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Radial excitations of mesons and nucleons from QCD sum rules

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Within the framework of QCD sum rules, we use the least-squares fitting method to investigate the first radial excitations of the nucleon and light mesons such as ρ, K^*, π, and φ. The extracted masses of these radial excitations are consistent with the experimental data. In particular, we find that the decay constant of $\pi(1300)$, which is the first radial excitation of π, is tiny and is strongly suppressed as a consequence of chiral symmetry.

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I. INTRODUCTION

The QCD sum rules method has been widely used to extract resonance information in hadron physics [1]. This formalism is usually applied to studying the ground state in a specific channel due to the limitation of theoretical accuracy and the difficulty of numerical analysis. The excitation of mesons has been studied within finite energy sum rules in the literature [2–4]. Recently, there have been some attempts to study the excitation of heavy-light mesons using the QCD sum rules method [5].

The radial excitations have the same spin-parity as the ground state. Experimentally, many radial excitations of mesons and baryons have been established [6]. Sometimes it is quite difficult to identify the radial excitations of hadrons. For example, the situation involving radial excitations of the vector charmonium above 4 GeV becomes quite unclear after so many charmoniumlike XYZ states have been reported experimentally in the past decade. Theoretical investigations of the radial excitations are also very challenging.

In this work, we shall study the first radial excitations of the light mesons and the nucleon within the framework of the QCD sum rule formalism. We explicitly keep two poles in the usual spectrum representation. Then, we employ the least-squares method in the numerical analysis to extract the resonance information of the first radial excited state. The extracted masses of the radial excitations of the light mesons and the nucleon agree with the experimental data quite well.

The paper is organized as follows. In Sec. II, we introduce the QCD sum rule formalism and our least-squares method. The numerical results are presented in Secs. III–VII. The last section is a short summary.

II. FORMALISM

Within the framework of the QCD sum rule approach, we study the correlation function at the quark level

$$\Pi(q) = \mathrm{i} \int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}qx} \langle 0|T\{j(x) j^\dagger(0)\}|0\rangle, \tag{1}$$

where $j(x)$ is the interpolating current with the same quantum numbers as the hadrons. The above correlation function satisfies the dispersion relation

$$\Pi(q^2) = \frac{1}{\pi} \int_{s_{\min}} ds \frac{\mathrm{Im}\Pi(s)}{s - q^2 - \mathrm{i}\epsilon}. \tag{2}$$

At the quark-gluon level, the correlation function can be calculated with the operator product expansion. The gluon and quark condensates appear as higher dimensional operators in this expansion. At the hadron level, the spectral density of the correlation function can be expressed in terms of the hadron masses and couplings. Because of the quark hadron duality, we get an equation called the QCD sum rule which relates the correlation function at the quark-gluon level to the physical states. After making a Borel transformation to the sum rule in the momentum space, one gets

$$\Pi'(M^2) = \frac{1}{\pi} \int \mathrm{e}^{-s/M^2} \mathrm{Im}\Pi(s) \mathrm{d}s, \tag{3}$$

where M is the Borel parameter.

The spectral density usually takes the one-pole approximation

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$$\rho(s) \equiv \frac{1}{\pi}\mathrm{Im}\Pi(s) = f\delta(s-m^2) + \rho_{\mathrm{continuum}}\theta(s-s_0), \tag{4}$$

where m is the mass of the ground state and s_0 is the threshold parameter. Above s_0, the spectral density at the hadron level is replaced by the spectral density derived at the quark-gluon level. Now the sum rule reads

$$f\mathrm{e}^{-m^2/M^2} = \Pi'(M^2) - \int_{s_0}^{\infty} \mathrm{e}^{-s/M^2}\rho^{\mathrm{OPE}}(s)\mathrm{d}s. \tag{5}$$

The usual numerical method in QCD sum rule analysis is to differentiate Eq. (5) with respect to $1/M^2$ and divide the resulting equation by Eq. (5):

$$m^2 = \frac{\int_0^{s_0} \mathrm{e}^{-s/M^2} s\rho^{\mathrm{OPE}}(s)\mathrm{d}s}{\int_0^{s_0} \mathrm{e}^{-s/M^2}\rho^{\mathrm{OPE}}(s)\mathrm{d}s}. \tag{6}$$

One usually plots the variation of the mass versus M^2 and s_0 to find a working window.

However, the method described above can only be applied to the ground states. In order to extract the resonance information of the first radial excitation, we modify the above spectral density and explicitly keep the pole of the first radial excitation in the spectrum. Now the modified spectral density reads

$$\rho(s) \equiv \frac{1}{\pi}\mathrm{Im}\Pi(s) = f_1\delta(s-m^2) + f_2\delta(s-m'^2) + \rho_{\mathrm{continuum}}\theta(s-s_0'). \tag{7}$$

To simply the numerical analysis, we use the zero width approximation for both the ground state and first radial excitation. The parameters f_1 and f_2 are related to the coupling parameters, while m and m' are the masses of the ground state and the first radial excitation, respectively. Now the sum rules read

$$\int \mathrm{e}^{-s/M^2}\rho_{\mathrm{ground}}(s)\mathrm{d}s + \int \mathrm{e}^{-s/M^2}\rho_{\mathrm{excitation}}(s)\mathrm{d}s + \int_{s_0}^{\infty} \mathrm{e}^{-s/M^2}\rho_{\mathrm{continuum}}(s)\mathrm{d}s = \Pi'(M^2) = \Pi'^{\mathrm{perturbation}}(M^2) + \Pi'^{\mathrm{condensates}}(M^2).$$

The usual numerical method cannot be applied here because the modified spectrum has two mass parameters. We use the least-squares method [7] to fit these masses and decay parameters. The details of the method are described below.

As usual in the sum rule analysis, one has to find an optimal working interval of the Borel parameter M^2. The lower boundary of M^2 is chosen to ensure the convergence of the operator product expansion, while the upper boundary is chosen to make the continuum contribution remain subleading.

To get an optimal interval of the Borel parameter M^2, we set

$$\left|\frac{\int_{s_0}^{\infty} \mathrm{e}^{-s/M^2}\rho_{\mathrm{continuum}}(s)\mathrm{d}s}{\Pi'(M^2)}\right| \le \alpha_1, \tag{8}$$

which ensures that the continuum contribution remains subleading and determines the upper boundary and

$$\left|\frac{\Pi'^{\mathrm{condensates}}(M^2)}{\Pi'(M^2)}\right| \le \alpha_2, \tag{9}$$

which ensures that the operator product expansion (OPE) is reliable and determines the lower boundary. The two boundaries determine the optimal interval of M^2 for our numerical analysis.

The numbers α_1 and α_2 are chosen to ensure a rational contribution of continuum and higher order OPE terms. For the meson case, we set $\alpha_1 = \alpha_2 = \alpha$ to get a reasonable interval of M^2. We use different values for α_1 and α_2 in the nucleon case. Note that we always try a smaller α in the excitation case since the continuum contribution decreases as the threshold parameter s_0 increases. If no reasonable interval of M^2 can be gotten in any way, the sum rule may not be appropriate in our numerical method.

We rewrite the sum rule as

$$\int \mathrm{e}^{-s/M^2}\rho_{\mathrm{ground}}(s)\mathrm{d}s + \int \mathrm{e}^{-s/M^2}\rho_{\mathrm{excitation}}(s)\mathrm{d}s = g(M^2, s_0) = \Pi'(M^2) - \int_{s_0}^{\infty} \mathrm{e}^{-s/M^2}\rho_{\mathrm{continuum}}(s)\mathrm{d}s,$$

which separates the part of expression with physical parameters from the part with just the Borel parameter M^2 and the threshold s_0.

With the above expression of $g(M^2, s_0)$, we can generate a series of points $\{(M_i^2, g(M_i^2, s_0))\}$ by choosing a set $\{M_i^2\}$ within the optimal interval of M^2. We uniformly choose N points in the optimal interval of M^2. The number N is chosen to be 20 or even larger.

With the sets $\{(M_i^2, g(M_i^2, s_0))\}$, we use the least-squares method which minimizes the sum of the squares of the difference between the two sides of the sum rules,

$$\sum_{i=1}^{N} \frac{\left|f_1\mathrm{e}^{-\frac{m^2}{M_i^2}} + f_2\mathrm{e}^{-\frac{m'^2}{M_i^2}} - g(M_i^2, s_0)\right|^2}{N} = \min, \tag{10}$$

to get the best fit of the resonance parameters of the ground state and the first radial excitation.

The masses of the ground states of the light mesons and the nucleon are measured precisely experimentally. The extracted masses from the traditional QCD sum rule formalism with the one-pole approximation agree with the experimental data very well. In our analysis we first use the least-squares method to reproduce the resonance parameters of the ground states. As expected, the resulting masses are consistent with the experimental data and the data extracted from the traditional QCD sum rule analysis.

Then we use the extracted masses of the ground states as inputs to extract the resonance parameters of the radial excited states since fewer parameters in the fitting will require fewer computing resources and will lead to relatively more stable results. Moreover, we do not fix the masses of the ground states in Eq. (10) in our numerical analysis. Instead, we allow them to vary around the experimental central value within ±5%. In this way, we extract the resonance parameters of the first radial excited states numerically.

We analyze several light mesons and nucleons in the following section. The sum rules of the light mesons can be found in the pioneer paper [1]. The nucleon sum rule with the radiative corrections can be found in Ref. [8]. We collect these sum rules in the Appendix.

In our analysis we use the following values for the various condensates and parameters [1,6,9]: $\langle\bar{q}q\rangle(2\text{ GeV}) = -(277^{+12}_{-10}\text{ MeV})^3$, $\langle 0|m_u\bar{u}u + m_d\bar{d}d|0\rangle = -\frac{1}{2}f_\pi^2 m_\pi^2 = -1.7\times 10^{-4}\text{ GeV}^4$, $m_s(2\text{GeV}) = (95\pm 5)\text{MeV}$, $\langle\bar{s}s\rangle/\langle\bar{q}q\rangle = 0.8\pm 0.3$, $\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle = 0.012^{+0.006}_{-0.012}\text{GeV}^4$, $\langle 0|\alpha_s(\bar{u}\gamma_\alpha\gamma_5 t^a u - \bar{d}\gamma_\alpha\gamma_5 t^a d)^2|0\rangle = \frac{32}{9}\alpha_s\langle 0|\bar{q}q|0\rangle^2 \simeq 6.5\times 10^{-4}\text{ GeV}^4$, $\langle 0|\alpha_s(\bar{u}\gamma_\alpha\gamma_5 t^a u - \bar{d}\gamma_\alpha\gamma_5 t^a d)\times\sum_{q=u,d,s}\bar{q}\gamma_\alpha t^a q|0\rangle \simeq -\frac{32}{9}\alpha_s\langle 0|\bar{q}q|0\rangle^2 \simeq -6.5\times 10^{-4}\text{ GeV}^4$, $\alpha_s(Q^2) = 4\pi/(b\ln(Q^2/\Lambda^2))$, $\Lambda = 0.1$ GeV, $\alpha_s(m_Z) = 0.1184\pm 0.0007$, and $\alpha_s(1.5\text{ GeV}) = 0.353\pm 0.006$.

III. THE ρ MESON

The interpolating current for the ρ meson is

$$j_\mu^{(\rho)} = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d), \tag{11}$$

and the resulting sum rule can be found in the Appendix. The usual single-pole spectral density reads

$$\rho^{(\rho)}(s) = 6\pi^2 f_\rho^2\delta(s-m_\rho^2) + \frac{3}{2}\left(1+\frac{\alpha_s(s)}{\pi}\right)\theta(s-s_0). \tag{12}$$

We also need the double-pole spectral density

$$\rho^{(\rho)}(s) = 6\pi^2 f_\rho^2\delta(s-m_\rho^2) + 6\pi^2 f_{\rho'}^2\delta(s-m_{\rho'}^2) + \frac{3}{2}\left(1+\frac{\alpha_s(s)}{\pi}\right)\theta(s-s_0), \tag{13}$$

where f_ρ and $f_{\rho'}$ are defined as

$$\langle 0|\bar{q}\gamma_\mu q|\rho\rangle = m_\rho f_\rho\epsilon_\mu, \qquad \langle 0|\bar{q}\gamma_\mu q|\rho'\rangle = m_{\rho'}f_{\rho'}\epsilon'_\mu, \tag{14}$$

where $q = u, d$.

We first use the least-squares method and the traditional one-pole spectrum representation with $\alpha = 0.2$ and $N = 40$ to extract the mass and decay constant of the ρ meson. The results are listed in Table I. The parameter f_1 is related to the decay constant in Eq. (4). The values of "min" are the sum of the squares of the differences in Eq. (10). Only when the value of min is much smaller than the parameters f_1^2, f_2^2, etc., are the fit and the extracted decay constants reliable.

We collect the fitting results with the double-pole spectrum in Table II. Note that the parameter m in Table II is the input to extract the information of the excited state. We use $\alpha = 0.1$ in this case. The threshold s_0 plays the role of including the first radial excitation in the spectrum while excluding the contribution from the higher excitations. To check the consistency of our fitting and the dependence of our results on s_0, we vary s_0 in a range. A reliable fitting requires that the mass m' and the decay constant $f_{\rho'}$ of the first radial excitation should not vary too much with s_0.

From Table I we have

$$m = (0.76\pm 0.01)\text{ GeV}, \quad f_\rho = (194\pm 6)\text{ MeV}, \tag{15}$$

TABLE I. The mass and decay constant of the ρ ground state with $\alpha = 0.2$ and $N = 40$.

s_0 [GeV^2]	1.2	1.3	1.4	1.5	1.6
M^2_{min} [GeV^2]	0.43	0.43	0.43	0.43	0.43
M^2_{max} [GeV^2]	0.74	0.82	0.88	0.94	1.00
m [GeV]	0.74	0.75	0.75	0.76	0.77
f_ρ [MeV]	187	190	193	197	201
f_1 [GeV^2]	2.06	2.13	2.22	2.30	2.39
min [GeV^4]	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}

TABLE II. Masses and decay constants of the ρ ground state and first radial excitation, with $\alpha = 0.1$ and $N = 40$.

s_0 [GeV^2]	2.3	2.4	2.5	2.6	2.7
M^2_{min} [GeV^2]	0.50	0.50	0.50	0.50	0.50
M^2_{max} [GeV^2]	1.00	1.04	1.10	1.14	1.18
m [GeV]	0.76	0.76	0.76	0.76	0.76
m' [GeV]	1.24	1.29	1.35	1.38	1.40
f_ρ [MeV]	196	197	198	198	198
$f_{\rho'}$ [MeV]	130	141	152	161	170
f_1 [GeV^2]	2.3	2.3	2.3	2.3	2.3
f_2 [GeV^2]	1.0	1.2	1.4	1.5	1.7
min [GeV^4]	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}

which agrees with the ρ meson mass from PDG, $m = 0.77$ GeV [6], and the experimental measurement of the ρ meson decay constant [10]

$$f_\rho^{\rm exp} \simeq 216(5)\ \text{MeV}. \quad (16)$$

In order to reduce the dependence on the threshold parameter s_0, the extracted values of m and f_ρ are the average values of the numerical values in Table I. From Table II we have

$$m' = (1.33 \pm 0.07)\ \text{GeV},$$
$$f_\rho = (197 \pm 1)\ \text{MeV}, f_{\rho'} = (151 \pm 16)\ \text{MeV}. \quad (17)$$

From PDG, the mass of the first radial excitation is $m' = 1.47$ GeV and its width is $\Gamma = 0.40$ GeV. Our extracted ρ' mass is consistent with the experimental data. At present, the decay constant of ρ' has not been measured yet.

IV. THE π AND A_1 MESONS

We adopt the axial current for the pion and A_1 mesons

$$j_{5\mu}^{A_1} = \bar{u}\gamma_\mu\gamma_5 d, \quad (18)$$

and the resulting sum rule can be found in the Appendix. Besides the a_1 pole, the pion also contributes to this sum rule due to the partial conservation of the axial vector current. As a Goldstone boson, the pion mass is tiny. Especially in the sum rule analysis, m_π^2 is much, much less than the Borel parameter M^2. We can safely ignore the pion mass and let it be zero in the numerical analysis.

The usual spectrum representation is

$$\rho(s) = \pi f_\pi^2 \delta(s) + \pi f_{A_1}^2 \delta(s - m_{A_1}^2) + \frac{1}{4\pi}\left(1 + \frac{\alpha_s(s)}{\pi}\right)\theta(s - s_0). \quad (19)$$

Our modified spectrum representation reads

$$\rho^{(\pi)}(s) = \pi f_\pi^2 \delta(s) + \pi f_{\pi'}^2 \delta(s - m_{\pi'}^2) + \pi f_{A_1}^2 \delta(s - m_{A_1}^2) + \frac{1}{4\pi}\left(1 + \frac{\alpha_s(s)}{\pi}\right)\theta(s - s_0), \quad (20)$$

where f_π, $f_{\pi'}$ f_{A_1} are defined as

$$\langle 0|j_\mu^\pi|\pi\rangle = i f_\pi p_\mu, \qquad \langle 0|j_\mu^\pi|\pi'\rangle = i f_{\pi'} p'_\mu,$$
$$\langle 0|j_\mu^\pi|A_1\rangle = m_{A_1} f_{A_1} \epsilon'_\mu. \quad (21)$$

In the fitting, we use the least-squares method and the traditional spectrum representation with $\alpha = 0.3$ and $N = 80$ to extract the A_1 mass and decay constant. The results are listed in Table III.

In order to extract the resonance parameters of the first excitation of the pion meson, we employ the modified spectrum and allow f_{A_1} and m_{A_1} to vary around the experimental data within $\pm 5\%$. The numerical results are listed in Table IV.

From Table III, we have

$$m_{A_1} = (1.22 \pm 0.06)\ \text{GeV}, \qquad f_\pi = (135 \pm 1)\ \text{MeV},$$
$$f_{A_1} = (151 \pm 20)\ \text{MeV}. \quad (22)$$

From PDG, we have $m_{A_1} = 1.23$ GeV and $\Gamma_{A_1} = 0.40$ GeV. We note that the A_1 mass from the fitting is in rough agreement with the experimental data. The extracted pion decay constant agrees with the experimental data [6]:

$$f_\pi^{\rm exp} = 130\ \text{MeV}. \quad (23)$$

However, the extracted A_1 decay constant is only half of the experimental data [11]:

TABLE III. The mass and decay constant of the A_1 meson. We use the least-squares method and the traditional spectrum representation with $\alpha = 0.3$ and $N = 80$.

s_0 [GeV^2]	1.3	1.40	1.50	1.60	1.70
$M^2_{\rm min}$ [GeV^2]	0.52	0.52	0.52	0.52	0.52
$M^2_{\rm max}$ [GeV^2]	1.16	1.20	1.28	1.36	1.44
m_{A_1} [GeV]	1.14	1.18	1.22	1.26	1.28
f_π [MeV]	134	135	136	137	137
f_{A_1} [MeV]	124	139	153	166	175
f_1 [GeV^2]	0057	0.057	0.058	0.059	0.059
f_2 [GeV^2]	0.048	0.060	0.074	0.087	0.096
min [GeV^4]	10^{-7}	10^{-7}	10^{-7}	10^{-8}	10^{-8}

TABLE IV. Masses and decay constants of the A_1 ground state and the first radial excitation of the pion with $\alpha = 0.2$ and $N = 80$.

s_0 [GeV^2]	2.0	2.1	2.2	2.3	2.4	2.5	2.6
$M^2_{\rm min}$ [GeV^2]	0.63	0.63	0.63	0.63	0.63	0.63	0.63
$M^2_{\rm max}$ [GeV^2]	1.28	1.34	1.38	1.44	1.50	1.56	1.62
m_{A_1} [GeV]	1.29	1.29	1.29	1.29	1.29	1.29	1.29
m'_π [GeV]	1.34	1.36	1.31	1.34	1.41	1.43	1.46
f_π [MeV]	121	122	123	123	124	125	126
f_{A_1} [MeV]	248	248	248	248	248	248	248
$f_{\pi'}$ [MeV]	0.2	0.3	0.1	2.3	0.2	0.7	0.1
f_1 [GeV^2]	0.05	0.05	0.05	0.05	0.05	0.05	0.05
f_2 [GeV^2]	0.19	0.19	0.19	0.19	0.19	0.19	0.19
f_3 [GeV^2]	10^{-7}	10^{-7}	10^{-8}	10^{-5}	10^{-7}	10^{-6}	10^{-8}
min [GeV^4]	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}

$$f_{A_1}^{\exp} = 254(20)\ \text{MeV}. \tag{24}$$

To extract the first radial excitation of the pion meson, we use the experimental data of the A_1 decay constant as input in the numerical analysis. The results are collected in Table IV. We have

$$m_{\pi'} = (1.38 \pm 0.06)\ \text{GeV}, \qquad f_\pi = (123 \pm 1)\ \text{MeV},$$
$$f_{\pi'} = (0.6 \pm 0.8)\ \text{MeV}. \tag{25}$$

The resulting mass of the pion radial excitation agrees with the PDG value very nicely: $m_{\pi'} = 1.30$ GeV and $\Gamma_{\pi'} = 0.40$ GeV [6]. Note that the extracted numerical value of $f_{\pi'}$ is not reliable since the parameter f_3^2 is even smaller than the min. In this case, we may get an upper bound

$$|f_3| < \sqrt{\text{min}} \sim 0.0032\ \text{GeV}^2. \tag{26}$$

Accordingly, we get the upper bound for $f_{\pi'}$,

$$f_{\pi'} < 0.032\ \text{GeV}. \tag{27}$$

If the value of $f_{\pi'}$ is larger than 0.032 GeV, we should be able to extract its value through the least-squares fitting method.

In other words, our numerical analysis demonstrates that the decay constant of the pion radial excitation π' is much smaller than the pion decay constant around 130 MeV. This interesting fact was also noticed in previous theoretical work including lattice simulations [3,4,12–20]. In fact, the suppression of the π' decay constant is a consequence of the chiral symmetry breaking. In the chiral limit, the decay constants of the pion and its radial excitations satisfy the following relation [21]:

$$f_{\pi_n} m_{\pi_n}^2 = 0, \tag{28}$$

where m_{π_n} $(n \geq 1)$ is the mass of the pion radial excitation. The pion ground state is massless in the chiral limit as a Goldstone boson; hence, its decay constant can be large and nonzero. For the pion radial excitation, its mass is large and nonzero. Therefore, its decay constant has to vanish, i.e., $f_{\pi_1} = 0$.

V. THE K^* MESON

The interpolating current for the K^* meson is

$$j_\mu^{(K^*)} = \bar{u}\gamma_\mu s \tag{29}$$

and the resulting sum rule can be found in the Appendix. The usual single-pole spectral density reads

$$\rho(s) = \pi f_{K^*}^2 \delta(s - m_{K^*}^2) + \frac{1}{4\pi}\left(1 + \frac{\alpha_s(s)}{\pi}\right)\theta(s - s_0). \tag{30}$$

Our modified spectrum representation reads

$$\rho^{(K^*)}(s) = \pi f_{K^*}^2 \delta(s - m_{K^*}^2) + \pi f_{K^{*\prime}}^2 \delta(s - m_{K^{*\prime}}^2) + \frac{1}{4\pi}\left(1 + \frac{\alpha_s(s)}{\pi}\right)\theta(s - s_0), \tag{31}$$

where f_{K^*} and $f_{K^{*\prime}}$ are defined as

$$\langle 0|j_\mu^{(K^*)}|K^*\rangle = m_{K^*} f_{K^*} \epsilon_\mu, \qquad \langle 0|j_\mu^{(K^*)}|K^{*\prime}\rangle = m_{K^{*\prime}} f_{K^{*\prime}} \epsilon'_\mu. \tag{32}$$

The results from the first spectrum representation are listed in Table V and those from the modified spectrum are listed in Table VI. From Table V we have

$$m = (0.89 \pm 0.01)\ \text{GeV}, \quad f_{K^*} = (210 \pm 7)\ \text{MeV}. \tag{33}$$

From Table VI, we have

$$m' = (1.28 \pm 0.06)\ \text{GeV}, \qquad f_{K^*} = (203 \pm 3)\ \text{MeV},$$
$$f_{K^{*\prime}} = (155 \pm 11)\ \text{MeV}, \tag{34}$$

where m is an input parameter in Table VI. The decay constant of K^* was measured to be [10]

TABLE V. The mass and decay constant of the K^* ground state. We use the least-squares method and the traditional spectrum representation with $\alpha = 0.3$ and $N = 20$.

s_0 [GeV^2]	1.4	1.5	1.6	1.6	1.8
$M^2_{\min}$ [GeV^2]	0.63	0.63	0.63	0.63	0.63
$M^2_{\max}$ [GeV^2]	1.10	1.18	1.28	1.36	1.44
m [GeV]	0.88	0.89	0.90	0.90	0.91
f_{K^*} [MeV]	202	206	210	215	219
f_1 [GeV^2]	0.13	0.13	0.14	0.14	0.15
min [GeV^4]	10^{-7}	10^{-7}	10^{-7}	10^{-7}	10^{-7}

TABLE VI. Masses and decay constants of the K^* ground state and the first radial excitation with $\alpha = 0.2$ and $N = 20$.

s_0 [GeV^2]	2.3	2.4	2.5	2.6	2.7	2.8
$M^2_{\min}$ [GeV^2]	0.63	0.63	0.63	0.63	0.63	0.63
$M^2_{\max}$ [GeV^2]	1.40	1.46	1.52	1.58	1.64	1.70
m [GeV]	0.89	0.89	0.89	0.89	0.89	0.89
m' [GeV]	1.22	1.25	1.27	1.33	1.29	1.37
f_{K^*} [MeV]	200	201	202	200	207	207
$f_{K^{*\prime}}$ [MeV]	139	146	153	162	159	172
f_1 [GeV^2]	0.13	0.13	0.13	0.12	013	0.13
f_2 [GeV^2]	0.06	0.07	0.08	0.08	0.08	0.09
min [GeV^4]	10^{-8}	10^{-8}	10^{-8}	10^{-8}	10^{-8}	10^{-8}

$$f_{K^*}^{\rm exp} \simeq 217 \text{ MeV}. \tag{35}$$

From PDG, the mass and the width of $K^{*\prime}$ are $m' = 1.41$ GeV and $\Gamma = 0.232$ GeV, respectively. Clearly our extracted f_{K^*} from both fittings agrees with the data. The extracted m' is also consistent with the data.

VI. THE φ MESON

The interpolating current for the φ meson is

$$j_\mu^{(\varphi)} = -\frac{1}{3}\bar{s}\gamma_\mu s, \tag{36}$$

and the resulting sum rule can be found in the Appendix. The usual spectrum representation is

$$\rho(s) = \frac{1}{9}\pi f_\varphi^2 \delta(s - m_\varphi^2) + \frac{1}{36\pi}\left(1 + \frac{\alpha_s(s)}{\pi}\right)\theta(s - s_0). \tag{37}$$

We also use the modified spectrum representation

$$\rho^{(\varphi)}(s) = \frac{1}{9}\pi f_\varphi^2 \delta(s - m_\varphi^2) + \frac{1}{9}\pi f_{\varphi'}^2 \delta(s - m_{\varphi'}^2) + \frac{1}{36\pi}\left(1 + \frac{\alpha_s(s)}{\pi}\right)\theta(s - s_0), \tag{38}$$

where f_φ and $f_{\varphi'}$ are defined as

$$\langle 0|\bar{s}\gamma^\mu s|\varphi\rangle = m_\varphi f_\varphi \epsilon_\mu, \qquad \langle 0|\bar{s}\gamma^\mu s|\varphi'\rangle = m_{\varphi'} f_{\varphi'} \epsilon'_\mu. \tag{39}$$

We use the least-squares method and the traditional spectrum representation with $N = 20$. Note that there does not exist a working interval of M^2 for $\alpha = 0.2$. So we use $\alpha = 0.3$ here. The results from the first spectrum representation are listed in Table VII and those from the modified spectrum are listed in Table VIII, where m is the input parameter in Table VIII.

From PDG, the mass and the width of the φ ground state are $m = 1.020$ GeV and $\Gamma = 0.004$ GeV, while $m' = 1.68$ GeV, $\Gamma = 0.20$ GeV for the first radial excitation. The decay constant of ground state was measured to be [10]

$$f_\varphi^{\rm exp} = 233 \text{ MeV}. \tag{40}$$

From Table VII we have

$$m = (1.04 \pm 0.02) \text{ GeV}, \qquad f_\varphi = (229 \pm 9) \text{ MeV}. \tag{41}$$

From Table VIII we have

$$m' = (1.54 \pm 0.07) \text{ GeV}, \qquad f_\varphi = (210 \pm 8) \text{ MeV}, \qquad f_{\varphi'} = (228 \pm 11) \text{ MeV}. \tag{42}$$

The decay constant of the φ meson from both fittings agrees with the data very well, while the extracted mass of the first radial excitation is in rough agreement with the data.

TABLE VII. The mass and decay constant of the φ ground state. We use the least-squares method and the traditional spectrum representation with $N = 20$ and $\alpha = 0.3$ here.

s_0 [GeV^2]	1.7	1.8	1.9	2.0	2.1	2.2
$M^2_{\rm min}$ [GeV^2]	0.87	0.87	0.87	0.87	0.87	0.87
$M^2_{\rm max}$ [GeV^2]	1.66	1.78	1.90	2.00	2.12	2.24
m [GeV]	1.02	1.03	1.03	1.04	1.05	1.06
f_φ [Mev]	217	221	226	231	236	240
f_1 [GeV^2]	0.016	0.017	0.018	0.019	0.019	0.020
min [GeV^4]	10^{-9}	10^{-9}	10^{-9}	10^{-9}	10^{-9}	10^{-9}

TABLE VIII. Masses and decay constants of the φ ground state and the first radial excitation with $\alpha = 0.2$ and $N = 20$.

s_0 [GeV^2]	3.4	3.5	3.6	3.7	3.8	3.9	4.0
$M^2_{\rm min}$ [GeV^2]	1.15	1.15	1.15	1.15	1.15	1.15	1.15
$M^2_{\rm max}$ [GeV^2]	2.02	2.08	2.16	2.22	2.28	2.36	2.42
m [GeV]	1.00	1.00	1.00	1.00	1.00	1.00	1.00
m' [GeV]	1.45	1.64	1.52	1.55	1.62	1.50	1.51
f_φ [MeV]	203	221	210	213	218	203	202
$f_{\varphi'}$ [MeV]	215	215	222	226	230	240	246
f_1 [GeV^2]	0.014	0.017	0.015	0.016	0.017	0.014	0.014
f_2 [GeV^2]	0.016	0.016	0.017	0.018	0.018	0.020	0.021
min [GeV^4]	10^{-10}	10^{-10}	10^{-10}	10^{-10}	10^{-10}	10^{-10}	10^{-9}

VII. THE NUCLEON

The interpolating current for the nucleon is

$$\eta = \epsilon^{abc}[u^{aT}Cd^b]\gamma^5 u^c - \epsilon^{abc}[u^{aT}C\gamma^5 d^b]u^c \tag{43}$$

and the resulting sum rule [8] can be found in the Appendix. The usual spectrum representation for the nucleon is

$$\rho^{(N)}(s) = \beta_N^2 \delta(s - m^2) + \rho_{\rm continuum}(s)\theta(s - s_0), \tag{44}$$

where

$$\rho_{\rm continuum}(s) = \frac{1}{\pi}{\rm Im}\Pi(s) = \frac{s^2}{4(2\pi)^4}\left(1 + \frac{71}{12}\frac{\alpha_s}{\pi} - \frac{\alpha_s}{\pi}\ln\frac{s}{\mu^2}\right) + \frac{1}{(2\pi)^2}\frac{1}{8}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle - \frac{2\langle\bar{q}q\rangle^2}{9}\frac{\alpha_s}{\pi}\frac{1}{s}. \tag{45}$$

We also use the modified spectrum representation

$$\rho(s) = \beta_N^2 \delta(s - m^2) + \beta_{N'}^2 \delta(s - m'^2) + \rho_{\rm continuum}(s)\theta(s - s_0), \tag{46}$$

TABLE IX. The mass of the nucleon ground state with $\alpha_1 = 0.8$, $\alpha_2 = 0.4$, and $N = 20$.

s_0 [GeV^2]	1.80	1.85	1.90	1.95	2.0
$M^2_{\min}$ [GeV^2]	0.7	0.7	0.7	0.7	0.7
$M^2_{\max}$ [GeV^2]	1.52	1.54	1.58	1.62	1.64
m [GeV]	0.89	0.91	0.93	0.94	0.96
β_N^2	1.9	2.0	2.1	2.2	2.4
min	10^{-4}	10^{-4}	10^{-4}	10^{-4}	10^{-4}

TABLE X. The masses of the nucleon ground state and the first radial excitation with $\alpha_1 = 0.7$, $\alpha_2 = 0.3$, and $N = 20$.

s_0 [GeV^2]	2.1	2.15	2.20	2.25	2.30	2.35	2.40
$M^2_{\min}$ [GeV^2]	0.83	0.83	0.83	0.83	0.83	0.83	0.83
$M^2_{\max}$ [GeV^2]	1.36	1.38	1.40	1.42	1.44	1.48	1.50
m [GeV]	0.929	0.929	0.929	0.929	0.929	0.929	0.929
m' [GeV]	1.45	1.47	1.48	1.50	1.52	1.53	1.55
β_N^2 input	2.1	2.1	2.1	2.2	2.2	2.2	2.2
$\beta_{N'}^2$	0.67	0.86	1.06	1.28	1.50	1.76	2.00
min	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}

where $\beta_N^2 = 32\pi^4\lambda_N^2$, $\beta_{N'}^2 = 32\pi^4\lambda_{N'}^2$ and λ_N is the overlapping amplitude of the interpolating current with the nucleon state.

The results from the first spectrum representation are listed in Table IX and those from the modified spectrum are listed in Table X. To get stable results, we have used the nucleon mass and $\beta_N^2 = 2.1$ from Table IX as an input in the numerical analysis of the first radial excitation. From PDG, the nucleon mass is $m = 0.938$ GeV, while the mass and the width of its first radial excitation are $m' = 1.44$ GeV and $\Gamma' = 0.300$ GeV. From Table IX, we have

$$m = (0.93 \pm 0.03)\ \text{GeV}. \tag{47}$$

From Table X, we have

$$m' = (1.50 \pm 0.04)\ \text{GeV}, \tag{48}$$

which is in rough agreement with the data.

VIII. SUMMARY

To summarize, we attempted to extract the masses of the first radial excited states of the light mesons and the nucleon. In our modified hadronic spectral density, we explicitly kept the pole of the first radial excited states together with the ground state. Requiring that the operator product expansion converge and that the continuum contribution be subleading led to the optimal working interval of the Borel parameter M^2. Then a series of "data" points (or pseudo data points) were produced within this working interval of M^2. Using the usual one-pole spectral density, we were able to extract the mass of the ground state with the least-squares fitting method, which agreed with the experimental data. Then we used these data points and the mass of the ground state as input parameters to extract the mass and the decay constant of the first radial excited state by the least-squares method, which was in good agreement with the available data.

The QCD sum rule method has its inherent accuracy limit due to the various approximations adopted within this framework, such as the truncation of the OPE series of the correlation function, the assumption of the quark-hadron duality, the omission of the decay width in the spectral density, the factorization of the four quark condensates, the uncertainties of the values of the various condensates, etc. In our analysis we only included the uncertainty from the fitting using the least-squares method itself. The least-squares method with the modified spectrum representation allows us to extract useful information from the first radial excitations, which depends on the accuracy of the sum rules. It will be very interesting to explore whether such a formalism can be applied to other hadrons.

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APPENDIX: QCD SUM RULES OF THE LIGHT MESONS AND NUCLEON

For the ρ meson,

$$\begin{aligned}\int \mathrm{d}s e^{-s/M^2}\rho(s) = \frac{3}{2}M^2\Bigg[&1 + \frac{\alpha_s(M)}{\pi} + \frac{4\pi^2\langle 0|m_u\bar{u}u + m_d\bar{d}d|0\rangle}{M^4} \\ &+ \frac{1}{3}\pi^2\frac{\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle}{M^4} - 2\pi^3\frac{\langle 0|\alpha_s(\bar{u}\gamma_\alpha\gamma_5 t^a u - \bar{d}\gamma_\alpha\gamma_5 t^a d)^2|0\rangle}{M^6} \\ &- \frac{4}{9}\pi^3\frac{\langle 0|\alpha_s(\bar{u}\gamma_\alpha t^a u + \bar{d}\gamma_\alpha t^a d\sum_{q=u,d,s}\bar{q}\,\gamma_\alpha t^a q)|0\rangle}{M^6}\Bigg].\end{aligned} \tag{A1}$$

For the π meson,

$$\int e^{-s/M^2}\rho(s)ds = \frac{M^2}{4\pi}\left[1+\frac{\alpha_s(M)}{\pi}+\frac{1}{3}\pi^2\frac{\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle}{M^4}+\frac{4\pi^3\alpha_s\langle 0|\bar{u}\gamma_\alpha\gamma_5 t^a d\bar{d}\gamma_\alpha\gamma_5 t^a u|0\rangle}{M^6}\right.$$
$$\left.-\frac{4}{9}\pi^3\alpha_s\frac{\langle 0|(\bar{u}\gamma_\alpha t^a u+\bar{d}\gamma_\alpha t^a d\sum_{q=u,d,s}\bar{q}\gamma_\alpha t^a q)|0\rangle}{M^6}\right]. \tag{A2}$$

For the K^* meson,

$$\int ds e^{-s/M^2}\rho^{(K^*)}(s) = \frac{M^2}{4\pi}\left[1+\frac{\alpha_s(M)}{\pi}+4\frac{\pi^2\langle 0|m_u\bar{u}u+m_s\bar{s}s|0\rangle}{M^4}\right.$$
$$+\frac{1}{3}\pi^2\frac{\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle}{M^4}-2\pi^3\frac{\langle 0|\alpha_s(\bar{u}\gamma_\alpha\gamma_5 t^a u-\bar{s}\gamma_\alpha\gamma_5 t^a s)^2|0\rangle}{M^6}$$
$$\left.-\frac{4}{9}\pi^3\frac{\langle 0|\alpha_s(\bar{u}\gamma_\alpha t^a u+\bar{s}\gamma_\alpha t^a s\sum_{q=u,d,s}\bar{q}\gamma_\alpha t^a q)|0\rangle}{M^6}\right]. \tag{A3}$$

For the φ meson,

$$\int e^{-s/M^2}\rho^{(\varphi)}(s)ds = \frac{M^2}{36\pi}\left[1+\frac{\alpha_s(M)}{\pi}-\frac{6m_s^2(M)}{M^2}+\frac{8\pi^2\langle 0|m_s\bar{s}s|0\rangle}{M^4}+\frac{1}{3}\pi^2\frac{\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle}{M^4}-\frac{448}{81}\pi^3\alpha_s(\mu)\frac{\langle 0|\bar{q}q|0\rangle^2}{M^6}\right]. \tag{A4}$$

For the nucleon [8],

$$\tilde{A}_0+\tilde{A}_4+\tilde{A}_6+\tilde{A}_8 = \beta_N^2 e^{-m^2/M^2}+\beta_{N'}^2 e^{-m'^2/M^2}, \tag{A5}$$

where

$$\tilde{A}_0(M^2,W^2) = M^6E_2\left[1+\frac{\alpha_s}{\pi}\left(\frac{53}{13}-\ln\frac{W^2}{\mu^2}\right)\right]-\frac{\alpha_s}{\pi}\left[M^4W^2\left(1+\frac{3W^2}{4M^2}\right)e^{-\frac{W^2}{M^2}}+M^6\varepsilon\left(-\frac{W^2}{M^2}\right)\right]$$
$$\tilde{A}_4(M^2,W^2) = \frac{bM^2E_0}{4L}$$
$$\tilde{A}_6(M^2,W^2) = \frac{4}{3}a^2\left[1-\frac{\alpha_s}{\pi}\left(\frac{5}{6}+\frac{1}{3}\left(\ln\frac{W^2}{\mu^2}+\varepsilon\left(-\frac{W^2}{M^2}\right)\right)\right)\right]$$
$$a = -(2\pi)^2\langle\bar{q}q\rangle,\qquad b=(2\pi)^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle,\qquad \beta_N=(2\pi)^4\lambda_N^2,\qquad \alpha_s(1\ \text{GeV})\approx 0.37$$
$$E_0 = 1-e^{-x},\qquad E_2 = 1-\left(1+x+\frac{1}{2}x^2\right)e^{-x},$$

with $x=W^2/M^2$, $\varepsilon(x)=\sum_n\frac{x^n}{n\cdot n!}$,

$$L = \frac{\ln(M^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}.$$

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