Search for pseudoscalar-mediated WIMPs in $t \to c$ transitions with missing energy

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The recent astronomical observation of GeV gamma-ray excess from the Galactic center was suggested due to a $b\bar{b}$ mode in the tens GeV of weakly interacting massive particle (WIMP) pair annihilations, and this mode was also explored by the new dwarf galaxy observation. Considering the case where the WIMP pair mass is below the top quark mass, a pseudoscalar ϕ is studied in this article, which mediates the interactions between the standard model fermions and fermionic WIMPs and the neutral flavor-changing interactions in standard model fermion sectors. The $b\bar{b}$ mode is favored in WIMP pair annihilations, while the WIMPnucleus scattering is highly suppressed in direct detection. Alternative schemes of $t \rightarrow c$ decay and single top production are employed to search the WIMPs. Assuming the mass of the WIMPs is around 5–60 GeV and with reasonable inputs by the constraints, the branching ratio $\mathcal{B}_{t\to c\bar{\chi}\chi}$ of a top quark decaying into a charm quark and a WIMP pair is derived of order 10[−]⁸–10[−]⁵; thus, careful studies in the future on top physics may help to gain a better understanding of WIMPs.

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I. INTRODUCTION

It is believed that the major constituent of the matter in the Universe is dark matter (DM), and one of the most favored DM candidates are WIMPs (weakly interacting massive particles). The recent results from LUX [\[1\]](#page-6-0) and SuperCDMS [\[2,3\]](#page-6-1) set stringent constraints on WIMPs in a direct search, and meanwhile the collider physics is providing complementary conditions to narrow possible survival ranges for WIMP models. We definitely know that the WIMP-nucleus scattering cross section may be suppressed by q^2 , v^2 , or J, where q is the momentum transfer, v
is the WIMP relative velocity and L is the total spin of the is the WIMP relative velocity, and J is the total spin of the target nucleus; thus, so far no firm signal has been observed in DM direct detection. We expect that the WIMP signals will be detected above the neutrino irreducible background in the next decades by the ultimate DM direct detectors (see, e.g., [\[4](#page-6-2)–9] for more). As a matter of fact, even though the signals are below the neutrino background, with today's sophisticated facilities and understanding of neutrinos, it is still possible to distinguish them from the background.

The highly suppressed WIMP-nucleus interaction may make the WIMPs evade our direct detection, so some alternative ways to detect WIMPs are needed. One important means is obviously accelerator physics. If at the highenergy accelerators, such as the LHC, the WIMPs which are definitely not the standard fermions or bosons are produced as missing energy, we could identify them and declare a success. But since the background of the LHC is too complicated, it would be hard to dig out missing energy from the messy products and reestablish the concerned events. An alternative detection proposed by the authors of Ref. [\[10\]](#page-6-3) suggested that rare decays of the B meson might be feasible for searching WIMPs of a few GeV and sub-GeV in $b \rightarrow s$ transitions with missing energy. If the masses of WIMPs are to heavy to be detected in B decays, a natural extension of the idea is to the top quark decays [\[11,12\]](#page-6-4).

The gamma-ray emissions from DM dense regions may provide the information about WIMPs, e.g., the Galactic center gamma rays [\[13](#page-6-5)–19]. Recent studies of the excess of $∼1-3$ GeV gamma rays [20–[25\]](#page-6-6) and the antiproton spectrum [\[26\]](#page-6-7) from the region surrounding the Galactic center indicate that the excess can be interpreted as annihilating WIMPs for WIMPs of $35 \sim 51$ GeV mainly annihilating to $b\bar{b}$ pairs with a cross section of ~10⁻²⁶ cm³/s ($b\bar{b}$). These results are at the order of the expectation values of the thermal freeze-out WIMPs [\[27\]](#page-6-8) and are consistent with the prediction of the model where a pseudoscalar boson is the interaction messenger [28–[34\]](#page-6-9) (for general discussions see, e.g., Refs. [\[35,36\]\)](#page-6-10). Moreover, as pointed out in Ref. [\[37\],](#page-6-11) the interpretation of the gamma-ray excess can be tolerated by the limits from the new dwarf galaxy [\[38\]](#page-6-12) given by the Fermi-LAT Collaboration. That is an encouraging piece of information and may hint at a direction to search for WIMPs if the allegation about WIMP annihilation causing the excess is valid. In this work, the limits from the new dwarf galaxy will be taken into consideration.

An optimistically alternative scheme to detect the WIMPs is via the flavor-changing processes of $t \to c$ decay and single top production (see, e.g., Refs. [\[11,12,39](#page-6-4)–41] about the missing energy in $t - c$, u transitions). When a WIMP pair has mass below the top quark mass (e.g., tens GeV), the [*](#page-0-1)

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 $t\bar{c}$ ($\bar{t}c$) mode in WIMP pair annihilations will be negligible, and the main SM products are from other modes, e.g., the bb mode as indicated by the observation of anomalous gammaray excess. This is the thing of concern and study in this paper. In this case, the channel of $t \to c$ decay may be ideal to search for WIMPs.

Now, the crucial point is to reasonably estimate the rate of the rare decay because it is the key to whether the channel can be observed at the LHC and future facilities, such as the planned ILC [\[42\]](#page-6-13) or another proposed top factory. Indeed, the estimate of the rate depends on the models of WIMPs; namely, for different models, the estimated rate might be quite different. It is also well known that DM cannot interact with the regular particles via the Standard Model (SM), but requires something beyond SM. What that is, nobody knows for sure; however, there are many plausible models available, and each of them undergoes a stringent test of astronomical observation and accelerator experiments, as the corresponding parameter space is rigorously constrained.

In this work, we carry out our calculation based on the WIMP model where a new pseudoscalar mediates the interaction between DM and SM, and neutral flavorchanging interactions are introduced for SM fermions.

This work is organized as follows. After this Introduction, we introduce the Lagrangian for the interactions where a new pseudoscalar boson is responsible for the annihilation of WIMPs and the production of the SM partlees (say, bb), and then formulate its contributions to $t - c$ transitions with the missing energy. We also evaluate the annihilation cross section of WIMPs. Next, the numerical results are presented along with all the input parameters. The last section is devoted to our conclusion and a brief discussion.

II. THE MISSING ENERGY IN $t - c$ TRANSITIONS

As discussed in the Introduction, in this work we employ the model where a flavor-changing pseudoscalar bridges between the SM fermions and fermionic WIMPs. In this section, we will consider a few neutral flavor-changing processes where the top quark and the new pseudosclar boson are involved.

A. The interaction between the SM and WIMPs

Here we present the interaction of interest, by which the SM FCNC (flavor-changing neutral current) processes are realized via exchanging a neutral pseudoscalar boson ϕ at tree level. The effective couplings of ϕ to quarks and leptons are depicted as

$$
\mathcal{L}_{\text{SM}}^{i} = -\lambda_{q'q} \bar{q}' i\gamma_5 q\phi - \lambda_{l'l} \bar{l}' i\gamma_5 l\phi + \text{H.c.},\qquad(1)
$$

where $\lambda_{q'q}$, $\lambda_{l'l}$ are dimensionless parameters of ϕ corre-
conording to quarks and laptons, respectively. In the Chang sponding to quarks and leptons, respectively. In the Cheng and Sher ansatz [\[43\]](#page-6-14), the flavor-changing interactions are correlated with the geometric mean values of the two relevant fermion masses. Following Ref. [\[43\]](#page-6-14), we set the flavor-changing parameters in similar forms; that is,

$$
\lambda_{q'q} \sim \theta \frac{\sqrt{2m_{q'}m_q}}{v}, \qquad \lambda_{l'l} \sim \theta \frac{\sqrt{2m_{l'}m_l}}{v}, \qquad (2)
$$

where $v/\sqrt{2}$ is the electroweak vacuum expectation value,
as $v \approx 246$ GeV and θ is a dimensionless parameter which as $v \approx 246 \text{ GeV}$, and θ is a dimensionless parameter which is much smaller than unity. A solution to the mass and mixing hierarchies in quark and lepton sectors was explored in Ref. [\[44\]](#page-6-15) in the Froggatt-Nielsen mechanism (see, e.g., Ref. [\[45\]](#page-6-16) for a recent discussion about flavor violations).

The vertex for the fermionic WIMP pseudoscalar is $-i\lambda_D\gamma₅$, and λ_D is the effective coupling constant in the perturbative limit $\lambda_D < \sqrt{4\pi}$. Thus, the total Yukawa
countings between a pseudoscalar field ϕ and SM fermions couplings between a pseudoscalar field ϕ and SM fermions and fermionic WIMPs are

$$
\mathcal{L}_{\text{SM+DM}}^{i} = -i\lambda_{q'q}\bar{q}'\gamma_{5}q\phi - i\lambda_{l'l}\bar{l}'\gamma_{5}l\phi - i\lambda_{D}\bar{\chi}\gamma_{5}\chi\phi + \text{H.c.},\tag{3}
$$

where χ is the WIMP field.

The scattering operator between the WIMPs and the target nucleus is of the form $\bar{\chi}\gamma_5\chi\bar{q}'\gamma_5q$. This interaction
structure is spin dependent, and the scattering cross section structure is spin dependent, and the scattering cross section is suppressed by q^4 , as discussed in [\[4,35\].](#page-6-2) Thus, for WIMPs interacting via this type of Lagrangian, the WIMPs WIMPs interacting via this type of Lagrangian, the WIMPs would evade the direct detection at the present underground experiment. Therefore, if it is the case, a possible way to identify them is by evaluating the contributions of ϕ to the processes of $t \to c$ decay or single top production.

B. The flavor-changing neutral transitions of t

By Eq. [\(2\),](#page-1-0) it is found that the coupling between the field ϕ and SM fermions is suppressed by the small θ (e.g., of order 10^{-2}) and fermion masses, unless there is a top quark involved. As no pseudoscalar bosons beyond the SM Higgs were observed at LEP, Tevatron, and LHC until now, it implies that the mass of ϕ may be too heavy to be observed at present collider energies or be within the present collider reach but very difficult to be observed. Here we would lean toward the latter case. The production rate of ϕ at e^+e^- , $p\bar{p}$, and $p\bar{p}$ colliders is much smaller than that of the SM Higgs boson due to the suppression of θ . If the mass of ϕ is above a WIMP pair production threshold, but not heavier than the top quark, the decay rates of SM fermions would all be suppressed in ϕ decay. Even though ϕ is produced at the high-energy collider, the missing energy from $\phi \rightarrow \chi \bar{\chi}$ will be swamped by the SM background. Thus, an undiscovered pseudoscalar ϕ can be consistent with the present collider experiment.

Moreover, low-energy neutral flavor-changing transitions in quark and lepton sectors will give constraints on new flavor-changing interactions. In Ref. [\[46\]](#page-6-17), the Yukawatype flavor-changing couplings are discussed in terms of the Higgs boson, and the corresponding bounds are taken here in our analysis. The ϕ -mediated flavor-changing interactions can be allowed by quark and lepton sectors when $|\lambda_i \lambda_j|/m_\phi^2 \lesssim |Y_i Y_j|/m_h^2$, where m_ϕ , m_h is the ϕ 's mass and Higgs mass, respectively, λ_i , λ_j and Y_i , Y_j are the corresponding couplings of the two vertexes connected by a ϕ boson and a Higgs boson, respectively. For ϕ 's mass above tens GeV, the case $\theta \lesssim 10^{-2}$ is allowed by the flavorchanging interactions in quark and lepton sectors, e.g., an approximate upper limit θ/m_{ϕ} (GeV) $\lesssim 10^{-2}/20$ with m_{ϕ} in units of GeV.

Now let us evaluate the contribution of ϕ to the top quark FCNC decays. Considering the mass of ϕ to be in the range $2m_{\chi} < m_{\phi} < m_{t}$, where m_{χ} is the WIMP mass, according to Eq. [\(3\),](#page-1-1) the leading-order decay width of $t \to c\phi$ is

$$
\Gamma_{t \to c\phi} = \frac{\bar{\beta}_f}{16\pi} m_t |\lambda_{tc}|^2 \left[\left(1 - \frac{m_c}{m_t} \right)^2 - \frac{m_\phi^2}{m_t^2} \right],\tag{4}
$$

where

$$
\bar{\beta}_f = \sqrt{1 - \frac{2(m_c^2 + m_\phi^2)}{m_t^2} + \frac{(m_c^2 - m_\phi^2)^2}{m_t^4}}.
$$
 (5)

As long as the parameter λ_D is not extremely tiny and the mass of ϕ is larger than the threshold of the WIMP pair, the channel of $\phi \rightarrow \chi \bar{\chi}$ should be overwhelming, and $B_{t\to c\bar{\chi}\chi} \simeq B_{t\to c\phi}$. Meanwhile, in the SM the branching ratio of the system power pointing also manifest as missing approximately of $t \to c\bar{\nu}$ where neutrinos also manifest as missing energy is of order \sim 3 × 10⁻¹⁴ [\[47\];](#page-6-18) thus, it should be possible to identify the WIMP final state in $t \to c$ decays with missing energy.

The new interaction also contributes to the single top production which is a FCNC process. The fusion of a gluon and an up-type quark u_i (c or u quark) into a top quark and ϕ , i.e. $u_i g \rightarrow t \phi$, at a hadron collider (see Refs. [\[39,40,48,49\]](#page-6-19) for more) is depicted in Fig. [1](#page-2-0). From Eq. [\(3\),](#page-1-1) the flavor-changing interaction of $t - u_i - \phi$ is

$$
\mathcal{L}_{tu_i}^i = -i\lambda_{tu_i}\bar{t}\gamma_5 u_i\phi. \tag{6}
$$

As the pseudoscalar ϕ is invisible, and the final product is a single top quark and large missing energy. The recent

monotop production data of the CMS Collaboration [\[50\]](#page-6-20) would set a constraint on the parameter for this flavorchanging coupling, and we will discuss it later.

III. WIMP ABUNDANCE AND ANNIHILATION

Even though the relic density of DM sets constraints on the transitions between WIMPs and SM particles, in DM dense regions the observed cosmic ray excess may shed light on the properties of WIMP annihilations. Here we are going to formulate the WIMP abundance and WIMP annihilation.

A. WIMP abundance

In the concerned model, the process of WIMP pairs annihilating into SM heavy fermions dominates. As the width of the intermediate boson ϕ is smaller compared with its mass, the WIMP pair annihilation cross section can be written as

$$
\sigma_{\text{ann}} = \frac{1}{2} \sigma_{\text{ann}}^{\text{Dirac}} = \frac{1}{2} \frac{1}{\beta_i (2s_\chi + 1)^2} \frac{\beta_f}{4\pi} \lambda_{f'f}^2 \lambda_D^2
$$

$$
\times \frac{N_c [s - (m_f - m_{f'})^2]}{(s - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2}.
$$
(7)

The factor $\frac{1}{2}$ in the expression is for the Dirac fermionic WIMPs, whereas it is 1 for the Majorana fermionic WIMPs. *s* is the square of the total invariant mass, and s_x is the WIMP spin projection. Γ_{ϕ} is the total width of ϕ , and N_c is the color factor of the final state fermions. The kinematic factors β_i , β_f are

$$
\beta_i = \sqrt{1 - \frac{4m_{\chi}^2}{s}},
$$
\n
$$
\beta_f = \sqrt{1 - \frac{2(m_f^2 + m_{f'}^2)}{s} + \frac{(m_f^2 - m_{f'}^2)^2}{s^2}}.
$$
\n(8)

We get the WIMP annihilation cross section by Eq. (7) . The DM relic density Ω_D depends on the thermal evolution after the big bang. The values of the relic density and the freeze-out temperature can be approximately written as [\[51,52\]](#page-6-21)

$$
\Omega_D h^2 \simeq \frac{1.07 \times 10^9 x_f}{\sqrt{g_* m_{\text{Pl}}} \text{GeV} \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle},\tag{9}
$$

$$
x_f \simeq \ln \frac{0.038gm_{\rm Pl}m_\chi \langle \sigma_{\rm ann} v_{\rm rel} \rangle}{\sqrt{g_* x_f}}.
$$
 (10)

Here h is the Hubble constant in units of 100 km/(s·Mpc). x_f is equal to m_γ/T_f with T_f being the freeze-out temperature, and g_* is the number of the FIG. 1. The process $u_i g \to t \phi$. relativistic degrees of freedom for masses being less than

 T_f . $m_{\text{Pl}} = 1.22 \times 10^{19}$ GeV is the Planck mass, and the number g is the degrees of freedom of DM. $\langle \sigma_{ann} v_{rel} \rangle$ is the thermal-averaged annihilation cross section of WIMPs \rightarrow SM particles, and v_{rel} is the relative speed of the annihilating WIMP pair. The thermal average of the annihilation cross section is [\[53\]](#page-6-22)

$$
\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle = \frac{1}{8m_{\chi}^4 T K_2^2 \left(\frac{m_{\chi}}{T} \right)} \int_{4m_{\chi}^2}^{\infty} \text{ds}
$$

$$
\times \sigma_{\text{ann}} \sqrt{s} (s - 4m_{\chi}^2) K_1 \left(\frac{\sqrt{s}}{T} \right), \qquad (11)
$$

where $K_i(x)$ is the *i*th-order modified Bessel function.

The value of x_f can be obtained by solving the Eq. [\(10\)](#page-2-2) iteratively. g_* varies with the freeze-out temperature T_f , and we will adopt the MicrOMEGAs 3.1 data [\[54\]](#page-6-23) about the Gondolo-Gelmini effective degrees of freedom at $T_{\text{OCD}} =$ 150 MeV in numerical calculations.

B. Present WIMP annihilation rate

As today's environmental temperature is negligible compared with the WIMP mass, the annihilation rate of WIMPs is derived in the $T = 0$ limit. From Eq. [\(7\),](#page-2-1) the present WIMP annihilation rate is

$$
\langle \sigma_{\text{ann}} v_r \rangle = \left\langle \frac{1}{2} \sigma_{\text{ann}}^{\text{Dirac}} v_r \right\rangle_{T=0}
$$

=
$$
\frac{1}{2} \frac{1}{(2s_\chi + 1)^2} \frac{\beta_f}{2\pi} \lambda_{f'f}^2 \lambda_D^2
$$

$$
\times \frac{N_c [4m_\chi^2 - (m_f - m_{f'})^2]}{(4m_\chi^2 - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2}, \qquad (12)
$$

with the same notations as given in Eq. [\(7\)](#page-2-1). v_r is the relative speed of the WIMP pair, and here $s = 4m_{\chi}^2$ has been taken
due to the small velocity of the WIMPs [55] due to the small velocity of the WIMPs [\[55\].](#page-6-24)

In the range $2m_\gamma < m_\phi < m_t$, if the mass m_γ is heaver than the b quark, the dominant products of WIMP pairs annihilating into SM particles are $b\bar{b}$ quarks, and this coincides with the observed GeV gamma-ray excess and the corresponding interpretation.

The present WIMP annihilation rate $\langle \sigma_{ann} v_r \rangle$ is not equal to the value of $\langle \sigma_{\rm ann} v_{\rm rel} \rangle$ at the freeze-out temperature (see, e.g., [\[33\]\)](#page-6-25), especially when the value of $2m_\gamma$ is around m_ϕ . Generally, when the WIMP pair mass $2m_\gamma$ is below the mass of ϕ , we have $\langle \sigma_{ann} v_r \rangle < \langle \sigma_{ann} v_{rel} \rangle$, while $\langle \sigma_{ann} v_r \rangle >$ $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle$ for $2m_{\chi} > m_{\phi}$. In the next section, we will numerically analyze the WIMP pair annihilation in detail.

IV. NUMERICAL ANALYSIS OF $t - c$ TRANSITIONS

Here, we first briefly analyze the single top production, then focus on the top quark decay.

A. Single top production

The data on single top production at the hadron collider set certain constraints on $c \rightarrow t$ transitions. The CMS Collaboration's results [\[50\]](#page-6-20) set an upper limit on the flavor-changing couplings a_{FC}^0 in the process $u_i g \rightarrow t \phi_0$,
with ϕ_0 (we deliberately add a subscript 0 to distinguished with ϕ_0 (we deliberately add a subscript 0 to distinguished it from ϕ) being a spin-0 invisible boson. If only considering ϕ_0 contributing to the *u*-*t* transition with the coupling $a_{\text{FC}}^0 = 0.1$, the observed data well fit the SM expectations
with no excess signal being observed at the 95% confidence with no excess signal being observed at the 95% confidence level, and a new particle ϕ_0 with mass below 330 GeV is excluded.

This constraint can be relaxed for a pseudoscalar ϕ which mediates flavor-changing interaction. If $\theta \sim 10^{-2}$ is taken, the *c*-t coupling λ_{tc} is of order 10⁻³. The *u*-t coupling

² should be even smaller and is about 3.6×10^{-5} with a λ_{tu} should be even smaller and is about 3.6×10^{-5} with a small current mass of m \approx 2.3 MeV. Thus, the results small current mass of $m_u \sim 2.3$ MeV. Thus, the results obtained by the CMS Collaboration (see Fig. 3 of Ref. [\[50\]\)](#page-6-20) imply that the contribution of ϕ to the single top production is very small compared with that of the SM, and the pseudoscalar ϕ -relevant signals are swamped by the SM background.

In the energy range of concern, the monotop production at the LHC may not be employed for determining the new flavor-changing interactions, and no solid information about WIMPs can be extracted from the measurements on the corresponding missing energy.

B. $t \rightarrow c$ decay

Let us now turn to the $t \to c$ decay which was discussed in the Introduction as an optimistic channel for detecting WIMPs and here consider the mass of WIMPs in the range 5–60 GeV. The $b\bar{b}$ pairs are the dominant products in WIMP annihilation into SM, and very few $c\bar{c}$, $\tau\bar{\tau}$, $b\bar{s}$, etc., may take a small fraction. The relic abundance of cold DM observed today is employed to restrict the parameter spaces.

1. Constraints set by the WIMP annihilations

The relic density of cold DM today is $\Omega_c h^2 = 0.1197 \pm$ 0.0022 [\[56\],](#page-6-26) and this value can be taken to restrict the parameter spaces. We set $m_t = 173.21$, $m_b(\overline{MS}) = 4.18$, $m_c = 1.275$, $m_s = 0.095$, $m_\tau = 1.7768$ GeV according to the Particle Data Group values [\[57\]](#page-6-27).

Let us define $2m_\chi/m_\phi = \xi$, with $\xi < 1$ here. When the mass m_{ϕ} is slightly above $2m_{\gamma}$, the WIMP pair annihilation rate will be enhanced. Using WIMP relic density, we can derive that the thermal-averaged annihilation cross section $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle$ of concern is about $(1.63 - 1.76) \times$ 10^{-9} GeV⁻²(~(1.91 – 2.06) × 10⁻²⁶ cm³/s). There is a critical value ξ_c , when $\xi \gtrsim \xi_c$, the annihilation cross section $\langle \sigma_{\rm ann} v_r \rangle_{b\bar{b}}$ (the dominant products $b\bar{b}$ in SM) at $T = 0$ will not be smaller than $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle$ at $T = T_f$. The numerical

TABLE I. The numerical results of ξ_c , for $\lambda_D = 0.5, 1.0,$ $m_{\gamma} = 5, 20, 35, 50, 60$ GeV.

m_{χ} (GeV)	5	20	35	50	60
$\xi_c(\lambda_D=0.5)$	0.9971	0.9971	0.9972	0.9972	0.9973
$\theta_c(\xi_c) \times 10^{-2}$	0.107	0.330	0.562	0.795	0.950
$\xi_c(\lambda_D=1.0)$	0.9940	0.9939	0.9940	0.9941	0.9941
$\theta_c(\xi_c) \times 10^{-2}$	0.120	0.374	0.638	0.903	1.08

results of ξ_c are shown in Table [I,](#page-4-0) for $\lambda_D = 0.5, 1.0,$ $m_{\gamma} = 5$, 20, 35, 50, 60 GeV. The new dwarf galaxy $b\bar{b}$ channel annihilation limits [\[38\]](#page-6-12) show that, $\langle \sigma_{ann} v_r \rangle_{b\bar{b}} \lesssim$ $\langle \sigma_{\rm ann} v_{\rm rel} \rangle$, for WIMPs with masses below a hundred GeV. Thus, ξ_c can be taken as an upper bound of ξ , i.e. $\xi \lesssim \xi_c$.

To meet the DM relic density observed, there is a ξ_c dependent parameter θ_c , and θ_c is determined by the case of $\xi \sim \xi_c$, $\theta \sim \theta_c$. The results of $\theta_c(\xi_c)$ are also given in Table [I,](#page-4-0) for the set WIMP masses and λ_D . It can be seen that θ_c varies within a range of 10^{-3} – 10^{-2} , and θ_c is taken as the lower bound of θ i.e. $\theta > \theta$. Meanwhile θ needs to be as lower bound of θ , i.e. $\theta \gtrsim \theta_c$. Meanwhile, θ needs to be as small as possible associated with SM. An upper limit $\theta \lesssim \theta_u$ with θ_u of order 10⁻² is of concern, and correspondingly, there is a θ_u -dependent lower limit ξ_l for ξ , $\xi \gtrsim \xi_l$. Considering the low-energy flavor-changing constraints in quark and lepton sectors discussed above, we have

$$
\frac{\xi \theta}{2m_{\chi}(\text{GeV})} \lesssim \frac{10^{-2}}{20}.
$$
 (13)

Noting a m_{χ} -dependent parameter $\theta_{m_{\chi}}$, with

$$
\theta_{m_{\chi}} = 10^{-2} \times \frac{m_{\chi}(\text{GeV})}{10};\tag{14}
$$

thus, $\theta_u \sim \theta_{m_u}$ can be taken as an upper limit. The numerical results of ξ_l are given in Table [II](#page-4-1) for $\theta_u = \theta_{m_x}$, along with $\theta_u = 1.0 \times 10^{-2}$ as comparison.

When the parameters ξ , θ vary in the range $\xi_c \gtrsim \xi \gtrsim \xi_l$, $\theta_c \lesssim \theta \lesssim \theta_u$, respectively, to satisfy the observed DM relic
density the WIMP application cross section $\langle \tau, u \rangle$, is density, the WIMP annihilation cross section $\langle \sigma_{ann} v_r \rangle_{b\bar{b}}$ is also determined. The result of $\langle \sigma_{ann} v_r \rangle_{b\bar{b}}$ is depicted in Fig. [2](#page-4-2), for WIMP masses varying from 5 to 60 GeV, and $\theta = \theta_u \sim \theta_{m_v}$, 1.0×10^{-2} , $\lambda_D = 1.0$, 0.5, respectively. The coupling λ_D dominates the width Γ_{ϕ} , which is important

TABLE II. The numerical results of ξ_l , for $\theta_u = \theta_{m_v}$, 1.0×10^{-2} , $\lambda_D = 0.5$, 1.0, $m_{\gamma} = 5$, 20, 35, 50, 60 GeV. The 'null' in the table means no parameter space.

	m_{γ} (GeV)	5 ⁵	20	35	50	60
$\theta_u = \theta_{m_v}$	$\xi_l(\lambda_D = 0.5)$ 0.9164 0.9126 0.9135 0.9144 0.9181 $\xi_l(\lambda_D = 1.0)$ 0.9147 0.9095 0.9103 0.9111 0.9117 $\theta_u = 0.010 \xi_l(\lambda_D = 0.5)$ 0.8838 0.9436 0.9694 0.9862 0.9948					
	$\xi_l(\lambda_D = 1.0)$ 0.8775 0.9439 0.9716 0.9892 null					

FIG. 2 (color online). The WIMP annihilation cross section $\langle \sigma_{\rm ann} v_r \rangle_{b\bar{b}}$ as a function of m_χ , for m_χ varying in the range 5–60 GeV. The dashed-dotted, solid-dotted curves are for the case of $\theta = \theta_u \sim \theta_{m_y}$, 1.0 × 10⁻², respectively, with the upper one and lower one in each type two curves corresponding to $\lambda_D = 1.0$ and 0.5, respectively. The solid curve is the WIMP annihilation cross section $\langle \sigma_{ann} v_{rel} \rangle$. The dashed curves are the Pass 8 dSphs upper limits of bb from the Fermi-LAT Collaboration [\[38\]](#page-6-12) for the combined limit of 68% containment and 95% containment, respectively, from bottom to top.

near the resonance in WIMP annihilations. In the case $\theta \sim \theta_c$, $\langle \sigma_{\rm ann} v_r \rangle_{h\bar{h}}$ approximately equals $\langle \sigma_{\rm ann} v_{\rm rel} \rangle$. It can be seen that, considering the Pass 8 dwarf spheroidal satellite galaxies (dSphs) upper limits of bb from the Fermi-LAT Collaboration [\[38\],](#page-6-12) there are still parameter spaces allowed.

As pointed by the analysis of Ref. [\[37\],](#page-6-11) the new dwarf galaxy limits [\[38\]](#page-6-12) can be compatible with the galaxy center excess. In this case, when the value of θ is about θ_c , the corresponding $\langle \sigma_{ann} v_r \rangle_{b\bar{b}}$ is ~10⁻²⁶ cm³/s, which can fit the GeV gamma-ray excess.

2. The decay rate

Now we turn to the WIMP production in $t \to c$ decay with the allowed parameter spaces. The ranges of parameters θ , ξ determine the decay width $\Gamma_{t\rightarrow c\bar{\nu}\gamma}$. According to Eq. [\(4\)](#page-2-3) and neglecting m_c/m_t , the decay width is rewritten as

$$
\Gamma_{t \to c\bar{\chi}\chi} \simeq \frac{1}{16\pi} m_t |\lambda_{tc}|^2 \left(1 - \frac{4m_\chi^2}{\xi^2 m_t^2}\right)^2.
$$
 (15)

The total top quark decay width is 2.0 ± 0.5 GeV [\[57\]](#page-6-27). Substituting the ranges of θ , ξ , we can obtain the branching ratio $\mathcal{B}_{t\to c\bar{\chi}\chi}$, as shown in Fig. [3](#page-5-0), where m_{χ} varies within the range 5 GeV $\leq m_{\gamma} \leq 60$ GeV, and $\Gamma_t = 2.0$ GeV is taken. The case of $\theta = \theta_c$ is adopted as the lower limit, with $\lambda_D = 0.5, 1.0.$ The case of $\theta = \theta_u = \theta_{m_x}$, 1.0×10^{-2} is taken as the upper limit, and for each θ_u the curves with $\lambda_D = 0.5$, 1.0 are overlapped. There is no parameter space
for $m \ge 55$ GeV, with $\theta = 1.0 \times 10^{-2}$ and $\lambda = 1.0$ for $m_\chi \gtrsim 55$ GeV with $\theta_u = 1.0 \times 10^{-2}$ and $\lambda_D = 1.0$. When θ varies between θ_c and θ_u , the branching ratio $\mathcal{B}_{t\to c\bar{\chi}\chi}$ is of order 10⁻⁸–10⁻⁵. There is about 25%

FIG. 3 (color online). The branching ratio $\mathcal{B}_{t\to c\bar{\chi}\chi}$ as a function of m_{γ} , for m_{γ} varying from 5 to 60 GeV. $\Gamma_t = 2.0$ GeV is taken. The dashed, solid curves correspond to $\lambda_D = 0.5$ and 1.0, respectively, and for both cases we set $\theta = \theta_c$. The dashed and solid overlapping curves are for the case of $\theta = \theta_u$ (for the dashed, solid part, $\lambda_D = 0.5$ and 1.0, respectively), with the upward bending and the downward bending curves corresponding to $\theta_u = \theta_{m_v}$, 1.0 × 10⁻², respectively.

uncertainty for the branching ratio, which is mainly coming from the value of the top quark decay width.

The $t \to c\bar{\chi}\chi$ decay can be searched at LHC, e.g., by the process $pp \rightarrow t\bar{t} \rightarrow c\phi + \bar{b}W^-$ with missing energy, which was discussed in Ref. [\[12\]](#page-6-28). There are about 10⁶ order $t\bar{t}$ pairs production [\[58,59\]](#page-6-29) at 8 TeV center-of-mass energy at LHC run I, and maybe a few events produced. For the LHC run at 14 TeV, the top quark pair production cross section is about 953.6 pb [\[60\].](#page-6-30) As indicated in Ref. [\[12\]](#page-6-28), for the branching ratio $t \to c\bar{\chi}\chi$ of order 10⁻⁵, the decay mode seems challenging to observe at the LHC's next run.

The WIMP signature may appear in $t \to c$ decay with missing energy at the high-energy collider or the corresponding upper limit of the θ is set by experiments. The observation of cosmic-ray excess from DM dense region indirectly gives the property of WIMP annihilations or raises the lower limit of θ (meanwhile, it may also give constraints on the upper limit of θ as well). Thus, the pseudoscalar ϕ -mediated WIMPs will be examined by the joint effort of the collider search and cosmic-ray observation.

V. CONCLUSION AND DISCUSSION

The scenario of a pseudoscalar ϕ -mediating FCNC Yukawa-type interaction which results in a transition of $t \rightarrow c$ is discussed in this work. In this model, the WIMPnucleus scattering is highly suppressed, so that one cannot expect to "see" them via direct detection, and the small θ value and fermion masses (except the top quark) let the new interactions be tolerated by the present observation at the LHC. In the case of $2m_\gamma < m_\phi < m_t$, the SM channels in ϕ decay are suppressed, and the $\bar{\chi}\chi$ channel is dominant. The missing energy in the process of the single top production and $t \rightarrow c$ decay is employed to search for WIMPs. For the single top production with missing energy at the hadron collider, we notice that the contribution of ϕ is swamped in the SM background when $\theta \sim 10^{-2}$.

We consider the mass of WIMPs in the range 5–60 GeV and the dominant products of WIMPs annihilating into SM particles are $b\bar{b}$ pairs, and this scenario is favored by the discovery of a gamma-ray excess from the galaxy center. The WIMP relic abundance is employed to restrict the parameter spaces. We find that considering the constraints of Pass 8 dSphs upper limits of $b\bar{b}$ [\[38\]](#page-6-12), there is parameter space left for $\langle \sigma_{ann} v_r \rangle_{b\bar{b}} \lesssim \langle \sigma_{ann} v_{rel} \rangle$, and the GeV gamma-ray excess can be fitted within the parameter space. Thus, our numerical analysis about WIMP pair annihilations is still effective even if the bb mode no longer fits the Galactic center gamma-ray excess. With the parameter spaces allowed, we derive the branching ratio $\mathcal{B}_{t\to c\bar{\chi}\chi}$ which is of order $10^{-8}-10^{-5}$.
The t solution is a explored at the LHC a

The $t \rightarrow c\bar{\chi}\chi$ decay can be explored at the LHC and the proposed ILC, while its observation seems challenging for the next run at LHC. The very clean environment for $t\bar{t}$ production at ILC [\[42\]](#page-6-13) and FCC-ee (TLEP) [\[61\]](#page-6-31) will offer an incredible opportunity to measure the $t \to c\bar{\chi}\chi$ decay. For the e^+e^- collider at 500 GeV, the $t\bar{t}$ production cross section is about 0.572 pb [\[62\],](#page-6-32) and thus the branching ratio $t \to c\bar{\chi}\chi$ of order 10⁻⁵ can be searched with an integrated luminosity of 1000 fb⁻¹. Thus, the excess favored WIMPs from the galaxy center could be explored at a future $e^+e^$ collider. Even though one cannot expect to gain a definite conclusion, measurement on $t \to c$ transitions with missing energy at the collider can help to reduce the upper limit of θ , whereas the indirect observation of WIMP annihilations will raise the lower limit of θ .

Moreover, when the WIMP pair mass is above the top quark mass, the $t\bar{c}(\bar{t}c)$ mode is opened in WIMP pair annihilations, and another alternative interpretation of the gamma-ray excess is the WIMP pair annihilating into the top-charm quark mode [\[63\].](#page-6-33) In this case, there is still parameter space for the case $m_t + m_c < 2m_\gamma < 2m_t$ to satisfy the constraints, while searching for the WIMPs in the top quark decay is impossible.

Though the pseudoscalar ϕ -mediated WIMPs could evade our direct detection for the highly suppressed WIMP-nucleus scattering, with the joint efforts of the collider search, cosmic-ray observation, and other means, the scenario where the pseudoscalar ϕ mediates interaction between WIMPs and SM particles will be tested.

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SEARCH FOR PSEUDOSCALAR-MEDIATED WIMPS IN ... PHYSICAL REVIEW D 92, 074006 (2015)

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