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$J = 0$ fixed pole and D-term form factor in deeply virtual Compton scattering

D. Mueller[1] and K. M. Semenov-Tian-Shansky[2,3]

[1]*Department of Physics, University of Cape Town, Private Bag X3 7701 Rondebosch, South Africa*
[2]*IFPA, département AGO, Université de Liége, BE-4000 Liége, Belgium*
[3]*CPhT, École Polytechnique, CNRS, FR-91128 Palaiseau, France*

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Brodsky, Llanes-Estrada, and Szczepaniak emphasized the importance of the $J = 0$ fixed-pole manifestation in real and (deeply) virtual Compton scattering measurements and argued that the $J = 0$ fixed pole is universal, i.e., independent on the photon virtualities [Phys. Rev. D 79, 033012 (2009)]. In this paper we review the $J = 0$ fixed-pole issue in deeply virtual Compton scattering. We employ the dispersive approach to derive the sum rule that connects the $J = 0$ fixed-pole contribution and the subtraction constant, called the D-term form factor for deeply virtual Compton scattering. We show that in the Bjorken limit the $J = 0$ fixed-pole universality hypothesis is equivalent to the conjecture that the D-term form factor is given by the inverse moment sum rule for the Compton form factor. This implies that the D-term is an inherent part of the corresponding generalized parton distribution (GPD). Any supplementary D-term added to a GPD results in an additional $J = 0$ fixed-pole contribution and implies the violation of the universality hypothesis. We argue that there exists no theoretical proof for the $J = 0$ fixed-pole universality conjecture.

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I. INTRODUCTION

Compton scattering off a nucleon

$$\gamma^{(*)}(q_1) + N(p_1) \to \gamma^{(*)}(q_2) + N(p_2) \tag{1.1}$$

with real photons ($q_1^2 = q_2^2 = 0$), with one virtual space-like and one real photon ($q_1^2 = -Q_1^2 < 0,\ q_2^2 = 0$), and with two virtual space-like photons ($q_1^2 = -Q_1^2 < 0,\ q_2^2 = -Q_2^2 < 0$) are important processes to address the internal structure of nucleons from the low-energy to the high-energy regime. Depending on the resolution scale which is set up experimentally, different theoretical frameworks are appropriate to analyze experimental data and to provide interpretation in terms of the proper degrees of freedom.

Experimental and theoretical investigations of real Compton scattering date back to the pre-QCD era and were mainly based on the dispersive approach and the SO(3) partial-wave (PW) expansion in terms of the cross-channel angular momentum J. In particular, in the high-energy region Regge theory was extensively employed. This implies that the high-energy asymptotic behavior $s^{\alpha(t)}$ of the amplitude is determined by the (leading) Regge trajectory $\alpha(t)$, which depends on the momentum transfer squared $t = (p_2 - p_1)^2$. It is assumed that the corresponding partial-wave amplitudes are analytic functions of J. Leading Regge behavior then originates from moving poles in the complex J plane. Besides such moving poles there also might exist so-called *fixed-pole* singularities (see e.g. Chapter I of Ref. [1]) which

(i) do not move with the change of t, and
(ii) cannot be revealed by means of the analytic continuation in J.

A $J = J_0$ fixed-pole singularity may arise from a cross-channel exchange with a non-Reggeized (elementary) particle of spin J_0 in the cross channel (or from a contact interaction term). It is then manifest as the Kronecker-δ singularity in the complex J plane. Its t-channel quantum numbers might be exemplified e.g. by means of the Froissart-Gribov projection [2,3].

To our best knowledge, a $J = 0$ fixed pole in the context of Compton scattering off a proton first arose in Ref. [4] by Creutz, Drell, and Pashos as a constant, denoted here as C_∞, in the Regge-pole representation of the real forward Compton scattering amplitude

$$f_1(\nu) = \sum_{\substack{\alpha\neq 0\\ \alpha\leq 1}} \frac{\beta_\alpha \nu^\alpha}{4\pi} \frac{-1 - e^{-i\pi\alpha}}{\sin(\pi\alpha)} + C_\infty, \tag{1.2}$$

where the energy variable is $\nu = \frac{s-u}{4M}$. The representation (1.2) is supposed to be valid for the high-energy region, while for low energy the Compton amplitude $f_1(\nu)$ is known to satisfy the Thomson limit

$$f_1(0) = \lim_{\nu\to 0} f_1(\nu) = -\frac{e_p^2}{4\pi M}, \tag{1.3}$$

where e_p is the electric charge and M is the proton mass. Therefore, in a loose sense, the value of C_∞ in Eq. (1.2) characterizes how much from the Thomson limit (1.3) survives in the high-energy regime. In Ref. [5], employing the subtracted dispersion relation of Gell-Mann, Goldberger, and Thirring [6], this was equivalently formulated in a more abstractly mathematical manner as the $J=0$ *fixed-pole sum rule* expressed in terms of an analytically regularized inverse moment

$$C_\infty = f_1(0) - \frac{2}{\pi}\int_{\nu_{\rm thr}}^{(\infty)} \frac{d\nu}{\nu}{\rm Im} f_1(\nu), \qquad (1.4)$$

where the absorptive part is given by the total photoabsorption cross section ${\rm Im} f_1(\nu) = \frac{\nu}{4\pi}\sigma_T(\nu)$. First attempts [5,7] to extract the value of $J=0$ fixed-pole contribution at $t=0$ from experimental measurements employing finite-energy sum rules based on Eq. (1.4) found its value to be roughly consistent with the Thomson limit.

The manifestation of the $J=0$ fixed-pole contribution for virtual Compton scattering, i.e. of the constant contribution in the high-energy asymptotic limit, was the subject of a broad discussion in the early 1970s. Brodsky, Close and Gunion [8,9] provided field-theoretical arguments in favor of such a contribution originating from a local two-photon interaction corresponding at the diagrammatic level to the so-called "seagull" diagrams. A. Zee in Ref. [10] argued that the $J=0$ fixed pole is an inherent consequence of the scaling behavior of the Compton amplitude in the Bjorken limit. However, this reasoning was criticized by Creutz [11], who disclaimed the existence of any theoretical argument in favor of such a singularity independent of specific models.

The importance of a $J=0$ fixed-pole contribution has been emphasized more recently by Brodsky, Llanes-Estrada and Szczepaniak [12]. They argued that this contribution possesses unique features that are absent in amplitudes of other processes such as meson production:

(i) The $J=0$ fixed-pole contribution is a t-dependent constant that is *independent* of the photon virtualities and is therefore universal.

(ii) In the parton model its value is given by the inverse moment of the corresponding t-dependent parton distribution function (PDF).

On the other hand, within the partonic picture, the subtraction constant, which appears in the transverse nonflip deeply virtual Compton scattering (DVCS) amplitude, originates from the so-called D-term. Originally, the D-term was introduced in Ref. [13] as a separate addendum to a generalized parton distribution (GPD) that complements the polynomiality condition for the unpolarized charge-even GPD $H^{(+)}$ within the double distribution representation [14,15]. The existence of the D-term has also been justified from chiral dynamics. The first Mellin moment of the D-term contributes to the hadronic matrix elements of both the quark and gluon parts of the QCD energy-momentum tensor. The negative value of this specific moment has been argued to be a necessity for the stability of the nucleon [16]. It was realized that the D-term can be implemented as an inherent part of the GPD within the modified double distribution representation [17–19]. The D-term also turns out to be a natural GPD ingredient within the GPD representation based on the double partial-wave expansion [in conformal and in the cross-channel SO(3) partial waves]. This representation is known in two versions (the approach based on the Mellin-Barnes integral techniques of Ref. [20], and the so-called dual parametrization approach [21–23]) that were recently found to be completely equivalent [24]. Within this approach it was first realized that the problem of universality of a $J=0$ fixed pole is related to the analytic properties of GPD moments in the complex conformal spin j. The analyticity assumption requiring the absence of a $j=-1$ fixed-pole singularity in the Mellin space of spectral functions allows one to express the subtraction constant through the analytically regularized inverse moment sum rule and turns out to be equivalent to the $J=0$ fixed-pole conjecture of Ref. [12].

In this paper we restrict ourselves to Compton scattering in the generalized Bjorken limit and provide a pedagogical presentation of the issue of the $J=0$ fixed-pole conjecture and the D-term representation. In Sec. II we review the derivation of fixed-t dispersion relations for the Compton amplitude. We introduce a pair of equivalent dispersion relations: the standard subtracted one and the analytically regularized one. This provides the $J=0$ fixed-pole sum rule in terms of the analytically regularized inverse moment. In Sec. III we employ these findings within the parton model to express the corresponding sum rule in terms of GPDs. We discuss the mathematical subtleties in taking the high-energy limit of the D-term sum rule. In Sec. IV we show that the $J=0$ fixed-pole conjecture holds true if the D-term is an inherent part of the GPD. This statement is illustrated with a toy GPD model example in the Appendix. Finally, in Sec. V we draw our conclusions.

II. DISPERSION APPROACH FOR COMPTON SCATTERING

A. Subtracted and unsubtracted dispersion relations for the Compton amplitude

To parametrize the photon helicity amplitudes of Compton scattering (1.1) we adopt the notations and conventions of Ref. [25]. In particular, the transverse nonflip photon helicity amplitude reads

$$\mathcal{T}_{++} = \bar{u}(p_2)\left[\frac{P}{P\cdot q}\mathcal{H}(\nu,t|Q_1^2,Q_2^2) + i\sigma^{\alpha\beta}\frac{P_\alpha\Delta_\beta}{2MP\cdot q} \times \mathcal{E}(\nu,t|Q_1^2,Q_2^2)\right]u(p_1) + \text{parity odd part}, \qquad (2.1)$$

where $\Delta = p_2 - p_1$ and ν stands for the energy variable

$$\nu = \frac{P \cdot q}{2M} = \frac{s-u}{4M}, \quad \text{with} \quad P = p_1 + p_2,$$
$$q = \frac{1}{2}(q_1 + q_2). \tag{2.2}$$

In what follows we mainly focus on the Compton form factor (CFF) $\mathcal{H}(\nu, t|Q_1^2, Q_2^2)$, the analog of the Dirac form factor, which has even signature (even parity and even charge-conjugation parity), i.e., it is symmetric under the interchange of $\nu \to -\nu$.

(i) In the forward kinematics ($Q_1^2 = Q_2^2 = Q^2,\ t = 0$) its imaginary part corresponds to the deep inelastic scattering structure function F_1

$$\mathrm{Im}\mathcal{H}(\nu, t = 0|Q_1^2 = Q_2^2 = Q^2) = 2\pi F_1(x_\mathrm{B}, Q^2), \tag{2.3}$$

where $x_\mathrm{B} = \frac{Q^2}{2M\nu}$.

(ii) For real Compton scattering it can be expressed through the transverse photoabsorption cross section

$$\mathrm{Im}\mathcal{H}(\nu, t = 0|Q_1^2 = Q_2^2 = 0) = 4\pi M \mathrm{Im} f_1(\nu) = M\nu\sigma_T(\nu). \tag{2.4}$$

The derivation of the fixed-t dispersion relation (DR) for real photons or fixed space-like photon virtualities is based on the Cauchy theorem,

$$\mathcal{H}(\nu, t|Q_1^2, Q_2^2) = \frac{1}{2\pi i}\oint d\nu' \frac{1}{\nu' - \nu}\mathcal{H}(\nu', t|Q_1^2, Q_2^2), \tag{2.5}$$

(see left panel in Fig. 1) and standard assumptions on the analytic structure of the CFF. In the following we concentrate on the Bjorken limit. Therefore, the Born term can be safely neglected and only the cuts along the real axis $[-\infty, -\nu_\mathrm{cut}]$ and $[\nu_\mathrm{cut}, \infty]$, which start at the pion production threshold,

$$\nu_\mathrm{cut} = \frac{Q_1^2 + Q_2^2 + t + (M + 2m_\pi)^2 - M^2}{4M}$$

are to be accounted for. Deforming the integration contour in Eq. (2.5) as shown in the right panel of Fig. 1, and assuming that $\mathcal{H}$ vanishes at infinity ($\lim_{|\nu|\to\infty}\mathcal{H}(\nu, t|Q_1^2, Q_2^2) = 0$), we work out the unsubtracted DR in the standard form,

$$\mathcal{H}(\nu, t|Q_1^2, Q_2^2) = \frac{1}{\pi}\int_{\nu_\mathrm{cut}}^{\infty} d\nu' \frac{2\nu' \mathrm{Im}\mathcal{H}(\nu', t|Q_1^2, Q_2^2)}{\nu'^2 - \nu^2 - i\epsilon}. \tag{2.6}$$

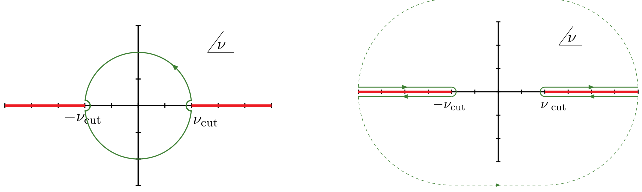

FIG. 1 (color online). Left panel: Integration contour in the complex ν plane in Eq. (2.5). Right panel: Deformation of the integration contour in Eq. (2.6).

If $\mathcal{H}$ does not vanish at infinity, the unsubtracted DR (2.5) still formally provides the correct result once the contributions from the large semicircles are retained. However, it is practically of little use, since both the dispersive integral along the cuts and the contribution from the large semicircles are divergent. Therefore, if one prefers to work with the unsubtracted DR, e.g., as done in Ref. [12], it is indispensable to specify a regularization procedure at the point $\nu = \infty$.

A possible choice, which was already briefly discussed in the Introduction, is the analytic regularization. Here, the integration contour of the dispersive integral is deformed in a way that the integral along the real axis is replaced by the loop integral in the complex plane that includes the point $\nu = \infty$, denoted as (∞); for details see, e.g., Ref. [26]. The unsubtracted DR (2.5) then reads

$$\mathcal{H}(\nu, t|Q_1^2, Q_2^2) = \mathcal{H}_\infty(t|Q_1^2, Q_2^2) + \frac{1}{\pi}\int_{\nu_\mathrm{cut}}^{(\infty)} d\nu' \frac{2\nu' \mathrm{Im}\mathcal{H}(\nu', t|Q_1^2, Q_2^2)}{\nu'^2 - \nu^2 - i\epsilon}, \tag{2.7}$$

where the constant $\mathcal{H}_\infty$, arising from the analytic regularization at $\nu = \infty$, turns out to be the analog of C_∞ in the expansion of the real forward Compton scattering amplitude (1.2). Within the Regge-pole expansion of the amplitude it is interpreted as the $J = 0$ fixed-pole contribution.

However, the analytically regularized DRs can be employed only once the analytic form of the spectral function is explicitly known. Therefore, the conventional form of the DR employed within the deeply virtual (d.v.) regime is the subtracted DR with the subtraction taken at the unphysical point $\nu = 0$:

$$\mathcal{H}(\nu, t|Q_1^2, Q_2^2) \overset{\text{d.v.}}{=} \mathcal{H}_0(t|Q_1^2, Q_2^2) + \frac{1}{\pi}\int_{\nu_\mathrm{cut}}^{\infty} \frac{d\nu'}{\nu'} \frac{2\nu^2 \mathrm{Im}\mathcal{H}(\nu', t|Q_1^2, Q_2^2)}{\nu'^2 - \nu^2 - i\epsilon}. \tag{2.8}$$

The detailed derivation of Eq. (2.8) is given, e.g., in Sec. 2.2 of Ref. [27].

The dispersion relations (2.7) and (2.8) are supposed to represent the same function. Therefore, the $J = 0$ fixed-pole contribution $\mathcal{H}_\infty$ could be related to the subtraction constant $\mathcal{H}_0$. Plugging the algebraic decomposition

$$\frac{2\nu'}{\nu'^2-\nu^2-i\epsilon}=\frac{1}{\nu'}\frac{2\nu^2}{\nu'^2-\nu^2-i\epsilon}+\frac{2}{\nu'}$$

of the Cauchy kernel into Eq. (2.7) and comparing it with Eq. (2.8), we read off the sum rule

$$\mathcal{H}_\infty(t|Q_1^2,Q_2^2)=\mathcal{H}_0(t|Q_1^2,Q_2^2)-\frac{2}{\pi}\int_{\nu_{\rm cut}}^{(\infty)}\frac{d\nu}{\nu}\mathrm{Im}\mathcal{H}(\nu,t|Q_1^2,Q_2^2), \tag{2.9}$$

expressing the $J=0$ fixed-pole contribution through the subtraction constant and the analytically regularized inverse moment of the absorptive part of the amplitude.

B. Dispersive approach in the scaling regime

In general, the subtraction constant $\mathcal{H}_0(t|Q_1^2,Q_2^2)$ of the DR (2.8) represents an unknown quantity. However, in the deeply virtual regime one can rely on the operator product expansion and formulate the external principle allowing one to fix the value of the subtraction constant from the absorptive part. In particular, within the leading twist-two approximation current conservation ensures that for *equal* photon virtualities the subtraction constant vanishes, $\mathcal{H}_0(t|Q_1^2=Q_2^2=\mathcal{Q}^2)\overset{\rm d.v.}{=}0$ (see Sec. 3.2.2 of Ref. [27] for a detailed discussion), while in the DVCS kinematics the subtraction constant corresponds to the D-term form factor $\mathcal{H}_0(t|Q_1^2=\mathcal{Q}^2,Q_2^2=0)\overset{\rm d.v.}{=}4\mathcal{D}(t)$.

Furthermore, in the framework of the operator product expansion it has been conjectured in Ref. [27] that in the absence of the $\delta_{j,-1}$ Kronecker singularity (also called the $j=-1$ fixed-pole contribution) in the Mellin space of moments of the spectral function, the subtraction constant $\mathcal{H}_0(t|Q_1^2,Q_2^2)$ for nonequal photon virtualities can be evaluated from the analytically regularized inverse moment of the spectral function to leading twist accuracy to any order of perturbation theory. Within the convention used here, Eq. (47) of Ref. [27] reads

$$\mathcal{H}_0(t|Q_1^2,Q_2^2)=\frac{2}{\pi}\int_{\nu_{\rm cut}}^{(\infty)}\frac{d\nu}{\nu}\times\mathrm{Im}[\mathcal{H}(\nu,t|Q_1^2,Q_2^2)-\mathcal{H}(\nu,t|Q_1^2=Q_2^2)], \tag{2.10}$$

where the inverse ν moment is computed by the analytic continuation of ν-Mellin moments.

Plugging this conjectured inverse moment sum rule (2.10) into the expression (2.9) for the $J=0$ fixed-pole contribution, one realizes that the $J=0$ fixed pole is independent of the ratio of photon virtualities and can be calculated from the equal photon virtuality case, yielding the conjecture of Ref. [12]:

$$\mathcal{H}_\infty(t|Q_1^2,Q_2^2)=-\frac{2}{\pi}\int_{\nu_{\rm cut}}^{(\infty)}\frac{d\nu}{\nu}\mathrm{Im}\mathcal{H}(\nu,t|Q_1^2=Q_2^2). \tag{2.11}$$

Within the deeply virtual kinematics regime it is convenient to rewrite the DRs of the previous subsection in terms of scaling variables. A natural choice is to use the Bjorken-like variable ξ and the skewness-related scaling variable η:

$$\xi=\frac{Q^2}{P\cdot q}=\frac{Q^2}{2M\nu};\qquad \eta=-\frac{\Delta\cdot q}{P\cdot q}=-\frac{\Delta\cdot q}{2M\nu}, \tag{2.12}$$

where $Q^2=-q^2\equiv-\frac{(q_1+q_2)^2}{4}$. Here, instead of the scaling variable η, we employ the photon asymmetry parameter

$$\vartheta\equiv\eta/\xi=\frac{q_1^2-q_2^2}{q_1^2+q_2^2}+\mathcal{O}(t/Q^2), \tag{2.13}$$

which does not depend on the energy variable ν [28].

(i) For $t=0$, the $\vartheta=0$ case corresponds to the usual deep inelastic scattering kinematics.

(ii) The case $\vartheta=1$ corresponds to the DVCS kinematics.

Within the scaling variables (2.12) the analytically regularized DR (2.7) and the subtracted one (2.8) read as follows:

$$\mathcal{H}(\xi,t|\vartheta)=\frac{1}{\pi}\int_{(0)}^{1}\frac{d\xi'}{\xi'}\frac{2\xi^2\mathrm{Im}\mathcal{H}(\xi',t|\vartheta)}{\xi^2-\xi'^2-i\epsilon}+\mathcal{H}_\infty(t|\vartheta), \tag{2.14}$$

$$\mathcal{H}(\xi,t|\vartheta)=\frac{1}{\pi}\int_{0}^{1}d\xi'\frac{2\xi'\mathrm{Im}\mathcal{H}(\xi',t|\vartheta)}{\xi^2-\xi'^2-i\epsilon}+\mathcal{H}_0(t|\vartheta). \tag{2.15}$$

Here, the upper integration limit, given by $\xi_{\rm cut}=\frac{Q^2}{2M\nu_{\rm cut}}$, has been set in the (generalized) Bjorken limit to $\xi_{\rm cut}=1$ and the lower integration limit, $\xi=0$, corresponds to $\nu=\infty$. We emphasize that although the spectral function grows with increasing ξ', the analytically regularized DR (2.14) can be evaluated as long as its small-ξ' asymptotic is analytically known. The equivalence of the two DRs (2.14) and (2.15) is ensured by the sum rule (2.9), which now reads

$$\mathcal{H}_\infty(t|\vartheta)=\mathcal{H}_0(t|\vartheta)-\frac{2}{\pi}\int_{(0)}^{1}\frac{d\xi}{\xi}\mathrm{Im}\mathcal{H}(\xi,t|\vartheta). \tag{2.16}$$

III. DISPERSIVE VERSUS PERTURBATIVE QCD APPROACH

In this section, within the GPD framework set up in the familiar momentum fraction representation, we point out the origin of the additional fixed-pole contribution $\Delta\mathcal{H}_\infty$, which eventually violates the $J=0$ fixed-pole universality conjecture (2.11). In this approach, to the leading order (LO) accuracy, the CFF $\mathcal{H}(\xi, t|\vartheta)$ arises from the elementary amplitude

$$\mathcal{H}(\xi, t|\vartheta) \overset{\text{LO}}{=} \int_0^1 dx \frac{2x}{\xi^2 - x^2 - i\epsilon} H^{(+)}(x, \eta = \vartheta\xi, t), \quad (3.1)$$

where $H^{(+)}(x,\eta,t) = H(x,\eta,t) - H(-x,\eta,t)$ stands for the antisymmetric charge-even quark GPD combination. The imaginary part of the CFF is given by the GPD value in the outer region $\xi \geq \eta = \xi\vartheta$ for all allowed values $|\vartheta| \leq 1$,

$$\frac{1}{\pi}\mathrm{Im}\mathcal{H}(\xi, t|\vartheta) \overset{\text{LO}}{=} H^{(+)}(x = \xi, \eta = \xi\vartheta, t). \quad (3.2)$$

Inserting the imaginary part (3.2) into the sum rule (2.16) allows one to express the $J = 0$ fixed-pole contribution $\mathcal{H}_\infty(t|\vartheta)$ to LO accuracy by the GPD in the outer region:

$$\mathcal{H}_\infty(t|\vartheta) \overset{\text{LO}}{=} \mathcal{H}_0(t|\vartheta) - 2\int_{(0)}^1 \frac{dx}{x} H^{(+)}(x, \vartheta x, t). \quad (3.3)$$

Now, by plugging the imaginary part (3.2) into the subtracted DR (2.15) and equating it with the LO convolution formula (3.1) for the CFF, we obtain the GPD sum rule [29], which was originally worked out within the double distribution representation [30,31]:

$$4\mathcal{D}(t|\vartheta) \overset{\text{LO}}{=} \int_0^1 dx \frac{2x}{x^2 - \xi^2}[H^{(+)}(x, \vartheta x, t) - H^{(+)}(x, \vartheta\xi, t)] \quad (3.4)$$

for the D-term form factor. Note that for $\xi \neq 0$ the integrand in Eq. (3.4) has an integrable singularity at $x = \xi$. The spectral function $H^{(+)}(x, \vartheta x, t)$ has a branch point at $x = 0$, while the GPD $H^{(+)}(x, \vartheta\xi, t)$ has a branch point at $x = \vartheta\xi$ and vanishes at $x = 0$ due to antisymmetry in x. The sum rule (3.4) is valid for all values of ξ [32], where the special values $\xi \in \{0, 1, 1/\vartheta, \infty\}$ should be approached in special limiting procedures (see Ref. [29] for a more detailed discussion).

(i) The low-energy limit $\xi \to \infty$ of Eq. (3.4) is rather uncritical: the DR integral drops out and the D-term form factor is given in terms of the D-term $d(x, t)$ that is here defined as a limiting value of the GPD:

$$4\mathcal{D}(t|\vartheta) \overset{\text{LO}}{=} \int_0^1 dx \frac{2x\vartheta^2}{1 - x^2\vartheta^2} d(x, t) \quad \text{with} \quad d(x, t) = \lim_{\xi\to\infty} H^{(+)}(\xi x, \xi, t). \quad (3.5)$$

(ii) Contrarily, the high-energy limit $\xi \to 0$ of Eq. (3.4) requires special attention. At first glance this limit looks tempting to provide a proof for the $J = 0$ fixed-pole conjecture of Ref. [12]. However, we would like to stress that interchanging the integration and limiting procedure can render a wrong result, since a squeezed contribution from the central GPD region might be missed.

Let us consider the popular GPD representation in which the D-term (denoted as $d^{\text{f.p.}}$) is an addendum that completes polynomiality [13]:

$$H^{(+)}(x, \eta, t) = H^{(+)}_{\text{DD}}(x, \eta, t) + \theta(|\eta| - |x|)d^{\text{f.p.}}(x/|\eta|, t), \quad (3.6)$$

where $H^{(+)}_{\text{DD}}$ has the rather common double distribution representation; see below Eq. (4.1) for $a = 0$. In the Nth Mellin moment $\int_{-1}^1 dx x^N H^{(+)}_{\text{DD}}(x, \eta, t)$ of the GPD the highest possible power in η^{N+1} for odd N is missing. We would like to show that the D-term addendum in Eq. (3.6) can be interpreted as the $J = 0$ fixed-pole contribution violating the $J = 0$ fixed-pole sum rule conjectured in Ref. [12]. For simplicity, let us suppose that both the CFF spectral function $H^{(+)}_{\text{DD}}(x, \vartheta x, t)$ and the GPD $H^{(+)}_{\text{DD}}(x, \vartheta\xi, t)$ vanish at $x = 0$, allowing us to interchange safely the integration and $\xi \to 0$ limiting procedure in Eq. (3.4). We plug the GPD (3.6) into the D-term form factor sum rule (3.4), and separate the integration region into the central, $x \in [0, \vartheta\xi]$, and outer, $x \in [\vartheta\xi, 1]$, GPD regions. Then taking the high-energy limit $\xi \to 0$, we find that the corresponding sum rule reads

$$4\mathcal{D}(t|\vartheta) \overset{\text{LO}}{=} 2\int_0^1 \frac{dx}{x}[H^{(+)}_{\text{DD}}(x, \vartheta x, t) - q^{(+)}(x, t)] + 4\mathcal{D}^{\text{f.p.}}(t|\vartheta), \quad (3.7)$$

where $q^{(+)}(x, t) \equiv H^{(+)}_{\text{DD}}(x, 0, t)$ stands for the corresponding t-dependent PDF [we assume that $q^{(+)}(x = 0, t) = 0$ to ensure convergence of the integral] and

$$4\mathcal{D}^{\text{f.p.}}(t|\vartheta) \overset{\text{LO}}{=} \lim_{\xi\to 0}\int_0^{\vartheta\xi} dx \frac{2x}{\xi^2 - x^2} d^{\text{f.p.}}\left(\frac{x}{\vartheta\xi}, t\right) \overset{\text{LO}}{=} \int_0^1 dx \frac{2x\vartheta^2}{1 - \vartheta^2 x^2} d^{\text{f.p.}}(x, t). \quad (3.8)$$

Note that since by construction the $d^{\text{f.p.}}$. term provides the complete contribution to the D-term form factor, the inverse moment of the GPD/PDF combination in Eq. (3.7) vanishes.

Now, inserting the D-term form factor (3.7) into Eq. (3.3), we conclude that in addition to the universal inverse PDF moment the subtraction constant $\mathcal{H}_\infty(t|\vartheta)$ receives an additional nonuniversal contribution from the D-term $d^{\text{f.p.}}(x)$, defined solely within the GPD central region:

$$\mathcal{H}_\infty(t|\vartheta) \overset{\text{LO}}{=} -2\int_0^1 \frac{dx}{x} q^{(+)}(x,t) + \underbrace{4\mathcal{D}^{\text{f.p.}}(t|\vartheta)}_{\Delta\mathcal{H}_\infty(t|\vartheta)}. \quad (3.9)$$

Note that the additional $J=0$ fixed-pole contribution $\Delta\mathcal{H}_\infty(t|\vartheta)$, depends on the photon virtualities and is therefore nonuniversal.

(i) Therefore, we conclude that on general grounds the GPD sum rule (3.4) cannot deliver a proof for the conjecture (2.10) that the subtraction constant (D-term form factor) can be evaluated from an inverse moment of the spectral function and so the $J=0$ fixed-pole (2.11) universality conjecture remains also unproved.

(ii) We also add that the high-energy limit $\xi \to 0$ and integration procedure in the GPD sum rule (3.4) cannot be interchanged in the presence of Regge behavior. Neglecting the central GPD region now implies that one also throws away divergent terms that are needed to render a finite D-term form factor result.

A particular example of a GPD model with a nonzero $j=-1$ fixed-pole contribution is provided by the calculation [33] of pion GPDs in the nonlocal chiral quark model [34]. In this model the universality conjecture (2.11) is not valid due to a supplementary $J=0$ fixed-pole contribution originating from the D-term $d^{\text{f.p.}}(x,t)$, which has to be added to make the GPD satisfy the soft pion theorem [35] fixing pion GPDs in the limit $\eta \to 1$.

Now we are about to spell out the relation between the $J=0$ fixed-pole contribution and the D-term form factor $\mathcal{D}(t|\vartheta)$ making special emphasis on the two kinds of analytical properties relevant for GPDs and associated CFFs:

(i) analyticity of CFFs in the cross-channel angular momentum J;

(ii) analyticity of GPD Gegenbauer/Mellin moments in the variable j, labeling the conformal spin $j+2$ of twist-two quark conformal basis operators [36]

$$\mathcal{O}_j^a = \frac{\Gamma(3/2)\Gamma(1+j)}{2^j\Gamma(3/2+j)}(i\overleftrightarrow{\partial}_+)^j \bar{\psi}\lambda^a\gamma_+ C_j^{3/2}\left(\frac{\overleftrightarrow{D}_+}{\overleftrightarrow{\partial}_+}\right)\psi. \quad (3.10)$$

To deal with J analytical properties of CFFs, following Sec. 6.3 of Ref. [24], it is instructive to consider the Froissart-Gribov projection [2,3] of the cross-channel SO(3) PWs of the CFF $\mathcal{H}(\xi,t|\vartheta)$:

$$a_J(t|\vartheta) \equiv \frac{1}{2}\int_{-1}^1 d(\cos\theta_t) P_J(\cos\theta_t)\mathcal{H}^{(+)}(\cos\theta_t, t|\vartheta), \quad (3.11)$$

where, neglecting the threshold corrections $\sim\sqrt{1-\frac{4M^2}{t}}$,

$$\cos\theta_t = -\frac{1}{\vartheta\xi} + \mathcal{O}(1/Q^2).$$

For $J>0$ PWs the Froissart-Gribov projection provides to LO accuracy

$$a_{J>0}(t|\vartheta) \overset{\text{LO}}{=} 2\int_0^1 dx \frac{\mathcal{Q}_J(1/x)}{x^2} H^{(+)}(x,\vartheta x,t), \quad (3.12)$$

where $\mathcal{Q}_J(1/x)$ stand for the Legendre functions of the second kind. For $J=0$ one obtains

$$a_{J=0}(t|\vartheta) \overset{\text{LO}}{=} 2\int_0^1 dx\left[\frac{\mathcal{Q}_0(1/x)}{x^2} - \frac{1}{x}\right] H^{(+)}(x,\vartheta x,t) + 4\mathcal{D}(t|\vartheta). \quad (3.13)$$

Indeed, as is clearly seen from Eqs. (3.12) and (3.13), the $J=0$ PW $a_{J=0}(t|\vartheta)$ might not be obtained from analytic continuation of $a_{J>0}(t|\vartheta)$ to $J=0$. Therefore, analyticity in the cross-channel angular momentum J turns out to be "spoiled" by the presence of a $J=0$ fixed-pole contribution

$$a_{J=0}^{\text{f.p.}}(t|\vartheta) \overset{\text{LO}}{=} 4\mathcal{D}(t|\vartheta) - 2\int_{(0)}^1 \frac{dx}{x} H^{(+)}(x,\vartheta x,t). \quad (3.14)$$

Since the rhs of Eqs. (3.3) and (3.14) coincide, one immediately recognizes that the constant $\mathcal{H}_\infty = a_{J=0}^{\text{f.p.}}$ is indeed the $J=0$ fixed-pole contribution.

Note, that in the operator product expansion approach, e.g., based on the conformal operator basis [27], the presence of a $J=0$ fixed-pole contribution (3.14) to the CFF $\mathcal{H}$ can be understood from the absence of conformal operators with Lorentz spin $J \equiv j+1 = 0$. Such a $j=-1$ contribution is effectively subtracted from the $J=0$ partial wave; see the $1/x$ moment in the integral of Eq. (3.13). The analogous cancellation appears also in the framework of the dual parametrization of GPDs [24].

As pointed out in Refs. [24,27], the analytic properties in j of GPD Gegenbauer/Mellin moments control the validity of the internal duality principle for GPDs (see also the discussion in Ref. [29]). This principle relies on the underlying Lorentz covariance and establishes the relation between the inner and outer support regions for a GPD. The absence of the $j=-1$ fixed-pole contribution, violating analyticity in j, results in a complete correspondence between the inner and outer GPD support regions. This excludes the possibility to add a supplementary fixed-pole D-term contribution $d^{\text{f.p.}}(x,\eta,t)$, defined solely in the central GPD support region. In its turn, as explained above, the absence of the $j=-1$ fixed-pole D-term contribution leads to the validity of the $J=0$ fixed-pole universality conjecture of Ref. [12] [Eq. (2.11)]:

$$\mathcal{H}_\infty(t|\vartheta) \overset{\rm LO}{=} -2\int_{(0)}^1 \frac{dx}{x} H^{(+)}(x,0,t). \tag{3.15}$$

This statement is further illustrated within the double distribution representation of GPDs in the next section.

Moreover, we would like to emphasize that the inverse PDF moment (3.15) cannot be extracted from the D-term form factor. The corresponding inverse moment is exactly canceled within the GPD sum rule (3.4) for the D-term form factor. This statement is obvious within the framework based on the conformal partial-wave expansion. Indeed, once the operator with the corresponding quantum numbers ($j=-1,\ J=0$) does not appear within the conformal basis (3.10), the inverse PDF moment cannot show up in the final expression for the CFF. Its somewhat artificial separation within the expression for the D-term form factor (as the universal $J=0$ fixed-pole contribution) suggests that it is exactly canceled against the same term coming from the inverse moment of the absorptive part of the amplitude. This issue is illustrated within the dual parametrization framework in Sec. 6.2 of Ref. [24]. In other words, experimental data turn out to be directly sensitive only to a possible additional nonuniversal contribution $\Delta\mathcal{H}_\infty(t|\vartheta)$ to $\mathcal{H}_\infty(t|\vartheta)$ [cf. eq. (3.9)].

IV. $J=0$ FIXED-POLE PROBLEM AND GPD DOUBLE DISTRIBUTION REPRESENTATION

According to the Mellin-space analysis of Ref. [27], a $J=0$ fixed-pole contribution originating from the D-term should be absent if the D-term is the inherent part of a GPD. To illustrate this statement, let us employ the double distribution (DD) representation for the charge-even GPD combination $H^{(+)}(x,\eta)$ (for simplicity we omit the t dependence and still adopt a specific form of the DD representation)

$$H^{(+)}(x,\eta) = \int_0^1 dy \int_{-1+y}^{1-y} dz[(1-ax)\delta(x-y-z\eta) - \{x\to -x\}]h(y,z). \tag{4.1}$$

Here the DD $h(y,z)$ is symmetric in z and antisymmetric in y. The factor $(1-ax)$ is included in a way that for $a=0$ the GPD polynomiality condition is not respected in its complete form (see Ref. [13]), while for $a\neq 0$ polynomiality is complete. In the following we need to restrict the admissible class of functions for the DD $h(y,z)$. We assume that $h(y,z)$ has a "smooth" asymptotic behavior in the limit $y\to 0$; in particular contributions concentrated in $y=0$ [$\sim\delta(y)$ and its derivatives] are absent [37]. In order to employ the analytic regularization prescription for the relevant integrals we need to specify explicitly the analytic behavior of the DD for $y\sim 0$. We assume the usual Regge-like behavior for the DD

$$h(y,z) = \sum_{\alpha>0} y^{-\alpha} h_\alpha(z) + \{\text{terms regular at}\, y\sim 0\} \tag{4.2}$$

with $h_\alpha(z) = h_\alpha(-z)$.

The GPD spectral function (3.2), given by the GPD in the outer region, reads in terms of the DD as

$$H^{(+)}(x,\vartheta x) = (1-ax)\int_{\frac{1-x}{1-\vartheta x}}^{\frac{1-x}{1+\vartheta x}} dz h([1+\vartheta z]x, z). \tag{4.3}$$

For $\vartheta=0$ it reduces to the corresponding (t-dependent) PDF,

$$q^{(+)}(x) = H^{(+)}(x,\vartheta x)|_{\vartheta=0} = (1-ax)\int_{-1+x}^{1-x} dz h(x,z). \tag{4.4}$$

The D-term form factor can be calculated from the limit $\eta\to\infty$ [Eq. (3.5)] in which the y dependence in the δ-function drops out and only the a proportional term survives,

$$4\mathcal{D}(\vartheta) = -a\int_0^1 dy\int_0^{1-y} dz \frac{4z^2\vartheta^2}{1-z^2\vartheta^2} h(y,z). \tag{4.5}$$

First, let us show that the D-term form factor sum rule (3.4) holds true for the DD representation (4.1). Plugging the latter into the rhs of the former, we get

$$\begin{aligned}&\int_0^1 dx \frac{2x}{x^2-\xi^2}[H^{(+)}(x,\vartheta x,t) - H^{(+)}(x,\vartheta\xi,t)] \\ &= \int_0^1 dx\int_0^1 dy\int_{-1+y}^{1-y} dz \frac{2x(1-ax)}{x^2-\xi^2} \\ &\quad\times[\delta(x(1-\vartheta z)-y) - \delta(x-y-z\vartheta\xi)]h(y,z).\end{aligned}$$

Performing the x integration and dropping into the resulting integrand its antisymmetric part in z, which is proportional to $zh(y,z)$,

$$\begin{aligned}&\frac{2\vartheta z h(y,z)}{(y+\xi)^2-(\vartheta\xi z)^2}\left[\xi - \frac{ay(2\xi+y)}{1-\vartheta^2 z^2}\right] - \frac{2a\vartheta^2 z^2}{1-\vartheta^2 z^2} h(y,z) \\ &\Rightarrow -\frac{2az^2\vartheta^2}{1-\vartheta^2 z^2} h(y,z),\end{aligned}$$

we immediately recover the D-term form factor expression (4.5) in terms of the DD.

Next, we calculate the inverse moment of the GPD spectral function in terms of the DD,

$$\int_{(0)}^{1} \frac{dx}{x} H^{(+)}(x, \vartheta x) = \int_{(0)}^{1} dx \int_{0}^{1} dy \int_{-1+y}^{1-y} dz \times \frac{1-ax}{x} \delta(x(1-\vartheta z) - y) h(y,z), \tag{4.6}$$

where the small-x behavior of the GPD spectral function inherits the small-y behavior of the DD. Therefore, we regularize the y integral analytically, which allows us to perform the x integration. This renders a well-defined inverse moment in terms of the DD,

$$\int_{(0)}^{1} dx \frac{2}{x} H^{(+)}(x, \vartheta x) = \int_{(0)}^{1} dy \int_{-1+y}^{1-y} dz \left[\frac{2}{y} - \frac{2a}{1-\vartheta^2 z^2}\right] h(y,z). \tag{4.7}$$

The inverse moment of the DD on the rhs of Eq. (4.7) can be rewritten employing the value of the inverse moment at $\vartheta = 0$, which yields

$$\int_{(0)}^{1} dx \frac{2}{x} H^{(+)}(x, \vartheta x) = \int_{(0)}^{1} dx \frac{2}{x} q^{(+)}(x) - a \int_{0}^{1} dy \int_{0}^{1-y} dz \frac{4\vartheta^2 z^2}{1-\vartheta^2 z^2} h(y,z). \tag{4.8}$$

The second term on the rhs is nothing but the D-term form factor (4.5) and, thus, we conclude that the sum rule (2.10) holds for the GPD (4.1).

Consequently, the $J = 0$ fixed-pole universality conjecture (2.11) [or equivalently Eq. (3.9) with $\mathcal{D}^{\text{f.p.}} = 0$] of Ref. [12] is valid. However, adding a separate D-term contribution $d^{\text{f.p.}}$. to the spectral representation (4.1)

$$H^{(+)}(x, \vartheta x) \to H^{(+)}(x, \vartheta x) + \theta(x \le \eta) d^{\text{f.p.}}(x/\eta) \tag{4.9}$$

leads to the breakdown of the $J = 0$ fixed-pole universality conjecture [see Eq. (3.9)] and results in the fixed-pole contribution to the D-term form factor which cannot be computed from the inverse moment of the GPD spectral function.

V. CONCLUSIONS

In this paper we addressed the $J = 0$ fixed-pole universality conjecture and the related analyticity principle allowing us to fix the subtraction constant in the standard DR for the Compton scattering amplitude from the absorptive part of the amplitude. The latter, formulated within the GPD framework by adopting the operator product expansion, holds true if a $j = -1$ fixed-pole singularity in Mellin space is absent. This turns to be equivalent to the existence of the GPD spectral representation in which the D-term is an inherent part of the GPD. In this paper we reduced ourselves to considering the LO GPD framework, although it was already demonstrated that the result is more general and is valid to all orders of perturbation theory.

In particular, we clarified that the $J = 0$ fixed-pole universality conjecture cannot be proven by merely taking the high-energy limit of the D-term sum rule (3.4). A D-term associated fixed-pole contribution may arise from a supplementary D-term added in the central GPD region. This contribution is overlooked by the naive version of the aforementioned limiting procedure. Generally, it may lead to the breakup of the $J = 0$ fixed-pole universality conjecture (2.11).

Instead, the relation between the $J = 0$ fixed-pole contribution and the D-term form factor only can be viewed as a manifestation of the equivalence between analytic properties of CFFs in the cross-channel angular momentum J and the spectral properties of GPDs. Although the relevant analyticity principle ensuring the validity of the $J = 0$ fixed-pole universality conjecture looks quite appealing, we cannot provide reliable theoretical arguments in its favor. Moreover, examples of field theoretical GPD models for which this analyticity principle is violated are well known in the literature.

Therefore, we confirm our pessimistic conclusion from Ref. [24] that the absence of a D-term-related $J = 0$ fixed pole (or the validity of the $J = 0$ fixed-pole universality conjecture) remains an external assumption, which can probably never be proved theoretically.

In principle one may try to address the $J = 0$ fixed-pole universality conjecture phenomenologically by verifying the GPD sum rule (3.4) for the D-term form factor. This task certainly provides further motivation to build up a unique framework for Compton scattering from the real to the deeply virtual regime, launched in Ref. [25]. However, employing the GPD sum rule for the D-term form factor requires the theoretical extrapolation of experimental measurements into the high-energy asymptotic regime. This might imply a general problem, namely a phenomenological test will be biased by the theory framework and/or the utilized model. Even the first step—the reliable extraction of the D-term form factor from experimental data—represents a considerable challenge (see e.g. Ref. [38]).

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APPENDIX: A TOY GPD EXAMPLE

To illustrate the general reasoning of Sec. IV we consider a simple toy GPD model that arises from the DD

$$h^{\text{toy}}(y,z) = (N/2)y^{-\alpha}, \tag{A1}$$

where $N = M_2 \frac{\Gamma(5-\alpha)}{\Gamma(2-\alpha)[2+(1-a)(2-\alpha)]}$ is the convenient overall normalization factor expressed in terms of the averaged parton momentum fraction M_2; see below Eq. (A4). We take x and η to be positive and restrict [39] ourselves to the case $\alpha < 1$. For illustration we ambiguously add to the spectral representation (4.1) a supplementary D-term contribution $d^{\text{f.p.}}(x)$, which vanishes at the boundaries $d^{\text{f.p.}}(z=0) = d^{\text{f.p.}}(z=1) = 0$.

The GPD is calculated from the DD representation (4.1) and (4.9)

$$\begin{aligned} H^{\text{toy}}(x,\eta|\alpha) = \frac{N}{2(1-\alpha)\eta}\Bigg\{&\theta(x\le\eta)\left[(1-ax)\left(\frac{x+\eta}{1+\eta}\right)^{1-\alpha} - (1+ax)\left(\frac{\eta-x}{1+\eta}\right)^{1-\alpha}\right] \\ &+ \theta(x\ge\eta)(1-ax)\left[\left(\frac{x+\eta}{1+\eta}\right)^{1-\alpha} - \left(\frac{x-\eta}{1-\eta}\right)^{1-\alpha}\right]\Bigg\} + \theta(x\le\eta)d^{\text{f.p.}}(x/\eta). \end{aligned} \tag{A2}$$

For $\eta > 0$ the GPD vanishes at $x = 0$ and has branch points at $x = \eta$ and $x = 1$.

(i) For $a \neq 0$ the polynomiality condition is implemented in its full form irrespective of the absence or presence of the fixed-pole contribution.

(ii) For $a = 0$ the highest possible power of η for a given Mellin moment of the GPD entirely arises from $d^{\text{f.p.}}$..

The GPD spectral function (3.2) is easily calculated from the GPD (A2) by setting $\eta = x\vartheta$ in the outer region

$$H^{\text{toy}}(x,\vartheta x) = \frac{Nx^{-\alpha}(1-ax)}{2\vartheta(1-\alpha)} \times \left[\left(\frac{1+\vartheta}{1+x\vartheta}\right)^{1-\alpha} - \left(\frac{1-\vartheta}{1-x\vartheta}\right)^{1-\alpha}\right]. \tag{A3}$$

In particular, the PDF ($\vartheta = 0$) and the GPD on the cross-over line ($\vartheta = 1$) read as follows:

$$q^{\text{toy}}(x) = Nx^{-\alpha}(1-ax)(1-x), \tag{A4}$$

$$H^{\text{toy}}(x,x) = \frac{N}{1-\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha}\frac{1-ax}{1+x}, \tag{A5}$$

where $\int_0^1 dx x q^{\text{toy}}(x) = M_2$.

The D-term consists of the integral GPD part, calculated from the low-energy limit (3.5), and the fixed-pole piece:

$$d^{\text{toy}}(x) = -\frac{aN}{2(1-\alpha)}x(1-x)^{1-\alpha} + d^{\text{f.p.}}(x). \tag{A6}$$

Now, the D-term form factor (3.5) might be directly calculated by means of the complete D-term (A6), where it contains the integral and the fixed-pole part

$$\begin{aligned} &4\mathcal{D}^{\text{toy}}(\vartheta) = 4\mathcal{D}^{\text{int}}(\vartheta) + 4\mathcal{D}^{\text{f.p.}}(\vartheta) \quad \text{with} \\ &4\mathcal{D}^{\text{int}}(\vartheta) = \frac{aN}{1-\alpha}\left[\frac{2}{2-\alpha} + \frac{1-\vartheta}{(1-\alpha)\vartheta}\,{}_2F_1\left(1,\frac{1}{2}-\alpha\Big|\vartheta\right) - \frac{1+\vartheta}{(1-\alpha)\vartheta}\,{}_2F_1\left(1,\frac{1}{2}-\alpha\Big|-\vartheta\right)\right]; \\ &4\mathcal{D}^{\text{f.p.}}(\vartheta) = \int_0^1 dx\frac{4x\vartheta^2}{1-x^2\vartheta^2}d^{\text{f.p.}}(x). \end{aligned} \tag{A7}$$

The individual contributions $\mathcal{D}^{\cdots}$ satisfy $\mathcal{D}^{\cdots}(\vartheta = 0) = 0$.

The direct evaluation of the inverse moment from the GPD spectral function (A3) yields

$$\int_{(0)}^1 dx\frac{2}{x}H^{\text{toy}}(x,\vartheta x) = -\frac{2N}{1-\alpha}\left[\frac{1}{\alpha} + \frac{a}{2-\alpha}\right] + 4\mathcal{D}^{\text{int}}(\vartheta). \tag{A8}$$

It contains a ϑ-independent term and the ϑ dependence is entirely contained in the GPD integral part of the D-term while the fixed-pole contribution is missing.

Consequently, the conjecture that the D-term form factor can be calculated from the inverse moment sum rule,

$$\begin{aligned} 4\mathcal{D}^{\text{int}}(\vartheta) &= \int_{(0)}^1 dx\frac{2}{x}[H^{\text{toy}}(x,\vartheta x) - H^{\text{toy}}(x,0)] \\ &\neq 4\mathcal{D}^{\text{toy}}(\vartheta) = 4\mathcal{D}^{\text{int}}(\vartheta) + 4\mathcal{D}^{\text{f.p.}}(\vartheta) \end{aligned}$$

is spoiled by the D-term-related fixed-pole contribution $\mathcal{D}^{\text{f.p.}}(\vartheta)$. In accordance with that, in the $J = 0$ fixed pole (3.3), build from the net D-term (A7) and the inverse moment (A8), only the GPD integral part of the D-term cancels out while the fixed-pole-related one induces a ϑ dependence:

$$\mathcal{H}_\infty(\vartheta) = 4\mathcal{D}^{\text{f.p.}}(\vartheta) + \frac{2N}{1-\alpha}\left[\frac{1}{\alpha} + \frac{a}{2-\alpha}\right].$$

Hence, our simple toy model with an ambiguous non-vanishing D-term-related fixed-pole contribution contradicts the conjecture of Ref. [12] that the $J = 0$ fixed pole is independent of the photon virtualities.

[1] V. de Alfaro, S. Fubini, G. Furlan, and C. Rossetti, *Currents in Hadron Physics* (North-Holland, Amsterdam, 1973).
[2] V. N. Gribov, Zh. Eksp. Teor. Fiz. **41**, 667 (1961) [Sov. Phys. JETP **14**, 478 (1962)].
[3] M. Froissart, Phys. Rev. **123**, 1053 (1961).
[4] M. J. Creutz, S. D. Drell, and E. A. Paschos, Phys. Rev. **178**, 2300 (1969).
[5] M. Damashek and F. J. Gilman, Phys. Rev. D **1**, 1319 (1970).
[6] M. Gell-Mann, M. Goldberger, and W. E. Thirring, Phys. Rev. **95**, 1612 (1954).
[7] C. A. Dominguez, C. Ferro Fontan, and R. Suaya, Phys. Lett. B **31**, 365 (1970).
[8] S. J. Brodsky, F. E. Close, and J. F. Gunion, Phys. Rev. D **5**, 1384 (1972).
[9] S. J. Brodsky, F. E. Close, and J. F. Gunion, Phys. Rev. D **6**, 177 (1972).
[10] A. Zee, Phys. Rev. D **5**, 2829 (1972).
[11] M. Creutz, Phys. Rev. D **7**, 1539 (1973).
[12] S. J. Brodsky, F. J. Llanes-Estrada, and A. P. Szczepaniak, Phys. Rev. D **79**, 033012 (2009).
[13] M. V. Polyakov and C. Weiss, Phys. Rev. D **60**, 114017 (1999).
[14] D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, and J. Hořejši, Fortschr. Phys. **42**, 101 (1994).
[15] A. V. Radyushkin, Phys. Rev. D **56**, 5524 (1997).
[16] M. V. Polyakov, Phys. Lett. B **555**, 57 (2003).
[17] A. V. Belitsky, D. Mueller, A. Kirchner, and A. Schafer, Phys. Rev. D **64**, 116002 (2001).
[18] O. V. Teryaev, Phys. Lett. B **510**, 125 (2001).
[19] A. V. Radyushkin, Phys. Rev. D **83**, 076006 (2011).
[20] D. Müller and A. Schäfer, Nucl. Phys. **B739**, 1 (2006).
[21] M. V. Polyakov and A. G. Shuvaev, arXiv:hep-ph/0207153.
[22] M. V. Polyakov and K. M. Semenov-Tian-Shansky, Eur. Phys. J. A **40**, 181 (2009).
[23] K. M. Semenov-Tian-Shansky, Eur. Phys. J. A **45**, 217 (2010).
[24] D. Müller, M. V. Polyakov, and K. M. Semenov-Tian-Shansky, J. High Energy Phys. 03 (2015) 052.
[25] A. V. Belitsky, D. Müller, and Y. Ji, Nucl. Phys. **B878**, 214 (2014).
[26] I. M. Gelfand and G. E. Shilov, *Generalized Functions Vol. I* (Academic Press, New York, 1964).
[27] K. Kumerički, D. Müller, and K. Passek-Kumerički, Nucl. Phys. **B794**, 244 (2008).
[28] These variables are not uniquely defined in the literature, i.e., the various definitions differ by $1/Q^2$-suppressed terms, that vanish in the generalized Bjorken limit.
[29] K. Kumerički, D. Müller, and K. Passek-Kumerički, Eur. Phys. J. C **58**, 193 (2008).
[30] O. V. Teryaev, arXiv:hep-ph/0510031.
[31] I. V. Anikin and O. V. Teryaev, Phys. Rev. D **76**, 056007 (2007).
[32] For $|\xi| > 1$, where one might rely on the GPD crossing properties [18].
[33] K. M. Semenov-Tian-Shansky, Eur. Phys. J. A **36**, 303 (2008).
[34] M. Praszalowicz and A. Rostworowski, Acta Phys. Pol. B **34**, 2699 (2003).
[35] P. Pobylitsa, M. V. Polyakov, and M. Strikman, Phys. Rev. Lett. **87**, 022001 (2001).
[36] Here the covariant derivative $\overleftrightarrow{D}$ and the total derivative $\overleftrightarrow{\partial}$ are contracted with the light-cone vector n, and $C_j^{\frac{3}{2}}$ stand for the usual Gegenbauer polynomials with the index $3/2$.
[37] To avoid confusion, let us add that the DD representation is not uniquely defined and the representation can be changed by a "gauge" transformation [18]. In this way one can generate also representations with a common D-term; however, the analytic properties of the spectral function might also be changed. These are more intricate technicalities since the GPD remains unchanged under "gauge" transformations.
[38] B. Pasquini, M. V. Polyakov, and M. Vanderhaeghen, Phys. Lett. B **739**, 133 (2014).
[39] The "Pomeron" case $\alpha \sim 1$ can be treated in a similar fashion; however, within the considered DD representation a residue function would require that $h_\alpha(z)$ vanishes at the boundary $z = \pm 1$.