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How to reveal the exotic nature of the $P_c(4450)$

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The LHCb Collaboration announced two pentaquark-like structures in the $J/\psi p$ invariant mass distribution. We show that the current information on the narrow structure at 4.45 GeV is compatible with kinematical effects of the rescattering from $\chi_{c1}p$ to $J/\psi p$: First, it is located exactly at the $\chi_{c1}p$ threshold. Second, the mass of the four-star well-established $\Lambda(1890)$ is such that a leading Landau singularity from a triangle diagram can coincidentally appear at the $\chi_{c1}p$ threshold, and third, there is a narrow structure at the $\chi_{c1}p$ threshold but not at the $\chi_{c0}p$ and $\chi_{c2}p$ thresholds. In order to check whether that structure corresponds to a real exotic resonance, one can measure the process $\Lambda_b^0 \to K^-\chi_{c1}p$. If the $P_c(4450)$ structure exists in the $\chi_{c1}p$ invariant mass distribution as well, then the structure cannot be just a kinematical effect but is a real resonance; otherwise, one cannot conclude that $P_c(4450)$ is another exotic hadron. In addition, it is also worthwhile to measure the decay $\Upsilon(1S) \to J/\psi p\bar{p}$: a narrow structure at 4.45 GeV but not at the $\chi_{c0}p$ and $\chi_{c2}p$ thresholds would exclude the possibility of a pure kinematical effect.

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The observation of many different hadrons half a century ago stimulated the proposal of the quark model as a classification scheme [1], and helped to establish quantum chromodynamics (QCD) as the fundamental theory of the strong interactions. Since then, hundreds of more hadrons were discovered. A renaissance of hadron spectroscopy studies started in 2003, and since then a central topic has been the identification of the so-called exotic hadrons. These are states beyond the naive quark model scheme, in which mesons and baryons are composed of a quark-antiquark pair and three quarks, respectively. Most of the new interesting structures were observed in the mass region of heavy quarkonium and are called XYZ states (for a list of these particles and a review up to 2014, see Ref. [2]). In particular, the $X(3872)$ [3] extremely close to the $D^0\bar{D}^{*0}$ threshold is widely regarded as an exotic meson, and the charged structures with a hidden pair of heavy quark and heavy antiquark such as the $Z_c(4430)$ [4,5], $Z_c^\pm(3900)$ [6,7], $Z_c^\pm(4020)$ [8], and $Z_b^\pm(10610, 10650)$ [9] would be explicitly exotic were they really resonances, i.e. poles of the S matrix. Candidates for explicitly exotic hadrons were extended to the pentaquark sector by the new LHCb observations of two structures, denoted as P_c, in the $J/\psi p$ invariant mass distribution with masses (widths) $(4380 \pm 8 \pm 29)$ MeV $[(205 \pm 18 \pm 86)$ MeV$]$ and $(4449.8 \pm 1.7 \pm 2.5)$ MeV $[(39 \pm 5 \pm 19)$ MeV$]$, respectively [10]. They were suggested to be hadronic molecules composed of an anticharm meson and a charmed baryon [11–13], the existence of which was already predicted in Refs. [14–17]. They were also discussed as a pentaquark doublet in Ref. [18].

Normally, such structures are observed as peaks in invariant mass distributions of certain final states, and fitted by using the Breit-Wigner parametrization to extract the masses and widths. However, such a procedure is problematic. On the one hand, many of these structures are very close to certain thresholds to which they couple strongly. In this case, the use of Breit-Wigner is questionable, and one needs to account for the thresholds. This can be achieved using the Flatté parametrization [19] (a method in this spirit for near-threshold states with coupled channels and unitarity was recently proposed in Ref. [20]). On the other hand, not every peak should be attributed to the existence of a resonance. In particular, kinematical effects may also show up as peaks. Such kinematical effects correspond to singularities of the S matrix as well, but they are not poles. In general, they are the so-called Landau singularities including branch points at thresholds and more complicated ones such as the triangle singularity, also called the anomalous threshold (detailed discussions of these singularities can be found in the textbooks [21,22]). The observability of the triangle singularity was extensively discussed in the 1960s (see Refs. [23–25] and references

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therein) and was recently used to explain some structures including the $\eta(1405)$, $a_1(1420)$ and $\phi(2170)$ [26–31]. In fact, there were suggestions that some of the Z_c and Z_b states were threshold effects [32–36] and the threshold effects might be enhanced by triangle singularities [37]. For a general discussion of S-wave threshold effects, see also Ref. [38]. Therefore, in order to establish a structure as a resonance, one has to discriminate it from such kinematical effects. Indeed, this is possible. As discussed in Ref. [39], a resonance can be distinguished from threshold kinematical effects only in the elastic channel which is the channel with that threshold. The purpose of this paper is to discuss the possible kinematical effects for the narrower structure at 4.45 GeV in the LHCb observations and suggest measurements to check whether it is a real exotic resonance or not.

We first notice that the $P_c(4450)$ structure is exactly located at the threshold of a χ_{c1} and proton pair, (4448.93 ± 0.07) MeV, and

$$M_{P_c(4050)} - M_{\chi_{c1}} - M_p = (0.9 \pm 3.1)\ \text{MeV}. \tag{1}$$

If the angular momentum between the χ_{c1} and the proton is a P wave, then the two-body system can have quantum numbers $J^P = (1/2, 3/2, 5/2)^-$, compatible with the favored possibilities $5/2^+$, $5/2^-$ and $3/2^-$ [10]. The $\chi_{c1}p$ can rescatter into the observed $J/\psi p$ by exchanging soft gluons. Two possible diagrams for such a mechanism are shown in Fig. 1, where (a) is a two-point loop with a prompt three-body production $\Lambda_b^0 \to K^- \chi_{c1} p$ followed by the rescattering process $\chi_{c1} p \to J/\psi p$, and in (b) the $K^- p$ pair is produced from an intermediate Λ^* state and the proton rescatters with the χ_{c1} into the $J/\psi p$. We discuss these two diagrams subsequently.

It is worthwhile to notice that the χ_{c1} can be produced in the weak decays of the Λ_b with a similar magnitude as that for the J/ψ. In the bottom quark decays, the charm quark is produced via the mediation of the W boson. After integrating out the off-shell mediators, one arrives at two effective operators for the $b \to c\bar{c}s$ transition:

$$\begin{aligned} \mathcal{O}_1 &= [\bar{c}^\alpha \gamma^\mu (1-\gamma_5) c^\alpha][\bar{s}^\beta \gamma_\mu (1-\gamma_5) b^\beta], \\ \mathcal{O}_2 &= [\bar{c}^\alpha \gamma^\mu (1-\gamma_5) c^\beta][\bar{s}^\beta \gamma_\mu (1-\gamma_5) b^\alpha], \end{aligned} \tag{2}$$

where one-loop QCD corrections have been taken into account to form $\mathcal{O}_1$. Here, α, β are color indices, and they

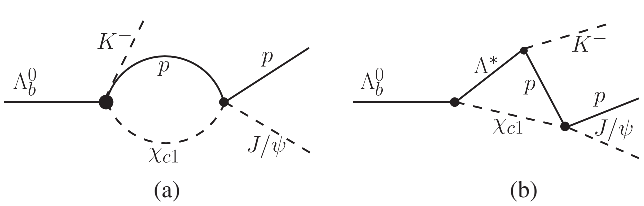

FIG. 1. Two-point and three-point loops for the mechanism of the $\chi_{c1} p \to J/\psi p$ rescattering in the decay $\Lambda_b^0 \to K^- J/\psi p$.

should be set to be the same in $\mathcal{O}_2$ in order to form a color-singlet charmonium state. The quark fields, $[\bar{c}\gamma^\mu(1-\gamma_5)c]$, will directly generate the charmonium state. A charmonium with $J^{PC} = 1^{--}$ like the J/ψ is produced by the vector current, while the axial-vector current tends to produce the χ_{c1} with $J^{PC} = 1^{++}$ and the η_c state with $J^{PC} = 0^{+-}$. Since the vector and axial-vector currents have the same strength in the weak operators, one would expect that the production rates for the J/ψ and χ_{c1} are of the same order in b quark decays. Corrections to this expectation come from higher-order QCD contributions but are subleading [40]. In fact, such an expectation is supported by the B-meson decay data [2]:

$$\begin{aligned} \mathcal{B}(B^+ \to J/\psi K^+) &= (10.27 \pm 0.31) \times 10^{-4}, \\ \mathcal{B}(B^+ \to \chi_{c1} K^+) &= (4.79 \pm 0.23) \times 10^{-4}. \end{aligned} \tag{3}$$

Having made these general observations, we return to the discussion of the Λ_b^0 decays measured by LHCb. We first focus on the two-point loop diagram whose singularity is a branch point at the $\chi_{c1}p$ threshold on the real axis of the complex s plane, where in the following $\sqrt{s}$ denotes the invariant mass of the $J/\psi p$ or $\chi_{c1}p$ system. It manifests itself as a cusp at the threshold if the $\chi_{c1}p$ is in an S wave. For higher partial waves, the threshold behavior of the amplitude is more smooth and a cusp becomes evident in derivatives of the amplitude with respect to s. Since we are only interested in the near-threshold region, both the χ_{c1} and the proton are nonrelativistic. Thus, the amplitude for Fig. 1(a) is proportional to the nonrelativistic two-point loop integral

$$G_\Lambda(E) = \int \frac{d^3q}{(2\pi)^3} \frac{\vec{q}^{\,2} f_\Lambda(\vec{q}^{\,2})}{E - m_1 - m_2 - \vec{q}^{\,2}/(2\mu)}, \tag{4}$$

where $m_{1,2}$ denote the masses of the intermediate states in the loop, μ is the reduced mass and E is the total energy. Here, we consider the case for the P wave $\chi_{c1}p$ which has quantum numbers compatible with the possibilities of the $P_c(4450)$ reported by the LHCb Collaboration, though one should be conservative in taking these determinations for granted as none of the singularities discussed here was taken into account in the LHCb amplitude analysis. If we take a Gaussian form factor, $f_\Lambda(\vec{q}^{\,2}) = \exp(-2\vec{q}^{\,2}/\Lambda^2)$, to regularize the loop integral, the analytic expression for the loop integral is then given by

$$\begin{aligned} G_\Lambda(E) = &-\frac{\mu\Lambda}{(2\pi)^{3/2}} \left(k^2 + \frac{\Lambda^2}{4}\right) \\ &+ \frac{\mu k^3}{2\pi} e^{-2k^2/\Lambda^2} \left[\operatorname{erfi}\left(\frac{\sqrt{2}k}{\Lambda}\right) - i\right], \end{aligned} \tag{5}$$

with $k = \sqrt{2\mu(E - m_1 - m_2 + i\epsilon)}$, and the imaginary error function $\operatorname{erfi}(z) = (2/\sqrt{\pi}) \int_0^z e^{t^2} dt$. A better regularization method should be applied in the future, but for our present study such an approach is fine.

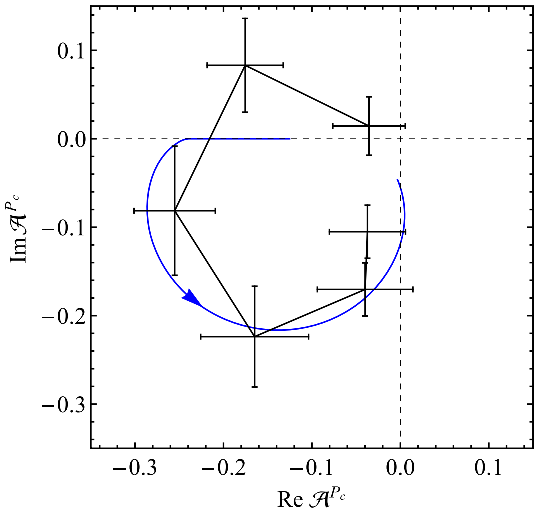

FIG. 2 (color online). Fit to the real and imaginary parts of the $P_c(4450)$ amplitude shown in Fig. 9 of Ref. [10] with Eq. (6). The blue curve represents the best fit. It is counterclockwise with increasing $J\psi p$ invariant mass from 4.41 GeV to 4.49 GeV, the same range as for the LHCb diagram.

Using an amplitude with the loop function given in Eq. (5), one can get a peak around the $\chi_{c1}p$ threshold. In order to have a more quantitative description of the effect of Fig. 1(a), we fit to the Argand plot for the $P_c(4450)$ amplitude depicted in Fig. 9(a) of Ref. [10] with an amplitude

$$\mathcal{A}_{(a)} = N[b + G_\Lambda(E)], \tag{6}$$

where b is a constant background term which may originate from direct production of the $K^- J/\psi p$, and N is an overall normalization. We fit to both the real and imaginary parts of the $P_c(4450)$ amplitude by minimizing the sum of the chi-squared values for both the real and imaginary parts. The best fit with a real background term has $\chi^2/\text{d.o.f.} = 1.75$ and is given by $N = 3144$, $b = -2.9 \times 10^{-4}$ GeV^4 and $\Lambda = 0.16$ GeV. With a real background term, the amplitude in Eq. (6) can only be complex when the energy is larger than the $\chi_{c1}p$ threshold, as is evident in Fig. 2. The background is, in general, complex as a result of the fact that the $K, J/\psi$ and p can go on shell and many Λ resonances can contribute to the Kp state. One sees from the figure that the counterclockwise feature of the LHCb amplitude is reproduced, and the overall agreement is good. The absolute value of the amplitude in Eq. (6) with these determined parameters has a narrow peak around the $\chi_{c1}p$ threshold as shown in Fig. 3(a). We have checked that using a different form factor $\Lambda^4/(\vec{q}^2 + \Lambda^2)^2$ gives a similar result. In both cases, the peak is asymmetric, unlike the Breit-Wigner form.

There can be further enhancement around the $\chi_{c1}p$ threshold due to the presence of nearby triangle singularities, also called leading Landau singularities of a triangle diagram, from Fig. 1(b). The leading Landau singularities for a triangle diagram are solutions of the Landau equation [41]

$$1 + 2y_{12}y_{23}y_{13} = y_{12}^2 + y_{23}^2 + y_{13}^2, \tag{7}$$

where $y_{ij} = (m_i^2 + m_j^2 - p_{ij}^2)/(2m_i m_j)$ with $m_i (i = 1,2,3)$ masses of the intermediate particles, and $p_{ij} = p_i + p_j$ being the four-momentum of the ij pair. To be specific, we let m_1, m_2 and m_3 correspond to the masses of the Λ^*, J/ψ and proton, respectively. Then, $p_{12}^2 = M_{\Lambda_b}^2$, $p_{13}^2 = M_{K^-}^2$, and $p_{23}^2 = s$ is the invariant mass squared of the $J/\psi p$ pair. It is easy to solve this equation for any given variable. We solve it as an equation of s, which has two solutions. For an easy visualization, we plot in the left panel of Fig. 4 the motion of the solutions in the complex $\sqrt{s}$ plane. As discussed in 1960s (see e.g. Ref. [23]), only one of the singularities can have an impact on the amplitude in the physical region defined on the upper edge of the real axis on the first Riemann sheet of the complex s plane, and it is effective only in a limited region of one of these

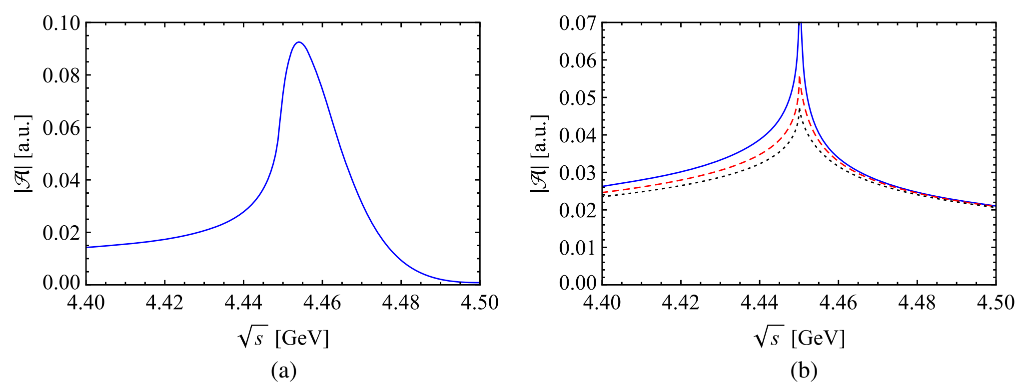

FIG. 3 (color online). Absolute values of amplitudes in arbitrary units: Panel (a) is for the amplitude in Eq. (6) fitted to the Argand plot; (b) is for the triangle loop integral with the $\chi_{c1}p$ vertex in a P wave. In (b), we assume the $\Lambda(1890)$ with a mass of 1.89 GeV is exchanged in the triangle diagram. The solid, dashed and dotted lines correspond to a width of the $\Lambda(1890)$ of 10, 60 and 100 MeV, in order.

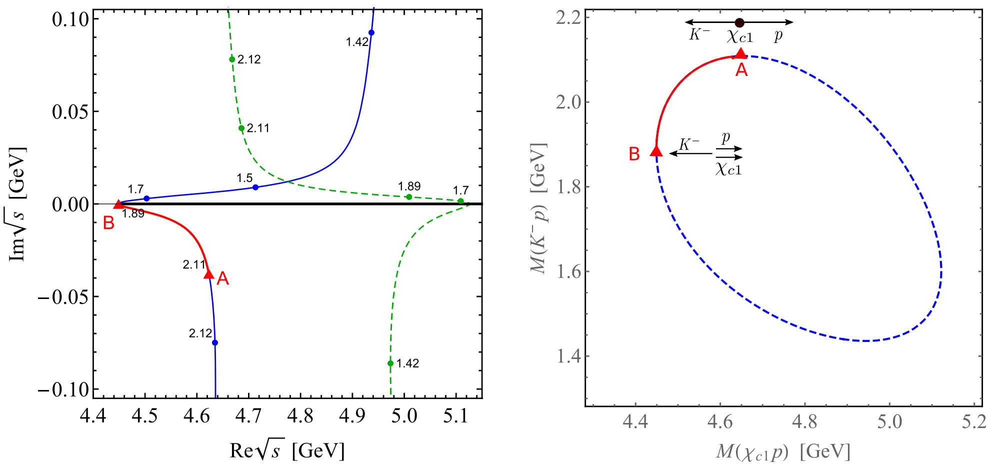

FIG. 4 (color online). Left panel: Motion of the two triangle singularities in the complex plane of $\sqrt{s} = M_{\chi_{c1} p} = M_{J/\psi p}$ with respect to changing the mass of the exchanged Λ^* baryon (several values are labeled in the plot in units of GeV). In order to distinguish the trajectories from the real axis, we put a small imaginary part, −5 MeV corresponding to a width of 10 MeV, to M_{Λ^*}. Only the part between the two filled triangles, labeled as A and B, has a large impact on the physical amplitude. The thick solid straight line represents the unitary cut starting from the $\chi_{c1} p$ threshold. Right panel: The corresponding Dalitz plot which shows the region between A and B.

variables. Here we want to investigate which values the Λ^* mass can take so that there can be an evident singularity effect in the $J/\psi p$ invariant mass, $\sqrt{s}$. According to the Coleman-Norton theorem [42], the singularity is in the physical region only when the process can happen classically, which means that all the intermediate states are on shell, and the proton emitted from the decay of the Λ^* moves along the same direction as the χ_{c1} and can catch up with it to rescatter. Let us start from a very large mass for the Λ^* so that it cannot go on shell in Fig. 1(b). Decreasing this mass, when it has a value

$$m_{1,\text{high}} = \sqrt{p_{12}^2} - m_2, \tag{8}$$

it can go on shell. At this point, the χ_{c1} is at rest in the rest frame of the decaying particle Λ_b, and the proton emitted from the decay $\Lambda^* \to K^- p$ can definitely rescatter with the χ_{c1} classically. This is the point shown as a filled triangle with $M_{\Lambda^*} = 2.11$ GeV, labeled as A, on the solid curves in Fig. 4. If we decrease m_1 further, the χ_{c1} will speed up and the proton will slow down. Thus, the lower bound of m_1 for the rescattering process that happens classically is given by the case when the χ_{c1} and the proton are at a relative rest, i.e. when the $\chi_{c1} p$ invariant mass is equal to their threshold. Thus, at this point the triangle singularity coincides with the normal threshold, and one gets

$$m_{1,\text{low}} = \sqrt{\frac{p_{12}^2 m_3 + p_{13}^2 m_2}{m_2 + m_3} - m_2 m_3}. \tag{9}$$

If m_1 is smaller than $m_{1,\text{low}}$, the proton would not be able to catch up with the χ_{c1} and the triangle diagram can only be a quantum process. For the case of Fig. 1(b), $m_{1,\text{low}}$ is given by $M_{\Lambda^*} = 1.89$ GeV, labeled as B, and also shown as a filled triangle in Fig. 1. In the left panel of Fig. 4, in order to move the singularity trajectories away from the real axis, we give a 10 MeV width to the Λ^*. For a vanishing width, the solid and dashed trajectories would pinch the real axis at $m_1 = m_{1,\text{high}}$. We can now know on which Riemann sheet of the complex s plane the singularities are located. Since only when m_1 is between $m_{1,\text{low}}$ and $m_{1,\text{high}}$ (the part between the two filled triangles in the figure), the process can happen classically and the singularity can be on the physical boundary (if the Λ^* width vanishes), we conclude that the singularity shown as the solid curve is always on the second Riemann sheet. On the contrary, the singularity whose trajectory is shown as the dashed curve in the left panel of Fig. 4 is on the second Riemann sheet when it is above the real axis, and it moves into the lower half-plane of the first Riemann sheet otherwise. Thus, it is always far away from the physical boundary and does not have any visible impact on the physical amplitude. For an easy visualization of the kinematical region between A and B, we show the corresponding Dalitz plot in the right panel of Fig. 4.

An intriguing observation for the case of interest is that within the range between 1.89 GeV and 2.11 GeV, there is a four-star baryon $\Lambda(1890)$ with $3/2^+$. Taking $M_{\Lambda^*} = 1.89$ GeV, the triangle singularity is just at the $\chi_{c1} p$ threshold which can provide a further threshold enhancement.[1] Giving a finite width to the $\Lambda(1890)$, the singularity moves away from the real axis into the lower half-plane of the second Riemann sheet [it is located at

[1]The mechanism of enhanced threshold effect due to the triangle singularity was recently discussed for the case of Z_c and Z_b states [37].

$(4447 - i0.2)$ MeV for $M_{\Lambda^*} = (1.89 - i0.03)$ GeV], and the enhancement is reduced. The $\Lambda(1890)$ has a relatively small width (60 to 100 MeV [2]) so that there can still be an important enhancement. In Fig. 3(b), we show the absolute value of the triangle loop integral with the $\chi_{c1}p$ in a P wave for three different widths (for a discussion of the triangle singularities in the nonrelativistic triangle loop integral, see Ref. [43]). There is clearly an enhancement near 4.45 GeV even when the width is taken to be 100 MeV.

In the above, we have shown that kinematical effects can result in a narrow structure around the $\chi_{c1}p$ threshold in the $J/\psi p$ invariant mass of the $\Lambda_b^0 \to K^- J/\psi p$ decay. Consequently, a natural question is whether such an effect happens at other thresholds, in particular, those related to the $\chi_{c1}p$ through heavy quark spin symmetry (HQSS). As a result of the HQSS, the operator for annihilating a χ_{c1} and creating a J/ψ is contained in

$$\frac{1}{2}\langle J^\dagger \chi^i\rangle = -\psi^{j\dagger}\chi_{c2}^{ij} - \frac{1}{\sqrt{2}}\epsilon^{ijk}\psi^{j\dagger}\chi_{c1}^k + \frac{1}{\sqrt{3}}\psi^{i\dagger}\chi_{c0} + \eta_c^\dagger h_c^i, \tag{10}$$

where the fields $J = \vec{\psi}\cdot\vec{\sigma} + \eta_c$ and $\vec{\chi} = \sigma^j(-\chi_{c2}^{ij} - \frac{1}{\sqrt{2}}\epsilon^{ijk}\chi_{c1}^k + \frac{1}{\sqrt{3}}\delta^{ij}\chi_{c0}) + h_c^i$ [44,45] annihilate the S-wave and P-wave charmonium states, respectively. This means that the rescattering interaction strength for $\chi_{c2}p \to J/\psi p$ or $\chi_{c0}p \to J/\psi p$ is of similar size as that for the $\chi_{c1}p \to J/\psi p$. One might naively expect enhancements at both the $\chi_{c2}p$ and $\chi_{c0}p$ thresholds in the $J/\psi p$ invariant mass as well. However, this is not the case. As we have shown in Eq. (2), at leading order in α_s, the charmonium is produced by the $[\bar{c}\gamma^\mu(1-\gamma_5)c]$ current. This current has no projection onto the χ_{c0} or χ_{c2}. The production of the $\chi_{c0,c2}$ in the b decays can come only from higher-order QCD corrections which are suppressed. Indeed, there is no enhancement at the $\chi_{c2}p$ and $\chi_{c0}p$ thresholds in Λ_b decays, which is consistent with our expectation.

The above analysis is applicable to any b quark decay in which the $\chi_{c0,c2}$ is directly generated by the weak interaction. But it would be different if the initial decay heavy particle contained a charm or anticharm quark in addition to the bottom quark. Processes of this type include the decays of the B_c meson and the doubly-heavy baryon Ξ_{bc}. An explicit calculation of B_c decays [46] indicates that the $\chi_{c0,c2}$ can have similar production rates with the χ_{c1}. Considering the large amount of data on the B_c to be accumulated by the LHCb Collaboration [47], it appears very promising to investigate the $\chi_{c0}p$ and $\chi_{c2}p$ threshold effects in the future. In addition, one can study the threshold effects in the prompt production of the $J/\psi p$ at the LHC, or in the $\Upsilon(1S)$ decays into the $\chi_{cJ}p\bar{p}$ and $J/\psi p\bar{p}$.

In conclusion, what we have shown here is that the present information on the narrow structure around 4.45 GeV observed by the LHCb Collaboration is compatible with kinematical effects around the $\chi_{c1}p$ threshold: First, it is located exactly at the $\chi_{c1}p$ threshold. Second, the mass of the four-star well-established $\Lambda(1890)$ coincidentally makes the triangle singularity on the physical boundary located at the $\chi_{c1}p$ threshold, despite a small shift into the complex plane due to the finite width of the $\Lambda(1890)$, and third, the χ_{c1}, instead of the χ_{c0} or χ_{c2}, can be easily produced in the weak decays of the Λ_b by the $V - A$ current so that there can be an evident effect at the $\chi_{c1}p$, but not the $\chi_{c0}p$ or $\chi_{c2}p$, threshold.

Therefore, the most important question regarding the structure around 4.45 GeV is whether it is just a kinematical effect or a real resonance. As discussed in Ref. [39], kinematical singularities, including both the normal threshold and the triangle singularity, cannot produce a narrow near-threshold peak in the elastic channel, which is the $\chi_{c1}p$ in this case. The reason is that the interaction strength in the elastic channel controls the threshold behavior, and there can be a narrow near-threshold peak only when the interaction in the elastic channel is strong enough to produce a pole in the S-matrix which corresponds to a real resonance. On the contrary, one cannot simply determine the interaction strength for the inelastic channel ($\chi_{c1}p \to J/\psi p$ in our case) because it can always interfere with a direct production of the final state. Thus, the question can be answered by analyzing the process $\Lambda_b^0 \to K^-\chi_{c1}p$: If there is a narrow structure just above threshold in the $\chi_{c1}p$ invariant mass distribution, then the structure cannot be just a kinematical effect and it calls for the existence of a real pentaquark-like exotic resonance; otherwise, one cannot conclude the $P_c(4450)$ to be another exotic hadron.

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