# Fragmentation contributions to $J/\psi$ photoproduction at HERA

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(Received 3 August 2015; published 28 October 2015)

We compute leading-power fragmentation corrections to  $J/\psi$  photoproduction at DESY HERA, making use of the nonrelativistic QCD factorization approach. Our calculations include parton production cross sections through order  $\alpha_s^3$ , fragmentation functions though order  $\alpha_s^2$ , and leading logarithms of the transverse momentum divided by the charm-quark mass to all orders in  $\alpha_s$ . We find that the leading-power fragmentation corrections, beyond those that are included through next-to-leading order in  $\alpha_s$ , are small relative to the fixed-order contributions through next-to-leading order in  $\alpha_s$ . Consequently, an important discrepancy remains between the experimental measurements of the  $J/\psi$  photoproduction cross section and predictions that make use of nonrelativistic-QCD long-distance matrix elements that are extracted from the  $J/\psi$  hadroproduction cross-section and polarization data.

DOI: 10.1103/PhysRevD.92.074042

PACS numbers: 12.38.Bx, 12.39.St, 14.40.Pq, 13.60.-r

### I. INTRODUCTION

According to the nonrelativistic QCD (NRQCD) factorization conjecture [1], the inclusive production cross section to produce a quarkonium H in a collision of particles A and B can be written as

$$d\sigma_{A+B\to H+X} = \sum_{n} d\sigma_{A+B\to Q\bar{Q}(n)+X} \langle \mathcal{O}^{H}(n) \rangle, \quad (1)$$

where the  $d\sigma_{A+B\to Q\bar{Q}(n)+X}$  are the short-distance coefficients (SDCs), which are essentially the production cross section of the heavy quark-antiquark pair  $Q\bar{Q}(n)$  in a specific color and angular momentum state n, and the  $\langle \mathcal{O}^H(n) \rangle$  are the corresponding NRQCD long-distance matrix elements (LDMEs), which account for the evolution of the  $Q\bar{Q}(n)$  pair into the quarkonium.

The LDMEs have a known scaling with v, the heavyquark velocity in the quarkonium rest frame [1]. Usually, in heavy-quarkonium production phenomenology, v is considered to be a small parameter ( $v^2 \approx 0.3$  for the  $J/\psi$ ), and the sum over n in Eq. (1) is truncated at relative order  $v^4$ . For  $H = J/\psi$ , the sum over n, truncated at order  $v^4$ , involves four  $Q\bar{Q}$  LDMEs:  $\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle$ ,  $\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle$ ,  $\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle$ , and  $\langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]})\rangle$ , where the expressions in parentheses give the color state of the  $Q\bar{Q}$  pair (singlet or octet) and the angular-momentum state in spectroscopic notation. The color-singlet LDME  $\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle$  can be measured in lattice QCD, can be determined from potential models, or can be computed from the  $J/\psi$  decay rate into lepton pairs. It is not known how to calculate the color-octet LDMEs from first principles. As a result, they are usually obtained by comparing Eq. (1) to measured cross sections.

SDCs for the  $J/\psi$  production cross sections and polarizations have been computed through next-to-leading order (NLO) in the strong-coupling constant  $\alpha_s$  for hadroproduction [2–7] and photoproduction [8–10]. The LDMEs that are obtained by fitting the resulting cross-section and polarization predictions to the experimental data vary considerably, depending on the data that are used in the fits. It is possible to fit the hadroproduction cross-section data [11,12] and polarization data [13–15] simultaneously [5], but the resulting LDMEs give a prediction for the photoproduction cross section that overshoots the data by factors of 4-6 at the highest value of the quarkonium transverse momentum  $p_T$  at which the cross section has been measured [16]. Alternatively, one can extract the LDMEs by comparing the NLO predictions for the hadroproduction and photoproduction cross sections with the experimental data [6], but the resulting LDMEs lead to predictions of large transverse polarization in hadroproduction at large  $p_T$  that are at odds with the experimental data [6].

Recently, it was found that fragmentation contributions at leading power (LP) in  $p_T$  that go beyond NLO in  $\alpha_s$  are important in  $J/\psi$  hadroproduction [17]. With the inclusion of these contributions it was possible, for the first time, to use the LDMEs that have been extracted from the hadroproduction cross sections alone to obtain predictions for the  $J/\psi$  polarization at large  $p_T$  that are near zero and are in agreement with the experimental data [17]. However, these LDMEs, when combined with the NLO SDCs for the photoproduction cross section, lead to a prediction that also overshoots the HERA data from the H1 Collaboration [18,19] by about a factor of 8 in the highest  $p_T$  bin for which the cross section has been measured.

Motivated by this discrepancy between theory and experiment and by the large LP fragmentation contributions to  $J/\psi$  hadroproduction beyond NLO in  $\alpha_s$ , we compute in this paper LP fragmentation contributions to  $J/\psi$ 

photoproduction. Our approach is based on the method that was described in Ref. [17].

The remainder of this paper is organized as follows. In Sec. II, we briefly review the general method for computing the LP fragmentation contributions that was given in Ref. [17]. Section III contains a description of the details of the computation of the LP fragmentation contribution to photoproduction. In Sec. IV, we present our numerical results. Finally, in Sec. V, we summarize and discuss our results.

### **II. LP FRAGMENTATION**

At large transverse momentum  $p_T$ , the contribution at LP in  $p_T$  to the cross section to produce a  $Q\bar{Q}$  pair in two-body collisions is given by [20,21]

$$d\sigma_{A+B\to Q\bar{Q}(n)+X}^{\text{LP}} = \sum_{i} d\hat{\sigma}_{A+B\to i+X} \otimes D_{i\to Q\bar{Q}(n)}, \quad (2)$$

where the  $d\hat{\sigma}_{A+B\to i+X}$  are the inclusive parton production cross sections (PPCSs) to produce a parton *i*, and the  $D_{i\to Q\bar{Q}(n)}$  are the fragmentation functions (FFs) for a parton *i* to fragment into a  $Q\bar{Q}$  pair in color and angularmomentum state *n*. At the parton level, before convolution with parton distribution functions, the LP contribution to  $d\sigma/dp_T^2$  depends asymptotically on  $p_T$  as  $1/p_T^4$ .

In this paper, we consider gluon fragmentation and lightquark fragmentation. The gluon FFs  $D_{g \rightarrow Q\bar{Q}(n)}$  in Eq. (2) are given for the  ${}^{1}S_{0}^{[8]}$  channel at order  $\alpha_{s}^{2}$  (LO) in Refs. [22,23], for the  ${}^{3}S_{1}^{[8]}$  channel at order  $\alpha_{s}$  (LO) in Ref. [24] and at order  $\alpha_s^2$  (NLO) in Refs. [25,26], and for the  ${}^{3}P_{J}^{[8]}$  channels at order  $\alpha_{s}^{2}$  (LO) in Refs. [23,24]. The lightquark FF  $D_{a \to Q\bar{Q}(n)}$  in the  ${}^{3}S_{1}^{[8]}$  channel is given at order  $\alpha_{s}^{2}$ (LO) in Refs. [26-30]. Light-quark fragmentation in the other color-octet channels vanishes through order  $\alpha_s^2$ . In the color-singlet channel, gluon fragmentation occurs at order  $\alpha_s^3$  (LO) [31]. Light-quark fragmentation in the colorsinglet channel vanishes through order  $\alpha_s^2$ , but charm-quark fragmentation occurs at order  $\alpha_s^2$  [29,30]. As we will explain in more detail below, an estimate of the size of the color-singlet LP fragmentation contribution shows that it is negligible in comparison with the overall theoretical uncertainties. Therefore, we will ignore the color-singlet LP fragmentation contributions in our numerical analyses.

The FFs depend on the factorization scale  $\mu_f$ . The dependence on  $\mu_f$  is governed by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation [32–35]. At leading order in  $\alpha_s$ , the DGLAP equation reads

$$\frac{d}{d\log\mu_f^2} \begin{pmatrix} D_S \\ D_g \end{pmatrix} = \frac{\alpha_s(\mu_f)}{2\pi} \begin{pmatrix} P_{qq} & 2n_f P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} D_S \\ D_g \end{pmatrix},$$
(3)

where  $D_g = D_{g \to Q\bar{Q}(n)}$ ,  $D_S = \sum_f [D_{q_f \to Q\bar{Q}(n)} + D_{\bar{q}_f \to Q\bar{Q}(n)}]$ , f is the light-quark or light-antiquark flavor, the  $P_{ij}$  are the splitting functions for the FFs, and  $n_f$  is the number of active light-quark flavors. In order to match what was done in the NLO calculations of Refs. [8,9], we take  $n_f = 3$ . We resum the leading logarithms of  $p_T/m_c$  by solving Eq. (3) to evolve the FFs from  $\mu_f = 2m_c$  to  $\mu_f = m_T = \sqrt{p_T^2 + 4m_c^2}$ .

Following Ref. [17], we combine the LO-plus-NLO SDCs with the LP fragmentation contributions according to the formula

$$\frac{d\sigma^{\text{LP+NLO}}}{dp_T} = \frac{d\sigma^{\text{LP}}}{dp_T} - \frac{d\sigma^{\text{LP}}_{\text{NLO}}}{dp_T} + \frac{d\sigma_{\text{NLO}}}{dp_T}.$$
 (4)

Here,  $d\sigma^{\text{LP}}/dp_T$  is the DGLAP-evolved LP fragmentation contribution,  $d\sigma_{\text{NLO}}/dp_T$  is the contribution that arises from the LO-plus-NLO SDCs, and  $d\sigma^{\text{LP}}_{\text{NLO}}/dp_T$  is the contribution that is contained in both  $d\sigma^{\text{LP}}/dp_T$  and  $d\sigma_{\text{NLO}}/dp_T$ .

## **III. COMPUTATION OF LP SDCS**

In photoproduction at HERA, the incoming electron or positron emits a virtual photon that is nearly on mass shell and that subsequently interacts with the incoming proton. There are two types of photon-induced processes that contribute to photoproduction cross sections. The first is the *direct* process, in which the virtual photon interacts with a parton in the proton electromagnetically. The second is the *resolved* process, in which the virtual photon emits a parton, which then interacts strongly with a parton in the proton. The probability for the photon to emit a parton is given by the parton distribution function (PDF) of the photon.

We compute the PPCSs  $d\hat{\sigma}_{e+p \to i+X}$  for direct and resolved photoproduction to NLO accuracy in  $\alpha_s$  by making use of the EPHOX Fortran code [36–39]. We use the AFG04\_BF photon PDFs and the CTEQ6M proton PDFs at scale  $m_T$ . We carry out the computation using the same kinematics and cuts as were used by the H1 Collaboration in their most recent cross-section measurements [19]. The center-of-momentum energy of the ep system is  $\sqrt{s} = 319$  GeV. The cuts on the  $\gamma p$ -invariant mass  $W = \sqrt{(p_{\gamma} + p_p)^2}$  and elasticity  $z = p_{J/\psi} \cdot p_p / p_{\gamma} \cdot p_p$  are given by 60 GeV < W < 240 GeV and 0.3 < z < 0.9. Here,  $p_{\gamma}$ ,  $p_{p}$ , and  $p_{J/\psi}$  are the momenta of the photon, proton, and  $J/\psi$ , respectively. There is also a cut on the invariant mass of the virtual photon  $Q^2$ , which is  $Q^2 < Q_{\text{max}}^2 = 2.5 \text{ GeV}^2$ . The photon flux is calculated in EPHOX by making use of the Weizsäcker-Williams formula for quasireal photons:

$$f_{\gamma/e}(x) = \frac{\alpha}{2\pi} \left[ \frac{1 + (1-x)^2}{x} \log \frac{Q_{\max}^2(1-x)}{m_e^2 x^2} - \frac{2(1-x)}{x} \right].$$
(5)

Here,  $x = E_{\gamma}/E_e$ , where  $E_{\gamma}$  and  $E_e$  are the energy of the photon and the electron, respectively;  $\alpha$  is the quantum electrodynamics coupling constant; and  $m_e$  is the electron mass.

#### **IV. NUMERICAL RESULTS**

In Fig. 1, we compare our results for  $d\sigma_{\text{NLO}}^{\text{LP}}/dp_T$  for the direct process with the NLO calculation for the direct process. In Fig. 2, we make a similar comparison for the resolved process. Here, and throughout the remainder of this paper, we make use of the SDCs through NLO that were computed in Refs. [8,9,16].

For the direct process, LP fragmentation has a sizable contribution only for the  ${}^{3}S_{1}^{[8]}$  channel. In the  ${}^{1}S_{0}^{[8]}$  channel,  $d\sigma_{\rm NLO}^{\rm LP}/dp_{T}$  is very small in comparison to  $d\sigma_{\rm NLO}/dp_{T}$ 



FIG. 1 (color online). The ratio  $(d\sigma_{\rm NLO}^{\rm LP}/dp_T)/(d\sigma_{\rm NLO}/dp_T)$  for the  ${}^{1}S_{0}^{[8]}$ ,  ${}^{3}P_{J}^{[8]}$ , and  ${}^{3}S_{1}^{[8]}$  channels in the direct process  $\gamma p \rightarrow J/\psi + X$ .



FIG. 2 (color online). The ratio  $(d\sigma_{\rm NLO}^{\rm LP}/dp_T)/(d\sigma_{\rm NLO}/dp_T)$  for the  ${}^{1}S_{0}^{[8]}$ ,  ${}^{3}P_{J}^{[8]}$ , and  ${}^{3}S_{1}^{[8]}$  channels in the resolved process  $\gamma p \rightarrow J/\psi + X$ .

because the PPCSs involve only light-quark-initiated processes, which have much smaller partonic fluxes than do gluon-initiated processes, and because the FFs contain no distributions that emphasize the region near z = 1. In the  ${}^{3}P_{J}^{[8]}$  channels,  $d\sigma_{\text{NLO}}^{\text{LP}}/dp_{T}$  is also small in comparison to  $d\sigma_{\text{NLO}}/dp_{T}$  (less than 1% for  $p_{T} \leq 10$  GeV) because the PPCSs involve only light-quark-initiated processes. In the  ${}^{3}P_{J}^{[8]}$  channels,  $d\sigma_{\rm NLO}^{\rm LP}/dp_{T}$  is opposite in sign to the non-LP part of  $d\sigma_{\rm NLO}/dp_T$  and, as expected, grows in magnitude relative to the non-LP part of  $d\sigma_{\rm NLO}/dp_T$  as  $p_T$ increases. However, even at  $p_T = 10$  GeV,  $d\sigma_{\text{NLO}}^{\text{LP}}/dp_T$  is much smaller than the non-LP part of  $d\sigma_{\rm NLO}/dp_T$ . Consequently, the ratio  $(d\sigma_{\rm NLO}^{\rm LP}/dp_T)/(d\sigma_{\rm NLO}/dp_T)$  becomes increasingly negative as  $p_T$  increases. At very large values of  $p_T$ , at which the magnitude of  $d\sigma_{\rm NLO}^{\rm LP}/dp_T$  becomes comparable to or larger than the magnitude of the non-LP part of  $d\sigma_{\rm NLO}/dp_T$ , we would expect this ratio to change sign discontinuously and to approach unity. We do not show results for the  ${}^{3}S_{1}^{[1]}$  channel. We defer the discussion of that channel until we discuss the sum of the direct and resolved contributions to the cross section.

For the resolved process, LP fragmentation in the  ${}^{3}S_{1}^{[8]}$  and  ${}^{3}P_{J}^{[8]}$  channels approaches the NLO calculation as  $p_{T}$  rises. In the  ${}^{1}S_{0}^{[8]}$  channel, the LP fragmentation contribution is small because, unlike the FFs for the  ${}^{3}S_{1}^{[8]}$  and  ${}^{3}P_{J}^{[8]}$  channels, the FF for the  ${}^{1}S_{0}^{[8]}$  channel does not involve distributions that emphasize the region near z = 1. Again, we defer the discussion of the  ${}^{3}S_{1}^{[1]}$  channel until we discuss the sum of the direct and resolved contributions to the cross section.

In Figs. 3 and 4, we compare  $d\sigma^{\text{LP+NLO}}/dp_T$  with  $d\sigma_{\text{NLO}}/dp_T$  in each channel for the direct and the resolved process, respectively. For the direct process,  $d\sigma^{\text{LP+NLO}}/dp_T$  is larger than  $d\sigma_{\text{NLO}}/dp_T$  in the  ${}^{3}S_{1}^{[8]}$ 



FIG. 3 (color online). The ratio  $(d\sigma^{\text{LP+NLO}}/dp_T)/(d\sigma_{\text{NLO}}/dp_T)$  for the  ${}^{1}S_{0}^{[8]}$ ,  ${}^{3}P_{J}^{[8]}$ , and  ${}^{3}S_{1}^{[8]}$  channels in the direct process  $\gamma p \rightarrow J/\psi + X$ .



FIG. 4 (color online). The ratio  $(d\sigma^{\text{LP+NLO}}/dp_T)/(d\sigma_{\text{NLO}}/dp_T)$  for the  ${}^{1}S_{0}^{[8]}$ ,  ${}^{3}P_{J}^{[8]}$ , and  ${}^{3}S_{1}^{[8]}$  channels in the resolved process  $\gamma p \rightarrow J/\psi + X$ .

channel, while in the  ${}^{3}P_{J}^{[8]}$  and  ${}^{1}S_{0}^{[8]}$  channels, the differences between  $d\sigma^{\text{LP+NLO}}/dp_{T}$  and  $d\sigma_{\text{NLO}}/dp_{T}$  are less than 1% and less than 0.02%, respectively. For the resolved process, the difference between  $d\sigma^{\text{LP+NLO}}/dp_{T}$  and  $d\sigma_{\text{NLO}}/dp_{T}$  is substantial in the  ${}^{3}S_{1}^{[8]}$  and  ${}^{3}P_{J}^{[8]}$  channels. In the  ${}^{1}S_{0}^{[8]}$  channel,  $d\sigma^{\text{LP+NLO}}/dp_{T}$  is larger than  $d\sigma_{\text{NLO}}/dp_{T}$  by only 7% at  $p_{T} = 7$  GeV and by only 10% at  $p_{T} = 10$  GeV.

In Fig. 5, we compare, for each channel, the sum of the direct and resolved contributions to  $d\sigma^{\text{LP+NLO}}/dp_T$  with the sum of the direct and resolved contributions to  $d\sigma_{\text{NLO}}/dp_T$ . In the  ${}^3S_1^{[8]}$  channel,  $d\sigma^{\text{LP+NLO}}/dp_T$  is smaller than  $d\sigma_{\text{NLO}}/dp_T$  by 40% for  $p_T \leq 10$  GeV. On the other hand, in the  ${}^1S_0^{[8]}$  and  ${}^3P_J^{[8]}$  channels, the differences between  $d\sigma^{\text{LP+NLO}}/dp_T$  and  $d\sigma_{\text{NLO}}/dp_T$  are negligible. The reason for this is that, owing to the experimental kinematic constraints on W and z, the direct process dominates over the resolved process in these channels, and the differences between  $d\sigma^{\text{LP+NLO}}/dp_T$  and  $d\sigma_{\text{NLO}}/dp_T$  are small in direct production in these channels [16]. We do not

show the contribution of the  ${}^{3}S_{1}^{[1]}$  channel. We have made a rough estimate of the size of the LP fragmentation correction in that channel by making use of the gluon and charm-quark FFs at LO in  $\alpha_s$ . We estimate that, in the  ${}^{3}S_{1}^{[1]}$  channel, the LP fragmentation contributions for 9 GeV  $\leq p_T \leq 10$  GeV are less than 5% of the sum of the color-singlet direct and resolved contributions.

Finally, we can examine the effect of the additional LP fragmentation contributions on the complete cross section. As we have seen, the additional LP fragmentation contribution is a sizable fraction of the rate only for the  ${}^{3}S_{1}^{[8]}$ channel. However, the SDC through NLO for the  ${}^{3}S_{1}^{[8]}$ channel is much smaller than the SDCs through NLO for the  ${}^{1}S_{0}^{[8]}$  and  ${}^{3}P_{J}^{[8]}$  channels. Hence, the correction from the additional LP fragmentation contributions to the  $J/\psi$ photoproduction cross section is very small. For example, if we use the color-octet LDMEs that were obtained in Ref. [17] by fitting the LP + NLO SDCs to CMS [12] and CDF data [11] and if we use the color-singlet LDME that was obtained in Ref. [40] by making use of the  $J/\psi$ leptonic decay-rate data, then we find that the difference between  $d\sigma^{\text{LP+NLO}}/dp_T$  and  $d\sigma_{\text{NLO}}/dp_T$  for  $J/\psi$  photoproduction is less than 1% for  $p_T$  between 7 GeV and 10 GeV. If we use the LDMEs from Ref. [16], then the difference between  $d\sigma^{\text{LP+NLO}}/dp_T$  and  $d\sigma_{\text{NLO}}/dp_T$  is less than 3% for the same  $p_T$  range.

In Fig. 6, we compare the H1 data [18,19] for the  $J/\psi$  photoproduction cross section with the LP + NLO prediction that results from the use of the color-octet LDMEs of Ref. [17] and the color-singlet LDME of Ref. [40]. The uncertainty band in the LP + NLO prediction comes from the uncertainties in the LDMEs, combined in quadrature with the uncertainties in the SDCs that were obtained in Ref. [16] by varying the factorization scale  $\mu_f$  between  $2m_T$  and  $m_T/2$ . As we have already mentioned, the additional LP



FIG. 5 (color online). The ratio  $(d\sigma^{\text{LP+NLO}}/dp_T)/(d\sigma_{\text{NLO}}/dp_T)$  for the  ${}^{1}S_{0}^{[8]}$ ,  ${}^{3}P_{J}^{[8]}$ , and  ${}^{3}S_{1}^{[8]}$  channels in the sum of the direct and resolved processes  $\gamma p \rightarrow J/\psi + X$ .



FIG. 6 (color online). LP + NLO predictions for the  $J/\psi$  differential cross section at HERA compared with the H1 data [18,19].

fragmentation contributions do not significantly change the prediction from that of the NLO calculation. Consequently, the LP + NLO prediction overshoots the data by about a factor of 8 at the highest value of  $p_T$  at which the photoproduction cross section has been measured.

In order to suppress next-to-leading-power (NLP) contributions, we have compared the theoretical predictions with data only for values of  $p_T$  that are considerably larger than  $m_{J/\psi}$ . In Ref. [41], it was suggested that a criterion  $p_T > 3m_{J/\psi}$  be used in comparing data with theory. In Ref. [17], a criterion  $p_T \ge 10$  GeV was used in fitting the predictions to the data. The highest value of  $p_T$  at which the photoproduction cross section has been measured falls slightly short of these criteria. It is possible that NLP corrections and/or power-suppressed violations of NRQCD factorization could account for some of the differences between the LP + NLO prediction and the experimental data. However, the data and the prediction do not seem to be trending toward each other as  $p_T$  increases.

The color-octet LDMEs of Ref. [17] were extracted from the prompt  $J/\psi$  production data, which include feeddown from the  $\chi_{cJ}$  and  $\psi(2S)$  states. We would expect the corrections to the LDMEs from the removal of the feeddown contributions to prompt hadroproduction to be of order -30%[42], and we would expect the corrections from the inclusion of feeddown contributions to prompt photoproduction to be of order +15% [19]. This still leaves a substantial discrepancy between the data and theoretical predictions.

## V. SUMMARY AND DISCUSSION

In this paper, we have computed additional leading-power (LP) fragmentation contributions to  $J/\psi$  photoproduction at HERA that go beyond the existing fixed-order calculations through NLO in  $\alpha_s$ . Our computation has made use of parton production cross sections through NLO in  $\alpha_s$ , as implemented in EPHOX, for both direct and resolved photoproduction. We have included gluon and light-quark fragmentation processes and have used the current stateof-the-art fragmentation functions for the various color-octet  $Q\bar{Q}$  channels. For gluon fragmentation, the fragmentation function for the  ${}^{3}S_{1}^{[8]}$  channel is available through NLO in  $\alpha_{s}$ , and the fragmentation functions for the  ${}^{1}S_{0}^{[8]}$  and  ${}^{3}P_{J}^{[8]}$  channels are available at LO in  $\alpha_{s}$ . For light-quark fragmentation, only the LO fragmentation function for the  ${}^3S_{\scriptscriptstyle 1}^{[8]}$ channel is available. In addition to making use of the available fixed-order fragmentation functions, we have resummed leading logarithms of  $p_T^2/m_c^2$  to all orders in  $\alpha_s$ for the channels in which fragmentation functions are available. We have estimated the LP fragmentation corrections in the  ${}^{3}S_{1}^{[1]}$  channel by making use of the gluon and charm-quark fragmentation functions at LO in  $\alpha_s$ .

We find that the additional LP fragmentation contributions are important, relative to the fixed-order contributions through NLO in  $\alpha_s$ , only in the  ${}^{3}S_{1}^{[8]}$  channel in direct production and only in the  ${}^{3}P_{J}^{[8]}$  channel in resolved production. However, the fixed-order contributions from direct production in the  ${}^{3}S_{1}^{[8]}$  channel are themselves small, and the contributions from resolved production are small in the  ${}^{3}P_{J}^{[8]}$  channel (and the  ${}^{1}S_{0}^{[8]}$ channel) for the kinematics and cuts of the H1 crosssection measurement. Furthermore, all of the additional LP fragmentation contributions in the  ${}^{3}S_{1}^{[1]}$  channel are negligible compared to the theoretical uncertainty in the sum of the contributions of all of the channels. Consequently, the additional LP fragmentation contributions that we have computed have little effect on the cross-section prediction.

If one predicts the photoproduction cross section by using NRQCD long-distance matrix elements that are consistent with both the hadroproduction cross-section and polarization data, then there is a sizable discrepancy between theory and experiment at the highest values of  $p_T$ at which the photoproduction cross section has been measured. If NRQCD factorization is correct, then one would expect it to hold at values of  $p_T$  that are considerably larger than  $m_{J/\psi}$ . As we have mentioned, the highest value of  $p_T$  at which the photoproduction cross section has been measured is not very large in comparison with  $m_{J/\psi}$ . Hence, it is possible that NLP corrections and/or powersuppressed violations of NRQCD factorization could account for some of the discrepancy between theory and the experimental data. However, the shapes of the data and the LP + NLO prediction versus  $p_T$  do not suggest a resolution of the discrepancy at larger values of  $p_T$ . Hence, the discrepancy between theory and experiment in photoproduction of the  $J/\psi$  seems to challenge the validity of NROCD factorization.

The calculations in this paper include some, but not all, of the nonlogarithmic LP fragmentation contributions at next-to-next-to-leading order (NNLO) in  $\alpha_s$  (order  $\alpha_s^5$ ). A complete calculation of the nonlogarithmic LP fragmentation contributions through NNLO in  $\alpha_s$  would require the calculation of additional QCD corrections to fragmentation functions and parton production cross sections. In the case of gluon fragmentation, which dominates in hadroproduction, a complete NNLO calculation in the  ${}^{1}S_{0}^{[8]}$  and  ${}^{3}P_{J}^{[8]}$ channels would require the use of fragmentation functions through NLO and parton production cross sections through NLO, and a complete calculation in the  ${}^{3}S_{1}^{[8]}$  channel would require the use of the fragmentation function through NNLO and parton production cross sections through NNLO. In the case of light-quark fragmentation, a complete NNLO calculation in the  ${}^{1}S_{0}^{[8]}$  and  ${}^{3}P_{J}^{[8]}$  channels would require the use of fragmentation functions through NLO and the LO parton production cross sections, and a complete calculation in the  ${}^{3}S_{1}^{[8]}$  channel would require the use of the fragmentation function through NLO and parton

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production cross sections through NLO. As we have mentioned, the fragmentation functions are all known only at LO, except in the case of gluon fragmentation in the  ${}^{3}S_{1}^{[8]}$  channel, in which case the fragmentation function is known through NLO. Parton production cross sections are publicly available through NLO.

A complete NNLO calculation of the LP fragmentation contributions might be important for hadroproduction cross sections and polarizations, and, hence, could affect the extractions of the long-distance matrix elements from the hadroproduction data. However, given the small sizes of the additional LP fragmentation contributions that we have found in this paper, it seems unlikely that these further LP fragmentation contributions would be important for photoproduction.

## ACKNOWLEDGMENTS

We thank Mathias Butenschön and Bernd Kniehl for supplying detailed numerical results from their NLO calculations of the photoproduction SDCs. We also thank Michel Fontannaz for advice regarding the use of the EPHOX code and Michel Fontannaz and Jean-Philippe Guillet for providing a version of the EPHOX code that contains the AFG04 photon parton distributions. The work of G.T.B. and H.S.C. is supported by the U.S. Department of Energy, Division of High Energy Physics, under Contract No. DE-AC02-06CH11357. The work of U-R. K. is supported by the National Research Foundation of Korea under Contract No. NRF-2012R1A1A2008983. J. L. and U-R. K. thank APCTP for its hospitality through the CAT program. The submitted manuscript has been created in part by UChicago Argonne, LLC, Operator of Argonne National Laboratory. Argonne, a U.S. Department of Energy Office of Science laboratory, is operated under Contract No. DE-AC02-06CH11357. The U.S. Government retains for itself, and others acting on its behalf, a paid-up nonexclusive, irrevocable worldwide license in said article to reproduce, prepare derivative works, distribute copies to the public, and perform publicly and display publicly, by or on behalf of the Government.

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