Chiral loops in the isospin violating decays of $D_{s1}(2460)^+$ and $D_{s0}^*(2317)^+$

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Positive parity meson states $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$ have masses slightly below the DK threshold. Both states can strongly decay only into isospin violating decays $D_{s1}(2460)^+ \to D_s^+ \pi\pi$, $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$, and $D_{s0}^*(2317)^+ \to D_s^+\pi^0$. The π states have rather small energies, which makes these decays appropriate to study within heavy meson chiral perturbation theory and to calculate loop contributions. The $D_{s1}(2460)^+ \to D_s^+ \pi\pi$ decays occur only at the loop level. Amplitude is a result of chiral loop contributions, which then have to be finite. However, in the case of $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ decays, there is a tree-level contribution. We find that chiral loop contributions might be important in both cases. The calculated amplitudes are sensitive on the coupling constant describing the interaction of positive and negative parity heavy meson multiplets with the light pseudoscalars. The counterterm contributions are also present in the amplitudes $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+\pi^0$. We explore an experimentally known ratio of the decay widths for these two decay modes to estimate the size of counterterm contributions. We determine decay widths for both decay modes to be $\Gamma(D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-) \simeq 0.25$ keV and $\Gamma(D_{s1}(2460)^+ \to D_s^+\pi^0\pi^0) \simeq 0.15$ keV.

I. INTRODUCTION

It has been more than a decade since $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$ mesons were observed by the *BABAR* and CLEO collaborations, respectively. Their existence was confirmed by several experiments [1–5]. The measurement of $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$ quantum numbers, as well as decay widths and decay rates, has continued since that time [1–9]. The experimental data support the interpretation of the $D_{s0}^*(2317)^+$ meson as a positive parity scalar ($J^P = 0^+$), while $D_{s1}(2460)^+$ appears to be positive parity axial vector ($J^P = 1^+$). Both states behave as $c\bar{s}$ systems, although their masses turned out to be 100 MeV smaller than expected by calculation based on the quark models (for a review see [10]). Many proposals have suggested that these states are tetraquarks or DK molecules [10–14]. There is also a suggestion that these states can be a result of the mixing of $\bar{s}c$ and $\bar{s}c\bar{q}q$ states (see, e.g., [15]). Recently, lattice studies indicated that $D_{s0}^*(2317)^+$ is a combined state of $\bar{s}c$ and DK molecules [16–19]. However, multiple lattice volumes will be needed [18] to resolve the structure of $D_{s0}^*(2317)^+$. On the other hand, the total decay widths of $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$ are still unmeasured. The upper bound on the total decay width of the $D_{s0}^*(2317)^+$ meson is 3.5 MeV at 95% confidence level, while the upper bound on the total decay width of the $D_{s1}(2460)^+$ meson is

3.8 MeV at 95% confidence level [7]. Nevertheless, some branching rations for $D_{s1}(2460)^+$ decays were determined. The masses of $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$ mesons are slightly below the threshold for the decay into a D and a K meson. Therefore, only strong isospin violating decays of $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$ are kinematically allowed, as well as radiative decays of $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$. The decay $D_{s0}^*(2317) \to D_s^+\pi^0$ was observed by the *BABAR* Collaboration [1], but the branching ratio or partial decay width was not determined. However, the branching ratio of $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ was found to be $(48 \pm 11)\%$ [7,20]. The *BABAR* Collaboration also observed $D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ [6], with the branching ratio of $(4.3 \pm 1.3)\%$ [20]. There is no measurement of the branching fraction for the decay mode $D_{s1}(2460)^+ \to D_s^+\pi^0\pi^0$ yet. In the case that $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$ are only $c\bar{s}$ mesons, with quantum numbers 0^+ and 1^+, the decay of $D_{s0}^*(2317)^+$ into $D_s^+\pi^0$ is possible, while the decays of $D_{s0}^*(2317)^+$ into $D_s^{*+}\pi^0$ and $D_s^+\pi\pi$ cannot occur. Similarly, the decay of $D_{s1}(2460)^+$ into $D_s^+\pi^0$ is not possible, while the decays of $D_{s1}(2460)^+$ into $D_s^{*+}\pi^0$ and $D_s^+\pi\pi$ are allowed. Therefore, it seems that existing experimental results favor the treatment of both states $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$ as a $c\bar{s}$ states with the positive parity.

The $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ decays were considered within a variety of approaches [21–33]. The quark models were mostly exploited for the decays $D_{s0}^*(2317)^+ \to D_s^+\eta \to D_s^+\pi^0$ and $D_{s1}(2460)^+ \to D_s^{*+}\eta \to D_s^{*+}\pi^0$,

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assuming isospin symmetry violation via $\eta - \pi$ mixing. In Ref. [22] states $D_{s1}(2460)^+$ and $D_{s0}^*(2317)^+$ are treated in the DK molecule picture. Most of these studies predict the partial decay widths of $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ decay modes to be in the range (10–30) keV, although higher values (about 100 keV) were suggested in Ref. [24] too.

The three-body decays $D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ and $D_{s1}(2460)^+ \to D_s^+\pi^0\pi^0$ were studied in Refs. [34,35]. The authors of [34] assumed that the decay occurs through the intermediate $\tilde{\sigma}$ fields, which then convert to pions through the $\bar{s}s$ component of $\tilde{\sigma}$ fields mixing with the $\bar{u}u$ and $\bar{d}d$ components. Two relevant σ states (σ_0 and σ_8) were considered with masses set to 1 GeV and 1.5 GeV. In Ref. [34] high sensitivity of the amplitude on the mass of the lighter sigma meson state was found out and the variation of its mass in the range 0.8 GeV and 1.2 GeV can change the predicted result for an order of magnitude. In [35], the σ states were replaced by the scalar $f_0(980)$ state. However, the decay width was estimated to be $\Gamma(D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-) < 25$ keV.

In this paper, we determine chiral loop contributions to the isospin violating decay amplitudes of $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$ mesons. For two-body decays, there is a tree-level contribution to decay amplitude arising from the η-π mixing. However, chiral loops even in these cases might give significant contributions. This was indicated already in Ref. [30], where some of the loop contributions to the decay amplitude were determined. In our analysis, we rely on the heavy meson chiral perturbation theory (HMχPT). Within HMχPT the $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$ states have quantum numbers of $c\bar{s}$. The use of HMχPT in both decay modes is fully justified by the fact that the pions in the final set have rather small energies. The loop contributions within this framework arise from the light pseudoscalar meson exchanges. The light resonances, as light vector mesons (ρ, K^*) in the amplitudes at tree level, give the contributions of the same order in the chiral counting [36] as light pseudoscalar meson loops [37]. In comparison with the approach of [30], we find that there are additional Feynman diagrams leading to the relevant contribution to the two-body decay amplitudes. The three-body decay amplitude within this framework arises from chiral loops. The energy release in both two-body and three-body decays is very small. Both negative and positive parity intermediate D states are taken into account within this framework [38–40].

The heavy meson Lagrangian formalism will be introduced in Sec. II. In Sec. III, the analysis of the $D_{s1}(2460)^+ \to D_s^+\pi\pi$ decay channels will be presented, while in Sec. IV we will calculate the decay width of the two-body $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ decay modes while a short conclusion will be given in Sec. V.

II. FRAMEWORK

The framework of heavy meson chiral perturbation theory combines the heavy quark effective theory with the chiral perturbation theory [41,42]. Heavy quark effective theory is used to describe mesons composed of one heavy quark (c or b) and one light quark (u, d, or s) [41,42]. In such mesons, the heavy quark moves almost on shell with the velocity v^μ and the momenta of the heavy meson can be written as $p^\mu = mv^\mu + k^\mu$, where m is a heavy meson mass and k^μ is of the order of Λ_{QCD} and much smaller then mv^μ. In a limit, when the mass of the heavy quark becomes infinite, pseudoscalar and vector meson states become degenerate, as well as scalar and axial vector meson states. The negative parity states are described by the field H, while the positive parity states are described by the field S:

$$H = \frac{1}{2}(1 + v \cdot \gamma)[P_\mu^*\gamma^\mu - P\gamma_5],$$

$$S = \frac{1}{2}(1 + v \cdot \gamma)[D_\mu^*\gamma^\mu\gamma_5 - D], \quad (1)$$

where P_μ^* and P annihilate the vector and pseudoscalar mesons, respectively, while D_μ^* and D annihilate the axial-vector and scalar mesons, respectively.

Within chiral perturbation theory, the light pseudoscalar mesons are accommodated into the octet $\Sigma = \xi^2 = e^{(2i\Pi/f)}$ with

$$\Pi = \begin{pmatrix} \pi^0/\sqrt{2}+\eta_8/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta_8/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta_8/\sqrt{6} \end{pmatrix} \quad (2)$$

and $f \sim 120$ MeV at one loop [43]. The leading order of the HMχPT Lagrangian, that describes the interaction of heavy and light mesons, can be written as

$$\mathcal{L} = -\text{Tr}[\bar{H}_a(iv \cdot \mathcal{D}_{ab} - \delta_{ab}\Delta_H)H_b] + g\text{Tr}[\bar{H}_bH_a\gamma \cdot \mathcal{A}_{ab}\gamma_5]$$
$$+ \text{Tr}[\bar{S}_a(iv \cdot \mathcal{D}_{ab}) - \delta_{ab}\Delta_S)S_b] + \tilde{g}\text{Tr}[\bar{S}_bS_a\gamma \cdot \mathcal{A}_{ab}\gamma_5] + h\text{Tr}[\bar{H}_bS_a\gamma \cdot \mathcal{A}_{ab}\gamma_5], \quad (3)$$

where $\mathcal{D}_{ab}^\mu = \delta_{ab}\partial^\mu - \mathcal{V}_{ab}^\mu$ is a heavy meson covariant derivative, $\mathcal{V}_\mu = 1/2(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger)$ is the light meson vector current, and $\mathcal{A}_\mu = i/2(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)$ is the light meson axial current. A trace is taken over spin matrices and repeated light quark flavor indices. All terms in (3) are of the order $\mathcal{O}(p)$ in the chiral power counting (see, e.g., [39]). As in [39] we assign for $\Delta_{SH} = \Delta_S - \Delta_H \sim \mathcal{O}(p)$ in order to maintain a well-behaved chiral expansion. Light mesons are described by the Lagrangian [41,42], which is of the order $\mathcal{O}(p^2)$ in the chiral expansion:

$$\mathcal{L}_0 = \frac{f^2}{8}\mathrm{Tr}[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger] + \frac{f^2 \lambda_0}{4}\mathrm{Tr}[m_q^\xi \Sigma + \Sigma m_q^\xi], \quad (4)$$

with $\lambda_0 = m_\pi^2/(m_u + m_d) = (m_{K^+}^2 - m_{K_0}^2)/(m_u + m_d) = (m_K^2 - m_\pi^2/2)/m_s$. The above Lagrangians lead to Feynman rules, as given in [39]. The scalar (pseudoscalar) and vector (axial-vector) heavy meson propagators can be written in the forms

$$\frac{i}{2(k \cdot v - \Delta_i)} \quad \text{and} \quad \frac{-i(g^{\mu\nu} - v^\mu v^\nu)}{2(k \cdot v - \Delta_i)}, \quad (5)$$

respectively, where Δ_i in the propagator represents the residual mass of the corresponding field. Residual masses are responsible for mass splitting of heavy meson states. The difference $\Delta_S - \Delta_H$ splits the masses of positive and negative parity states. In addition, we also have to take into account mass splitting between D_s and D states as well as mass splitting between vector (axial-vector) and pseudoscalar (scalar) fields. These splittings arise due to the heavy meson Lagrangian correction of the order $\mathcal{O}(m_q)$ (m_q stands for the mass of light quarks). To account for all of the above mass splittings, we will follow the approach of [38,39] and set the values of Δ_i to the experimentally measured mass differences between D meson states. We use

the mass of the initial particle as a reference value, so all mass differences are defined as mass differences between the relevant D meson and initial state [38].

The coupling constants g, h, and $\tilde{g}$ were already discussed by several authors and determined by several methods: the QCD sum rules [44–48], the lattice QCD [49–55], and the extraction from the experimental data [38,39,56,57]. We will use recent results from the lattice QCD: $g = 0.54(3)\binom{+2}{-4}$ [49], $\tilde{g} = -0.122(8)(6)$, and $h = -0.84(3)(2)$ [55]. The values of h and $\tilde{g}$ were determined for the B meson sector, so $1/m_c$ corrections can make a slight difference in stated values. Although HMχPT relies on the expansion in the light pseudoscalar momentum and $1/m_c$ expansion, we do not consider $1/m_c$ corrections for at least two reasons: first, the number of additional terms in the Lagrangian becomes huge and impossible to estimate and second, lattice studies indicate that these contributions are rather small [53]. The authors of [55] made an estimation of h for the D meson sector and found that $h \simeq 0.74(8)$, which is still inside the error bars of $h = -0.84(3)(2)$.

In order to absorb divergences coming from loop integrals, one needs to include counterterms in the Lagrangian. Following [38,39], they can be written as

$$\begin{aligned} \mathcal{L}_{ct} &= \lambda_1[\bar{H}_b \bar{H}_a (m_q^\xi)_{ba}] + \lambda_1'[\bar{H}_a \bar{H}_a (m_q^\xi)_{bb}] - \tilde{\lambda}_1[\bar{S}_b \bar{S}_a (m_q^\xi)_{ba}] - \tilde{\lambda}_1'[\bar{S}_a \bar{S}_a (m_q^\xi)_{bb}] \\ &+ \frac{h\kappa_1' \lambda_0}{(4\pi f)^2}\mathrm{Tr}[(\bar{H}S\gamma_\mu \mathcal{A}^\mu \gamma_5)_{ab}(m_q^\xi)_{ba}] + \frac{h\kappa_3' \lambda_0}{(4\pi f)^2}\mathrm{Tr}[(\bar{H}S\gamma_\mu \mathcal{A}^\mu \gamma_5)_{aa}(m_q^\xi)_{bb}] \\ &+ \frac{h\kappa_5' \lambda_0}{(4\pi f)^2}\mathrm{Tr}[\bar{H}_a S_a \gamma_\mu \mathcal{A}^\mu_{bc} \gamma_5 (m_q^\xi)_{cb}] + \frac{h\kappa_9' \lambda_0}{(4\pi f)^2}\mathrm{Tr}[\bar{H}_c S_a \gamma_\mu \mathcal{A}^\mu_{bc} \gamma_5 (m_q^\xi)_{ab}] \\ &+ \frac{\delta_2'}{(4\pi f)^2}\mathrm{Tr}[\bar{H}_a S_b i v \cdot \mathcal{D}_{bc} \gamma_\mu \mathcal{A}^\mu_{ca} \gamma_5] + \frac{\delta_3'}{(4\pi f)^2}\mathrm{Tr}[\bar{H}_a S_b i\gamma_\mu \cdot \mathcal{D}^\mu_{bc} v \cdot \mathcal{A}^\mu_{ca} \gamma_5] + \text{H.c.} + \cdots, \quad (6) \end{aligned}$$

where $m^\xi = (\xi m_q \xi - \xi^\dagger m_q \xi^\dagger)/2$ and $D^\alpha_{bc} A^\beta_{ca} = \partial^\alpha A^\beta_{ba} + [v^\alpha A^\beta]_{ba}$. At the given scale, the finite part of κ_3' can be absorbed into the definition of h. Parameters λ_1' and $\tilde{\lambda}_1'$ can be absorbed into the definition of heavy meson masses by phase redefinition of H and S, while λ_1 and $\tilde{\lambda}_1$ split the masses of SU(3) flavor triplets of H_a and S_a [38,39]. Therefore, only contributions proportional to κ_1', κ_9', κ_5', δ_2', and δ_3' will be explicitly included in the amplitudes.

III. THE $D_{s1}(2460)^+ \to D_s^+ \pi^+ \pi^-$ AND $D_{s1}(2460)^+ \to D_s^+ \pi^0 \pi^0$ DECAY MODES

First, we will consider the chiral loop contributions to the $D_{s1}(2460)^+ \to D_s^+ \pi^+ \pi^-$ decay rate. The decay width for the $D_{s1}(2460)^+ \to D_s^+ \pi^+ \pi^-$, averaged over the $D_{s1}(2460)^+$ polarizations, can be written as

$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M_i^3}|\mathcal{M}|^2 dm_{12}^2 dm_{23}^2, \quad (7)$$

where M_i is the mass of initial particle $D_{s1}(2460)^+$, $dm_{12}^2 = (p_+ + p_-)^2$, and $dm_{23}^2 = (p_- + q)^2$. Here p_- and p_+ denote the momenta of π^+ and π^-, respectively, while q is the momentum of D_s^+. The decay amplitude $\mathcal{M}$, in general, can be written in the form

$$\mathcal{M} = \epsilon \cdot (\mathcal{A}_1 p_+ + \mathcal{A}_2 p_- + \mathcal{A}_3 q).$$

Within the heavy quark limit $P^\mu = M_i v^\mu$, $q^\mu = M_f v^\mu$, and $p_+^\mu + p_-^\mu = \Delta_M v^\mu$, where $\Delta_M = (M_i - M_f)$ and M_f is the mass of D_s^+ meson. As $\epsilon \cdot v = 0$, the amplitude simplifies to

$$\mathcal{M} = \mathcal{A}\epsilon \cdot (p_+ - p_-) = \mathcal{A}\epsilon \cdot \Delta p,$$
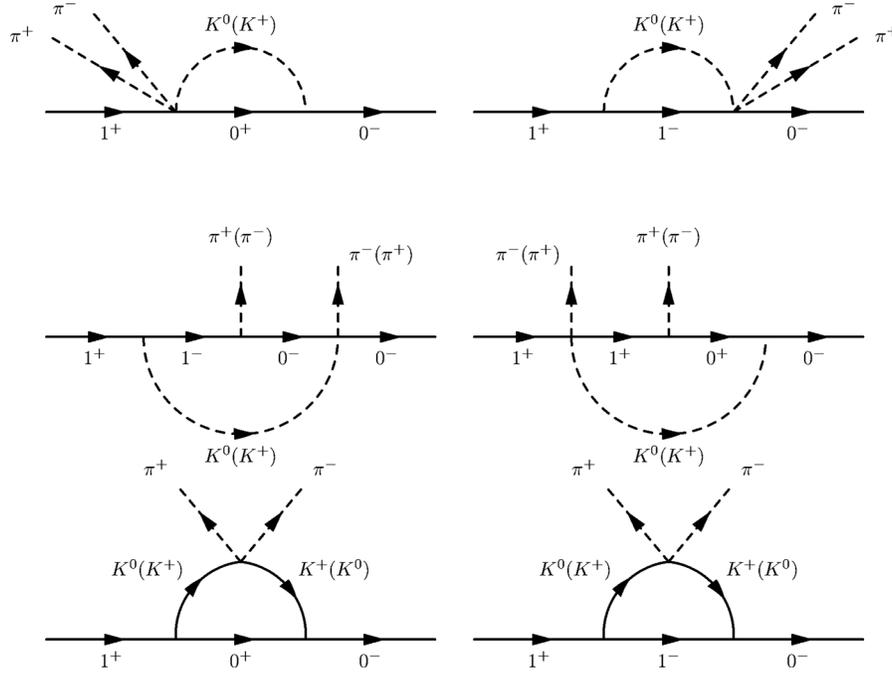
FIG. 1. Nonzero contributions to $D_{s1}(2460)^+ \to D_s^+ \pi^+ \pi^-$ decay amplitude.

where $\mathcal{A}$ can be calculated from the diagrams presented in Fig. 1. Only diagrams giving a nonzero contribution are shown. Note that there are no diagrams with the η meson in the loop on Fig. 1, as they all give a vanishing contribution.

The diagrams in Fig. 2 also give a nonzero contribution to the amplitude. Since they are next-to-leading order in HMχPT their contributions can be neglected. Note, also, that by taking $P^\mu = M_i v^\mu$ and $q^\mu = M_f v^\mu$, all scalar products of the momenta become independent of the phase space parameters: $p_+ \cdot v = p_- \cdot v = \Delta M/2$ and $p_- \cdot p_+ = \Delta M/2 - m^2$. Therefore, the spin averaging of the amplitude $\mathcal{M}$ is constant on the whole phase space region, implying that the calculation of the amplitude and the integration over the phase space in (7) can be done independently.

Now, we proceed to the calculation of the amplitude $\mathcal{A}$. Using Feynman rules derived from (3), we obtain

$$\mathcal{A} = \frac{h\sqrt{M_i M_f}}{16\pi^2 f^4}(a_1 + a_2 + b_1 + b_2 + c_1 + c_2), \quad (8)$$

where

$$a_1 = \frac{g}{2}(\bar{B}_1(\Delta_{P^*}, m_{K^0}) - \bar{B}_1(\Delta_{P^*}, m_{K^+})), \quad (9)$$

$$a_2 = \frac{\tilde{g}}{2}(\bar{B}_1(\Delta_D + \Delta_M, m_{K^0}) - \bar{B}_1(\Delta_D + \Delta_M, m_{K^+})), \quad (10)$$

$$b_1 = 2g((\bar{B}_2'(\Delta_{P^*}, \Delta_P + \Delta_M/2, m_{K^0}) - \Delta_M/2 \cdot \bar{B}_1'(\Delta_{P^*}, \Delta_P + \Delta_M/2, m_{K^0})) - (\bar{B}_2'(\Delta_{P^*}, \Delta_P + \Delta_M/2, m_{K^+}) - \Delta_M/2 \cdot \bar{B}_1'(\Delta_{P^*}, \Delta_P + \Delta_M/2, m_{K^+}))), \quad (11)$$

$$b_2 = 2\tilde{g}((\bar{B}_2'(\Delta_{D^*} + \Delta_M/2, \Delta_D + \Delta_M, m_{K^0}) + \Delta_M/2 \cdot \bar{B}_1'(\Delta_{D^*} + \Delta_M/2, \Delta_D + \Delta_M, m_{K^0})) - (\bar{B}_2'(\Delta_{D^*} + \Delta_M/2, \Delta_D + \Delta_M, m_{K^+}) + \Delta_M/2 \cdot \bar{B}_1'(\Delta_{D^*}, \Delta_D + \Delta_M, m_{K^+}))), \quad (12)$$

$$c_1 = -2g((B_{00}(m_{K^0}) + \Delta_{P^*}\bar{C}_{00}(\Delta_{P^*}, m_{K^0})) - (B_{00}(m_{K^+}) + \Delta_{P^*}\bar{C}_{00}(\Delta_{P^*}, m_{K^+})))$$
$$c_2 = -2\tilde{g}((B_{00}(m_{K^0}) + \Delta_D\bar{C}_{00}(\Delta_D, m_{K^0})) - (B_{00}(m_{K^+}) + \Delta_D\bar{C}_{00}(\Delta_D, m_{K^+}))). \quad (13)$$
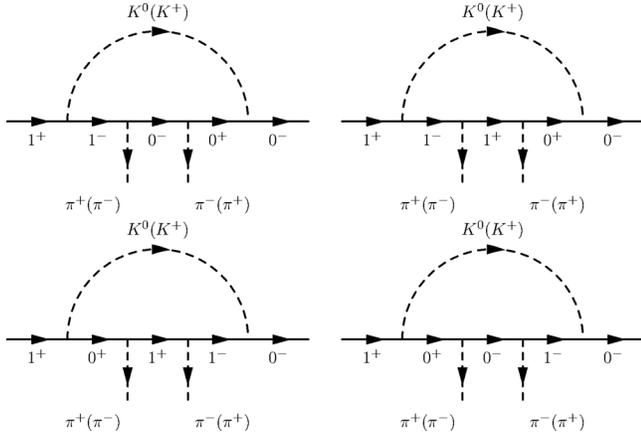
FIG. 2. Feynman diagrams contributing to $D_{s1}(2460)^+ \to D_s^+ \pi^+ \pi^-$ decay amplitude at higher order and which are not included in our calculations.

Here, $\bar{B}_1$, $\bar{B}_2$, B_{00}, and $\bar{C}_{00}$ are the Passarino-Veltman loop integrals defined in Appendix A. Note that these loop integrals, as well as the parts of the amplitude a_i, b_i, and c_i, are divergent in the $D \to 4$ limit. Nevertheless, by summing up all contributions, divergences cancel out, so both $\mathcal{A}$ and $\mathcal{M}$ are finite. As $D_{s1}(2460)^+ \to D_s^+ \pi^+ \pi^-$ does not have any tree-level contributions from the heavy meson Lagrangian, this was expected. Another interesting feature is that the amplitude $\mathcal{M}$ is nonzero due to the mass difference of the K^+ and K^0 mesons. Namely, if we put $m_{K^+} = m_{K^0}$, the amplitude would vanish. Finally, the decay width coming from these amplitudes is

$$\Gamma(D_{s1}(2460)^+ \to D_s^+ \pi^+ \pi^-) = 0.25(4)(7)\binom{+2}{-4} \text{ keV}.$$

The first error comes from the uncertainty in the coupling constant h, the second from uncertainty in the coupling constant g, and the last from the uncertainty in the mass of $D_{s1}(2460)^+$ meson. Uncertainties in the coupling constant $\tilde{g}$ and other meson masses are relatively small and therefore can be safely neglected. This result implies that the total decay width of $D_{s1}(2460)^+$ is found to be between 2 keV and 13 keV.

The decay amplitude for $D_{s1}(2460)^+ \to D_s^+ \pi^0 \pi^0$ can be easily found by replacing the $\pi^+ \pi^-$ state by $\pi^0 \pi^0$ in the final state. Note that one has to include the factor $1/2$ in the decay mode due to two identical bosons in the final state. This yields

$$\Gamma(D_{s1}(2460)^+ \to D_s^+ \pi^0 \pi^0) = 0.15(4) \text{ keV}.$$

IV. THE $D_{s1}(2460)^+ \to D_s^{*+} \pi^0$ AND $D_{s0}^*(2317)^+ \to D_s^+ \pi^0$ DECAY MODES

The tree-level contribution to $D_{s1}(2460)^+ \to D_s^{*+} \pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+ \pi^0$ decay amplitudes results from the $\eta - \pi$ mixing. In this scenario, the decays proceed
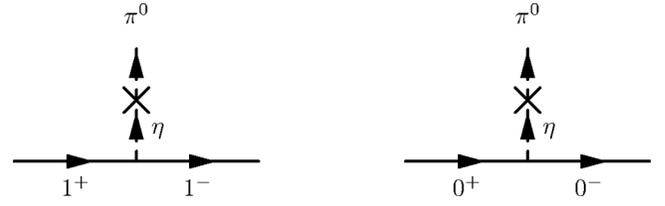
FIG. 3. Tree-level contribution to $D_{s1}(2460)^+ \to D_s^{*+} \pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+ \pi^0$ decay modes.

through the channels $D_{s1}(2460)^+ \to D_s^{*+} \eta \to D_s^{*+} \pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+ \eta \to D_s^+ \pi^0$ as presented in Fig. 3. The $\eta - \pi^0$ mixing can be approached by the mixing Lagrangian [58,59],

$$\mathcal{L}_{\eta - \pi_0} = \frac{m_\pi^2(m_u - m_d)}{\sqrt{3}(m_u + m_d)} \pi_0 \eta, \tag{14}$$

which comes from the second term in (4). The decay width for $D_{s1}(2460)^+ \to D_s^{*+} \pi^0$ coming from this mixing tree-level amplitude is

$$\Gamma(D_{s1}(2460)^+ \to D_s^{*+} \pi^0) = \frac{h^2}{2\pi f^2} |k_\pi| E_\pi \delta_{\text{mix}}^2 = 16 \text{ keV}, \tag{15}$$

where k_π and E_π are the momenta and energy of the outgoing pion, while mixing parameter δ_{mix} is defined as in [58,59]:

$$\delta_{\text{mix}} = \frac{1}{2\sqrt{2}} \frac{m_u - m_d}{m_s - (m_u + m_d)/2}. \tag{16}$$

Note that decay widths for $D_{s1}(2460)^+ \to D_s^{*+} \pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+ \pi^0$ decay modes differ only for a small difference in D_s^+ masses.

Next, we calculate chiral loop corrections to the above decay modes. By including chiral corrections to the $\eta - \pi$ mixing, the decay width becomes

$$\Gamma(D_{s1}(2460)^+ \to D_s^{*+} \pi^0) = \frac{h^2}{2\pi f^2} |k_\pi| E_\pi \delta_{\text{mix}}^2 \left| \frac{\sqrt{Z_f}\sqrt{Z_i}}{Z_v} \right|^2, \tag{17}$$

where Z_f and Z_i denote wave function renormalization of the initial and final heavy meson states,

$$Z_{i,f} = 1 - \frac{1}{2} \frac{\partial \Pi_{i,f}(v \cdot p)}{\partial v \cdot p} \Big|_{\text{on mass shell}}, \tag{18}$$

while Z_v represents the vertex corrections:

$$Z_v = 1 - \frac{\hat{\Gamma}(v \cdot p_i, v \cdot p_f, k^2)}{\hat{\Gamma}_0(v \cdot p_i, v \cdot p_f, k^2)} \Big|_{\text{on mass shell}}. \tag{19}$$
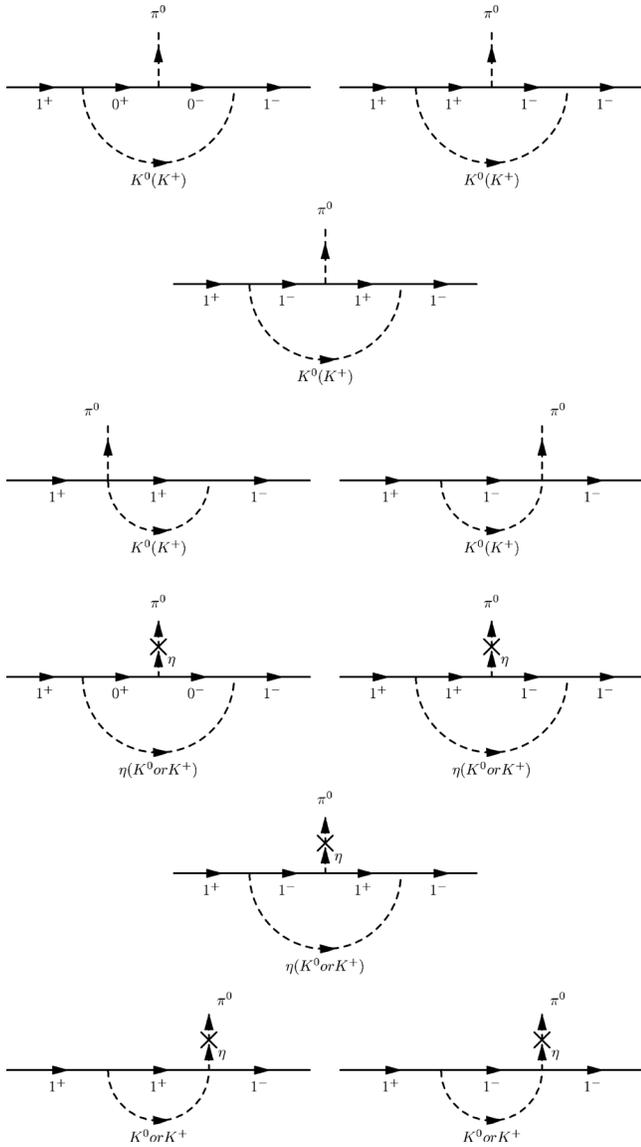
FIG. 4. Chiral corrections to the $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ decay mode.

FIG. 5. Chiral corrections to the $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ decay mode.
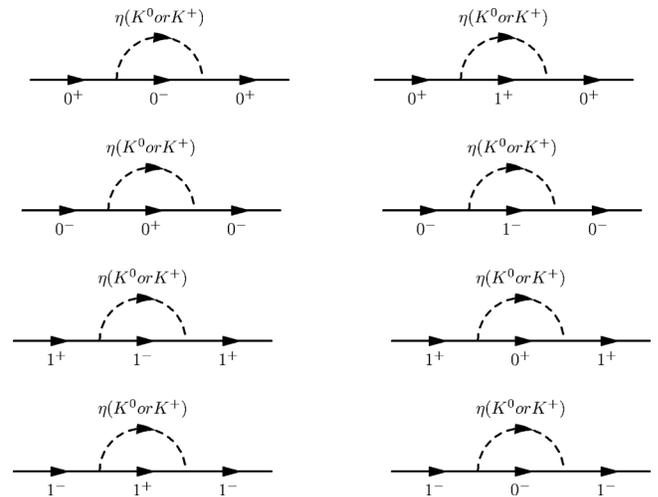
FIG. 6. Chiral corrections to the D meson wave functions.

Here $\hat{\Gamma}$ is the vertex amplitude calculated from the Feynman diagrams presented in Figs. 4 and 5, while $\hat{\Gamma}_0$ is the vertex amplitude resulting from the tree-level Feynman diagrams (see Fig. 3). Similarly, $\Pi(v \cdot p)$ is the heavy meson self-energy arising from the Feynman diagrams in Fig. 6.

The vertex corrections come from the Feynman diagrams presented in Figs. 4 and 5 and can be summarized into the expression:

$$Z_v = 1 - \left(\delta'_{\text{mix}} + \frac{2}{3}\mathcal{A}'_i(m_\eta) - \frac{1}{2}(\mathcal{A}_i(m_{K^+}) + \mathcal{A}_i(m_{K^0})) \right.$$
$$\left. + \frac{1}{\sqrt{2}\delta_{\text{mix}}}(\mathcal{A}_i(m_{K^+}) - \mathcal{A}_i(m_{K^0})) + \mathcal{A}_{ct} \right), \qquad (20)$$

where $\delta'_{\text{mix}} = 0.11$ includes corrections to the $\eta - \pi$ mixing angle beyond tree level [38,43] and the functions $\mathcal{A}_i$ are given in Appendix B.

The isospin violating nature of both decays are manifested either by proportionality of the amplitude to the mixing parameter δ_{mix}, or by vanishing of the amplitude in the case of the isospin limit $m_{K^0} = m_{K^+}$.

Finite parts of counterterms are included in the amplitude as $\mathcal{A}_{ct}$:

$$\mathcal{A}_{ct} = \frac{1}{32\pi^2 f^2}\left(\left(m_K^2 - \frac{m_\pi^2}{2}\right)(\kappa_1' + \kappa_9') + \left(m_K^2 - m_\pi^2 + \frac{\sqrt{2}(m_{K^+}^2 - m_{K^0}^2)}{\delta_{mix}}\right)\kappa_5' + \frac{E_\pi}{2}(\delta_2' + \delta_3')\right). \tag{21}$$

The values of the finite parts of counterterms, of course, depend on the renormalization scheme. We use dimensional regularization in the renormalization scheme in which the divergence $2/\epsilon$ contains the constant $-\gamma_E + \ln 4\pi + 1$, coming from the loop integrals. This has to be taken into account when discussing the numerical value of the $\mathcal{A}_{ct}$ term. The wave function renormalization terms $\mathcal{Z}_f$ and $\mathcal{Z}_i$ arise from sunrise diagrams presented in Fig. 6:

$$Z_j = 1 - \mathcal{B}_j(m_{K^+}) - \mathcal{B}_j(m_{K^0}) - \frac{2}{3}\mathcal{B}_j'(m_\eta). \tag{22}$$

The functions $\mathcal{B}_j$ are listed in Appendix B.

The calculated numerical values of the decay widths of $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ are very sensitive on the value of h and can be modified by 2 orders of magnitude when h varies form -0.65 to -0.9. The wave function corrections $\mathcal{Z}_{i,f}$ are source of that sensitivity. The decay widths are also moderately sensitive on the mass values of final and initial states and vary on the coupling constant g. The dependence on the decay width on g and the masses of the final and initial mesons are presented in Fig. 7, while the decay widths of $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ are presented in Fig. 8, for a range of values for counterterms $\mathcal{A}_{ct}$.
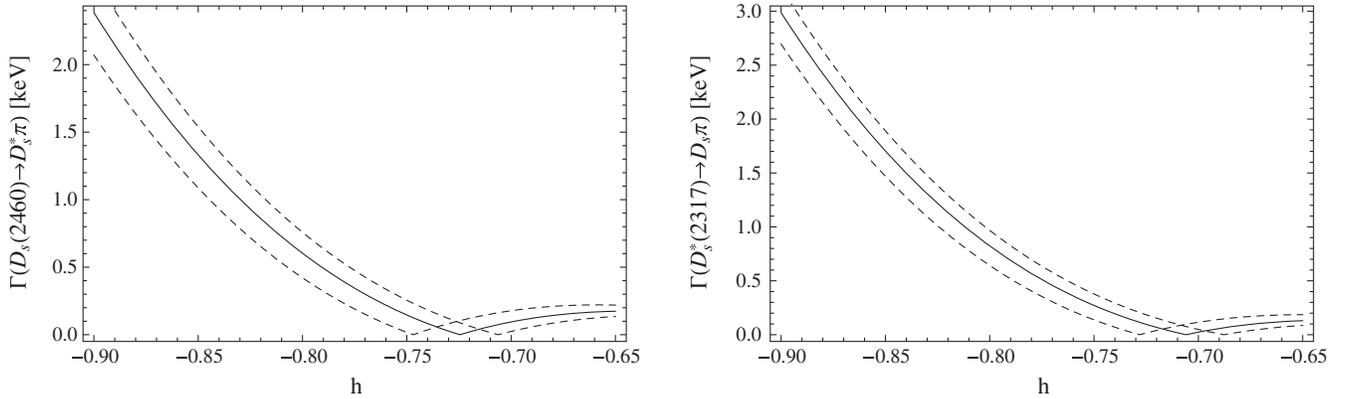
FIG. 7.   Dependence of $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ decay widths on input parameters at $\mathcal{A}_{ct} = 0$. Dashed lines present the uncertainty on the decay width due to the uncertainty in g and the mass of the initial meson.
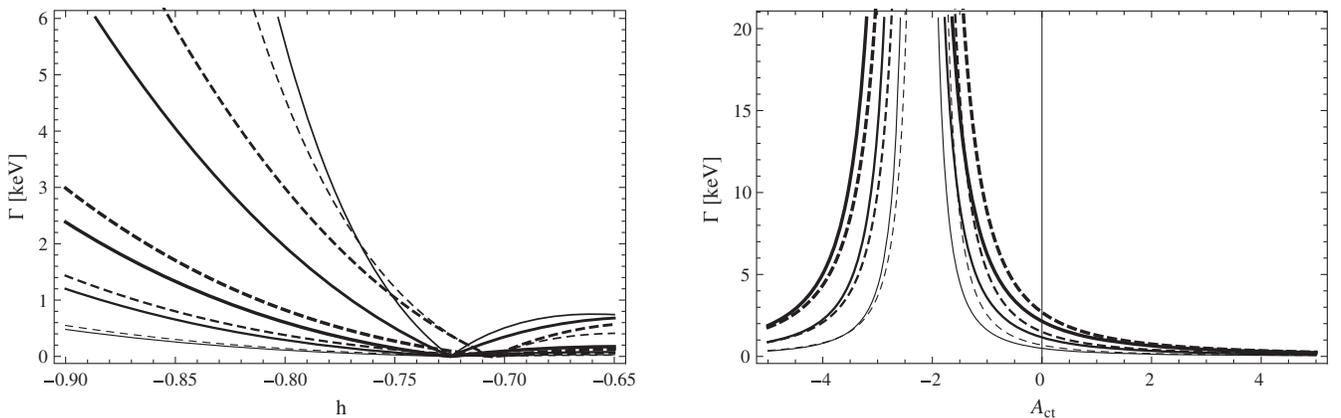
FIG. 8.   Left: Dependence of $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ decay widths on the decay constant h for five different values of counterterm amplitude. From thickest to thinnest line, $\mathcal{A}_{ct} = 0, -1, 1, -3, 3$. The $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ decay width is presented by the solid line, while the $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ decay width is presented by the dashed line. Right: Dependence of $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ decay widths on the value of counterterm amplitude for three different values of coupling constant h. From thickest to thinnest line, $h = -0.89, -0.84, -0.79$. The solid line denotes the decay width of $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$, while the dashed line is used for the decay width of $D_{s0}^*(2317)^+ \to D_s^+\pi^0$.
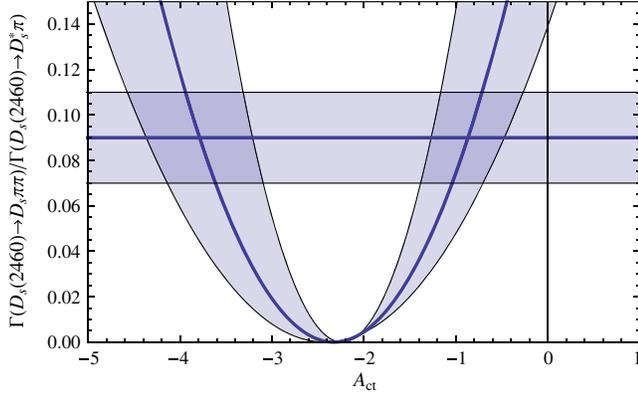
FIG. 9 (color online). Determination of the allowed regions for the counterterm size in the amplitude $\mathcal{A}_{ct}$. Horizontal lines present the experimental values of $\frac{BR(D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^0)}{BR(D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^+\pi^-)}$ with the one sigma error band. The "U" line presents the result of our calculation for the ratio $\frac{BR(D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^0)}{BR(D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^+\pi^-)}$ with the band region presenting the uncertainty coming from the variation of h in the range -0.79 and 0.84.

Since the numerical results are very sensitive on the value of coupling constant h and due to the unknown final parts of counterterms, we cannot make a definite prediction for the partial decay widths for both decay modes. Nevertheless, by using experimentally measured ratio of these rates,

$$\frac{BR(D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^0)}{BR(D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^+\pi^-)} = 0.09 \pm 0.02, \quad (23)$$

we can shed more light on the value of counterterm amplitude $\mathcal{A}_{ct}$. Our result, given in Fig. 9, indicates that $\mathcal{A}_{ct}$ can be accommodated within the ranges $(-4.8; -3.2)$ and $(-1.3; -0.1)$. Here, we only considered one σ experimental error on the decay widths ratio and the uncertainty in the theoretical prediction, by varying h between -0.79 and -0.89. Within these bounds, we obtain for the $D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^0$ and $D_{s0}^*(2317)^+ \rightarrow D_s^+\pi^0$ decay widths

$$\Gamma(D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^0) = 2.7 - 3.4 \text{ keV}, \quad (24)$$

$$\Gamma(D_{s0}^*(2317)^+ \rightarrow D_s^+\pi^0) = 2.4 - 4.7 \text{ keV}. \quad (25)$$

One can notice a slight difference between $\Gamma(D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^0)$ and $\Gamma(D_{s0}^*(2317)^+ \rightarrow D_s^+\pi^0)$, not present at the first order which arises from the $\eta - \pi^0$ mixing. This difference is a result of loop corrections, which were not considered in previous calculations [44].

V. DISCUSSION AND CONCLUSIONS

Within a HMχPT framework, we determine loop contributions to the strong isospin violating decay amplitudes

for $D_{s1}(2460)^+$ and $D_{s0}^*(2317)^+$. We have assumed that both states carry quantum numbers of the $\bar{c}s$ states. Since three-body decays of these states are forbidden at tree level, we calculate contributions to the decay amplitude at the loop level, which is consequently finite. Contrary to three-body decay amplitudes described by the finite loop contributions, two-body isospin violating decay amplitudes receive contributions at tree level, induced by the η–π^0 mixing. Therefore, the chiral loop contribution is not finite and in order to regularize it, one has to introduce counterterms. Our estimate of the size of counterterm in the amplitude $\mathcal{A}_{ct}$ relies on the result we derive for the $D_{s1}(2460)^+ \rightarrow D_s^+\pi^+\pi^-$ decay width.

Note that within chiral perturbation theory, only the light pseudoscalar mesons are present in the loops. Contributions of the light resonances with the spin $J^P = 1^-$, 0^+, 1^+ are already accounted by the chiral loop contributions of light pseudoscalar mesons [60,61]. Additional inclusion of light resonances with $J^P = 1^-$, 0^+, 1^+ in HMχPT is not consistent with the original framework. Nevertheless, one can roughly estimate contribution of light vector mesons in the loop. For example, the contribution of K^* vector mesons in loops as seen in [12] would correspond to two loop effects within our framework. Their contribution to the decay rates is therefore suppressed within HMχPT, in contrast with the K^* loop contribution in the D^*K molecule picture presented in [12].

If one assumes as in Ref. [34] that heavy mesons interact with spin $J^P = 0^+$ light resonances as σ [or $f_0(500)$ as in [20]] at tree level, then $D_{s1}(2460)^+ \rightarrow D_s^+\pi^+\pi^-$ decay can proceed through $D_{s1}(2460)^+ \rightarrow D_s^+\sigma \rightarrow D_s^+\pi^+\pi^-$. However, their contribution is proportional to the $\epsilon \cdot v$ which is then equal to 0 in HMχPT. On the other hand, one might think that since the mass of σ is close to 500 MeV, that state can be important in $D_{s1}(2460)^+ \rightarrow D_s^+\pi^+\pi^-$. This would be the case if σ contained the $\bar{s}s$ component. Recent lattice QCD study [62] does not support such an idea. In principle, higher order terms in the chiral expansion might contain terms which describe interactions of heavy meson states with $J^P = 0^+$ light resonances. At the same time that will mean that some of these f_0 states should contain the $s\bar{s}$ contribution. Unfortunately, the structure of light positive parity scalar mesons is not known yet and reliable consideration of this contribution is not possible at present.

Better understanding of the structure of $D_{s1}(2460)^+$ and $D_{s0}^*(2317)^+$, as well as light scalar mesons, might shed more light on the decay mechanism for two- and three-body strong isospin violating decays of $D_{s1}(2460)^+$ and $D_{s0}^*(2317)^+$.

ACKNOWLEDGMENTS

APPENDIX A: LOOP INTEGRALS

By employing dimensional regularization, in the renormalization scheme with $\delta = \frac{2}{4-D} - \gamma_E + \ln 4\pi + 1 = 0$, we have

$$A_0(m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^D k}{(k^2 - m^2 + i\epsilon)} = m^2 \left(\delta - \ln\frac{m^2}{\mu^2}\right) + \mathcal{O}(D-4),$$

$$B_0(p, m, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^D k}{(k^2 - m^2 + i\epsilon)((k+p)^2 - m^2 + i\epsilon)} = \delta - \int_0^1 \ln\frac{x^2 p^2 - x p^2 + m^2}{\mu^2} + \mathcal{O}(D-4),$$

$$B_{00}(p, m, m) = \frac{1}{2(D-1)}[A_0(m) + (2m^2 - p^2/2)B_0(p, m, m)],$$

which in the $D \to 4$ limit gives

$$B_{00}(p, m, m) = \frac{1}{6}[A_0(m) + (2m^2 - p^2/2)B_0(p, m, m) + 2m^2 - p^2/3],$$

$$B_{00}(m) = B_{00}(\Delta_M v, m, m).$$

Loop integrals with one light meson propagator are

$$\bar{B}_0(\Delta, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)}$$

$$= -2\Delta\left[\delta - \ln\frac{m^2}{\mu^2} - 2F\left(\frac{m}{\Delta}\right) + 1\right] + \mathcal{O}(D-4),$$

with

$$F(1/x) = \begin{cases} \frac{1}{x}\sqrt{x^2 - 1} \ln(x + \sqrt{x^2 - 1} + i\epsilon); & |x| > 1, \\ \frac{-1}{x}\sqrt{1 - x^2}\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right); & |x| \leq 1, \end{cases}$$

$$\bar{B}^\mu(\Delta, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k^\mu d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} = \bar{B}_1(\Delta, m)v^\mu,$$

$$\bar{B}_1(\Delta, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k \cdot v d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} = A_0(m) + \Delta\bar{B}_0(\Delta, m),$$

$$\bar{B}^{\mu\nu}(\Delta, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k^\mu k^\nu d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} = \bar{B}_{00}(\Delta, m)g^{\mu\nu} + \bar{B}_{11}(\Delta, m)v^\mu v^\nu,$$

$$\bar{B}_{00}(\Delta, m) = \frac{1}{D-1}[(m^2 - \Delta^2)\bar{B}_0(\Delta, m) - \Delta A_0(m)],$$

which in $D \to 4$ gives

$$\bar{B}_{00}(\Delta, m) = \frac{1}{3}[(m^2 - \Delta^2)\bar{B}_0(\Delta, m) - \Delta A_0(m) + 2\Delta/3(3m^2 - 2\Delta^2)],$$

$$\bar{B}_{11}(\Delta, m) = \frac{1}{D-1}[(D\Delta^2 - m^2)\bar{B}_0(\Delta, m) + D\Delta A_0(m)],$$

which in $D \to 4$ gives

$$\bar{B}_{11}(\Delta, m) = \frac{1}{3}[(4\Delta^2 - m^2)\bar{B}_0(\Delta, m) + 4\Delta A_0(m) - 2\Delta/3(3m^2 - 2\Delta^2)],$$

$$\bar{B}_2(\Delta, m) = \bar{B}_{00}(\Delta, m) + \bar{B}_{11}(\Delta, m),$$

$$\bar{B}'_0(\Delta_1, \Delta_2, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int\frac{d^D k}{(k^2 - m^2)(v\cdot k - \Delta_1)(v\cdot k - \Delta_2)} = \frac{1}{\Delta_1 - \Delta_2}[\bar{B}_0(\Delta_1, m) - \bar{B}_0(\Delta_2, m)],$$

$$\bar{B}^{\mu\prime}(\Delta_1, \Delta_2, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int\frac{k^\mu d^D k}{(k^2 - m^2)(v\cdot k - \Delta_1)(v\cdot k - \Delta_2)} = \bar{B}'_1(\Delta_1, \Delta_2, m)v^\mu,$$

$$\bar{B}'_1(\Delta_1, \Delta_2, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int\frac{k\cdot v d^D k}{(k^2 - m^2)(v\cdot k - \Delta_1)(v\cdot k - \Delta_2)} = \bar{B}_0(\Delta_2, m) + \Delta_1\bar{B}'_0(\Delta_1, \Delta_2, m),$$

$$\bar{B}'_2(\Delta_1, \Delta_2, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int\frac{(k\cdot v)^2 d^D k}{(k^2 - m^2)(v\cdot k - \Delta_1)(v\cdot k - \Delta_2)} = A_0(m) + (\Delta_1 + \Delta_2)\bar{B}_0(\Delta_2, m) + \Delta_1^2\bar{B}'_0(\Delta_1, \Delta_2, m),$$

$$\bar{B}^{\mu\nu\prime}(\Delta_1, \Delta_2, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int\frac{k^\mu k^\nu d^D k}{(k^2 - m^2)(v\cdot k - \Delta_1)(v\cdot k - \Delta_2)} = \bar{B}'_{00}(\Delta_1, \Delta_2, m)g^{\mu\nu} + \bar{B}'_{11}(\Delta_1, \Delta_2, m)v^\mu v^\nu,$$

$$\bar{B}'_{00}(\Delta_1, \Delta_2, m) = \frac{1}{D-1}[m^2\bar{B}'_0(\Delta_1, \Delta_2, m) - \Delta_1\bar{B}'_1(\Delta_1, \Delta_2, m) - \bar{B}_1(\Delta_2, m)],$$

which in $D \to 4$ gives

$$\frac{1}{3}[m^2\bar{B}'_0(\Delta_1, \Delta_2, m) - \Delta_1\bar{B}'_1(\Delta_1, \Delta_2, m) - \bar{B}_1(\Delta_2, m) + 2/3(3m^2 - 2(\Delta_1^2 + \Delta_2^2 + \Delta_1\Delta_2))],$$

$$\bar{B}'_{11}(\Delta_1, \Delta_2, m) = \frac{1}{D-1}[-m^2\bar{B}'_0(\Delta_1, \Delta_2, m) + D\Delta_1\bar{B}'_1(\Delta_1, \Delta_2, m) + D\bar{B}_1(\Delta_2, m)],$$

which in $D \to 4$ gives

$$\frac{1}{3}[-m^2\bar{B}'_0(\Delta_1, \Delta_2, m) + 4\Delta_1\bar{B}'_1(\Delta_1, \Delta_2, m) + 4\bar{B}_1(\Delta_2, m) - 2/3(3m^2 - 2(\Delta_1^2 + \Delta_2^2 + \Delta_1\Delta_2))].$$

Loop integrals with two light meson propagators are

$$\bar{C}^\mu(p, \Delta, m_1, m_2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int\frac{k^\mu d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v\cdot k - \Delta + i\epsilon)}$$

$$= \bar{C}_1(p, \Delta, m_1, m_2)v^\mu,$$

$$\bar{C}_1(p, \Delta, m_1, m_2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int\frac{k\cdot v d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v\cdot k - \Delta + i\epsilon)}$$

$$= B_0(p, m_1, m_2) + \Delta\bar{C}_0(p, \Delta, m_1, m_2),$$

$$\bar{C}^{\mu\nu}(p, \Delta, m_1, m_2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int\frac{k^\mu k^\nu d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v\cdot k - \Delta + i\epsilon)}$$

$$= \bar{C}_{00}(p, \Delta, m_1, m_2)g^{\mu\nu} + \bar{C}_{11}(p, \Delta, m_1, m_2)v^\mu v^\nu,$$

$$\bar{C}_{00}(\Delta, m) = \bar{C}_{00}(-\Delta_M v, \Delta, m, m) = \frac{1}{D-1}[\bar{B}_0(-\Delta_M + \Delta, m) - (\Delta_M/2 + \Delta)B_0(\Delta_m v, m, m)$$

$$+ (m^2 - \Delta^2)\bar{C}_0(\Delta_M v, \Delta, m, m)],$$

which in $D \to 4$ gives

$$\bar{C}_{00}(\Delta, m) = \bar{C}_{00}(-\Delta_M v, \Delta, m, m) = \frac{1}{3}[\bar{B}_0(-\Delta_M + \Delta, m) - (\Delta_M/2 + \Delta)B_0(\Delta_m v, m, m)$$

$$+ (m^2 - \Delta^2)\bar{C}_0(\Delta_M v, \Delta, m, m) - 2/3(3/2\Delta_M - \Delta)],$$

$$\bar{C}_{11}(\Delta, m) = \bar{C}_{11}(-\Delta_M v, \Delta, m, m) = \frac{1}{D-1}[-\bar{B}_0(-\Delta_M + \Delta, m) + D(\Delta_M/2 + \Delta)B_0(\Delta_m v, m, m)$$

$$- (m^2 - D\Delta^2)\bar{C}_0(\Delta_M v, \Delta, m, m)],$$

which in $D \to 4$ gives

$$\bar{C}_{11}(\Delta, m) = \bar{C}_{11}(-\Delta_M v, \Delta, m, m) = \frac{1}{3}[-\bar{B}_0(-\Delta_M + \Delta, m) + 4(\Delta_M/2 + \Delta)B_0(\Delta_m v, m, m)$$
$$- (m^2 - 4\Delta^2)\bar{C}_0(\Delta_M v, \Delta, m, m) + 2/3(3/2\Delta_M - \Delta)].$$

The calculation of the integral,

$$\bar{C}_0(p, \Delta, m_1, m_2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)},$$

is done in [63]. For some calculations, we used the program FeynCalc [64].

APPENDIX B: LOOP FUNCTIONS

Loop functions entering Eq. (20) are listed here:

$$\mathcal{A}_1(m_i) = \frac{1}{16\pi^2 f^2} ((\bar{B}_{00}(\Delta_{P^*}, m_i) + \bar{B}_{11}(\Delta_{P^*}, m_i) - \Delta_M \bar{B}_1(\Delta_{P^*}, m_i)$$
$$- \bar{B}_{00}(\Delta_{D^*} + \Delta_M, m_i) - \bar{B}_{11}(\Delta_{D^*} + \Delta_M, m_i) - \Delta_M \bar{B}_1(\Delta_{D^*} + \Delta_M, m_i))/2$$
$$- h^2(\bar{B}'_{00}(\Delta_{P^*}, \Delta_{D^*} + \Delta_M, m_i) + \bar{B}'_{11}(\Delta_{P^*}, \Delta_{D^*} + \Delta_M, m_i))$$
$$+ g\tilde{g}(\bar{B}'_{00}(\Delta_P, \Delta_D + \Delta_M, m_i) + 2\bar{B}'_{00}(\Delta_{D^*}, \Delta_{P^*} + \Delta_M, m_i))), \quad \text{(B1)}$$

$$\mathcal{A}'_1(m_i) = \frac{1}{16\pi^2 f^2} (-h^2(\bar{B}'_{00}(\Delta_{P^*}, \Delta_{D^*} + \Delta_M, m_i) + \bar{B}'_{11}(\Delta_{P^*}, \Delta_{D^*} + \Delta_M, m_i))$$
$$+ g\tilde{g}(\bar{B}'_{00}(\Delta_P, \Delta_D + \Delta_M, m_i) + 2\bar{B}'_{00}(\Delta_{D^*}, \Delta_{P^*} + \Delta_M, m_i))), \quad \text{(B2)}$$

for the $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ decay mode and

$$\mathcal{A}_2(m_i) = \frac{1}{16\pi^2 f^2} ((\bar{B}_{00}(\Delta_P, m_i) + \bar{B}_{11}(\Delta_P, m_i) - \Delta_M \bar{B}_1(\Delta_P, m_i)$$
$$- \bar{B}_{00}(\Delta_D + \Delta_M, m_i) - \bar{B}_{11}(\Delta_D + \Delta_M, m_i) - \Delta_M \bar{B}_1(\Delta_D + \Delta_M, m_i))/2$$
$$- h^2(\bar{B}'_{00}(\Delta_P, \Delta_D + \Delta_M, m_i) + \bar{B}'_{11}(\Delta_P, \Delta_D + \Delta_M, m_i)) + 3g\tilde{g}\bar{B}'_{00}(\Delta_{D^*}, \Delta_{P^*} + \Delta_M, m_i)), \quad \text{(B3)}$$

$$\mathcal{A}'_2(m_i) = \frac{1}{16\pi^2 f^2} (-h^2(\bar{B}'_{00}(\Delta_P, \Delta_D + \Delta_M, m_i) + \bar{B}'_{11}(\Delta_P, \Delta_D + \Delta_M, m_i)) + 3g\tilde{g}\bar{B}'_{00}(\Delta_{D^*}, \Delta_{P^*} + \Delta_M, m_i)), \quad \text{(B4)}$$

for the $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ decay mode.

The functions $\mathcal{B}_j$ which are present in Eq. (22) are

$$\mathcal{B}_{D_s^+}(m_i) = \frac{1}{16\pi^2 f^2} (3\tilde{g}^2\bar{B}'_{00}(m_{D^*} - m_{D_s^+}, m_i) - h^2(\bar{B}'_{00}(m_P - m_{D_s^+}, m_i) + \bar{B}'_{11}(m_P - m_{D_s^+}, m_i))), \quad \text{(B5)}$$

$$\mathcal{B}_{D_s^{*+}}(m_i) = \frac{1}{16\pi^2 f^2} (3\tilde{g}^2(-\bar{B}'_{00}(m_D - m_{D_s^{*+}}, m_i) + 2\bar{B}'_{00}(m_{D^*} - m_{D_s^{*+}}, m_i))$$
$$- h^2(\bar{B}'_{00}(m_{P^*} - m_{D_s^{*+}}, m_i) + \bar{B}'_{11}(m_{P^*} - m_{D_s^{*+}}, m_i))), \quad \text{(B6)}$$

$$\mathcal{B}_{P_s}(m_i) = \frac{1}{16\pi^2 f^2} (3g^2\bar{B}'_{00}(m_{P^*} - m_{P_s}, m_i) - h^2(\bar{B}'_{00}(m_D - m_{P_s}, m_i) + \bar{B}'_{11}(m_D - m_{P_s}, m_i))), \quad \text{(B7)}$$

$$\mathcal{B}_{P_s^*}(m_i) = \frac{1}{16\pi^2 f^2} (3g^2(-\bar{B}'_{00}(m_P - m_{P_s^*}, m_i) + 2\bar{B}'_{00}(m_{P^*} - m_{P_s^*}, m_i))$$
$$- h^2(\bar{B}'_{00}(m_{D^*} - m_{P_s^*}, m_i) + \bar{B}'_{11}(m_{D^*} - m_{P_s^*}, m_i))). \quad \text{(B8)}$$

TABLE I.   Mass differences.

In the case of	Δ_P (GeV)	Δ_{P^*} (GeV)	Δ_D (GeV)	Δ_{D^*} (GeV)	Δ_M (GeV)
$\mathcal{A}_1$	−0.59	−0.45	−0.06	−0.04	−0.35
$\mathcal{A}_2$	−0.48	−0.31	0.09	0.1	−0.33
$\mathcal{A}_1'$	−0.47	−0.35	−0.14	0	−0.35
$\mathcal{A}_2'$	−0.33	−0.21	0	−0.14	−0.33

The expressions for $\mathcal{B}_j'(m_i)$ can be obtained from the above expressions of $\mathcal{B}_j(m_i)$ by substituting masses of D mesons by the masses of D_s mesons.

Here, $\bar{B}_1$, $\bar{B}_{00}$ and $\bar{B}_{11}$, $\bar{B}_{00}'$, and $\bar{B}_{11}'$ are loop integrals defined in Appendix A. Mass differences Δ_D, Δ_P, Δ_{D^*},

Δ_{P^*} are defined as mass differences between the appropriate state and the initial state while Δ_M is the mass difference between the final and initial states. Therefore, mass differences entering different amplitudes are not the same. The values are given in Table I.

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