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Lepton mixing from the hidden sector

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Experimental results indicate a possible relation between the lepton and quark mixing matrices of the form $U_{\mathrm{PMNS}} \approx V_{\mathrm{CKM}}^{\dagger} U_X$, where U_X is a matrix with special structure related to the mechanism of neutrino mass generation. We propose a framework which can realize such a relation. The main ingredients of the framework are the double seesaw mechanism, SO(10) grand unification and a hidden sector of theory. The latter is composed of singlets (fermions and bosons) of the grand unified theory (GUT) symmetry with masses between the GUT and Planck scale. The interactions in this sector obey certain symmetries G_{hidden}. We explore the conditions under which symmetries G_{hidden} can produce flavor structures in the visible sector. Here the key elements are the basis-fixing symmetry and mediators which communicate information about properties of the hidden sector to the visible one. The interplay of SO(10) symmetry, basis-fixing symmetry identified as $\mathbb{Z}_2 \times \mathbb{Z}_2$ and G_{hidden} can lead to the required form of U_X. A different kind of new physics is responsible for generation of the CKM mixing. We present the simplest realizations of the framework which differ by nature of the mediators and by symmetries of the hidden sector.

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I. INTRODUCTION

There are various indications that in spite of their big difference the quark and lepton mixings are somehow related. One appealing possibility can be formulated as a relation between the lepton mixing matrix, U_{PMNS}, and the quark mixing matrix, V_{CKM}, of the following form [1–8]:

$$U_{\mathrm{PMNS}} = U_{\mathrm{CKM}}^{\dagger} U_X, \tag{1}$$

where

$$U_{\mathrm{CKM}} \sim V_{\mathrm{CKM}} \tag{2}$$

is a unitary matrix which may coincide with the CKM mixing matrix or, in general, has the same hierarchical structure as V_{CKM} with expansion parameter $\lambda = \sin\theta_C$. We use here the notation U_{CKM} to underline the close connection to (the same origin as) the quark mixing matrix V_{CKM}. The unitary matrix U_X is related to additional structures in the lepton sector which are responsible for the smallness of neutrino masses, and it may be of special form originating from certain symmetries. To be in agreement with the data, U_X should have vanishing (or small) 1-3 mixing and large (or even maximal) 2-3 mixing.

The relation (1) has been explored on pure phenomenological grounds in [1]. It has been proposed in the framework of quark-lepton complementarity [2] with $U_X = \Gamma_\alpha U_{\mathrm{BM}}$, where U_{BM} is the bimaximal mixing matrix [9,10] and $\Gamma_\alpha = \mathrm{diag}(e^{i\alpha_e}, 1, 1)$. Later variations of (1) have been explored, in particular the TBM-Cabibbo mixing scheme [11] with $U_{\mathrm{CKM}} = U_{12}(\theta_C)$ and $U_X = U_{\mathrm{TBM}}$, where U_{TBM} is the tri-bimaximal mixing matrix [12]. Also golden-ratio mixing [13] has been considered with $U_X = U_{\mathrm{GR}}$ [11]. All these cases have maximal 2-3 mixing, zero 1-3 mixing but differ by the values of 1-2 mixing.

The measured value of θ_{13} supports the relation (1). Indeed, for $U_{\mathrm{CKM}} = U_{12}(\theta_C)$ and $U_X = U_{23}^{\max} U_{12}$ (where U_{12} is arbitrary) the lepton mixing according to (1) becomes $U_{\mathrm{PMNS}} = U_{12}(\theta_C)^T U_{23}^{\max} U_{12}$. Reducing it to the standard parametrization form leads immediately to

$$\sin^2\theta_{13} = \frac{1}{2}\sin^2\theta_C. \tag{3}$$

Here the coefficient $1/2$ originates from maximal 2-3 mixing, i.e., $\sin^2\theta_{23}^X = 1/2$. Equation (3) was in agreement with data in the first approximation. However, recent precise measurements of the leptonic 1-3 mixing angle [14–16] show a deviation from (3) by about 3σ. Indeed, with Cabibbo mixing $\sin\theta_C = 0.22537 \pm 0.00061$ [17] we have

$$\frac{1}{2}\sin^2\theta_C = 0.02540 \pm 0.00014, \tag{4}$$

whereas the most accurate value of $\sin^2(2\theta_{13}) = 0.084 \pm 0.005$ [15] gives

$$\sin^2\theta_{13} = 0.0215 \pm 0.0013. \tag{5}$$

Notice that the relative difference between the values in (4) and (5), ~0.18, is of the order of the small elements of the

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CKM matrix, $2\lambda^2 \sim 0.1$, and can therefore be substantially reduced if the CKM corrections—due to the use of the complete $V_{\rm CKM}$ in Eq. (1)—are taken into account [2]. The remaining difference can be due to nonmaximal 2-3 mixing in U_X. It can originate from some difference between $U_{\rm CKM}$ and $V_{\rm CKM}$ which, in turn, can be related to the difference of the masses of the charged leptons and down-type quarks.

Various data sets indicate that apart from the "visible" sector of theory, a "hidden sector" exists which is composed of singlets of the standard model (or GUT) gauge symmetry group. The hidden sector can be responsible for the dark sector of the Universe which includes particles of the dark matter, fields needed for inflation and particles involved in the generation of the lepton and baryon asymmetries of the Universe. Sterile neutrinos [18] of different masses (very light $\sim 10^{-3}$ eV [19]; eV-scale, as indicated by LSND [20], MiniBooNE [21] as well as the reactor [22] and Gallium [23–26] anomalies; keV-scale for warm dark matter [27]) can be manifestations of the hidden sector. Finally, the hidden sector could be responsible for the generation of small neutrino masses.

The hidden sector may not follow a generation structure and the number of new fermions as well as bosons can be bigger or even much bigger than three. The hidden sector particles may have their own interactions including gauge (dark photons) and Yukawa interactions. Moreover, the hidden sector may have its own symmetries $G_{\rm hidden}$, but there can be also some common symmetries with the visible sector. The origin of these symmetries as well as the components of the hidden sector can be compactification of extra dimensions in string theory [28].

In this paper we will update on the relation (1). We argue that it suggests the double seesaw mechanism, grand unification, and the presence of a hidden sector of theory. We propose a framework in which the required form of the matrix U_X originates from symmetries of the hidden sector, whereas $V_{\rm CKM}$ is generated by another kind of new physics.

The paper is organized as follows. In Sec. II we consider the status of relation (1) and discuss its implications. In Sec. III we formulate a framework which allows to realize the relation (1). Here the main ingredients of the framework, and in particular the required symmetries, are considered. Several specific realizations are presented in Sec. IV. In Sec. V we consider effects of additional fermions from the hidden sector. Section VI is devoted to the new physics which is responsible for the CKM-type mixings in the lepton sector. Discussion and conclusions follow in Sec. VII.

II. THE RELATION BETWEEN $U_{\rm PMNS}$ AND $V_{\rm CKM}$ AND ITS IMPLICATIONS

Let us consider U_X of general form with the only restriction that the 1-3 mixing is vanishing or very small:

$$U_X = \Gamma U_{23}(\theta_{23}^X) U_{12}(\theta_{12}^X), \qquad \Gamma \equiv {\rm diag}(1, e^{i\varphi_2}, e^{i\varphi_3}). \tag{6}$$

Here we have omitted the Majorana phases of neutrinos and absorbed one overall phase of Γ into the charged-lepton fields. Then, with (6) and exact equality $U_{\rm CKM} = V_{\rm CKM}$, which we will use for definiteness, the relation (1) yields expressions for the mixing parameters of $U_{\rm PMNS}$ in terms of θ_{ij}^X and φ_k. Thus, the 1-3 mixing equals

$$\sin^2\theta_{13} = \lambda^2 \sin^2\theta_{23}^X \left\{1 + 2\frac{|V_{td}|}{V_{cd}} \cot\theta_{23}^X \cos(\alpha + {\rm Arg}V_{td}) + \mathcal{O}(\lambda^4)\right\}, \tag{7}$$

where

$$\alpha \equiv \varphi_2 - \varphi_3 \tag{8}$$

and ${\rm Arg}V_{td} = -21.8° = -0.12\pi$. Using the Wolfenstein parametrization [29] for $V_{\rm CKM}$, Eq. (7) can be rewritten as

$$\sin^2\theta_{13} = \lambda^2 \sin^2\theta_{23}^X \{1 - 2A\lambda^2 \sqrt{(1-\rho^2) + \eta^2} \cot\theta_{23}^X \cos(\alpha + {\rm Arg}V_{td})\} + \mathcal{O}(\lambda^6), \tag{9}$$

where ${\rm Arg}V_{td} = \arctan\frac{\eta}{\rho - 1} + \mathcal{O}(\lambda^2)$. For the 2-3 mixing we obtain

$$\tan^2\theta_{23} = \tan^2\theta_{23}^X (1 - \lambda^2)\left\{1 - \frac{4A\lambda^2 \cos\alpha}{\sin 2\theta_{23}^X}\right\} + \mathcal{O}(\lambda^4). \tag{10}$$

Eliminating θ_{23}^X from Eqs. (7) and (10) immediately yields a relation between the lepton mixing parameters $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ as a function of α (see Fig. 1). Approximate analytic expressions for this relation can be obtained taking into account that near maximum $\sin 2\theta_{23}^X$ only weakly depends on θ_{23}^X. Using $\sin 2\theta_{23}^X = 1$ in the denominator of (10) we find

$$\sin^2\theta_{13} \approx \lambda^2 \frac{\tan^2\theta_{23}}{\zeta^2 + \tan^2\theta_{23}} \left\{1 + 2\frac{|V_{td}|}{V_{cd}} \zeta \cot\theta_{23} \cos(\alpha + {\rm Arg}V_{td}) + \mathcal{O}(\lambda^4)\right\}, \tag{11}$$

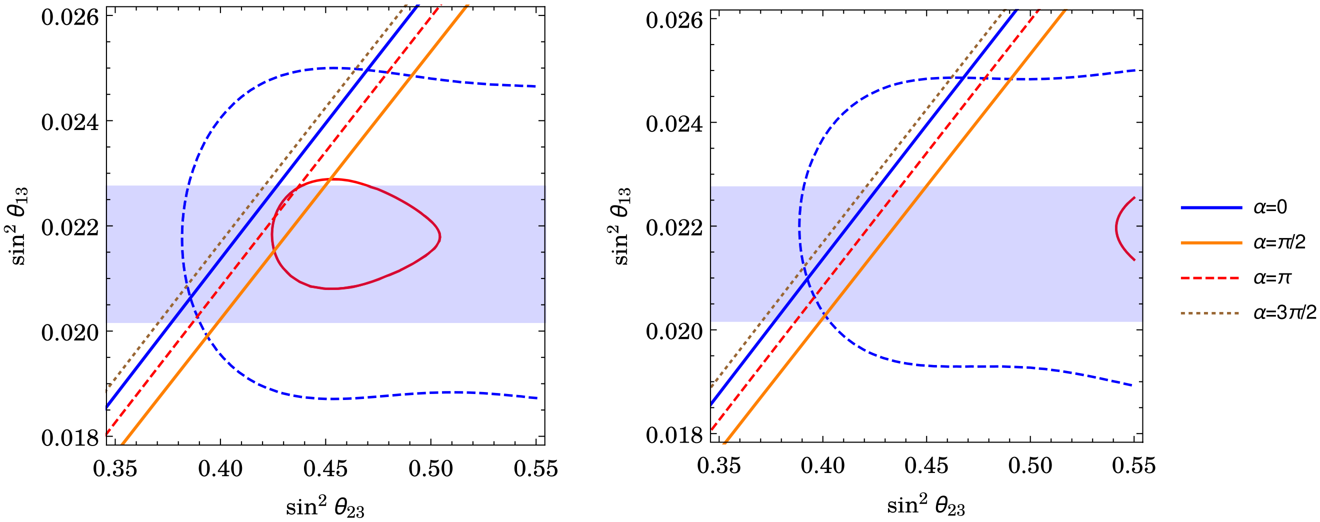

FIG. 1 (color online). Relation between $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ from Eq. (1) with $\theta_{13}^X = 0$ for different values of α. The CKM parameters have been set to the best-fit values of [17]. The 1σ (red solid line) and 3σ (blue dashed line) regions according to the global fit of [30,31] are shown for normal ordering (left plot) and inverted ordering (right plot). The blue band corresponds to the 1σ-range for $\sin^2\theta_{13}$ allowed by the recent results of the DayaBay experiment [15].

where

$$\zeta^2(\alpha) \equiv (1-\lambda^2)(1-4A\lambda^2\cos\alpha) \tag{12}$$

and we have used $\tan^2\theta_{23} \approx \zeta^2\tan^2\theta_{23}^X$. Figure 2 shows the values of θ_{23}^X and α allowed by experimental data. Notice that for $\theta_{23}^X = \pi/4$ (exactly maximal mixing) and $\alpha = -\mathrm{Arg}V_{td}$ we have $\sin^2\theta_{23} = 0.45$ and $\sin^2\theta_{13} = 0.0234$, which is just 1.5σ above the best value from experiment. As can be seen from Figs. 1 and 2, the relation (1) is in good agreement with experiment, especially for normal mass ordering and $\alpha \lesssim \pi$. Notice that these results should merely be considered as some orientation since in general U_{CKM} can deviate from V_{CKM}. Still the coincidence even at the correction level looks very appealing and we assume that it is not accidental.

Finally, the 1-2 mixing is determined by

$$\sin^2\theta_{12} = \sin^2\theta_{12}^X - \lambda\sin 2\theta_{12}^X\cos\theta_{23}^X\cos\varphi_2 + \lambda^2\cos 2\theta_{12}^X\cos^2\theta_{23}^X + \mathcal{O}(\lambda^3). \tag{13}$$

Fixing $\sin^2\theta_{12}$ to the best fit value of [30], and using the best fit value $\theta_{23}^X \approx 42°$ (see Fig. 2) we find from (13) the required range $\sin^2\theta_{12}^X \in [0.16, 0.47]$.

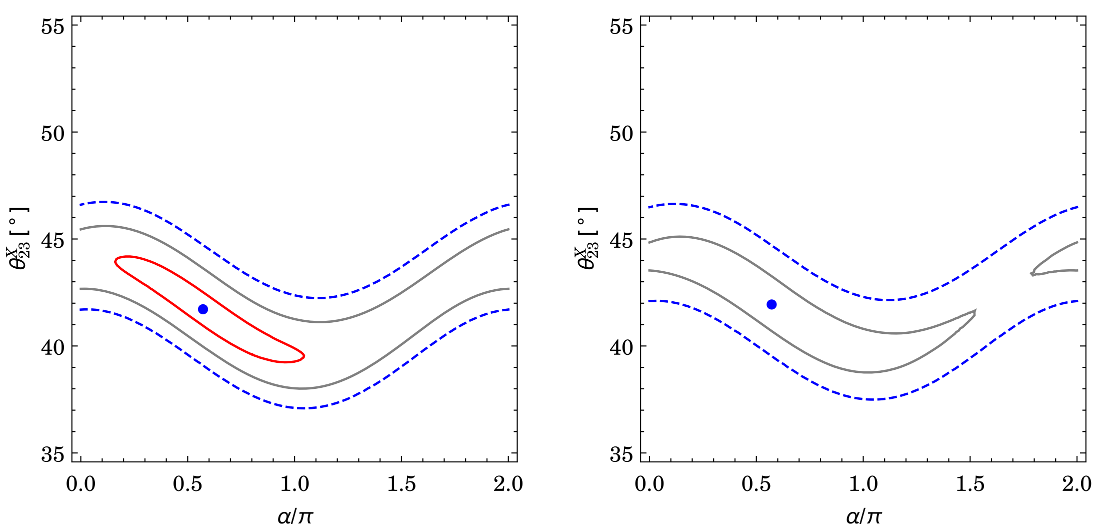

FIG. 2 (color online). The allowed regions for the parameters α and θ_{23}^X which reproduce relation (1). These regions have been computed from the two-dimensional projection of the χ^2-function of the global fit of [30,31] into the $(\sin^2\theta_{23}, \sin^2\theta_{13})$-plane. The red solid, grey solid and blue dashed lines are the boundaries of the 1σ, 2σ and 3σ regions, respectively. The blue dot corresponds to the best fit point. Left plot: normal mass ordering, right plot: inverted mass ordering.

The relation (1) means that information about the quark mixing is communicated somehow to the lepton mixing. In turn, this implies a kind of quark-lepton unification or/and common flavor symmetries in the quark and lepton sector [32]. Furthermore, Eq. (1) points toward the type-I seesaw mechanism or its extensions [33–37]. Indeed, the Dirac neutrino mass matrix m_D can be written as

$$m_D = U_L \hat{m}_D U_R^\dagger, \qquad \hat{m}_D \equiv \mathrm{diag}(m_1^D, m_2^D, m_3^D) \tag{14}$$

with U_L and U_R being unitary matrices of transformations of the left- and right-handed neutrino components, respectively. Then according to the seesaw mechanism the light-neutrino mass matrix is given by

$$m_\nu = -m_D M_R^{-1} m_D^T = U_L M_X U_L^T, \tag{15}$$

where

$$M_X \equiv -\hat{m}_D U_R^\dagger M_R^{-1} U_R^* \hat{m}_D. \tag{16}$$

If M_X is diagonalized by a unitary matrix U_X, i.e.,

$$U_X^T M_X U_X = \hat{M}_X, \tag{17}$$

the light-neutrino mass matrix m_ν is diagonalized by

$$U_\nu = U_L^* U_X. \tag{18}$$

The matrix U_L can be related to the quark mixing matrix V_{CKM} in grand unified theories [38,39]. An immediate realization is an SO(10)-GUT [40,41] with a dominant contribution of Higgs ten-plet fields $\underline{10}_H$ to the fermion mass terms. In this case all mass matrices are symmetric with $m_D \propto m_u$ and $m_\ell \propto m_d$ and thus, in the basis where $m_\ell \propto m_d$ is diagonal,

$$V_{\mathrm{CKM}} = U_u^\dagger = U_L^T \Rightarrow U_{\mathrm{PMNS}} = U_\nu = V_{\mathrm{CKM}}^\dagger U_X, \tag{19}$$

i.e., Eq. (1). For conditions allowing to realize the less restrictive relation $\theta_{13} \approx \theta_C/\sqrt{2}$—see Eq. (3)—in Pati-Salam and SU(5)-GUTs we refer the reader to [42].

According to our previous considerations, M_X should lead to vanishing or very small 1-3 mixing and close to maximal 2-3 mixing in U_X, i.e., M_X should be approximately invariant under the 2-3-permutation symmetry ($\mu\tau$-symmetry). Since we also need a sizeable $\sin^2\theta_{12}^X \gtrsim 0.16$, the matrix M_X should be close to the tri-bimaximal mass matrix

$$M_X \sim M_{\mathrm{TBM}}. \tag{20}$$

Furthermore, the light neutrinos have the weakest hierarchy among all known fermion species. Therefore, also M_X cannot be strongly hierarchical—see Eq. (15). On the other hand, the mentioned SO(10)-scenario, or generically the assumption of quark-lepton similarity $m_D \sim m_q \sim m_\ell$, suggests a strong hierarchy of m_D. From this it follows that

$$M_R = -U_R^* \hat{m}_D M_{\mathrm{TBM}}^{-1} \hat{m}_D U_R^\dagger \tag{21}$$

is extremely hierarchical (quadratical in the up-type quark mass hierarchy). This indicates that M_R itself is generated by a type-I seesaw mechanism, i.e., the double seesaw mechanism [43,44] for m_ν.

In order to implement the double seesaw, we add three heavy gauge-singlets S to the fermion sector, in which case the neutrino mass term reads

$$\mathcal{L}_\nu^{\mathrm{mass}} = -\frac{1}{2}\overline{n_L}\mathcal{M} n_L^c + \mathrm{H.c.}, \tag{22}$$

where

$$n_L = \begin{pmatrix} \nu_L \\ \nu_R^c \\ S \end{pmatrix} \tag{23}$$

and

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & m_{\nu S} \\ m_D^T & 0 & M_{RS} \\ m_{\nu S}^T & M_{RS}^T & M_S \end{pmatrix}. \tag{24}$$

Here M_S is the Majorana mass matrix of the new heavy fermions S and M_{RS} is a Dirac-type neutrino mass matrix of ν_R and S. In general also the mass matrix $m_{\nu S}$ connecting ν_L with S will be present. If M_S is invertible, the right-handed Majorana neutrino mass matrix M_R has the form

$$M_R \approx -M_{RS} M_S^{-1} M_{RS}^T, \tag{25}$$

so if M_{RS} is hierarchical, M_R will have the desired strong hierarchy. The light-neutrino mass matrix m_ν is approximately given by

$$m_\nu \approx m_\nu^{DS} + m_\nu^{LS}, \tag{26}$$

where

$$m_\nu^{DS} = m_D (M_{RS}^{-1T} M_S M_{RS}^{-1}) m_D^T \tag{27}$$

is the double seesaw contribution and

$$m_\nu^{LS} = -[m_D (m_{\nu S} M_{RS}^{-1})^T + (m_{\nu S} M_{RS}^{-1}) m_D^T] \tag{28}$$

is the linear seesaw [45] contribution to m_ν. If M_S is singular but M_{RS} has rank three (and is thus invertible), we find

TABLE I. The different scales involved in the presented double seesaw framework.

Mass matrix	Scale
M_S	$\sim 10^{18}$ GeV $\sim \frac{1}{10} M_{\rm Pl}$
M_{RS}	$\sim 10^{16}$ GeV $\sim M_{\rm GUT}$
$M_R \approx -M_{RS} M_S^{-1} M_{RS}^T$	$\sim 10^{14}$ GeV
$m_{\nu S}$	$\sim 10^2$ GeV $\sim M_{\rm EW}$
m_D	$\sim 10^2$ GeV $\sim M_{\rm EW}$
$m_\nu^{DS} = m_D (M_{RS}^{-1T} M_S M_{RS}^{-1}) m_D^T$	$\sim 10^{-1}$ eV
$m_\nu^{LS} = -[m_D (m_{\nu S} M_{RS}^{-1})^T + (m_{\nu S} M_{RS}^{-1}) m_D^T]$	$\sim 10^{-3}$ eV

$$\begin{pmatrix} 0 & M_{RS} \\ M_{RS}^T & M_S \end{pmatrix}^{-1} = \begin{pmatrix} -(M_{RS}^{-1})^T M_S M_{RS}^{-1} & (M_{RS}^{-1})^T \\ M_{RS}^{-1} & 0 \end{pmatrix} \qquad (29)$$

in which case Eqs. (27) and (28) still hold, while Eq. (25) does not.

If M_{RS} is at GUT-scale, and the new fermions S have masses at about one order of magnitude below the Planck scale we obtain $M_R \sim 10 M_{\rm GUT}^2 / M_{\rm Pl} \sim 10^{14}$ GeV. From the study of the ranges of the masses of the right-handed neutrinos in the case of $m_D \sim m_u$ [46] one can find that, depending on the values of the Dirac and Majorana phases of m_ν, a value of the mass of the heaviest right-handed neutrino $M_{R3} \sim 10^{14}$ GeV is possible for a smallest neutrino mass $m_0 \gtrsim (10^{-3} \div 10^{-2})$ eV. The mass ranges for the other two right-handed neutrinos in this case are $M_{R2} \sim (10^8 \div 10^{10})$ GeV and $M_{R1} \sim (10^4 \div 10^8)$ GeV [46]. If $m_0 \simeq (10^{-4} \div 10^{-3})$ eV, a higher scale $M_{R3} \simeq (10^{15} \div 10^{16})$ GeV is required. The relevant mass scales for the double seesaw scenario with $M_{R3} \sim 10^{14}$ GeV are shown in Table I. The linear seesaw term is dominated by the double seesaw contribution, but it may still play a subleading role in the phenomenology of m_ν.

To summarize, the realization of the relation (1) implies

(i) The seesaw mechanism,

(ii) quark-lepton unification (e.g., an SO(10)-GUT) or/and common flavor symmetries in the quark and lepton sector,

(iii) "CKM physics" leading to small quark mixing $V_{\rm CKM}$, and

(iv) new physics in the neutrino sector generating $U_X \sim U_{\rm TBM}$.

Moreover, quark-lepton similarity indicates the double seesaw mechanism for the generation of the light-neutrino mass matrix m_ν. The new fermionic singlets can be components of the hidden sector of theory.

III. FRAMEWORK

The main ingredients of the framework which can realize relation (1) include

(i) SO(10) grand unification (although other GUT symmetries can be considered).

(ii) The existence of a hidden sector composed of fermions and bosons, which are singlets of SO (10). The interactions in the hidden sector may have certain symmetries.

(iii) The basis-fixing symmetry and mediators which communicate information about the structure/interactions of the hidden sector to the visible one.

(iv) The double (or even more complicated) seesaw mechanism which ensures complete or partial screening of the Dirac structures.

(v) Separation of the physics responsible for the CKM mixing from the physics responsible for large neutrino mixing.

In the following we will discuss these ingredients in detail.

A. The visible and the hidden sector

We consider an SO(10)-GUT with three families of fermions in 16-plets $\underline{16}_F$. The dominant contribution to the fermion mass terms is generated by ten-plet Higgs-fields $\underline{10}_H$. Two or more $\underline{10}_H$ are needed to generate the different mass hierarchies of the up- and down-components of the doublets: Namely, one Higgs ten-plet field $\underline{10}_H^u$ gives rise to the up-type quark mass matrix m_u and the Dirac neutrino mass matrix m_D, and another Higgs field $\underline{10}_H^d$ is responsible for the mass matrices of the down-type quarks m_d and charged leptons m_ℓ and for CKM mixing [47,48]. Additional physics is required to generate the mass hierarchy of quarks and leptons. Further complication is needed to explain the difference of the masses of the charged leptons and down-type quarks. We refer to all this as "CKM new physics" which we will comment on in Sec. VI.

In the following we will call the set of particles which have nontrivial transformation properties under SO(10) the "visible sector" of theory. We refer to the hidden sector as to the system of particles (fermions and bosons) and fields which are singlets of SO(10). The interactions in this sector (Yukawa and new gauge interactions) may have a certain symmetry $G_{\rm hidden}$. The idea is that this hidden sector symmetry is responsible for the generation of U_X with the required properties. In general, the hidden symmetry can include several different factors and the hidden sector fields may have all possible charge assignments with respect to these factors. Also, there can be some common symmetry in the hidden and visible sector and the charges of multiplets in the visible sector can be such that they allow to couple them with only few components from the hidden sector.

For the remainder of this paper, the most important part of the hidden sector will be gauge singlet fermions S, which are needed in order to implement the double seesaw mechanism. However, also scalar gauge singlets $\underline{1}_\chi$ will play an important role for the realization of the hidden sector symmetry. *A priori*, we may add an arbitrary number of SO(10)-singlet fermions S with a mass scale of $M_S \sim 10^{18}$ GeV to the fermion content of our framework, which in the Lagrangian are denoted by $\underline{1}_S$. However, if there are less than three of these singlets coupling to the active neutrinos,[1] the matrix M_{RS} will have rank smaller than three, in which case the right-handed neutrino mass matrix M_R will be singular, a case we do not want to study here. Therefore, at least three singlets S contributing to M_{RS} should be introduced. In what follows we will consider only three singlets which couple with the visible sector directly. This is also needed to realize screening [49,50] of the Dirac structures. The case of more than three singlets will be considered in Sec. V.

To connect the visible and the hidden sector and generate M_{RS} we need to introduce scalar 16-plet(s) $\overline{\underline{16}}_H$. The generation of the neutrino masses via the double seesaw mechanism allows us to avoid introduction of the high-dimensional multiplets $\underline{120}_H$ and $\underline{126}_H$. The absence of $\underline{120}_H$ and $\underline{126}_H$ is in fact desirable, because including such high-dimensional scalar representations is known to give rise to Landau poles in the gauge coupling already before reaching the Planck scale $M_{\mathrm{Pl}} \sim 10^{19}$ GeV.

B. Yukawa interactions, the neutrino portal and screening

There are three types of Yukawa couplings in our framework. Their graphical representations are shown in Fig. 3.

(1) The visible sector couplings:

$$\mathcal{L}^{(FF)} = -Y^{(FF)}_{ab\alpha}\, \underline{16}_{Fa}\underline{16}_{Fb}\underline{10}^{\alpha}_H + \mathrm{H.c.}, \qquad (30)$$

$(a, b = 1, 2, 3,\ \alpha = u, d)$ which generate the Dirac mass matrices of fermions. Although the number of Higgs ten-plets $\underline{10}^u_H$ and $\underline{10}^d_H$ could be arbitrary, we will here consider only one $\underline{10}^u_H$ and one $\underline{10}^d_H$. These interactions are also responsible for quark mixing (see Sec. VI).

(2) The "portal interactions"

$$\mathcal{L}^{(FS)} = -Y^{(FS)}_{ajk}\, \underline{16}_{Fa}\underline{1}_{Sj}\overline{\underline{16}}_{Hk} + \mathrm{H.c.}, \qquad (31)$$

which couple fermions of the visible and hidden sector and thus provide the (neutrino) "portal" between the two sectors.

(3) The hidden sector interactions

$$\mathcal{L}^{(SS)} = -\frac{1}{2} Y^{(SS)}_{ijk}\, \underline{1}_{Si}\underline{1}_{Sj}\underline{1}_{\chi k} + \mathrm{H.c.}, \qquad (32)$$

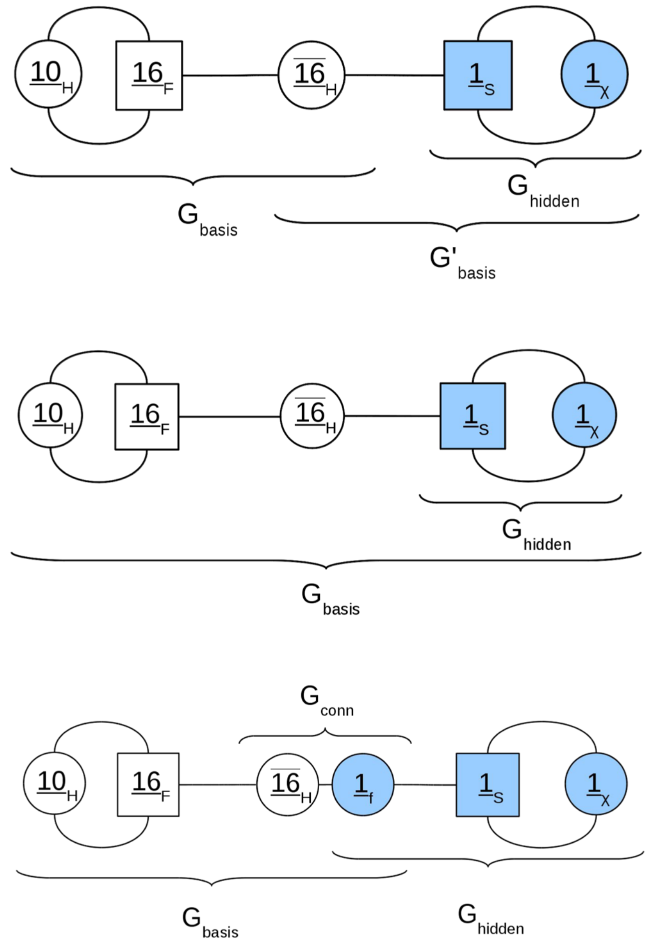

FIG. 3 (color online). Graphical representation of the Yukawa couplings and the basis-fixing symmetry. The visible sector interactions are shown on the left-hand side of the diagrams by a circle connecting $\underline{10}_H$ with $\underline{16}_F$. The portal interactions are symbolized by the lines connecting $\underline{1}_S$ and $\underline{16}_F$ to $\overline{\underline{16}}_H$. The right-hand parts of the diagrams show the hidden sector interactions between $\underline{1}_S$ and $\underline{1}_\chi$. In the upper plot the basis information is transferred to the hidden sector by G_{basis} and G'_{basis} with the mediator fields being $\overline{\underline{16}}_H$. The figure in the middle shows the direct communication of G_{basis} via the singlets $\underline{1}_S$. The lower part shows direct communication of G_{basis} to the hidden sector via hidden sector scalars $\underline{1}_f$.

where $\underline{1}_{\chi k}$ are scalar SO(10)-singlets. These interactions include only particles of the hidden sector. The neutrino mass term is given by Eqs. (22)–(24) with the mass matrices

$$m_D = Y^{(FF)}_u \langle \underline{10}^u_H \rangle, \qquad m_{\nu S} = Y^{(FS)} \langle \overline{\underline{16}}_H \rangle,$$
$$M_{RS} = Y^{(FS)} \langle \overline{\underline{16}}_H \rangle, \qquad M_S = Y^{(SS)} \langle \underline{1}_\chi \rangle. \qquad (33)$$

Note that $m_{\nu S}$ and M_{RS} are generated by different components of the VEVs of the $\overline{\underline{16}}_H$. If the VEVs of $\overline{\underline{16}}_H$ are at about GUT scale, the mass of the heaviest right-handed neutrino is given by $M_{R3} \sim 10^{14}$ GeV. Since we have not

[1] Since we have an SO(10)-GUT in mind, the term "active neutrinos" also includes ν_R.

introduced $\underline{126}_H$, there is no right-handed neutrino mass term.[2]

Let us introduce the matrix

$$D \equiv m_D (M_{RS}^{-1})^T \tag{34}$$

so that $m_\nu^{DS} = D M_S D^T$. Then the simplest way to obtain the correct hierarchical structures of M_R and m_ν is to generate a mild hierarchy in the matrices M_S and D. For this the strong hierarchies of m_D and M_{RS} should at least partially cancel each other in D, and consequently the light-neutrino mass matrix m_ν will have a hierarchy similar to M_S. If $m_D \propto M_{RS}$, we have $D \propto \mathbb{1}$, i.e., complete *screening* [49,50] of the Dirac neutrino mass matrix. In this case $m_\nu^{DS} \propto M_S$ and the difference between U_{PMNS} and $V_{\mathrm{CKM}}^\dagger$ directly reflects the structure of the mass matrix in the hidden sector. In the following we will show how screening and large neutrino mixing from the hidden sector can be obtained in our framework using symmetries in the visible and the hidden sector.

C. Basis-fixing symmetry and mediators

In this section we formulate conditions under which symmetries of the hidden sector can affect the flavor structure of the visible sector, and eventually lead to the required form of U_X.

In order to assure that the hidden sector symmetries can influence the form of m_ν, we must guarantee "communication" between the two sectors, which happens in the portal interaction $\mathcal{L}^{(FS)}$. In general the portal interaction is a sum over operators of the form

$$O_{\mathrm{vis}} \times O_{\mathrm{hidden}}, \tag{35}$$

where O_{vis} and O_{hidden} are operators containing only visible and hidden sector fields, respectively. If there are no visible sector fields transforming under G_{hidden} or hidden sector fields transforming under G_{vis} (the symmetry of the visible sector), then

$$\mathcal{L}^{(FS)} = \sum_{j,k} C_{jk} O_{\mathrm{vis}}^j \times O_{\mathrm{hidden}}^k + \mathrm{H.c.} \tag{36}$$

and the coefficients C_{jk} in front of the products of invariants O_{vis}^j of G_{vis} and O_{hidden}^k of G_{hidden} are unrestricted by both G_{vis} and G_{hidden}, and are thus free parameters of the theory. Consequently, there are no restrictions of the hidden sector symmetry G_{hidden} on the flavor structure of the visible sector. In order to have communication of information of the hidden sector to the visible sector, $\mathcal{L}^{(FS)}$ must not factorize as in Eq. (36). Therefore, some symmetry—G_{basis}—should exist which acts both in the visible and the hidden sector.

G_{basis} should at least fix a basis in both sectors and we call it the basis-fixing symmetry. We will call the fields, which in this case provide the communication between the two sectors, the mediators. In order to fix a basis, G_{basis} must be a symmetry which can differentiate among the three generations. The smallest potential candidates for G_{basis} are therefore $\mathbb{Z}_3$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$. In our examples we will always use $\mathbb{Z}_2 \times \mathbb{Z}_2$ which also makes $\mathcal{L}^{(FF)}$ diagonal.

According to the structure of $\mathcal{L}^{(FS)}$ there are three basic possibilities for the choice of the mediator fields:

(i) $\overline{\underline{16}}_H$ as mediators: For this at least three $\overline{\underline{16}}_H$ should be introduced that have to transform under a symmetry G_{basis} connecting it to the $\underline{16}_F$ and G'_{basis} connecting it to the $\underline{1}_S$. The full basis-fixing symmetry is $G_{\mathrm{basis}} \times G'_{\mathrm{basis}}$. This case is illustrated in the upper part of Fig. 3.

(ii) $\underline{1}_S$ as mediators: $\underline{1}_S$ have to transform under G_{basis} on the top of G_{hidden} (see the middle part of Fig. 3). In this case we have direct communication since $\underline{1}_S$ belong to the hidden sector.

(iii) The role of the mediators can be played by flavons $\underline{1}_f$, which will allow to reduce the number of $\overline{\underline{16}}_H$, and open more flexibility for the structure of the hidden sector. Namely, by means of an additional connecting symmetry G_{conn}, e.g., $G_{\mathrm{conn}} = \mathbb{Z}_m$, one can "bind" flavons $\underline{1}_f$ to $\overline{\underline{16}}_H$ or $\underline{1}_S$, i.e.,

$$\overline{\underline{16}}_{Hj} \to \overline{\underline{16}}_H \left(\frac{\underline{1}_{fj}}{\Lambda}\right)^q \qquad \text{invariant under } G_{\mathrm{conn}} \quad \text{or}$$
$$\underline{1}_{Sj} \to \underline{1}_S \left(\frac{\underline{1}_{fj}}{\Lambda}\right)^q \qquad \text{invariant under} G_{\mathrm{conn}}. \tag{37}$$

Here j is the index which corresponds to the symmetry G_{basis} and the power q is a positive integer. For example, if $\underline{1}_f$ transforms under G_{basis} and G_{hidden} and is connected to $\overline{\underline{16}}_H$ via G_{conn}, the combination $(\overline{\underline{16}}_H \frac{\underline{1}_f}{\Lambda})$ acts like a $\overline{\underline{16}}_H$-mediator. Thus only one field $\overline{\underline{16}}_H$ is sufficient and it does not need to obey any symmetries apart from G_{conn} and SO (10). Since the symmetry G_{basis} directly acts on the hidden sector fields $\underline{1}_f$, the communication of the basis information to the hidden sector is direct. The hidden sector symmetry G_{hidden} (under which the $\underline{1}_f$ have to be charged too) then transmits the basis information also to the $\underline{1}_S$. This is illustrated in the lower part of Fig. 3.

Let us now elaborate on the required properties of G_{hidden}. We differentiate two cases:

[2] The possible dimension-5 contribution to M_R stemming from $(1/\Lambda)\underline{16}_F \underline{16}_F \overline{\underline{16}}_H \overline{\underline{16}}_H$ can easily be forbidden by a discrete symmetry.

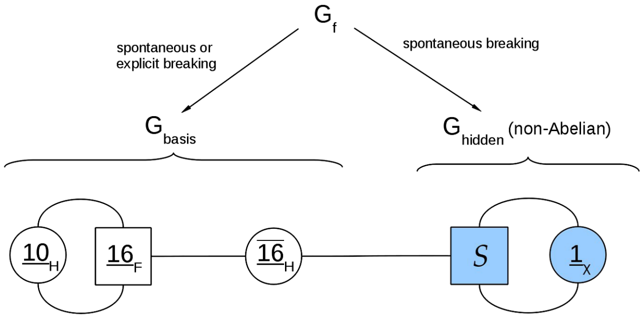

FIG. 4 (color online). G_{basis} and G_{hidden} as residual symmetries.

(A) G_{hidden} is an Abelian symmetry. In this case each individual field $\underline{1}_S$ transforms as a one-dimensional representation of G_{hidden} and the coupling to the $\underline{16}_F$ and $\overline{\underline{16}}_H$ in the portal interaction $\mathcal{L}^{(FS)}$ can be invariant with respect to all the symmetries.

(B) G_{hidden} is a non-Abelian symmetry, in which case some of the fermionic singlets $\underline{1}_S$ form a multiplet $\mathcal{S}$ of an irreducible representation of G_{hidden}. Then, all members of $\mathcal{S}$ have to transform in the same way under all other symmetries, in particular also $G^{(\prime)}_{\mathrm{basis}}$ which, consequently, does not distinguish different members of $\mathcal{S}$. Therefore, if G_{hidden} is an exact symmetry of $\mathcal{L}^{(FS)}$, communication of the basis-fixing symmetry to the hidden sector is excluded. The only way to have a non-Abelian symmetry G_{hidden} is thus to break G_{hidden} in the portal interaction $\mathcal{L}^{(FS)}$. This breaking can be explicit or spontaneous (through additional flavons). A scheme in which communication between the two sectors works can be conveniently arranged in the framework of residual symmetries [51–57]. There, G_{basis} and G_{hidden} originate from a larger symmetry G_f, which is broken to G_{basis} in $\mathcal{L}^{(FF)}$ and $\mathcal{L}^{(FS)}$ and to G_{hidden} in $\mathcal{L}^{(SS)}$—see Fig. 4. Since now G_{basis} and G_{hidden} stem from the same larger symmetry group G_f, even if G_{hidden} is explicitly broken in $\mathcal{L}^{(FS)}$, the hidden sector interaction $\mathcal{L}^{(SS)}$ "knows" about the basis fixed through G_{basis} and communication of flavor structures between the two sectors is possible. From another point of view: G_{basis} and G_{hidden} must be chosen in a way they can both be embedded in a finite group G_f.

IV. REALIZATIONS OF THE FRAMEWORK

In this section we present three realizations of the described framework. Note that we will not construct full models, but essentially focus on the effects of the different symmetries. The key elements are the same in all three cases:

(1) The symmetries G_{basis} (and G'_{basis}) make m_D and M_{RS} diagonal (which selects $G_{\mathrm{basis}} = \mathbb{Z}_2 \times \mathbb{Z}_2$), whereas M_S is not diagonal. The reason for this is that in the portal interaction $\mathcal{L}^{(FS)}$ only two fields carry hidden sector (or basis-fixing symmetry) charges, whereas in $\mathcal{L}^{(SS)}$ there are three.

(2) We introduce an additional symmetry G_{Yukawa} which is responsible for the large hierarchy in $m_u \propto m_D$.[3] For this, the symmetry G_{Yukawa} should distinguish three generations. If the same symmetry also gives rise to the hierarchy of Yukawa-couplings in $\mathcal{L}^{(FS)}$, cancellation between m_D and M_{RS} (complete or partial screening) is possible.

(3) The visible sector consists of three $\underline{16}_{Fa}$ $(a = 1, 2, 3)$, two (or more) Higgs fields $\underline{10}_H$ and one or three scalars $\overline{\underline{16}}_H$. All $\underline{10}_H$ have the same charges with respect to G_{basis}. Only one of them, 10^u_H, gives masses to the up-type quarks and neutrinos (see Sec. VI).

(4) The hidden sector includes (among other components) three $\underline{1}_{Sj}$ $(j = 1, 2, 3)$, one complex scalar $\underline{1}_Y$ responsible for the Yukawa-coupling hierarchy, and a set of complex scalars $\underline{1}_{\chi k}$.

(5) Since the Yukawa-coupling of the top-quark is $\mathcal{O}(1)$, we produce the masses of the third generation of fermions at the renormalizable level. The couplings for the first and second generation will be produced through effective operators of higher dimension. Therefore, the symmetries G_{basis} and G_{Yukawa} must be Abelian, or, if non-Abelian, act on the third generation with a one-dimensional representation. For models with three $\underline{16}_F$ transforming under a three-dimensional irreducible representation of a discrete group, and which realize screening, see, e.g., [58].

A. Realization I: $\underline{1}_S$ mediators with Abelian hidden symmetries

The fields communicating the basis-fixing symmetry to the hidden sector are chosen to be the heavy singlets $\underline{1}_S$ themselves. The charge assignments under the symmetries are shown in Fig. 5. The fact that all fermions transform in exactly the same way under the nongauged symmetries makes the present scenario particularly appealing. This could be a remnant of further unification beyond SO(10) [50]. If $\underline{16}_F$ and $\underline{1}_S$ stem from the decomposition of an E_6-multiplet [59–62] into SO(10)-multiplets

$$\underline{27} \to \underline{1} \oplus \underline{10} \oplus \underline{16}, \qquad (38)$$

we automatically obtain the same number of 16-plet and singlet fermions having the same transformation properties

[3] The large hierarchy can be achieved in two ways. Either the hierarchy is generated via flavon VEVs, or the couplings for the different generations are generated by operators of different dimensions ≥ 4. Also a combination of both mechanisms is possible.

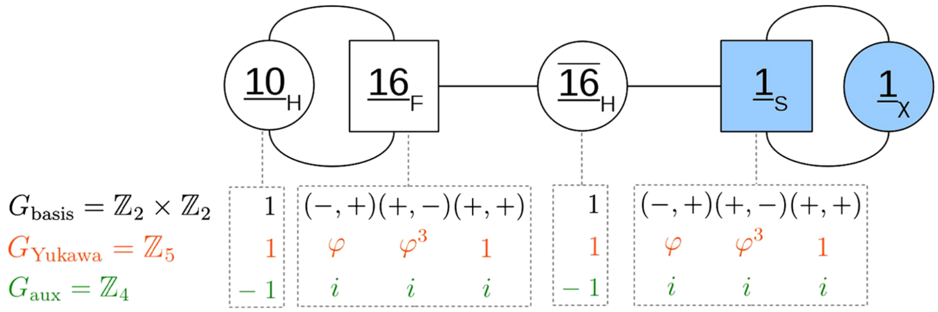

FIG. 5 (color online). Graphical representation of the charge assignments for realization I ($\varphi \equiv e^{2\pi i/5}$). The scalar $\underline{1}_Y$ is not shown here. It transforms as $\underline{1}_Y \to \varphi^4 \underline{1}_Y$ under $G_{\rm Yukawa}$ and trivially under $G_{\rm basis}$ and $G_{\rm aux}$. The charges of the flavon fields $\underline{1}_\chi$ are shown in Table II.

under all discrete groups.[4] Moreover, since $\mathcal{L}^{(FF)}$ and $\mathcal{L}^{(FS)}$ originate from the same coupling in the underlying E_6-theory,[5] also the Yukawa-coupling constants are the same, i.e., $Y_u^{(FF)} = Y^{(FS)}$.

The symmetry $G_{\rm basis}$ makes $Y_u^{(FF)}$ and $Y^{(FS)}$ diagonal. The strong mass hierarchy of quarks and charged leptons is achieved via effective operators of different dimensions for the different generations. The minimal group which can provide the required hierarchy is $G_{\rm Yukawa} = \mathbb{Z}_5$. The charges with respect to this group determine the dimension of the effective operator. Thus, taking into account operators up to dimension six, we obtain

$$Y_u^{(FF)} = Y^{(FS)} = {\rm diag}\left(y_1\left(\frac{\langle \underline{1}_Y\rangle}{\Lambda}\right)^2, y_2\frac{\langle \underline{1}_Y\rangle}{\Lambda}, y_3\right). \quad (39)$$

Here we have neglected the dimension-six contribution $\tilde{y}_3\frac{\langle \underline{1}_Y\rangle\langle \underline{1}_Y^*\rangle}{\Lambda^2}$ to the third-generation Yukawa coupling. The mass matrices m_D and M_{RS} are strictly proportional to each other and there is exact screening

$$D = m_D (M_{RS}^{-1})^T \propto \mathbb{1}_3. \quad (40)$$

The neutrino mass matrix is then given by

$$m_\nu^{DS} = D M_S D^T \propto M_S. \quad (41)$$

The auxiliary symmetry $G_{\rm aux}$ forbids a bare mass term for the singlets $\underline{1}_S$ and the dimension-5 contribution $\frac{1}{\Lambda}\underline{16}_F\underline{16}_F\overline{\underline{16}}_H\overline{\underline{16}}_H$ to the right-handed neutrino mass term M_R in the mass matrix (24). Introduction of flavons transforming as shown in Table II allows to generate independently any element of M_S, and therefore to obtain any set of texture zeros in M_S. For example, introduction of $\underline{1}_{\chi 22}$, $\underline{1}_{\chi 23}$ and $\underline{1}_{\chi 33}$ only, will produce a dominant 2-3-block. Notice that $G_{\rm Yukawa}$ plays an important role in structuring M_S. In order to get $\theta_{13}^X = 0$ without fine-tuning, a non-Abelian structure in M_S is needed (see Sec. IV C).

With more flavons an additional Abelian symmetry $G_{\rm hidden}$ can be realized, e.g., $G_{\rm hidden} = \mathbb{Z}_n$. If the singlets $\underline{1}_{Si}$ $(i = 1,2,3)$ transform with charges γ_i and flavons $\underline{1}_{fi}$ transform with charges $n - \gamma_i$, in $\mathcal{L}^{(FS)}$ the singlets $\underline{1}_{Si}$ can be substituted by operators

$$\underline{1}_{Si}\left(\frac{\underline{1}_{fi}}{\Lambda}\right) \quad (42)$$

without breaking of any other symmetry—see also Eq. (37). Then, even if all flavons of Table II are present in the theory, the extended hidden symmetry could for example be used to obtain a dominant 2-3-block in M_S, i.e.,

$$M_S \sim \begin{pmatrix} a & 0 & 0 \\ 0 & b & d \\ 0 & d & c \end{pmatrix} \quad (43)$$

giving large 2-3-mixing in U_X. For this already $G_{\rm hidden} = \mathbb{Z}_2$ with charge assignment $\underline{1}_S \sim (-,+,+)$ would be sufficient. Then nonzero 12 and 13 elements of M_S can

TABLE II. Transformation properties of the flavons generating M_S.

Flavon	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}_5$	$\mathbb{Z}_4^{\rm aux}$
$\underline{1}_{\chi 11}$	$(+,+)$	φ^3	-1
$\underline{1}_{\chi 12}$	$(-,-)$	φ	-1
$\underline{1}_{\chi 13}$	$(-,+)$	φ^4	-1
$\underline{1}_{\chi 22}$	$(+,+)$	φ^4	-1
$\underline{1}_{\chi 23}$	$(+,-)$	φ^2	-1
$\underline{1}_{\chi 33}$	$(+,+)$	1	-1

[4]In order not to generate unwanted new interactions, the tenplet fermions should be either at Planck scale or not realized in the theory at all.

[5]This requires that also the scalars $\underline{10}_H$ and $\overline{\underline{16}}_H$ are embedded in an E_6-multiplet via $\overline{\underline{27}} \to \underline{1} \oplus \underline{10} \oplus \overline{\underline{16}}$.

be generated by additional flavons or interactions with other hidden sector fermions (see Sec. V), or by higher order operators.

B. Realization II: $\underline{1}_S$ mediators with broken non-Abelian symmetry G_f

The field content and the symmetries G_{basis} and G_{Yukawa} are the same as in realization I. In addition, now we introduce a non-Abelian symmetry G_{hidden} in the basis fixed by G_{basis}. For this we embed G_{basis} and G_{hidden} into an extended flavor symmetry group $G_f \supset G_{\mathrm{basis}}, G_{\mathrm{hidden}}$. The embedding into the same group G_f ensures that G_{hidden} is introduced in the basis fixed by G_{basis}. This is necessary to communicate information about G_{hidden} to the visible sector. The flavons $\underline{1}_\chi$ break G_f spontaneously[6] to G_{hidden} in the hidden sector interaction $\mathcal{L}^{(SS)}$. In $\mathcal{L}^{(FS)}$ (as well as in $\mathcal{L}^{(FF)}$) G_f is broken down to G_{basis} explicitly or spontaneously (the latter would require a substantial complication of the model). In this way also G_{hidden} is broken explicitly in the low-energy interactions (see Fig. 4). Notice that G_{basis} is unbroken in $\mathcal{L}^{(FF)}$ and $\mathcal{L}^{(FS)}$. It will be broken by the mechanism which generates the CKM-mixing—see Sec. VI.

In what follows, we consider the 2-3-permutation symmetry ($\mu\tau$-symmetry) as G_{hidden}. The minimal group which realizes the embedding is[7] $G_f = \mathrm{D}_4 \times \mathbb{Z}_2$ with three generators A, B and C and the faithful three-dimensional reducible representation

$$\underline{3}\colon A \mapsto \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad B \mapsto \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad C \mapsto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{44}$$

We assign $\underline{16}_F$ and $\underline{1}_S$ to transform under reducible triplet representations $\underline{3}$ of G_f. A and B alone generate $G_{\mathrm{basis}} = \mathbb{Z}_2 \times \mathbb{Z}_2$, so that explicit breaking $G_f \to G_{\mathrm{basis}}$ leads to diagonal $Y_u^{(FF)}$ just as in the previous section. Equality $Y_u^{(FF)} = Y^{(FS)}$ can again be achieved by embedding of SO(10) into E_6. The generator C corresponds to the 2-3-permutation symmetry.

The irreducible representations of $\mathrm{D}_4 \times \mathbb{Z}_2$ are

$$\underline{\mathbf{1}}_{\mathbf{1}}\colon A \mapsto 1, \qquad B \mapsto 1, \qquad C \mapsto 1, \tag{45a}$$

$$\underline{\mathbf{1}}_{\mathbf{2}}\colon A \mapsto 1, \qquad B \mapsto -1, \qquad C \mapsto 1, \tag{45b}$$

$$\underline{\mathbf{1}}_{\mathbf{3}}\colon A \mapsto 1, \qquad B \mapsto 1, \qquad C \mapsto -1, \tag{45c}$$

$$\underline{\mathbf{1}}_{\mathbf{4}}\colon A \mapsto 1, \qquad B \mapsto -1, \qquad C \mapsto -1, \tag{45d}$$

$$\underline{\mathbf{1}}'_{\mathbf{1}}\colon A \mapsto -1, \qquad B \mapsto 1, \qquad C \mapsto 1, \tag{45e}$$

$$\underline{\mathbf{2}}\colon A \mapsto \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad B \mapsto \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad C \mapsto \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{45f}$$

and the products $\underline{\mathbf{2}}' \equiv \underline{\mathbf{1}}'_{\mathbf{1}} \otimes \underline{\mathbf{2}}$ and $\underline{\mathbf{1}}_{\mathbf{i}}' \equiv \underline{\mathbf{1}}'_{\mathbf{1}} \otimes \underline{\mathbf{1}}_{\mathbf{i}}$ $(i = 2, 3, 4)$. The relevant tensor product for the construction of the hidden sector interaction $\mathcal{L}^{(SS)}$ is thus

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} = (\underline{\mathbf{1}}'_{\mathbf{1}} \oplus \underline{\mathbf{2}}) \otimes (\underline{\mathbf{1}}'_{\mathbf{1}} \oplus \underline{\mathbf{2}}) = \underbrace{\underline{\mathbf{1}}'_{\mathbf{1}} \otimes \underline{\mathbf{1}}'_{\mathbf{1}}}_{\underline{\mathbf{1}}_{\mathbf{1}}} \oplus \underbrace{\underline{\mathbf{1}}'_{\mathbf{1}} \otimes \underline{\mathbf{2}}}_{\underline{\mathbf{2}}'} \oplus \underbrace{\underline{\mathbf{2}} \otimes \underline{\mathbf{1}}'_{\mathbf{1}}}_{\underline{\mathbf{2}}'} \oplus \underbrace{\underline{\mathbf{2}} \otimes \underline{\mathbf{2}}}_{\underline{\mathbf{1}}_{\mathbf{1}} \oplus \underline{\mathbf{1}}_{\mathbf{2}} \oplus \underline{\mathbf{1}}_{\mathbf{3}} \oplus \underline{\mathbf{1}}_{\mathbf{4}}}. \tag{46}$$

Introducing singlet flavons $\chi \sim \underline{\mathbf{1}}_{\mathbf{1}}$, $\rho \sim \underline{\mathbf{1}}_{\mathbf{2}}$ and a flavon doublet $\eta = (\eta_1, \eta_2)^T \sim \underline{\mathbf{2}}'$, we obtain the Yukawa-interaction[8] invariant with respect to G_f

[6]Since the breaking of G_f in $\mathcal{L}^{(SS)}$ happens at a very high energy scale of $\sim 10^{18}$ GeV, we want this breaking to be spontaneous through $\underline{1}_\chi$ rather than explicit.

[7]By D_4 we denote the dihedral group [63] of order eight. Sometimes in the literature this group is also denoted by D_8.

[8]Introduction of a singlet $\underline{\mathbf{1}}_{\mathbf{3}}$ would lead to nonequality of the 22 and 33 elements of M_S and $\underline{\mathbf{1}}_{\mathbf{4}}$ gives an antisymmetric contribution to M_S which vanishes due to the Majorana nature of $\underline{1}_S$.

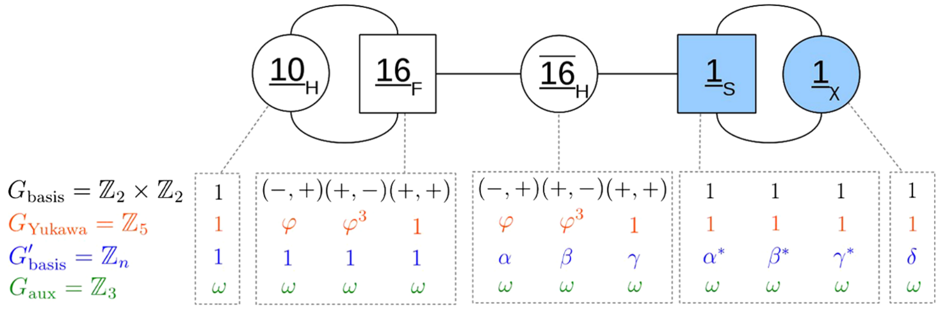

FIG. 6 (color online). Graphical representation of the couplings and charge assignments for realization III. The numbers α, β, γ, and δ are nth roots of unity. ($\varphi = e^{2\pi i/5}$, $\omega \equiv e^{2\pi i/3}$).

$$\mathcal{L}^{(SS)} = -\frac{1}{2}\left(\underline{1}_{S1} \quad \underline{1}_{S2} \quad \underline{1}_{S3}\right)\begin{pmatrix} y_\chi \chi & y_\eta \eta_1 & y_\eta \eta_2 \\ y_\eta \eta_1 & y'_\chi \chi & y_\rho \rho \\ y_\eta \eta_2 & y_\rho \rho & y'_\chi \chi \end{pmatrix} \times \begin{pmatrix} \underline{1}_{S1} \\ \underline{1}_{S2} \\ \underline{1}_{S3} \end{pmatrix} + \text{H.c.} \tag{47}$$

So, if $\langle\chi\rangle \neq 0$, $\langle\rho\rangle \neq 0$ and the doublet VEVs are aligned as $\langle\eta_1\rangle = \langle\eta_2\rangle$, m_ν^{DS} will be the most general 2-3-permutation symmetric matrix

$$m_\nu^{DS} \propto M_S = \begin{pmatrix} a & b & b \\ b & c & d \\ b & d & c \end{pmatrix}, \tag{48}$$

which is compatible with the neutrino mass squared-differences for both mass orderings and gives $\theta_{23}^X = 45°$ and $\theta_{13}^X = 0°$. Also the required size of 1-2-mixing can be obtained. Since in this paper we focus on the symmetries and do not construct models, we do not discuss mechanisms to get the required vacuum alignment. Such mechanisms have been elaborated in the literature, see e.g., [64,65], and can be realized here.

Also the group $G_{\text{Yukawa}} = \mathbb{Z}_5$ can be embedded into a discrete group $G_f \supset G_{\text{basis}} \times G_{\text{Yukawa}}$ by adding a fourth generator

$$D \mapsto \begin{pmatrix} \varphi & 0 & 0 \\ 0 & \varphi^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{49}$$

to Eq. (44). Using the computer algebra system GAP [66] we find that the resulting group has the structure $G_f = \mathbb{Z}_{10} \times (\mathbb{Z}_{10} \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2$. By construction, it contains $G_{\text{basis}} \times G_{\text{Yukawa}} = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_5$ as a subgroup. Since the three-dimensional representation of the extended group G_f is still reducible, i.e., $\underline{\mathbf{3}} = \underline{\mathbf{1}}' \oplus \underline{\mathbf{2}}$, generation of the 12 and 13 elements of M_S needs a scalar doublet $\eta \sim (\underline{\mathbf{1}}' \otimes \underline{\mathbf{2}})^*$. The 11-element can be generated by a coupling with a flavon $\chi \sim (\underline{\mathbf{1}}' \otimes \underline{\mathbf{1}}')^*$. Finally, the 2-3-block of M_S is determined by the tensor product

$$\underline{\mathbf{2}} \otimes \underline{\mathbf{2}} = \underline{\mathbf{1}}_{\mathbf{s}} \oplus \underline{\mathbf{1}}_{\mathbf{a}} \oplus \underline{\tilde{\mathbf{2}}}, \tag{50}$$

where

$$\underline{\mathbf{1}}_{\mathbf{s}}:\ A \mapsto 1, \quad B \mapsto -1, \quad C \mapsto 1, \quad D \mapsto \varphi^3, \tag{51a}$$

$$\underline{\mathbf{1}}_{\mathbf{a}}:\ A \mapsto 1, \quad B \mapsto -1, \quad C \mapsto -1, \quad D \mapsto \varphi^3, \tag{51b}$$

$$\underline{\tilde{\mathbf{2}}}:\ A \mapsto \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B \mapsto \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad C \mapsto \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad D \mapsto \begin{pmatrix} \varphi & 0 \\ 0 & 1 \end{pmatrix}. \tag{51c}$$

The representation $\underline{\mathbf{1}}_{\mathbf{a}}$ is the antisymmetric component of $\underline{\mathbf{2}} \otimes \underline{\mathbf{2}}$ and therefore does not contribute to $\mathcal{L}^{(SS)}$. The off-diagonal elements of the 2-3-block of M_S can be generated via the Yukawa interaction with a singlet $\rho \sim \underline{\mathbf{1}}_{\mathbf{s}}{}^*$ and the 22 and 33 elements need a flavon doublet $\chi' = (\chi'_1, \chi'_2)^T \sim \underline{\tilde{\mathbf{2}}}^*$. In this case, the hidden sector interactions obtain the form

$$\mathcal{L}^{(SS)} = -\frac{1}{2}\left(\underline{1}_{S1} \quad \underline{1}_{S2} \quad \underline{1}_{S3}\right)\begin{pmatrix} y_\chi \chi & y_\eta \eta_1 & y_\eta \eta_2 \\ y_\eta \eta_1 & y'_\chi \chi'_1 & y_\rho \rho \\ y_\eta \eta_2 & y_\rho \rho & y'_\chi \chi'_2 \end{pmatrix}\begin{pmatrix} \underline{1}_{S1} \\ \underline{1}_{S2} \\ \underline{1}_{S3} \end{pmatrix} + \text{H.c.} \tag{52}$$

If both doublet VEVs break G_f to 2-3-permutation symmetry, i.e., $\langle\eta_1\rangle = \langle\eta_2\rangle$ and $\langle\chi'_1\rangle = \langle\chi'_2\rangle$, M_S will be 2-3-permutation symmetric.

Instead of amending $\mathbb{Z}_2 \times \mathbb{Z}_2$ directly by the 2-3-permutation symmetry, we could also use the permutation symmetry

$$C \mapsto \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \tag{53}$$

which yields $G_f = \mathrm{A}_4 \times \mathbb{Z}_2$, i.e., a group with a three-dimensional irreducible representation. Embedding also $G_{\mathrm{Yukawa}} = \mathbb{Z}_5$ extends the flavor group to $G_f = \Delta(3 \times 10^2) \times \mathbb{Z}_{10}$, i.e., to a direct product of a cyclic group and a dihedral-like [67,68] subgroup of SU(3). In both cases the 2-3-permutation symmetry can be achieved by alignment of flavon triplet VEVs of G_f similar to the previous case.

Let us finally consider breaking of G_f in $\mathcal{L}^{(FS)}$ which implies the explicit breaking of G_{hidden}. This breaking will affect M_S and consequently m_ν^{DS}. We can estimate the corrections to M_S and m_ν using the general results of the type-I seesaw expansion [69]. According to [69] we expect that the effect of explicit breaking of G_{hidden} on M_S is of the order of magnitude of $\delta M_S \sim M_{\mathrm{GUT}}^2/m_S \sim 10^{-4} m_S$. Consequently, in the double seesaw expression, the effect will be of the order of $\delta m_\nu^{DS}/m_\nu^{DS} \sim 10^{-4}$, i.e., the corrections are negligible.

C. Realization III: Scalar fields $\overline{\mathbf{16}}_H$ as mediators

In this case we need to introduce three $\overline{16}_H$. The symmetries and charge assignments are shown in Fig. 6. The $\overline{16}_H$ have the same transformation properties under G_{basis} and G_{Yukawa} as $\underline{16}_F$. The symmetry G_{basis} makes the couplings $\underline{16}_F\underline{16}_F$ and $\underline{16}_F\overline{16}_H$ diagonal and G_{Yukawa} generates the strong hierarchy in m_D and M_{RS}. Thus, $\mathcal{L}^{(FF)}$ is given by (30) with the Yukawa coupling $Y_u^{(FF)}$ of (39). Note, however, that since embedding into E_6 is not possible here, the Yukawa couplings $Y_u^{(FF)}$ and $Y^{(FS)}$ are not equal and screening is in general only partial. D is still diagonal with, however, nonequal elements. This can be used to explain some features of mixing.

Notice that now communication between the visible and the hidden sector is not direct—it proceeds via $\overline{16}_H$. Furthermore, just $G_{\mathrm{basis}} = \mathbb{Z}_2 \times \mathbb{Z}_2$ is not enough since all hidden sector fields are singlets of G_{basis}. To fix a basis in both sectors, an additional symmetry G'_{basis} is required under which both $\overline{16}_H$ and hidden sector fields $\underline{1}_S$ are transformed. So, the information about the basis is transferred in two steps: from $\underline{16}_F$ to $\overline{16}_H$ by G_{basis} and from $\overline{16}_H$ to $\underline{1}_S$ by G'_{basis}. $\overline{16}_H$ is charged with respect to both G_{basis} and G'_{basis}.

As G'_{basis} we use an Abelian symmetry. For simplicity we choose $G'_{\mathrm{basis}} = \mathbb{Z}_n$ but the following arguments hold for any Abelian group. If the $\mathbb{Z}_n$ charges of $\underline{1}_S$ are $\alpha \neq \beta \neq \gamma \neq \alpha$, it makes also the couplings $\overline{16}_H \underline{1}_S$ diagonal—each $\overline{16}_H$ is uniquely connected to one $\underline{1}_S$. Therefore, due to the mediation by $\overline{16}_H$, the $\underline{1}_S$ "know" about the basis choice in the space of $\underline{16}_F$. For $\mathcal{L}^{(FS)}$ we obtain

$$\mathcal{L}^{(FS)} = -\left[y'_1 \left(\frac{\underline{1}_Y}{\Lambda}\right)^2 \underline{16}_{F1}\underline{1}_{S1}\overline{16}_{H1} + y'_2 \frac{\underline{1}_Y}{\Lambda} \underline{16}_{F2}\underline{1}_{S2}\overline{16}_{H2} + y'_3 \underline{16}_{F3}\underline{1}_{S3}\overline{16}_{H3} \right] + \mathrm{H.c.}, \tag{54}$$

and the matrix $D = m_D (M_{RS}^{-1})^T$ is given by

$$D = \mathrm{diag}\left(\frac{y_1 v_{10}^u}{y'_1 v_{16,1}}, \frac{y_2 v_{10}^u}{y'_2 v_{16,2}}, \frac{y_3 v_{10}^u}{y'_3 v_{16,3}} \right). \tag{55}$$

The simplest way to obtain partial screening is to assume that all Yukawa couplings are of similar size and that the VEVs of the three $\overline{16}_H$ are of the same size: $v_{16,1} \sim v_{16,2} \sim v_{16,3}$. In this case D is a diagonal matrix with elements of the same order. The dimension-three bare mass term for the $\underline{1}_S$ and dimension-six couplings of the form $\frac{1}{\Lambda}\underline{16}_F\underline{16}_F\overline{16}_H\overline{16}_H$, which would give rise to a right-handed neutrino mass term $M_R \neq 0$ in Eq. (24), are forbidden due to the auxiliary symmetry $G_{\mathrm{aux}} = \mathbb{Z}_3$.[9]

As discussed in Sec. III C, the role of $\overline{16}_{Hi}$ as the mediators could be given to new flavons $\underline{1}_{fi}$ having the transformation properties of $\overline{16}_{Hi}$. Then only one scalar 16-plet is needed which should transform trivially under all discrete groups except for a connecting symmetry G_{conn}. In the present example we can for instance choose

$$G_{\mathrm{conn}}\colon\ \underline{1}_f \to \omega \underline{1}_f, \qquad \overline{16}_H \to \omega^2 \overline{16}_H. \tag{56}$$

Then in Eq. (54) $\overline{16}_{Hi}$ should be replaced by $\overline{16}_H(\frac{\underline{1}_{fi}}{\Lambda})$. In order to achieve partial screening we should require $\langle \underline{1}_{f1} \rangle \sim \langle \underline{1}_{f2} \rangle \sim \langle \underline{1}_{f3} \rangle$.

As in realization I, by choosing appropriate $\mathbb{Z}_n$ charges δ_i for the flavons $\underline{1}_{\chi i}$, we can generate texture zeros in M_S. Since $G_{\mathrm{hidden}} = \mathbb{Z}_n$ can be extended to an arbitrary Abelian symmetry, all types of texture zeros in M_S can be obtained [70]. By means of texture zeros the exact 2-3-permutation symmetry in M_S can only be achieved for the matrix

$$M_S = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}. \tag{57}$$

Then for the light neutrinos we obtain

$$m_\nu^{DS} = \begin{pmatrix} aD_{11}^2 & 0 & 0 \\ 0 & 0 & bD_{22}D_{33} \\ 0 & bD_{22}D_{33} & 0 \end{pmatrix}. \tag{58}$$

[9] We could also have used $G_{\mathrm{aux}} = \mathbb{Z}_4$ as in realizations I and II. The main difference between these two choices is in the scalar potential, where $\mathbb{Z}_4$ forbids cubic scalar couplings, while $\mathbb{Z}_3$ does not.

This matrix can be experimentally feasible for a quasidegenerate neutrino mass spectrum only, i.e., when $|aD_{11}^2| \approx |bD_{22}D_{33}|$, and in addition corrections to M_S are needed to generate 1-2-mixing and mass splitting.

An approximate 2-3-permutation symmetric mass matrix M_S may for example be

$$M_S = \begin{pmatrix} a & c & d \\ c & 0 & b \\ d & b & 0 \end{pmatrix}. \qquad (59)$$

If $d = c(1+\epsilon)$, the eigenvector $(0,-1,1)^T$ of $m_\nu m_\nu^\dagger$ for an exactly 2-3-permutation symmetric m_ν will get corrections of order ϵ in all three entries. Consequently, $\sin\theta_{13}^X \sim |(c-d)/c|$, i.e., for $\sin\theta_{13}^X \ll \lambda \approx 0.2$ a fine-tuning at the few-percent level is necessary. Moreover, also the screening matrix D has to be close to $\mathbb{1}_3$ at the percent level to maintain the approximate 2-3-permutation symmetry in m_ν^{DS}.

To summarize, the scenario with a purely Abelian hidden sector symmetry, as expected, needs fine-tuning at the percent level to obtain the relation (1). Non-Abelian structures in M_S can be introduced with explicit symmetry breaking, as in realization II or, possibly, through the effects of additional SO(10)-singlet fermions, which are discussed in Sec. V.

V. EFFECTS OF ADDITIONAL SO(10)-SINGLET FERMIONS

Up to now we have considered three SO(10)-singlet fermions S which couple directly to the $\underline{16}_F$. In this section we will discuss the effects of additional fermionic singlets from the hidden sector. We differentiate two basic cases:

(1) Additional singlets which directly couple to the $\underline{16}_F$.
(2) Additional singlets which do not directly couple to the $\underline{16}_F$ but mix with the other fermionic singlets of the hidden sector.

In the first case, the number of singlets S directly coupled to the $\underline{16}_F$ in the portal interaction $\mathcal{L}^{(FS)}$ is larger than three. Thus, M_{RS} is not a square matrix and therefore not invertible. However, if M_S is invertible, M_R is given by the usual expression $M_R \approx -M_{RS}M_S^{-1}M_{RS}^T$ and the seesaw formulas change to

$$m_\nu \approx m_\nu^{DS} + m_\nu^{LS} + m_\nu' \qquad (60a)$$

$$m_\nu^{DS} = -m_D M_R^{-1} m_D^T, \qquad (60b)$$

$$m_\nu^{LS} = m_D M_R^{-1} M_{RS} M_S^{-1} m_{\nu S}^T + m_{\nu S} M_S^{-1} M_{RS}^T M_R^{-1} m_D^T, \qquad (60c)$$

$$m_\nu' = -m_{\nu S} M_S^{-1}\{\mathbb{1} + M_{RS}^T M_R^{-1} M_{RS} M_S^{-1}\} m_{\nu S}^T. \qquad (60d)$$

With the mass scales indicated in Table I, the new term m_ν' is expected to be of the order $\sim 10^{-5}$ eV and therefore negligible compared to all other contributions to m_ν. If M_S is singular and M_{RS} has rank three, the matrix

$$\begin{pmatrix} 0 & M_{RS} \\ M_{RS}^T & M_S \end{pmatrix} \qquad (61)$$

can still be invertible, but the formulas (25)–(28) and (60) will not hold any more. Notice that the case of more than three singlets coupled directly to the $\underline{16}_F$ is disfavored by the requirement of screening of the Dirac mass matrix. Indeed, in the basis where m_D is diagonal, exact screening in the sense that the structure of m_ν^{DS} is solely determined by M_S requires

$$M_{RS} \propto \begin{pmatrix} m_u & 0 & 0 & 0 & \dots & 0 \\ 0 & m_c & 0 & 0 & \dots & 0 \\ 0 & 0 & m_t & 0 & \dots & 0 \end{pmatrix}, \qquad (62)$$

i.e., a diagonal M_{RS}. However, if M_{RS} is a general matrix with hierarchy among its rows, partial screening is possible.

For case 2, which is favored by screening, we need vanishing of the couplings of the additional singlets $\underline{1}_S'$ to the $\underline{16}_F$. This can be achieved for example by introduction of an additional $\mathbb{Z}_2$ symmetry under which $\underline{16}_F$ and the three singlets $\underline{1}_S$ change the sign whereas $\underline{1}_S'$ do not change. In order to allow couplings between $\underline{1}_S$ and $\underline{1}_S'$ also new flavon fields charged under $\mathbb{Z}_2$ should be introduced. The $\mathbb{Z}_2$-symmetry thus acts as a kind of connecting symmetry. Among all hidden sector fields it selects three which can directly couple to the $\underline{16}_F$. In this case the neutrino mass matrix has the form

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & m_{\nu S} & 0 \\ m_D^T & 0 & M_{RS} & 0 \\ m_{\nu S}^T & M_{RS}^T & A & B \\ 0 & 0 & B^T & C \end{pmatrix} \qquad (63)$$

and the double seesaw formula becomes

$$m_\nu^{DS} = m_D M_{RS}^{-1T}(A - BC^{-1}B^T) M_{RS}^{-1} m_D^T, \qquad (64)$$

while the linear seesaw formula remains unchanged. Thus, in the double seesaw expression the former mass matrix M_S (A in the notation here) gets replaced by an effective mass matrix

$$M_S^{\text{eff}} \equiv A - BC^{-1}B^T. \qquad (65)$$

This looks like the first term in a seesaw expansion, but the derivation of Eq. (64) does not need the assumption of any hierarchy in the matrix

$$M \equiv \begin{pmatrix} A & B \\ B^T & C \end{pmatrix}. \tag{66}$$

The only condition is that C is invertible. Therefore, singlet fermions which do not couple directly to the $\underline{16}_F$ will have an impact on the neutrino mass matrix m_ν^{DS}.

The seesaw-like formula (65) may offer more possibilities to obtain interesting structures in M_S and therefore in m_ν^{DS}, in particular when non-Abelian symmetries of M are assumed (see Sec. IV B). If the symmetry G_{hidden} of M is Abelian, i.e., M is restricted by texture zeros only, there could still be "effective" non-Abelian structures in M_S^{eff}. For example, in case of a large hidden fermion sector, as could be motivated by string theory [28,71], M would have a large dimension of up to $\mathcal{O}(100)$, and therefore the elements of M_S^{eff} would get many contributions. Since M is close to string/Planck scale, the Yukawa couplings could be universal (nearly equal or with small spread). If also the flavon VEVs determining M are not completely random, some of the elements of M_S^{eff} could be approximately equal as is required by the 2-3-permutation symmetry. In this way specific structures in the 3×3 effective mass matrix M_S^{eff} are possible without non-Abelian symmetries in the hidden sector.

VI. CKM NEW PHYSICS

Up to now in our examples we discussed only the Higgs ten-plet $\underline{10}_H^u$ which has Higgs-doublet VEVs $v_u \neq 0$ and $v_d = 0$. This was enough for the symmetry considerations regarding m_D, but in order to have quark mixing we need $m_u \not\propto m_d$, which can be achieved by introduction of scalar ten-plets $\underline{10}_H^d$ with VEVs $v_u = 0$ and $v_d \neq 0$. Another possibility is the introduction of additional scalars $\underline{16}'_H$ and $\underline{16}''_H$ giving $m_u \not\propto m_d$ via the effective operator $(1/\Lambda)\underline{16}_F\underline{16}_F\underline{16}'_H\underline{16}''_H$ [72–75]. Here we will only discuss the scenario with additional ten-plets.

Generation of CKM mixing requires breaking of the basis-fixing symmetry $G_{\text{basis}} = \mathbb{Z}_2\times\mathbb{Z}_2$. In fact, G_{basis} is already broken spontaneously in the hidden sector by the flavon VEVs which generate the matrix M_S. This breaking leads to quark mixing via higher order operators

$$\underline{16}_F\underline{16}_F\underline{10}_H^d\frac{\underline{1}_{\chi ij}\underline{1}_{\chi kl}}{\Lambda^2}. \tag{67}$$

Here the singlet operator is built from the flavons of Table II. Invariance under G_{aux} requires it to be of second order. Note that it transforms nontrivially under G_{basis} and G_{Yukawa}. Hence, the operator of Eq. (67) gives corrections to the diagonal form of m_d of the order $\langle\underline{1}_\chi\rangle^2/\Lambda^2 \sim 10^{-2}$, which are certainly too small to generate the experimentally observed V_{CKM}. Therefore, we should introduce additional sources of G_{basis} breaking. For instance, breaking of G_{basis} can be achieved by introduction of several ten-plets $\underline{10}_H^d$ charged under G_{basis} and G_{Yukawa}. Instead, we can introduce only one $\underline{10}_H^d$, being a singlet of G_{basis}, which is connected by a symmetry $G_{\text{conn,d}}$ with additional flavons $\underline{1}'_Y$ charged under G_{basis} and G_{Yukawa}. As connecting symmetry we can use

$$G_{\text{conn,d}}\colon\ \underline{10}_H^d \to \alpha\underline{10}_H^d, \qquad (\underline{1}'_Y)^j \to \alpha^*(\underline{1}'_Y)^j, \tag{68}$$

where α is a root of unity and j is a positive integer. All elements of m_d can then be generated by effective operators of the form

$$\underline{16}_F\underline{16}_F\underline{10}_H^d\left(\frac{\underline{1}'_Y}{\Lambda}\right)^j. \tag{69}$$

The correct hierarchy of m_d can be produced by appropriate values of the VEVs and Yukawa-couplings of the different $\underline{1}'_Y$ as well as the powers j. Elaboration of this new physics is beyond the scope of this paper.

An important feature of this scenario is that $V_{\text{CKM}} \neq \mathbb{1}$ is generated solely by new physics that determines the structure of m_d and m_ℓ. Thus, the smallness of the 1-3 and 2-3 quark mixing angles is not related to the strong mass hierarchy of the up-type quarks. However, the size of the Cabibbo angle may still be a result of the down-type quark mass hierarchy [76,77].

Finally, to generate the correct mass hierarchy in m_ℓ, different from m_d, we have to introduce other representations for the Higgs fields. For example, adding 45-plet Higgs fields $\underline{45}_H$ one can effectively generate $m_\ell \not\propto m_d$ via the dimension-five operator [72]

$$\frac{1}{\Lambda}\underline{16}_F\underline{16}_F\underline{10}_H\underline{45}_H. \tag{70}$$

The problem is that inequality $m_\ell \neq m_d$ in general implies that $U_\ell \neq U_d$ and thus spoils relation (1). A possible solution is that the antisymmetric parts of m_d and m_ℓ coming from the interaction (70) are chosen in such a way that still $U_\ell \approx U_d$, but $U_{\ell R} \neq U_{dR}$ and $\hat{m}_d \not\propto \hat{m}_\ell$ [5,78].

VII. DISCUSSION AND CONCLUSIONS

The latest measurements of the lepton mixing parameters are in good agreement with the relation $U_{\text{PMNS}} \approx V_{\text{CKM}}^\dagger U_X$. In this paper we have proposed a framework which allows to realize such a relation. The framework is based on the double seesaw mechanism of neutrino mass generation, grand unification, and the existence of a hidden sector with certain symmetries.

The framework provides another way to unification of the quark and lepton mixings. The lepton mixing matrix has two contributions. The first one comes from the "CKM new physics," which is common for quarks and leptons and associated to interactions that generate the Dirac mass matrices. The other one is the contribution from structures responsible for the smallness of neutrino masses. These structures originate from the hidden sector. In this framework the CKM physics can be disentangled to a large extent from the "new neutrino physics." It allows to reconcile the strong mass hierarchy and small mixing of quarks with the mild hierarchy of light neutrinos and large lepton mixing.

The key elements of the framework are the basis-fixing symmetry and mediator fields which communicate information from the hidden sector to the visible one. An appropriate basis-fixing symmetry is the Abelian $\mathbb{Z}_2 \times \mathbb{Z}_2$ for which the symmetry basis coincides with the mass basis (in the first approximation). The mediator fields have charges of the basis-fixing symmetry, as well as charges of other symmetries in the hidden and/or visible sector. The basis-fixing symmetry is consistent with Abelian symmetries of the hidden sector which can lead to particular structures of M_X, e.g., a dominant 2-3-block.

To obtain U_X of the required form, non-Abelian symmetries in the hidden sector are required. These non-Abelian symmetries $G_{\rm hidden}$ (e.g., a 2-3 permutation symmetry) are broken in the portal interactions explicitly or spontaneously. Due to the lower scale of these interactions, the symmetry breaking produces only small corrections to the case of exact symmetry under $G_{\rm hidden}$. The groups $G_{\rm basis}$ and $G_{\rm hidden}$ (and also $G_{\rm Yukawa}$) can be embedded in a bigger group G_f. In this way one can realize the approach of residual symmetries such that G_f is broken down to $G_{\rm hidden}$ in the hidden sector and to $G_{\rm basis} \times G_{\rm Yukawa}$ in the portal and visible sector interactions.

The mediator fields have to appear in the portal interaction and can be $\overline{\underline{16}}_H$, or $\underline{1}_S$, or some new flavon fields associated to $\overline{\underline{16}}_H$ or $\underline{1}_S$. This association needs an additional connecting symmetry with respect to which the flavons and $\overline{\underline{16}}_H$ or $\underline{1}_S$ are charged. For each of these possibilities we provide illustrative examples.

An extended hidden sector with more than three singlets $\underline{1}_S$ may open up additional possibilities to explain features of U_X. The additional fermionic singlets $\underline{1}'_S$ should not participate directly in the portal interaction, otherwise screening will be destroyed. They can, however, couple with the three $\underline{1}_S$, thus modifying the matrix M_S, and in this way can, e.g., lead to an effective 2-3-permutation symmetry in M_S.

New physics and new symmetries are involved in the generation of the CKM mixing and the mass hierarchies of quarks and leptons. In the first approximation SO(10) allows to disentangle mixing and masses in a very simple way: if a single $\underline{10}_H$ gives the dominant contribution to the masses, no mixing is produced. So, we are forced to consider at least partly the CKM physics. In order to disentangle the generation of m_u and m_D from m_d and m_ℓ, we introduced two Higgs ten-plets $\underline{10}_H^u$ and $\underline{10}_H^d$ with identical charges under $G_{\rm basis}$. Additional flavons, charged under $G_{\rm basis}$ and $G_{\rm Yukawa}$ and connected to the field $\underline{10}_H^d$ by a symmetry $G_{\rm conn,d}$, allow to generate a nondiagonal $m_d \propto m_\ell$. The difference between m_d and m_ℓ should be obtained in such a way that the relation (1) is not destroyed.

A crucial question is how to test the proposed framework. Some possibilities are

(1) Further more accurate measurements of the leptonic 1-3 and 2-3 mixing, and especially the determination of the quadrant of the 2-3 mixing are important. The second quadrant for the 2-3 mixing angle would disfavor the framework.

(2) Specific realizations of the framework may lead to certain predictions for the Dirac as well as Majorana CP phases.

(3) The neutrino masses are generated at high scales which are not accessible to direct experimental studies. Therefore, one should not expect to see any new physics at the LHC associated directly to neutrino mass generation. Observation of such physics would probably exclude the framework.

(4) Indirect support of the approach can be provided by the observation of proton decay (which will be in favor of grand unification).

(5) A connection to leptogenesis and inflation may give another test of the high scale physics.

(6) Discoveries of other possible manifestations of the hidden sector are important. That includes identification of the dark matter and the discovery of sterile neutrinos.

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