Again the interactions of S with other bulk 27-plets can be forbidden by imposing the appropriate discrete Z_2 symmetry. This leads to the decoupling of the right-handed top quarks from the rest of the spectrum.

The extra fermionic state η, which appears in Eq. (2), can stem from the bulk supermultiplet that does not participate in the E_6 gauge interactions. This supermultiplet can decompose under the E_6 gauge group and Z_2, Z_2^I, Z_2^{II} and Z_2^{III} symmetries as follows $(1, +, +, +, +)$. As a result the field content of the weakly coupled elementary sector given by Eq. (2) is reproduced.

C. Anomaly cancellation and unification of gauge couplings

For the consistency of the orbifold GUT model it is crucial that all anomalies get canceled. In the 6D models there are two types of anomalies: 4D anomalies at orbifold fixed points [29] and bulk anomalies [30,31] which are induced by box diagrams with four gauge currents. The contributions of the anomalous box diagrams to the 6D anomalies are determined by the trace of four generators of gauge group. This trace contains a nonfactorizable part and a part which can be reduced to the product of traces of two generators. The first part corresponds to the irreducible gauge anomaly, while the second part is known as reducible anomaly. The reducible anomalies can be canceled by the Green–Schwarz mechanism [32]. On the other hand, the 6D orbifold GUT models based on the E_6 gauge group do not have an irreducible bulk anomaly [31]. At the fixed points, the brane anomaly reduces to the anomaly of the unbroken subgroup of E_6. It was shown that the sum of the contributions to the 4D anomalies at the fixed point is equal to the sum of the contributions of the zero modes localized at the brane [30,33]. In this context it is worth noting that in the orbifold GUT model under consideration the contributions of the elementary superfields, which are confined on each brane, to the corresponding brane anomalies get canceled automatically. Moreover the orbifold parity assignments are chosen so that the KK modes of the bulk 27-plets localized at the fixpoints always form pairs of $N = 1$ supermultiplets with opposite quantum numbers. This choice of parity assignments guarantees that the contributions of zero modes of the bulk superfields to the brane anomalies are canceled as well.

One should also mention that the orbifold GUT models do not lead to the exact gauge coupling unification at the scale M_X where E_6 gauge symmetry is broken. The gauge couplings at the scale M_X may not be identical, because of the sizable contributions to these couplings that can come from the branes where E_6 gauge symmetry is broken. However, if in the orbifold GUT model the bulk and brane gauge couplings have almost equal strength, then the gauge couplings, which are associated with the zero modes of gauge bosons, are dominated by the bulk contributions because of the spread of the wavefunction of the corresponding zero modes. Since the bulk contributions to the gauge couplings are necessarily E_6 symmetric, near the scale M_X an approximate unification of the gauge couplings is expected to take place. The gauge coupling unification within 5D and 6D orbifold GUT models was discussed in Refs. [18,19] and [23], respectively. As we do not require here exact gauge coupling unification in the vicinity of the scale where E_6 is broken, M_X can even be considerably larger than 10^{16} GeV, ensuring proton stability.

III. E_6 INSPIRED COMPOSITE HIGGS MODEL AND ITS PHENOMENOLOGICAL IMPLICATIONS

A. Global symmetries and constraints

Let us now consider the phenomenological implications of the composite Higgs model in which the weakly coupled elementary sector includes a set of states given by Eq. (2) at low energies, while the strongly interacting sector involves fields localized on the brane O where E_6 gauge symmetry is broken to $SU(6) \times SU(2)_N$. Because all fields of the strongly coupled sector reside on the O brane, the global symmetry in this sector can be $SU(6) \times SU(2)_N$ at high energies, even though local symmetry is broken down to the SM gauge group. The $SU(3)_C \times SU(2)_W \times U(1)_Y$

[3]The $SU(2)_N$ symmetry can be also broken spontaneously on the brane O. Then the VEVs of ν^c and $\bar{\nu}^c$ as well as s and $\bar{s}$ break the residual gauge symmetry down to the SM gauge group, inducing the mass terms (12).

gauge interactions break $SU(6)$ global symmetry. However, if the gauge couplings of the strongly coupled sector are substantially larger than the SM gauge couplings at any scale below M_X, then $SU(6)$ can still remain an approximate global symmetry of the strongly interacting sector, even at energies as low as say 10 TeV. To simplify our analysis, we further assume that the global $SU(2)_N$ symmetry is entirely broken so that the Lagrangian of the strongly coupled sector at low energies is just invariant under the transformations of the $SU(6)$ group only.

In this context it is worth noting that the minimal composite Higgs model (MCHM) possesses global $SO(5)$ symmetry which is broken down to $SO(4)$ at the scale f [7] (for a recent review, see [34]). The custodial symmetry $SU(2)_{\rm cust} \subset SO(4) \cong SU(2)_W \times SU(2)_R$ [35] allows one to protect the Peskin-Takeuchi $\hat{T}$ parameter [36], which is extremely constrained by present data [37], against new physics contributions. Within the composite Higgs models the contributions of new states to the electroweak precision observables, including the $\hat{S}$ and $\hat{T}$ parameters as well as the $Zb_L\bar{b}_L$ coupling, were analyzed in [14,38–45]. Experimental limits on the value of the parameter $|\hat{S}| \lesssim 0.002$ leads to the constraint $m_\rho = g_\rho f \gtrsim 2.5$ TeV where m_ρ is a scale associated with the masses of the set of spin-1 resonances that includes composite partners of the SM gauge bosons and g_ρ is a coupling of these ρ-like vector resonances [7].

Even more stringent bounds on f come from the observed suppression of the nondiagonal flavor transitions in the case when the matrices of effective Yukawa couplings in the strong sector, such as Y^u_{ij} and Y^d_{ij}, are structureless, i.e. anarchic matrices. Indeed, although the generalization of the GIM mechanism in the composite Higgs model associated with partial compositeness significantly reduces the new physics contributions to dangerous flavor-changing processes, this suppression is not sufficient to provide a fully realistic theory of flavor. The constraints that arise from the nondiagonal flavor transitions in the quark and lepton sectors were examined in Refs. [43–48] and [48–51], respectively. In particular, it was shown that in the case of anarchic partial compositeness f should be larger than 10 TeV, because of the constraints which stem from the measurements of CP violation in the Kaon system [43,44,46,47], as well as the measurements of the electron electric dipole moment and $\mu \to e\gamma$ transitions [50]. Large values of f imply that a substantial degree of tuning is required to get a 125 GeV Higgs state. The ratio $\xi = v^2/f^2$ constitutes a rough measure of the degree of fine–tuning and describes the departure from an elementary Higgs scenario in the composite Higgs models. The bound on f can be considerably alleviated in the composite Higgs models with flavor symmetries [42,43,46,48,49,52]. For instance, in the models with $U(2)^3 = U(2)_q \times U(2)_u \times U(2)_d$ symmetry, under which the first two generations of elementary quark states transform as doublets and the third generation as singlets, the bounds that originate from the Kaon and B systems can be satisfied even for relatively low values of f that correspond to $m_\rho \sim 3$ TeV [48,49]. Recently, the implications of the composite Higgs models were studied for Higgs physics [40,41,53–56], gauge coupling unification [57], dark matter [14,15,54,58] and collider phenomenology [39,40,42,46,49,56,59]. The nonminimal composite Higgs models were considered in [14,15,53,54,58,60].

The composite Higgs model under consideration (E_6CHM) does not possess $SU(2)_{\rm cust}$ symmetry mentioned above. As a consequence, the absolute value of the parameter $|\hat{T}|$ is expected to be of the order [14]

$$|\hat{T}| \sim \xi = \frac{v^2}{f^2}. \tag{13}$$

Since the electroweak precision measurements constrain $|\hat{T}| \lesssim 0.002$, Eq. (13) leads to the stringent lower bound on the scale $f \gtrsim 5$–6 TeV,[4] where the breakdown of the $SU(6)$ global symmetry takes place. Although the adequate suppression of the flavor-changing processes in general requires f to be even larger, i.e. $f \gtrsim 10$ TeV, the desirable suppression of the nondiagonal flavor transitions can also be achieved by imposing $U(2)^3$ or even larger flavor symmetry, just as in other composite Higgs models discussed above. Therefore, hereafter we assume that $f \gtrsim 5$–10 TeV. This means that a significant tuning, $\sim 0.1 - 0.01\%$, is needed to comply with the Higgs mass measurements. This tuning can be accomplished by canceling two different contributions associated with the exotic fermions and gauge fields that appear with different signs [15].

In contrast to the SM where there are two accidental $U(1)$ symmetries ($U(1)_B$ and $U(1)_L$) of the renormalizable Lagrangian that result in the conservation of baryon and lepton numbers, new interactions in the composite Higgs models in general give rise to baryon and lepton number violating processes. Indeed, because of the mixing between the elementary states and their composite partners, the four-fermion operators leading to proton decay can be generated through nonperturbative effects. Such operators are only suppressed by the scale f and the small fractions of compositeness of the first- and second-generation fermions. This suppression is not sufficient to prevent too rapid proton decay and other baryon number violating processes. Similarly, dimension-five operators of the form $\ell_i \ell_j HH/f$, where H is a composite Higgs doublet, can be induced resulting in lepton number violation and generating Majorana neutrino masses which are far too large with respect to the observed ones.

[4] A weaker bound was obtained in [61].

Thus, in the composite Higgs models one is forced to impose additional $U(1)_B$ and $U(1)_L$ symmetries to avoid violations of baryon and lepton numbers which are too large. These symmetries should be part of the global symmetries of the composite sector and should be extended consistently to the elementary sector so that baryon and lepton numbers are preserved to very good approximation up to scales $\sim M_X$. In particular, within the $\mathrm{E_6CHM}$ the interactions between the elementary states and their composite partners break $SU(6)$ global symmetry and its $SU(5)$ subgroup but must preserve its $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauged subgroup as well as global $U(1)_B$ and $U(1)_L$ symmetries. For this reason, the simplest possibility is to take $U(1)_B \times U(1)_L$ external to the group $SU(6)$, because $SU(6)$ and its $SU(5)$ subgroup always allow for baryon and lepton number violating operators in the elementary sector. In other words, at low energies the Lagrangian of the strongly coupled sector of the $\mathrm{E_6CHM}$ should be invariant under the transformations of an $SU(6) \times U(1)_B \times U(1)_L$ global symmetry, whereas the full effective Lagrangian of the $\mathrm{E_6CHM}$ respects $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_B \times U(1)_L$ symmetry.

The $U(1)_B$ and $U(1)_L$ symmetries can be incorporated into the orbifold GUT model considered in the previous section. Since this model is based on the $E_6 \times G_0$ gauge symmetry, $U(1)_B$ and $U(1)_L$ can be subgroups of the G_0 group associated with the strongly coupled sector. In principle the G_0 symmetry can be broken down to its subgroup G in such a way that the $U(1)_B$ and $U(1)_L$ symmetries remain intact on the brane O, where all composite sector fields reside. As a result the Lagrangian of the strongly coupled sector respects the $SU(6) \times U(1)_B \times U(1)_L$ global symmetry.

Nevertheless, both $U(1)_B$ and $U(1)_L$ symmetries are expected to get broken on the brane O_I. Note that the nearly exact conservation of the $U(1)_B$ and $U(1)_L$ charges at low energies implies that the elementary fermions with different baryon and/or lepton numbers should belong to different bulk 27-plets. In this sense the baryon and lepton numbers of the bulk supermultiplets are determined by the $U(1)_B$ and $U(1)_L$ charges (B and L) of the fermion components of these supermultiplets that survive to low energies. Thus, $\hat{\Phi}^u_i$ and $\hat{\Phi}^d_i$ have $B = -\frac{1}{3}$ and $L = 0$, $\hat{\Phi}^q_i$ carry $B = \frac{1}{3}$ and $L = 0$, $\hat{\Phi}^\ell_i$ have $B = 0$ and $L = 1$ whereas $\hat{\Phi}^e_i$ carry $B = 0$ and $L = -1$. Because all components of the bulk supermultiplets carry the same $U(1)_B$ and $U(1)_L$ charges, E_6 gauge interactions do not give rise to baryon and lepton number violating operators, in contrast with conventional GUTs. On the other hand, the breakdown of $U(1)_B$ occurs when the zero modes of the components h^c and h of the bulk supermultiplets $\hat{\Phi}^u_i$ and $\hat{\Phi}^d_i$ form vectorlike states. The corresponding mass terms in the Lagrangian are forbidden by the $U(1)_B$ symmetry. The Majorana mass terms associated with the zero modes of the ν^c and s components of the bulk supermultiplets $\hat{\Phi}^d_i$ also break this symmetry. Finally, the breakdown of both $U(1)_B$ and $U(1)_L$ symmetries takes place when the zero modes of the components h^u and h^d of the bulk 27-plets $\hat{\Phi}^q_i$ and $\hat{\Phi}^\ell_i$ form vectorlike states. All these mass terms are induced by the VEVs of the scalar components of the superfields which are localized on the brane O_I. Therefore, $U(1)_B$ and $U(1)_L$ should be broken on this brane. Then the phenomenological viability of the model under consideration requires that the masses of the elementary states, which cause the breakdown of $U(1)_B$ and $U(1)_L$ symmetries, should be sufficiently close to 10^{16} GeV to ensure the adequate suppression of the baryon and lepton number violating operators that give rise to proton decay. In the context of 5D and 6D orbifold GUT models, proton stability was discussed in Refs. [17,18] and [24], respectively.

B. Nonlinear realization of the Higgs mechanism

Below scale f ($f \gtrsim 5$–10 TeV) the global $SU(6)$ symmetry in the $\mathrm{E_6CHM}$ is broken down to $SU(5)$, which in turn contains the $SU(3)_C \times SU(2)_W \times U(1)_Y$ subgroup. Here we denote the unbroken generators of $SU(6)$, i.e. generators of its $SU(5)$ subgroup, by T^a, while the broken ones, i.e. generators from the coset $SU(6)/SU(5)$, are denoted by $T^{\hat{a}}$. The generators of the $SU(6)$ group are normalized here so that $\mathrm{Tr} T^a T^b = \frac{1}{2}\delta_{ab}$. There are eleven pNGB states in the $SU(6)/SU(5)$ coset space. These can be parametrized by

$$\Sigma = e^{i\Pi/f}, \qquad \Pi = \Pi^{\hat{a}} T^{\hat{a}}, \tag{14}$$

where f plays the role of a decay constant. The matrix Π is given by

$$\Pi = \begin{pmatrix} -\frac{\phi_0}{\sqrt{60}} & 0 & 0 & 0 & 0 & \frac{\phi_1}{\sqrt{2}} \\ 0 & -\frac{\phi_0}{\sqrt{60}} & 0 & 0 & 0 & \frac{\phi_2}{\sqrt{2}} \\ 0 & 0 & -\frac{\phi_0}{\sqrt{60}} & 0 & 0 & \frac{\phi_3}{\sqrt{2}} \\ 0 & 0 & 0 & -\frac{\phi_0}{\sqrt{60}} & 0 & \frac{\phi_4}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & -\frac{\phi_0}{\sqrt{60}} & \frac{\phi_5}{\sqrt{2}} \\ \frac{\phi_1^\dagger}{\sqrt{2}} & \frac{\phi_2^\dagger}{\sqrt{2}} & \frac{\phi_3^\dagger}{\sqrt{2}} & \frac{\phi_4^\dagger}{\sqrt{2}} & \frac{\phi_5^\dagger}{\sqrt{2}} & \frac{5\phi_0}{\sqrt{60}} \end{pmatrix}. \tag{15}$$

To write the nonlinear realization of the Higgs mechanism in the $\mathrm{E_6CHM}$, it is convenient to choose a specific direction for the vacuum Ω_0. In particular, the breaking $SU(6) \rightarrow SU(5)$ can be parametrized through the fundamental representation of $SU(6)$, i.e.

$$\Omega_0^T = (\,0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1\,). \tag{16}$$

Then the leading order Lagrangian that describes the interactions of the pNGB states can be written as

$$\mathcal{L}_{\mathrm{pNGB}} = \frac{f^2}{2}\left|\mathcal{D}_\mu \Omega\right|^2. \tag{17}$$

In Eq. (17) the nonlinear representation of the pNGB states is obtained in terms of a 6-component unit vector Ω that reads

$$\Omega = \Sigma\Omega_0,$$
$$\Omega^T = e^{i\frac{\phi_0}{\sqrt{15}f}}\left(C\phi_1\ C\phi_2\ C\phi_3\ C\phi_4\ C\phi_5\ \cos\frac{\tilde{\phi}}{\sqrt{2}f} + \sqrt{\frac{3}{10}}C\phi_0\right), \tag{18}$$

where

$$C = \frac{i}{\tilde{\phi}}\sin\frac{\tilde{\phi}}{\sqrt{2}f},$$
$$\tilde{\phi} = \sqrt{\frac{3}{10}\phi_0^2 + \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 + \phi_5^2}.$$

Since $\tilde{\phi}$ and ϕ_0 are invariant under the preserved $SU(5)$, Ω transforms as $\mathbf{5} + \mathbf{1}$ under the transformation of the $SU(5)$ group. Therefore, one can introduce a 5-component vector, $\tilde{H} \sim (\phi_1\phi_2\phi_3\phi_4\phi_5)$. The first two components of this vector transform as an $SU(2)_W$ doublet, $H \sim (\phi_1\phi_2)$, and, therefore, H is associated with the SM-like Higgs doublet. Three other components, $T \sim (\phi_3\phi_4\phi_5)$, correspond to an $SU(3)_C$ triplet. Because in the SM the Higgs doublet has $B = L = 0$, no components of Ω should carry any baryon and/or lepton numbers.

The low-energy effective Lagrangian of the $\mathrm{E_6}$CHM, that includes the interactions among the SM fields, the pNGB states and exotic fermions, can be obtained by integrating out the heavy resonances of the composite sector. However, only interactions, that break global $SU(6)$ symmetry, can induce the pNGB effective potential $V_{\mathrm{eff}}(\tilde{H}, T, \phi_0)$ which must vanish in the exact $SU(6)$ symmetry limit. As a consequence, the main contributions to $V_{\mathrm{eff}}(H, T, \phi_0)$ should come from the interactions of the elementary fermions and gauge bosons with their composite partners which explicitly break $SU(6)$ symmetry. The analysis of the structure of the pNGB effective potential within similar composite Higgs models, including the derivation of quadratic terms $m_H^2|H|^2$ and $m_T^2|T|^2$, shows that there is a substantial part of the parameter space where m_H^2 tends to be negative while m_T^2 remains positive [14,15]. In other words, in this parameter region EW symmetry is broken, whereas $SU(3)_C$ color is preserved. This happens when the contributions of the top quark and exotic fermions to m_H^2 are negative and sufficiently large to overcome the gauge boson contribution to m_H^2. At the same time, in this case m_T^2 can be positive due to the large contribution to m_T^2 generated by the interactions of gluons and their composite partners. Therefore, hereafter we just assume that the nonzero components of the vector Ω break $SU(6)$ symmetry so that $SU(2)_W \times U(1)_Y$ gauge symmetry gets broken down to $U(1)_{em}$, associated with electromagnetism, whereas $SU(3)_C$ symmetry remains intact.

C. Generation of masses of the SM fermions

As mentioned before, all elementary quark and lepton states gain masses through the mixing with their composite partners. Thus, it is important to ensure that the corresponding mixing can occur within the $\mathrm{E_6}$CHM. In the model under consideration different multiplets of elementary quarks and leptons stem from different representations of the $SU(6)$ subgroup of E_6. All other components of the corresponding $SU(6)$ representations are extremely heavy (see Sec. II). Thus, at low energies elementary quarks and leptons appear as incomplete multiplets of $SU(6)$, which decompose under the $SU(6) \times U(1)_B \times U(1)_L$ global symmetry as follows,

$$u^c_\alpha \in \mathbf{15}^u_\alpha = \left(\mathbf{15}, -\frac{1}{3}, 0\right)_\alpha \qquad q_i \in \mathbf{15}^q_i = \left(\mathbf{15}, \frac{1}{3}, 0\right)_i$$
$$d^c_i \in \bar{\mathbf{6}}^d_i = \left(\bar{\mathbf{6}}, -\frac{1}{3}, 0\right)_i$$
$$e^c_i \in \mathbf{15}^e_i = \left(\mathbf{15}, 0, -1\right)_i \qquad \ell_i \in \bar{\mathbf{6}}^\ell_i = \left(\bar{\mathbf{6}}, 0, 1\right)_i, \tag{19}$$

where the first, second and third quantities in brackets are the $SU(6)$ representation, $U(1)_B$ and $U(1)_L$ charges, respectively, while $\alpha = 1, 2$ and $i = 1, 2, 3$. The composite partners of the elementary fermions should be embedded into the $SU(6)$ representations so that all quark and lepton Yukawa interactions of the SM, which induce nonzero fermion masses, are allowed.

In our analysis we use the simplest $SU(5)$ GUT as a guideline. Below scale f, where $SU(6)$ global symmetry is broken down to $SU(5)$, the elementary quark and lepton states constitute the following incomplete $SU(5)$ multiplets,

$$u^c_\alpha \in \mathbf{10}^u_\alpha = \left(\mathbf{10}, -\frac{1}{3}, 0\right)_\alpha \qquad q_i \in \mathbf{10}^q_i = \left(\mathbf{10}, \frac{1}{3}, 0\right)_i$$
$$d^c_i \in \bar{\mathbf{5}}^d_i = \left(\bar{\mathbf{5}}, -\frac{1}{3}, 0\right)_i$$
$$e^c_i \in \mathbf{10}^e_i = \left(\mathbf{10}, 0, -1\right)_i \qquad \ell_i \in \bar{\mathbf{5}}^\ell_i = \left(\bar{\mathbf{5}}, 0, 1\right)_i, \tag{20}$$

where, as before, the second and third quantities in brackets correspond to the baryon and lepton numbers of these $SU(5)$ representations. If the particle content of this model involved an elementary Higgs boson, then the Higgs doublet h could be embedded into the fundamental representation of $SU(5)$, i.e. $h \in \mathbf{5}^h = (\mathbf{5}, 0, 0)$. In this case the Yukawa couplings of the up-type quarks have the following $SU(5)$ structure:

$$\mathcal{L}^u_{SU(5)} \simeq h^u_{\alpha i} \mathbf{10}^u_\alpha \mathbf{10}^q_i \mathbf{5}^h. \tag{21}$$

In order to reproduce the SM up-quark Yukawa couplings, one is forced to assume that interactions similar to those given by Eq. (21) are reproduced in the strongly coupled sector below the scale f. In the case of the $SU(6)$ symmetry, the Higgs multiplet $\mathbf{5}^h$ should be replaced by the unit vector Ω. Instead of two other $SU(5)$ representations $\mathbf{10}^u_\alpha$ and $\mathbf{10}^q_i$, that appear in Eq. (21), one should include two $SU(6)$ multiplets that contain an $SU(5)$ decuplet. The simplest $SU(6)$ representation of this type is an antisymmetric second-rank tensor field $\mathbf{15}$. The next-to-simplest $SU(6)$ representation, that involves an $SU(5)$ decuplet, is a totally antisymmetric third-rank tensor $\mathbf{20}$. These $SU(6)$ representations have the following decomposition in terms of $SU(5)$ representations: $\mathbf{15} = \mathbf{10} \oplus \mathbf{5}$ and $\mathbf{20} = \mathbf{10} \oplus \overline{\mathbf{10}}$. The presence of the $\mathbf{15}$-plet and $\mathbf{20}$-plet allows for the generalization of the $SU(5)$ structure of the up–quark Yukawa interactions (21) to the case of $SU(6)$ symmetry that results in

$$\mathcal{L}^u_{SU(6)} \sim \mathbf{20} \times \mathbf{15} \times \mathbf{6}, \tag{22}$$

where $\mathbf{6}$ should be identified with the unit vector Ω.

The structure of the interactions (22) leads to two different scenarios. In scenario A, the composite partners of u^c_α and q_i (U_α and Q_i) belong to $\mathbf{15}(U_\alpha)$ and $\mathbf{20}(Q_i)$ representations of $SU(6)$, whereas in scenario B the composite partners of u^c_α and q_i are components of $\mathbf{20}(U_\alpha)$ and $\mathbf{15}(Q_i)$, respectively. In principle, the $SU(6)$ symmetry forbids the mixing between the components of $\mathbf{20}$-plets, that contain the composite partners of quarks, and $\mathbf{15}$-plets which involve the elementary quark states. Nevertheless, such mixing can be induced below the scale f, where the $SU(6)$ global symmetry is broken down to $SU(5)$. To demonstrate this, let us focus on scenario A and assume that the strongly interacting sector includes not only $\mathbf{20}(Q_i)$ but also $\mathbf{15}(Q'_i)$ and $\overline{\mathbf{15}}(\bar{Q}'_i)$. Then the part of the Lagrangian that determines the mixing between the elementary quark states q_i and their composite partners Q_i can be written as

$$\begin{aligned}\mathcal{L}^q_{\text{mix}} = \sigma_Q f \mathbf{20}(Q_i)\mathbf{15}(Q'_i)\Omega + m_Q \mathbf{20}(Q_i)\mathbf{20}(Q_i)\\ + m_{Q'}\mathbf{15}(Q'_i)\overline{\mathbf{15}}(\bar{Q}'_i) + \mu_q \overline{\mathbf{15}}(\bar{Q}'_i)\mathbf{15}^q_i.\end{aligned} \tag{23}$$

When $\sigma_Q f \gg m_Q \sim m_{Q'} \gg \mu_q$, the $\overline{\mathbf{10}}$ from $\mathbf{20}(Q_i)$ and the 10 from $\mathbf{15}(Q'_i)$ form heavy vectorlike states with masses $\sim\sigma_Q f$, so that these states are almost decoupled from the rest of the particle spectrum. The remaining $\mathbf{10}$ from $\mathbf{20}(Q_i)$ and the components of $\mathbf{10}^q_i$ get mixed. If $\frac{m_Q m_{Q'}}{\sigma_Q f} \gg \mu_q$, then one superposition of these $\mathbf{10}$-plets, which is predominantly $\mathbf{10}$ from $\mathbf{20}(Q_i)$, and $\overline{\mathbf{10}}$-plet from $\overline{\mathbf{15}}(\bar{Q}'_i)$ get combined, forming vectorlike states that acquire masses of order of $\frac{m_Q m_{Q'}}{\sigma_Q f}$. Another superposition of $\mathbf{10}$ from $\mathbf{20}(Q_i)$ and $\mathbf{10}^q_i$, which is basically a superposition of elementary quark states q_i and their composite partners, gain masses after the EW symmetry breaking. Similarly, the mixing between the components of the incomplete $\mathbf{15}^u_\alpha$ and their composite partners from $\mathbf{20}(U_\alpha)$ can be induced in the case of the scenario B.

In the $SU(5)$ models the masses of the down-type quarks are induced through the Yukawa interactions,

$$\mathcal{L}^d_{SU(5)} \simeq h^d_{ij} \mathbf{10}^q_i \bar{\mathbf{5}}^d_j \bar{\mathbf{5}}^h. \tag{24}$$

The simplest $SU(6)$ generalization of the $SU(5)$ structure of the down-quark Yukawa interactions (24) takes the form

$$\mathcal{L}^d_{SU(6)} \sim \mathbf{15} \times \bar{\mathbf{6}} \times \bar{\mathbf{6}}'. \tag{25}$$

In scenario B the Yukawa couplings (25) can be used to generate the masses of the down-type quarks after EW symmetry breaking. In this case the $\bar{\mathbf{6}}'$ in Eq. (25) has to be identified with $\Omega^\dagger$, the $\mathbf{15}$ should be associated with $\mathbf{15}(Q_i)$ and the $\bar{\mathbf{6}}$ is expected to contain the composite partners of d^c_i (D_i), i.e. $\bar{\mathbf{6}} \equiv \bar{\mathbf{6}}(D_i)$. The $SU(6)$ symmetry does not forbid mixing between $\mathbf{15}^q_i$ and $\mathbf{15}(Q_i)$ or between $\bar{\mathbf{6}}^d_i$ and $\bar{\mathbf{6}}(D_i)$.

In the case of scenario A, the simplest $SU(6)$ generalization of the Yukawa interactions (24) that can give rise to the nonzero masses of the SM down-type quarks is given by

$$\mathcal{L}^d_{SU(6)} \sim \mathbf{20} \times \overline{\mathbf{15}} \times \bar{\mathbf{6}}', \tag{26}$$

where again $\bar{\mathbf{6}}' \equiv \Omega^\dagger$, while the $\mathbf{20}$-plet corresponds to the $SU(6)$ representations that involve composite partners of q_i, i.e. $\mathbf{20}(Q_i)$, and the $\overline{\mathbf{15}}$-plet should contain composite partners of d^c_i, i.e. $\overline{\mathbf{15}} \equiv \overline{\mathbf{15}}(D_i)$. As pointed out earlier, the mixing between components of the incomplete $\mathbf{15}^q_i$ multiplets and composite partners of q_i from $\mathbf{20}(Q_i)$ can be induced below scale f. The breakdown of $SU(6)$ symmetry can also give rise to the mixing between the corresponding components of the incomplete $\bar{\mathbf{6}}^d_i$ multiplet and $\overline{\mathbf{15}}(D_i)$. This happens, for example, when the strongly coupled sector involves $\overline{\mathbf{15}}(D_i)$ and $\mathbf{15}(\bar{D}_i)$ as well as $\bar{\mathbf{6}}(D'_i)$ and $\mathbf{6}(\bar{D}'_i)$. The part of the Lagrangian that leads to the mixing of the elementary quark states, d^c_i, and their composite partners, D_i, can be written in the following form:

$$\mathcal{L}^d_{\rm mix} = m_D\overline{\mathbf{15}}(D_i)\mathbf{15}(\bar{D}_i) + \sigma_d f\mathbf{15}(\bar{D}_i)\bar{\mathbf{6}}(D'_i)\Omega^\dagger + m_{D'}\bar{\mathbf{6}}(D'_i)\mathbf{6}(\bar{D}'_i) + \mu_d\mathbf{6}(\bar{D}'_i)\bar{\mathbf{6}}^d_i. \quad (27)$$

If $m_{D'} \gg m_D, \sigma_d f$ and μ_d the composite states $\bar{\mathbf{6}}(D'_i)$ and $\mathbf{6}(\bar{D}'_i)$ can be integrated out. Then the second term in the Lagrangian (27) results in mixing between the components of the incomplete $\bar{\mathbf{6}}^d_i$ multiplet and their composite partners from $\overline{\mathbf{15}}(D_i)$.

Since charged lepton and down-quark Yukawa interactions have the same $SU(5)$ structure in the simplest $SU(5)$ GUT, the $SU(6)$ generalization of these interactions (25) can be used in both scenarios A and B to generate the masses of charged leptons within the E$_6$CHM. Again one can set $\bar{\mathbf{6}}' \equiv \Omega^\dagger$ in Eq. (25). At the same time one can expect that the composite partners of e^c_i and ℓ_i are components of $\mathbf{15}(E_i)$ and $\bar{\mathbf{6}}(L_i)$, respectively. The mixing between the components of $\mathbf{15}^e_i$ and their composite partners from $\mathbf{15}(E_i)$, as well as the mixing between the corresponding components of $\bar{\mathbf{6}}^\ell_i$ and $\bar{\mathbf{6}}(L_i)$, are not forbidden by the $SU(6)$ symmetry. Therefore, such Yukawa interactions should lead to nonzero masses for the charged leptons after the breakdown of the EW symmetry.

The masses of the elementary left-handed neutrinos in the $SU(5)$ GUT are induced through the Yukawa interactions,

$$\mathcal{L}^\nu_{SU(5)} \simeq h^\nu_{ij}\bar{\mathbf{5}}^\ell_i\mathbf{5}^h\mathbf{1}_j, \quad (28)$$

where $\mathbf{1}_i$ correspond to Majorana right-handed neutrinos that do not participate in the SM gauge interactions. The simplest $SU(6)$ generalization of the Yukawa couplings (28) is given by

$$\mathcal{L}^\nu_{SU(6)} \sim \bar{\mathbf{6}} \times \mathbf{6}' \times \mathbf{1}. \quad (29)$$

In Eq. (29) $\mathbf{6}' \equiv \Omega$ and the $\bar{\mathbf{6}}$ should be associated with $\bar{\mathbf{6}}(L_i)$. The Yukawa interactions (29) imply that the dynamics of the strongly coupled sector should lead to the formation of a set of the $SU(6)$ singlet bound states with spin $1/2$. Because in the composite sector $U(1)_L$ symmetry is preserved, these fermion bound states N_i and $\bar{N}_i$ have to carry lepton number, so that $N_i = (\mathbf{1}, 0, -1)$ and $\bar{N}_i = (\mathbf{1}, 0, 1)$. To ensure the smallness of the masses of the elementary left-handed neutrinos one can include in the elementary sector a set of heavy Majorana states, S_i, that get mixed with $\bar{N}_i = (\mathbf{1}, 0, 1)$. Such fermionic states may come from the bulk supermultiplets that do not participate in the E_6 gauge interactions but carry lepton number. The Majorana masses of S_i can be generated after the breakdown of the $U(1)_L$ symmetry on the brane O_I. These masses can be somewhat lower than M_X.

In the case of one lepton flavor the simplest low-energy effective Lagrangian of the type discussed above can be written as

$$\mathcal{L}^{e\nu}_{\rm eff} = \mu_e(\bar{E}e^c) + \mu_\ell(\bar{L}\ell) + M_E(\bar{E}E) + M_L(\bar{L}L) + h_E(LH^\dagger)E + h_N(LH)N + M_N(N\bar{N}) + \mu_N(\bar{N}S) + M_S(SS) + \text{H.c.}, \quad (30)$$

where E, $\bar{E}$, L, $\bar{L}$, N and $\bar{N}$ are composite fermions, while H is a composite Higgs doublet. In the limit where lepton number is conserved, i.e. the parameter μ_N vanishes, the Lagrangian (30) results in a massless Majorana fermion that can be identified with the elementary left-handed neutrino if $\mu_\ell \to 0$. Assuming that the mixing between the elementary and composite states is rather small, i.e. $\mu_e, \mu_\ell, \mu_N \ll M_E, M_L, M_N$, one can obtain the approximate expressions for the masses of the elementary charged lepton and left-handed neutrino states (m_e and m_ν):

$$|m_e| \simeq h_E\left(\frac{\mu_e}{M_E}\right)\left(\frac{\mu_\ell}{M_L}\right)\frac{v}{\sqrt{2}}, \qquad |m_\nu| \simeq h_N^2\left(\frac{\mu_\ell}{M_L}\right)^2\left(\frac{\mu_N}{M_N}\right)^2\frac{v^2}{2M_S}. \quad (31)$$

From Eq. (31) it follows that $|m_\nu| \ll |m_e|$ if the mixing between the elementary and composite states is small and/or $M_S \gg v$.

D. Implications for collider phenomenology and dark matter

As pointed out in the introduction to this article, the composite Higgs model under consideration implies that in the exact $SU(6)$ symmetry limit the dynamics of the strongly interacting sector gives rise to massless $SU(6)$ representations that contain composite t^c. In scenario A the right-handed top quark state belongs to a **15**-plet that must carry the same baryon number as t^c, i.e. $B_{15} = -1/3$. In this case we assume that in addition to the **15**-plet, two $\bar{\mathbf{6}}$-plets ($\bar{\mathbf{6}}_1$ and $\bar{\mathbf{6}}_2$) with spin $1/2$ and opposite baryon numbers remain massless as well that leads to the $SU(6)$ anomaly cancellation in the massless sector. Moreover, we allow for interaction between vector Ω and multiplets **15** and $\bar{\mathbf{6}}_1$ of the type $\mathbf{15} \times \bar{\mathbf{6}}_1 \times \Omega^\dagger$ that do not violate the $U(1)_B$ symmetry if $B_{\bar{6}_1} = -B_{15} = 1/3$. Such a Yukawa coupling results in the formation of vectorlike states that involve a **5**-plet from **15** and a $\bar{\mathbf{5}}$-plet from $\bar{\mathbf{6}}_1$. The $SU(5)$ singlet components of $\bar{\mathbf{6}}_1$ and $\bar{\mathbf{6}}_2$ can also acquire mass through the interaction $(\bar{\mathbf{6}}_1\Omega)(\Omega\bar{\mathbf{6}}_2)$. As a consequence, only the **10**-plet from **15** and $\bar{\mathbf{5}}$-plet from $\bar{\mathbf{6}}_2$, that carry $B = -1/3$, do not acquire masses by interacting with Ω. Nevertheless, these **10**-plet and $\bar{\mathbf{5}}$-plet states get combined with elementary exotic states $\bar{q}$, $\bar{d^c}$, $\bar{\ell}$, $\bar{e^c}$, resulting in a set of vectorlike states with masses somewhat below f and a composite right-handed top quark.

In scenario B the right-handed top quark state belongs to the **20**-plet of $SU(6)$ with baryon number $B_{20} = -1/3$. We assume that in this case the dynamics of the strongly coupled sector results in a massless **20**-plet, **15**-plet ($\mathbf{15}'$) and two $\bar{\mathbf{6}}$-plets ($\bar{\mathbf{6}}'_1$ and $\bar{\mathbf{6}}'_2$) with spin 1/2 in the exact $SU(6)$ symmetry limit. The interaction between the **20**-plet, $\mathbf{15}'$ and vector Ω of the type $\mathbf{20}\times\mathbf{15}'\times\Omega$ gives rise to the formation of vectorlike states that involve the $\overline{\mathbf{10}}$-plet from **20** and the **10**-plet from $\mathbf{15}'$, whereas the coupling of $\mathbf{15}'$ to $\bar{\mathbf{6}}'_1$ and Ω, i.e. $\mathbf{15}'\times\bar{\mathbf{6}}'_1\times\Omega^\dagger$, leads to the massive states composed of a **5**-plet from $\mathbf{15}'$ and a $\bar{\mathbf{5}}$-plet from $\bar{\mathbf{6}}'_1$. Again the mass of the $SU(5)$ singlet components of $\bar{\mathbf{6}}'_1$ and $\bar{\mathbf{6}}'_2$ can be induced through the interaction $(\bar{\mathbf{6}}'_1\Omega)(\Omega\bar{\mathbf{6}}'_2)$. None of these interactions are forbidden by the $U(1)_B$ symmetry provided $B_{20}=-B_{15'}=B_{\bar{6}'_1}=-B_{\bar{6}'_2}=-1/3$. As before one **10**-plet from **20** and one $\bar{\mathbf{5}}$-plet from $\bar{\mathbf{6}}'_2$, that do not gain masses because of the interaction with vector Ω, as well as elementary exotic states $\bar{q}$, $\bar{d}^c$, $\bar{\ell}$, $\bar{e}^c$ form a set of vectorlike states and composite t^c. However, in contrast to scenario A, the **10**-plet from **20** and the $\bar{\mathbf{5}}$-plet from $\bar{\mathbf{6}}'_2$ have opposite baryon numbers, $-1/3$ and $1/3$, respectively.

Thus, in both scenarios the set of vectorlike fermion states that can have masses in the few TeV range includes color triplets $t'(\bar{t}')$ with electric charges $+2/3(-2/3)$, color triplets of quarks b'_1 and b'_2 ($\bar{b}'_1$ and $\bar{b}'_2$) with different masses but the same electric charge $-1/3(+1/3)$, colorless fermions e'_1 and e'_2 ($\bar{e}'_1$ and $\bar{e}'_2$) with different masses but the same electric charge $-1(+1)$, and a neutral fermion state ν' ($\bar{\nu}'$) which is formed by the components of the $SU(2)_W$ doublets. Baryon number conservation implies that all these fermion states carry nonzero $U(1)_B$ charges. In both cases t', b'_1 and $\bar{e}'_1$ have baryon number $-1/3$. In scenario A e'_2, ν' and $\bar{b}'_2$ carry baryon number $-1/3$, while in scenario B these states have opposite baryon number, $+1/3$. The set of the lightest exotic states should be supplemented by the scalar color triplet T ($T^\dagger$) with electric charge $-1/3$ ($+1/3$) and zero baryon number, that stem from the pNGB 5-plet, $\tilde{H}$, that also gives rise to the Higgs doublet, H.

One of the lightest exotic states in the E$_6$CHM should be stable. This can be understood in terms of the Z_3 symmetry which is known as baryon triality (see, for example [14,62]). The corresponding transformations can be defined as

$$\Psi \longrightarrow e^{2\pi i B_3/3}\Psi, \qquad B_3=(3B-n_C)_{\mathrm{mod}3}, \qquad (32)$$

where B is the baryon number of the given multiplet Ψ and n_C is the number of color indices ($n_C=1$ for the color triplet and $n_C=-1$ for $\bar{\mathbf{3}}$). Because baryon number is preserved to a very good approximation, the low-energy effective Lagrangian of the E$_6$CHM is invariant under the transformations of this discrete Z_3 symmetry. All bosons and fermions in the SM have $B_3=0$. On the other hand, in both scenarios $B_3(T)=2$ and $B_3(t')=B_3(b'_1)=B_3(e'_1)=1$. At the same time, in scenario A $B_3(b'_2)=0$ and $B_3(e'_2)=B_3(\nu')=2$, whereas in scenario B $B_3(b'_2)=B_3(e'_2)=B_3(\nu')=1$. Because of the invariance of the low-energy effective Lagrangian of the E$_6$CHM with respect to the transformations of baryon triality, the lightest exotic state with nonzero B_3 charge can not decay into SM particles and must, therefore, be stable. The decay of such an exotic state can be induced by baryon number violating operators and is, therefore, extremely strongly suppressed.

If the lightest states with nonzero B_3 charge are exotic color triplets or exotic charged fermions then these states would have been copiously produced during the very early epochs of the big bang. Those strong or electromagnetically interacting lightest exotic states which survive annihilation would subsequently have been confined in heavy hadrons which would annihilate further. The remaining heavy hadrons originating from the Big Bang should be present in terrestrial matter. On the other hand, there are very strong upper limits on the abundances of nuclear isotopes which contain such stable relics in the mass range from 1 GeV to 10 TeV. Different experiments set limits on their relative concentrations from 10^{-15} to 10^{-30} per nucleon [63]. At the same time, theoretical estimates show that if such remnant particles were to exist in nature today their concentration should be much higher than 10^{-15} per nucleon [64].

Therefore, the E$_6$CHM with stable exotic color triplets or stable exotic charged fermions is basically ruled out. In principle the set of exotic states in the E$_6$CHM also includes neutral fermion states ν' ($\bar{\nu}'$) that transform nontrivially under baryon triality. However, if these states are sufficiently light, i.e. they have masses in the few TeV range, to play the role of dark matter such states would need to couple to the Z boson and scatter on nuclei, resulting in a spin-independent cross section which is a few orders of magnitude larger than the upper bound from direct dark matter searches (for a recent analysis see [65]).

In order to ensure that the composite Higgs model under consideration is phenomenologically viable, we assume that the dynamics of the strongly interacting sector of the E$_6$CHM leads to the formation of the $SU(6)$ singlet state $\bar{\eta}$, with spin 1/2, which gains its mass through the mixing with the elementary state η. In scenario A we allow for an interaction between $\bar{\eta}$, vector Ω and $\bar{\mathbf{6}}_2$ of the type $\bar{\eta}\times\Omega\times\bar{\mathbf{6}}_2$. We also assume that a similar interaction between $\bar{\eta}$, Ω and $\bar{\mathbf{6}}'_2$ is allowed in the case of scenario B. This implies that $\bar{\eta}$ carries baryon number $+1/3$ in scenario A and $-1/3$ in scenario B. The breakdown of the EW symmetry gives rise to mixing between $\bar{\eta}$ and $\bar{\nu}'$ as well as η and ν', resulting in two mass eigenstates ζ_1 and ζ_2. When this mixing is rather small the lightest state, ζ_1, can be predominantly an $SU(2)_W$ singlet, so that its coupling to the Z boson can be strongly suppressed. As a consequence ζ_1 can play the role of dark matter if this state is the lightest exotic state with nonzero B_3 charge.

When ζ_1 is stable, some part of the baryon asymmetry can be stored in the dark matter sector, because ζ_1 carries baryon number. Indeed, if $\zeta_1\bar{\zeta}_1$ annihilation is efficient enough, the dark matter density in this model can be generated by the same mechanism that gives rise to the baryon asymmetry of the Universe. In this case one can estimate the ratio of the baryon charges B_{ζ_1} and B_n accumulated by ζ_1 states and nucleons as

$$\theta = \frac{B_{\zeta_1}}{B_n} \simeq \frac{1}{3}\left(\frac{\rho_{\zeta_1}}{\rho_n}\right)\left(\frac{m_n}{m_{\zeta_1}}\right), \tag{33}$$

where ρ_{ζ_1} and ρ_n are contributions of ζ_1 states and nucleons to the total energy density, while m_{ζ_1} and m_n are the masses of the ζ_1 states and nucleons, respectively. Taking into account that ρ_{ζ_1} does not exceed the total dark matter density, i.e. $\rho_{\zeta_1} \lesssim 5\rho_n$, the value of θ can be larger than 0.1% only when $m_{\zeta_1} \lesssim 2$ TeV.

The presence of exotic states with TeV scale masses can lead to remarkable signatures. Assuming that t', b'_1 and e'_1, which stem from the same $SU(6)$ multiplet as the right-handed top quark, couple most strongly to the third-generation fermions, the vectorlike exotic states b'_1 and e'_1 tend to decay into

$$b'_1 \to \bar{t} + \bar{b} + \zeta_1 + X, \qquad e'_1 \to \bar{t} + b + \zeta_1 + X. \tag{34}$$

The dominant decay channels of b'_1 and e'_1 are basically determined by the requirement of electromagnetic charge and baryon number conservation. Since the exotic quark t' can decay via $t' \to W^* + b'^*_1$, this exotic state results in a similar final state to b'_1. In scenarios A and B the exotic quark b'_2 carries baryon number $+1/3$ and $-1/3$, respectively. Thus, in scenario A the decay channel,

$$b'_2 \to Z + b, \tag{35}$$

is allowed, whereas in scenario B this exotic state decays like b'_1 in scenario A. The exotic state e'_2 decays either via $e'_2 \to W + \bar{\zeta}_1$ (scenario A) or via $e'_2 \to W + \zeta_1$ (scenario B).

If exotic quarks of the type described here do exist at sufficiently low scales, they can be accessed through direct pair hadroproduction at the LHC. The corresponding production processes are generated via gluon-induced QCD interactions. The exotic quarks b'_1 and t' are doubly produced and decay into a pair of third-generation quarks and ζ_1, resulting in the enhancement of the cross sections of

$$\begin{aligned} &pp \to t\bar{t}b\bar{b} + \not{E}_T + X \quad \text{and} \\ &pp \to b\bar{b}b\bar{b} + \not{E}_T + X. \end{aligned} \tag{36}$$

The final states (36) are similar to those associated with gluino pair production in the scenarios where the third-generation squarks are substantially lighter than the other sparticles, so that the gluino decays predominantly into a pair of third-generation quarks and a neutralino (for recent analysis see [66]). As compared with the exotic quarks, the direct production of e'_1, e'_2, ν' and ζ_1 is expected to be rather suppressed at the LHC. Nevertheless, it is worth noting that the pair production of $e'_1\bar{e}'_1$ can also lead to an enhancement of the cross sections for processes with the final states (36) if e'_1 is sufficiently light.

Finally, let us consider the collider signatures associated with the scalar color triplet T that comes from the same pNGB $SU(5)$ multiplet, $\tilde{H}$, as the composite Higgs doublet. In scenario A this scalar exotic state couples most strongly into b'_2 and ζ_1. Therefore, if T is heavier than b'_2 it decays predominantly as

$$T \to b'_2 + \bar{\zeta}_1. \tag{37}$$

Otherwise, it decays via

$$T \to b + \bar{\zeta}_1 + X. \tag{38}$$

In scenario B the scalar color triplet T couples not only to b'_2 and ζ_1 but also to t, t', b'_1 and e'_2. As a result, the following decay channels are allowed for this exotic state

$$\begin{aligned} &T \to b'_2(b'_1) + \zeta_1 \to \bar{t} + \bar{b} + \zeta_1 + \zeta_1 + X, \\ &T \to t' + e'_2 \to \bar{t} + \bar{b} + \zeta_1 + \zeta_1 + X, \\ &T \to \bar{t} + \bar{b}'_2 \to \bar{t} + t + b + \bar{\zeta}_1 + X. \end{aligned} \tag{39}$$

At the LHC, scalar color triplets can be pair-produced if these exotic states are light enough. Then from Eq. (39) it follows that the decays of $T\bar{T}$ may result in the enhancement of the cross sections for the processes (36), with the four third-generation quarks in the final states. Besides, as one can also see from Eq. (39), in some cases the $T\bar{T}$ production can lead to the enhancement of the cross sections that correspond to the processes with six third-generation quarks in the final states, i.e.

$$\begin{aligned} &pp \to T\bar{T} \to t\bar{t}t\bar{t}b\bar{b} + \not{E}_T + X, \\ &pp \to T\bar{T} \to b\bar{b}b\bar{b}b\bar{b} + \not{E}_T + X. \end{aligned} \tag{40}$$

IV. CONCLUSIONS

In this paper we have studied a composite Higgs model which can arise naturally after the breakdown of the E_6 gauge symmetry. Basically we focus on the GUT based on the $E_6 \times G_0$ gauge group, which is broken down to the $SU(3)_C \times SU(2)_W \times U(1)_Y \times G$ subgroup near some high energy scale M_X. The low-energy limit of this GUT comprises strongly interacting and weakly coupled sectors. Gauge groups G_0 and G are associated with the strongly coupled sector. Fields from this sector can be charged under both E_6 and G_0 (G) gauge symmetries. The weakly coupled sector involves elementary states that participate in the E_6 interactions only. In this E_6 inspired composite Higgs model (E$_6$CHM) all elementary quark and lepton fields can stem from the fundamental 27-dimensional representation of E_6.

In order to avoid rapid proton decay and to guarantee the smallness of the Majorana masses of the left-handed neutrino states, the Lagrangian of the strongly interacting sector of the E$_6$CHM has to be invariant under the transformations of global $U(1)_B$ and $U(1)_L$ symmetries which ensure the conservation of the baryon and lepton numbers to a very good approximation at low energies. Almost exact conservation of the $U(1)_B$ and $U(1)_L$ charges implies that elementary states with different baryon and/or lepton numbers must come from different 27-plets, while all other components of these multiplets gain masses of the order of M_X. Such a splitting of the E_6 fundamental representations can occur within the six-dimensional orbifold SUSY GUT model presented in this article. We consider the compactification of two extra dimensions on the orbifold $T^2/(Z_2 \times Z_2^I \times Z_2^{II})$ that allows us to reduce the physical region to a pillow with four branes as corners. In this model the elementary quark and lepton fields are components of different bulk 27-plets, while all fields from the strongly coupled sector are confined on the brane O, where E_6 symmetry is broken down to the $SU(6) \times SU(2)_N$ subgroup. The $SU(6)$ group, that remains intact on the brane O, contains an $SU(3)_C \times SU(2)_W \times U(1)_Y$ subgroup. We discuss the breakdown of the E_6 symmetry to the SM gauge group that results in the appropriate splitting of the bulk 27-plets. The 6D orbifold GUT models based on the E_6 gauge group do not have an irreducible bulk anomaly, whereas brane anomalies get canceled in the model under consideration.

In general, the SM gauge couplings in the orbifold GUT models may not be identical near the scale M_X where the GUT gauge symmetry is broken. This is because sizable contributions to these couplings can come from the branes where GUT symmetry is broken. Nevertheless, if the bulk contributions to the SM gauge couplings dominate, approximate gauge coupling unification can take place. Since in the E$_6$CHM all states in the strongly coupled sector fill complete $SU(6)$ representations, the convergence of the SM gauge couplings is determined by the matter content of the elementary sector in the leading approximation. Then the approximate unification of gauge couplings can be achieved if the right-handed top quark is entirely composite and the weakly coupled sector together with the SM fields (but without the right-handed top quark) contains a set of exotic states so that its field content is given by Eq. (2). The presence of extra exotic states also ensures anomaly cancellation in the elementary sector at low energies.

Since the strongly interacting sector is localized on the brane O, it can possess an $SU(6) \times SU(2)_N$ global symmetry at high energies, even though local symmetry is broken down to the SM gauge group. In order to simplify our consideration we assumed that $SU(2)_N$ symmetry is entirely broken. Thus, the Lagrangian of the strongly coupled sector respects $SU(6) \times U(1)_B \times U(1)_L$ global symmetry at high energies. The SM gauge interactions break $SU(6)$ global symmetry. Nonetheless, if the gauge couplings of the strongly interacting sector are considerably larger than the SM gauge couplings at any intermediate scale below M_X, then $SU(6)$ can be still an approximate global symmetry of the composite sector at low energies.

We assumed that below scale $f \gg v$ the global $SU(6)$ symmetry is broken down to $SU(5)$, which includes the $SU(3)_C \times SU(2)_W \times U(1)_Y$ subgroup. The $SU(6)/SU(5)$ coset space involves eleven pNGB states. One of these pNGB states does not participate in the SM gauge interactions. Ten others form a fundamental representation of $SU(5)$ with 5 components. Two of these components are associated with the SM-like Higgs doublet H, while three other components correspond to the $SU(3)_C$ triplet T. None of these pNGB states carry any baryon and/or lepton numbers. The pNGB effective potential is induced by radiative corrections caused by the interactions between elementary states and their composite partners that break $SU(6)$ symmetry. The structure of this scalar potential tends to be such that it can give rise to the spontaneous breakdown of the EW symmetry, whereas $SU(3)_C$ color is preserved. Since the pNGB Higgs potential arises from loops, the effective quartic Higgs coupling tends to be sufficiently small that it can lead to a 125 GeV Higgs mass.

As in most composite Higgs models, the elementary quarks and leptons in the E$_6$CHM acquire their masses through the mixing between these states and their composite partners. In particular, the corresponding masses can be generated if all quark and lepton Yukawa couplings of the SM, which result in nonzero fermion masses, are allowed in the E$_6$CHM. We argued that in the case of the quark sector this can happen in two different scenarios. Scenario A implies that the composite partners of the left-handed quarks, the right-handed up-type and down-type quarks are components of **20**, **15** and $\overline{\mathbf{15}}$ representations of $SU(6)$, respectively. In scenario B the composite partners of the right-handed up-type quarks, left-handed quarks and right-handed down-type quarks belong to **20**, **15** and $\bar{\mathbf{6}}$ representations of the $SU(6)$ group. We also explored the generation of lepton masses. In both scenarios these masses can be induced if the composite partners of the elementary left-handed leptons and right-handed charged leptons belong to $\bar{\mathbf{6}}$ and **15** representations of $SU(6)$, respectively, whereas the composite partners of the right-handed neutrinos are the $SU(6)$ singlet bound states. To ensure the smallness of the masses of the elementary left-handed neutrinos we assumed that the elementary sector includes a set of heavy Majorana right-handed neutrino states with masses somewhat below M_X, which do not participate in the E_6 gauge interactions but get mixed with the $SU(6)$ singlet bound states that carry lepton number.

Because E_6CHM does not possesses any custodial symmetry, the electroweak precision observables are not protected against the contributions of new composite states. As a result the stringent experimental constraint on the Peskin–Takeuchi $\hat{T}$ parameter pushes the $SU(6)$ symmetry breaking scale f above 5–6 TeV. Besides, in the most general case adequate suppression of the flavor-changing transitions in the E_6CHM requires $f \gtrsim 10$ TeV. The latter bound can be significantly relaxed if extra flavor symmetry is imposed. Nonetheless a significant fine-tuning, ~0.01%, is still needed to obtain the weak scale $v \ll f$. At first glance such a model might look a bit artificial. However, the fact that no indication of new physics phenomena or any significant deviation from the SM has been discovered at the LHC so far may suggest that the Higgs sector can be somewhat tuned. In other words, the scale of new physics might be higher than previously thought.

The large value of the $SU(6)$ symmetry breaking scale also implies that the composite partners of the SM particles have masses above 10 TeV, so that they are too heavy to be probed at the LHC. Moreover, since the deviations of the couplings of the composite Higgs to the SM particles are determined by v^2/f^2, the modifications of the Higgs branching fractions tend to be negligibly small in this model. So it seems rather problematic to test such small deviations at the LHC. These small modifications of the Higgs branching ratios are probably even beyond the reach of a future e^+e^- collider. The couplings of the top quark to other SM particles are also expected to be extremely close to the ones predicted by the SM.

On the other hand, the spectrum of the E_6CHM contains one scalar color triplet T ($T^\dagger$) with electric charge $-1/3$ ($+1/3$) and zero baryon number, as well as the set of vectorlike fermions. All these states can have masses in the few TeV range. The set of vectorlike fermions, in particular, involves color triplets $t'(\bar{t}')$ with electric charge $+2/3(-2/3)$, b'_1 and b'_2 ($\bar{b}'_1$ and $\bar{b}'_2$) with electric charge $-1/3(+1/3)$. The exotic quarks t' and b'_1 have baryon number $-1/3$. The color triplet $\bar{b}'_2$ carries baryon numbers $-1/3$ and $+1/3$ in the scenarios A and B, respectively. To ensure the phenomenological viability of the model under consideration, the set of exotic fermion states must include a Dirac fermion ζ_1 ($\bar{\zeta}_1$), related to the exotic state η in Eq. (2), with baryon number $+1/3(-1/3)$, which is predominantly a SM singlet state.[5] If such a fermion state were the lightest exotic particle, then it would tend to be stable and could play the role of dark matter. The production cross sections of the color triplet T and exotic vectorlike quarks t', b'_1 and b'_2 may not be negligibly small at the LHC provided these states are sufficiently light. Then the pair production of exotic quarks may lead to the enhancement of the cross sections for $pp \to t\bar{t}b\bar{b} + \not{E}_T + X$ and $pp \to b\bar{b}b\bar{b} + \not{E}_T + X$. We also argued that in some cases the $T\bar{T}$ pair production at the LHC can result in either similar final states with the four third-generation quarks and missing energy or even give rise to the enhancement of the cross sections that correspond to the processes with six third-generation quarks and missing energy in the final states, i.e. $pp \to T\bar{T} \to t\bar{t}t\bar{t}b\bar{b} + \not{E}_T + X$ and $pp \to T\bar{T} \to b\bar{b}b\bar{b}b\bar{b} + \not{E}_T + X$.

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[5] In this context it is important to take into account that, in general, the sets of light states predicted by the E_6CHM and MCHM involve particles with very different quantum numbers, thus allowing experimental discrimination between these models.

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