

**Supersymmetry across the light and heavy-light hadronic spectrum**Hans Günter Dosch<sup>\*</sup>*Institut für Theoretische Physik, Philosophenweg 16, 69120 Heidelberg, Germany*Guy F. de Téramond<sup>†</sup>*Universidad de Costa Rica, 11501 San Pedro de Montes de Oca, Costa Rica*Stanley J. Brodsky<sup>‡</sup>*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*

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Relativistic light-front bound-state equations for mesons and baryons can be constructed in the chiral limit from the supercharges of a superconformal algebra which connect baryon and meson spectra. Quark masses break the conformal invariance, but the basic underlying supersymmetric mechanism, which transforms meson and baryon wave functions into each other, still holds and gives remarkable connections across the entire spectrum of light and heavy-light hadrons. We also briefly examine the consequences of extending the supersymmetric relations to double-heavy mesons and baryons.

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**I. INTRODUCTION**

A symmetry relating the baryon and meson spectra was first proposed in Refs. [1,2], but it was found to be badly broken. The no-go theorem of Coleman and Mandula [3] seemed to put an end to such attempts, since it showed that internal degrees of freedom and space-time symmetries can only be connected to each other in a trivial way. However, mainly motivated by work on four-dimensional supersymmetric quantum field theories by Wess and Zumino [4], interest in relating particles by supersymmetry rose sharply in the 1970s, since it was shown [5] that this symmetry provides a way to unify space-time and internal symmetries which circumvent the no-go theorem [3]. Supersymmetry has remained an important underlying principle in particle physics, especially in connection with proposed extensions of the Standard Model, string theory and grand unification, where supersymmetry was introduced to solve the hierarchy problem [6]. Experimental limits require the superpartners of the field quanta to be very massive, and therefore supersymmetry is expected to be broken at the TeV scale.

In 1981, Witten [7] introduced supersymmetric quantum mechanics as a model to study nonperturbative supersymmetry breaking, but it was soon realized that this elegant complement to supersymmetric quantum field theory was compelling in its own right [8]. The simplest form of supersymmetric quantum mechanics is generated by two supercharges, the anticommutator of which is the Hamiltonian of the theory. In fact, as we have shown

recently [9], the striking empirical similarities of the Regge trajectories of baryons and mesons can be understood as supersymmetric algebraic relations underlying a light-front (LF) Hamiltonian formulation of confinement in the light quark sector. This effective theory follows from the clustering properties of the LF Hamiltonian and its holographic embedding in anti-de Sitter (AdS) space [10–12]. The resulting theory leads to supersymmetric one-dimensional hadronic LF bound-state equations for mesons and baryons, thus providing a semiclassical approximation to strongly coupled QCD dynamics [10].

In our previous papers [9,13,14], we have shown how conformal invariance, together with supersymmetric quantum mechanics [15,16], as expressed by holographic LF bound-state equations for light quarks [10,12], leads to remarkable *superconformal* relations which connect light meson to light baryon spectroscopy [17].

As shown in Ref. [13], an effective LF Hamiltonian for mesons as bound states of confined light quarks and antiquarks can be derived for arbitrary spin  $J$  based on light-front holography. The confining potential is determined to have the form of a harmonic oscillator in the boost-invariant transverse-impact LF variable  $\zeta$  [18]. In fact, the LF confining potential is unique if one requires that the action remain conformally invariant.

In Refs. [9,14], we showed that the extension of the conformal formalism to a superconformal algebra leads to a remarkable new set of supersymmetric relations for the spectra of light hadrons. In fact, the corresponding construction of the LF Hamiltonian derived from the generalized supercharges [16] dictates the form of the LF potential for light mesons and baryons, including constant terms which yield the correct spin dependence and notably a zero-mass pion. This new approach explains

<sup>\*</sup>h.g.dosch@thphys.uni-heidelberg.de<sup>†</sup>gdt@asterix.crnet.cr<sup>‡</sup>sjbth@slac.stanford.edu

the striking similarity of light-quark meson and baryon spectra. A crucial feature of the formalism is that the supermultiplets consist of a meson wave function with internal LF angular momentum  $L_M$  and the corresponding baryon wave function with angular momentum  $L_B = L_M - 1$  with the same mass. The  $L_M = 0$  meson has no supersymmetric partner.

In this paper we will show that supersymmetric relations between heavy mesons and baryons can also be derived from the supersymmetric algebra even though conformal invariance is explicitly broken by heavy quark masses. Thus, the new results are more general than the results derived in the previous papers [9,14], and they reveal deep relations in strong interaction dynamics, which hold even when conformal symmetry is strongly broken. We emphasize that the supersymmetric relations which are derived from supersymmetric quantum mechanics are not based on a supersymmetric Lagrangian in which QCD is embedded; instead, they are based on the fact that the supercharges of the supersymmetric algebra relate the wave functions of mesons and baryons in a Hilbert space in which the LF Hamiltonian acts. The properties of the supercharges predict specific constraints between mesonic and baryonic superpartners in agreement with measurements across the entire hadronic spectrum.

## II. SUPERSYMMETRIC QUANTUM MECHANICS

Supersymmetric quantum mechanics [7] can be constructed from the supercharges  $Q$  and  $Q^\dagger$  with the anti-commutation relations

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0, \quad (1)$$

and the Hamiltonian

$$H = \{Q, Q^\dagger\}, \quad (2)$$

which anticommutes with the fermionic generators  $[Q, H] = [Q^\dagger, H] = 0$ . Its minimal realization in matrix notation is

$$H = \{Q, Q^\dagger\} = \begin{pmatrix} -\frac{d^2}{dx^2} + \frac{4(L+1)^2-1}{4x^2} + U_1(x) & 0 \\ 0 & -\frac{d^2}{dx^2} + \frac{4L^2-1}{4x^2} + U_2(x) \end{pmatrix}, \quad (7)$$

with  $L = f - \frac{1}{2}$  and  $U_{1,2}(x) = \frac{2f}{x}V(x) + V^2(x) \mp V'(x)$ .

One can also explicitly break conformal symmetry without violating supersymmetry by adding to the Hamiltonian (7) a multiple of the unit matrix,  $\mu^2 \mathbf{I}$ , where the constant  $\mu$  has the dimension of a mass,

$$H_\mu = \{Q, Q^\dagger\} + \mu^2 \mathbf{I}. \quad (8)$$

$$Q = \begin{pmatrix} 0 & q \\ 0 & 0 \end{pmatrix}, \quad Q^\dagger = \begin{pmatrix} 0 & 0 \\ q^\dagger & 0 \end{pmatrix}, \quad (3)$$

with

$$q = -\frac{d}{dx} + W(x), \quad q^\dagger = \frac{d}{dx} + W(x). \quad (4)$$

For the special case  $W(x) = \frac{f}{x}$ , where  $f$  is a dimensionless constant, the resulting Hamiltonian is also invariant under conformal transformations, and one can extend the supersymmetric algebra to a superconformal algebra [15,16]. Furthermore, if one generalizes the supercharge  $Q$  to a superposition of fermion generators inside the superconformal algebra [16] by replacing

$$q \rightarrow -\frac{d}{dx} + \frac{f}{x} + \lambda x, \quad (5)$$

$$q^\dagger \rightarrow \frac{d}{dx} + \frac{f}{x} + \lambda x \quad (6)$$

in (3), then the resulting Hamiltonian can be identified with a semiclassical approximation to the QCD LF Hamiltonian of mesons (M) and baryons (B) in the limit of vanishing quark masses [9]. Remarkably, the dynamics in this case can be expressed in terms of a single variable, the invariant LF transverse coordinate  $\zeta$  [18] which is identified with the variable  $x$  in Eqs. (4)–(6). Additionally, we identify the mass-scale parameters  $\lambda = \lambda_M = \lambda_B$  and  $f = L_B + \frac{1}{2} = L_M - \frac{1}{2}$  in the Hamiltonian [9], from which follows the crucial relation  $L_M = L_B + 1$ .

The extension of conformal to superconformal symmetry plays an important role in fixing the effective LF potentials of both mesons and baryons; thus, it is interesting to examine the role of supersymmetry when conformal symmetry is broken explicitly by quark masses. In fact, conformal quantum mechanics was originally formulated by Witten [7] for any form of the superpotential  $W(x)$  (4): supersymmetry holds if one substitutes the term  $\lambda x$  in (5) and (6) by an arbitrary potential  $V(x)$ . The resulting Hamiltonian is

Interpreting, as in Ref. [9], the supercharges as transformation operators between the invariant transverse component of the meson and baryon wave functions [19], we obtain the same mass relations between mesons and baryons as in the conformal case, and thus their degeneracies remain. The absolute values of the hadron masses in the heavy quark case, however, cannot be computed in this

framework. This is in contrast to the massless case, where the construction principle uniquely determines the LF potential [9,14].

Quark masses appear in the LF kinetic energy [20]. In the case of small quark masses, one expects only a small effect on the LF potential. In fact, in Ref. [21] we have studied the effect of the strange quark mass in the meson sector. It was found that indeed the slopes of the trajectories remain unchanged, indicating that the LF potential is not modified to first order. Small quark masses only affect the longitudinal component of the LF wave function which allows one to compute perturbatively the shift of the squared meson masses. If supersymmetry holds, the same shift in the baryonic mass squared—with unchanged slope—then follows from (8).

Supersymmetry in the approach of this paper has a scope which is quite different from that in the original papers of Witten [7] and Fubini and Rabinovici [16] (see also the discussion in Ref. [9], Appendix A). In Ref. [7] and in Ref. [16] supersymmetric quantum mechanics was used as a method to construct a novel quantum field theory and to understand the mechanisms underlying supersymmetry breaking. In contrast, we are interested in supersymmetric relations between the spectroscopy and wave functions of physical mesons and baryons.

In our approach, the supercharges  $Q$ , as operators in  $\mathcal{L}_2(R_1)$ , transform baryon and meson wave functions into each other; this is in fact verified when one applies superconformal quantum mechanics to light-front holographic

QCD [9]. An essential feature of the resulting supersymmetric relations is the difference in the light-front angular momentum of the hadronic superpartners: The angular momentum of the baryon is one unit lower than that of the corresponding meson—the supercharge operator applied to a meson wave function with light-front angular momentum  $L_M$  yields a baryon wave function with  $L_B = L_M - 1$ .

A meson with angular momentum  $L_M = 0$  cannot have a baryonic partner, since the baryon would have negative angular momentum. This constraint is formally satisfied in superconformal quantum mechanics since the supercharge applied to the  $L = 0$  bosonic state yields zero. Since the Hamiltonian (7) commutes with the supercharges, the spectra of the mesons and corresponding baryons match. The modification of the Hamiltonian (7) by a constant term as in (8) does not affect the matching.

### III. COMPARISON WITH EXPERIMENT

We compare in Fig. 1 the measured masses for strange mesons and baryons with the predictions of the supersymmetric model. The squared masses are plotted against  $L_M = L_B + 1$ ; mesons and baryons with the same abscissa are predicted to have the same mass. The supersymmetric Hamiltonian (7) implies that the light-front angular momentum of the baryon is by one unit lower than that of its mesonic superpartner. Therefore, the lightest meson, which has LF angular momentum zero has no supersymmetric baryon partner (see Appendix B in Ref. [9]).

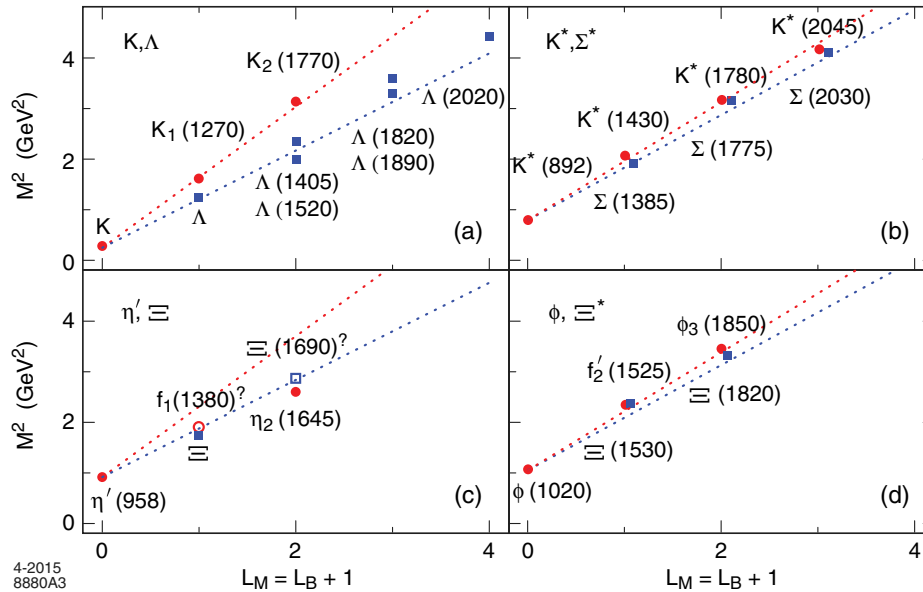


FIG. 1 (color online). Test of supersymmetric relations between the lowest radial excitations ( $n = 0$ ) of strange mesons and baryons. As an illustration we have included the dotted lines from Ref. [17] which show that the slopes are not affected by relatively small quark masses. Pseudoscalar meson masses are compared with their baryon partners in (a) and (c). Vector mesons are compared with their baryon partners in (b) and (d). Hadron masses and assignments are taken from the Particle Data Group (PDG) [22]. Confirmed states for the  $\eta'$ ,  $\Xi$  trajectories are indicated in (c) by a white square and white circle. When experimental points overlap, the value of the abscissa is displaced slightly from its integer value.

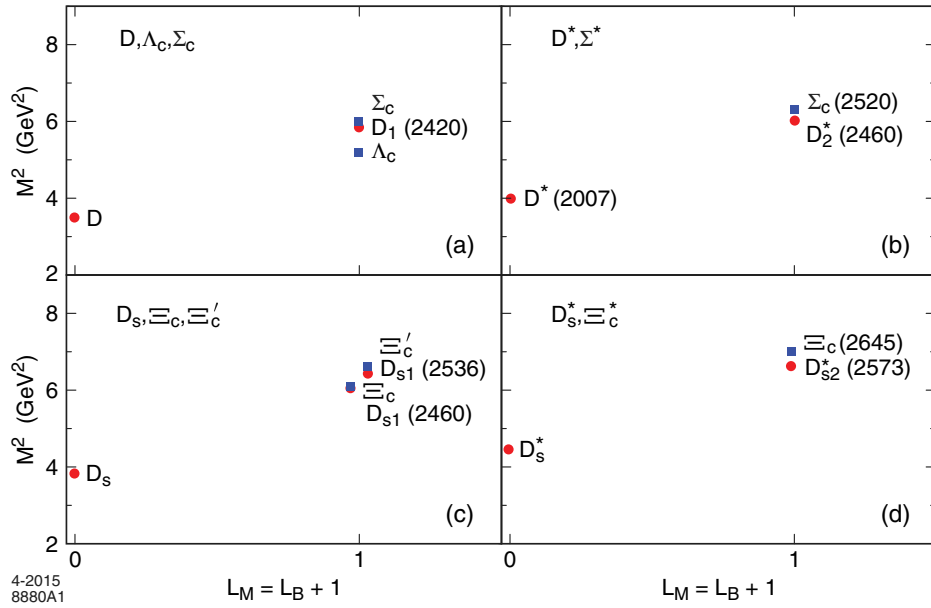


FIG. 2 (color online). Test of supersymmetric relations between mesons and baryons with charm. Pseudoscalar meson masses are compared with their baryon partners in (a) and (c). Vector mesons are compared with their baryon partners in (b) and (d). Hadron masses and assignments are taken from PDG [22].

The dotted lines in this figure are the trajectories with the slopes taken from the massless case [17]. The slopes which describe the Regge trajectories of the light-quark mass hadrons also fit the trajectories of the strange hadrons, which clearly indicates that the deviation from a common scale observed in the light (nonstrange) sector is faithfully replicated in the strange sector. The supersymmetric

relations are remarkably well satisfied for the trajectories of the  $K^*$  and  $\Sigma^*$  as well as for the  $\phi$  and  $\Xi^*$ . For the  $K$  and  $\Lambda$  hadrons, there are violations of supersymmetry, similar to those observed for the pion-nucleon system [9]. These violations can be traced back to an explicit breaking by the different mass-scale values for the mesons and the baryons; indeed, we have for the  $K$ -mesons

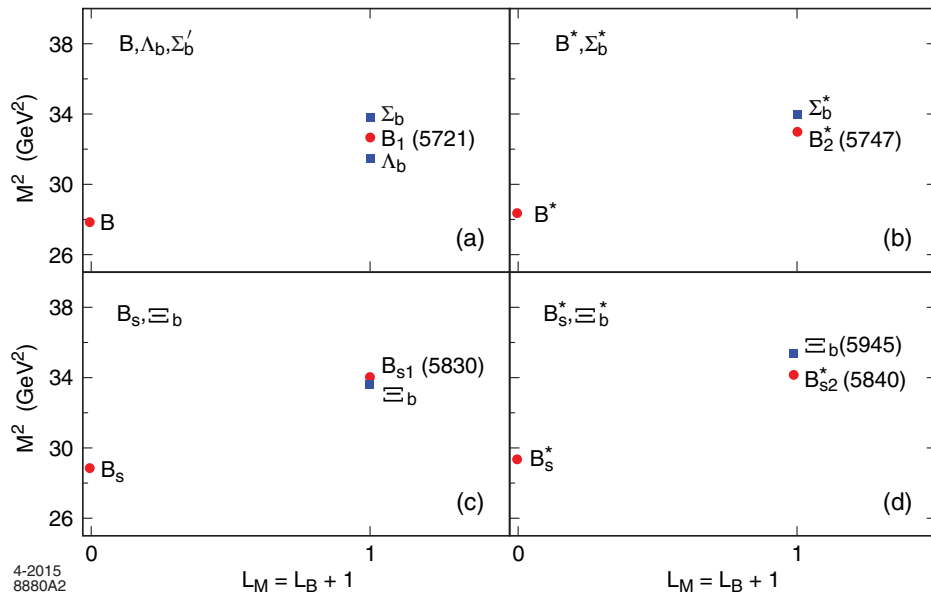


FIG. 3 (color online). Test of supersymmetric relations between mesons and baryons with beauty. Pseudoscalar meson masses are compared with their baryon partners in (a) and (c). Vector mesons are compared with their baryon partners in (b) and (d). Hadron masses and assignments are taken from PDG [22].

TABLE I. Double-heavy meson states and corresponding double-heavy baryons.

Double-heavy meson	Corresponding baryon
$h_c(1P)(3525)$	$\Xi_{ccq} \cdot \frac{1}{2}^+$
$\chi_{c2}(1P)(3556)$	$\Xi_{ccq}^* \cdot \frac{3}{2}^+$
$h_b(1P)(9899)$	$\Xi_{bbq} \cdot \frac{1}{2}^+$
$\chi_{b2}(1P)(9912)$	$\Xi_{bbq}^* \cdot \frac{3}{2}^+$

$\sqrt{\lambda_K} = \sqrt{\lambda_\pi} = 0.59$  GeV, whereas for the  $\Lambda$  we have the smaller value  $\sqrt{\lambda_\Lambda} = \sqrt{\lambda_N} = 0.49$  GeV. We also include in Fig. 1 the  $\eta'$  and  $\Xi$  trajectories, although the experimental situation is not yet completely settled.

Next we investigate the application of supersymmetry to heavy-light mesons and baryons, namely a meson with one heavy and one light or strange quark and its corresponding nucleon with one heavy and two light or strange quarks. Supersymmetry predicts the near degeneracy of mesons with angular momentum  $L_M$  and baryons with angular momentum  $L_B = L_M - 1$ . As can be seen in Figs. 2 and 3, the corresponding mass relations are fulfilled within the expected precision. It is remarkable that the small splitting of the  $\Xi_c$  and  $\Xi_c'$  is also observed for the corresponding  $D_{s1}(2460)$  and  $D_{s1}(2536)$  mesons.

One expects large effects from the breaking of conformal symmetry due to the heavy quark mass. For example, the mass scale  $\sqrt{\lambda}$  need not have a value similar to that of the light-quark conformal limit. In addition, the confining potential is not required to remain quadratic as prescribed by conformal symmetry. Indeed, the measured difference between the squared mass of the ground state and that of the first orbital excitation is significantly larger than the value obtained from the LF potential between massless quarks: For the  $D$ -mesons the discrepancy is a factor of 2 and for  $B$ -mesons a factor of 4 (see also Refs. [23,24]). The lack of confirmed states does not allow conclusions on the form of the heavy-light LF potential.

Finally, we extend our considerations to double-heavy hadrons, i.e., to mesons containing two heavy quarks and their supersymmetric baryon partners containing two heavy and one light quarks. This extension should be taken with care since the kinematical regime for the double-heavy hadronic states is significantly different from the heavy-light systems. With this proviso, we list in Table I the double-charm and double-beauty mesons together with the corresponding double-heavy baryons. Their masses should be degenerate with those of the mesons. These values are higher than the masses of the SELEX double-charm  $ccu$  and  $ccd$  candidates [25], but below the predictions from quark models and lattice computations (see Ref. [26] and references quoted therein).

## IV. SUMMARY AND CONCLUSIONS

In this paper we have described the consequences of the construction of semiclassical light-front bound-state equations based on supersymmetric quantum mechanics; it relates wave functions of mesons and baryons. These relations are possible since in the light-front holographic approach the baryon is described by the wave function of a quark-diquark cluster, and the supersymmetric relations reflect the transformation of the antiquark in the meson to the diquark cluster in the baryon [27]. The supersymmetric relations in hadron spectroscopy discussed in this paper are not consequences of new superpartners of known fields but are a property of the hadronic wave functions. It therefore must follow from specific properties of the dynamics of strong interactions. Indeed, when conformal symmetry is restored in the limit of massless quarks, the resulting spectrum of hadronic excitations accounts for essential aspects of the confining dynamics [9,13,14].

In the case of light quarks, the confining potential is determined uniquely from the underlying conformal invariance [9,13,14]. In the case of hadrons containing light and strange quarks, the superpotential has still the same form as for light quarks, but the trajectories undergo a common shift as can be seen in Fig. 1. For systems composed of one heavy and light or strange quarks, conformal symmetry is strongly broken, but the supersymmetric relations still hold in agreement with experimental observations, as shown in Figs. 2 and 3. Finally, we have extended the supersymmetric relations to systems containing two heavy quarks.

We have shown how supersymmetry, together with light-front holography, leads to new and unexpected connections between mesons and baryons across the hadronic spectrum, thus providing new perspectives for hadron spectroscopy and QCD. We also note that measurements of additional states in the heavy quark sector will provide important information on the modification of the superpotential due to the explicit breaking of conformal symmetry. This will allow the determination of the light-front confining potential in hadrons for both light and heavy quarks. This new approach may also give important insights into the role of conformal symmetry and its breaking in hadronic physics.

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