Chiral vortical wave and induced flavor charge transport in a rotating quark-gluon plasma

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We show the existence of a new gapless collective excitation in a rotating fluid system with chiral fermions, named the chiral vortical wave (CVW). The CVW has its microscopic origin at the quantum anomaly and macroscopically arises from interplay between vector and axial charge fluctuations induced by vortical effects. The wave equation is obtained both from hydrodynamic current equations and from chiral kinetic theory, and its solutions show nontrivial CVW-induced charge transport from different initial conditions. Using the rotating quark-gluon plasma in heavy ion collisions as a concrete example, we show the formation of an induced flavor quadrupole in quark-gluon plasma and estimate the elliptic flow splitting effect for Λ baryons that may be experimentally measured.

I. INTRODUCTION

Anomalous transport effects in many-body systems with chiral fermions have generated great interest recently. Such phenomena span a wide range of physical systems [1,2], from semimetals to cold atomic systems, and from hot quark-gluon plasma (QGP) created in heavy ion collisions to cold dense matter in neutron stars. These systems provide possible environments with nonzero macroscopic chirality and can manifest the microscopic chiral anomaly in macroscopic transport processes that would normally be forbidden by symmetries like parity invariance.

One way to induce the anomalous transport effects in such chiral systems is to apply external electromagnetic fields. A famous example is the so-called Chiral Magnetic Effect (CME) in which an electric current can be generated in parallel to an external magnetic field. The CME could lead to experimentally measurable effects both for the QGP in heavy ion collisions [3–15] and for certain Dirac and Weyl semimetals [16]. Other interesting examples include e.g. the Chiral Separation Effect (CSE) [17,18] and the Chiral Electric Separation Effect (CESE) [19,20]. For reviews, see e.g. Refs. [21–23].

The anomalous transport effects can also occur when the fluid undergoes a global rotation quantified by a vorticity $\boldsymbol{\omega} = \frac{1}{2} \boldsymbol{\nabla} \times \boldsymbol{v}$, where $\boldsymbol{v}$ is the flow velocity field. Such vortical effects were suggested in Ref. [4] and quantified in holographic models [24–26] and the anomalous hydrodynamic framework [27]. The so-called Chiral Vortical Effect (CVE) quantifies the generated vector current $\boldsymbol{J}_V$ as [24–27]

$$\boldsymbol{J}_V = \frac{1}{\pi^2} \mu \mu_5 \boldsymbol{\omega} \tag{1}$$

and the generated axial current $\boldsymbol{J}_A$ as [24–27]

$$\boldsymbol{J}_A = \left[\frac{1}{6} T^2 + \frac{1}{2\pi^2} (\mu^2 + \mu_5^2) \right] \boldsymbol{\omega}, \tag{2}$$

where T is temperature and μ and μ_5 are vector and axial chemical potentials. It was suggested that such a CVE may lead to baryon charge separation in heavy ion collisions [28,29].

The vorticity-driven anomalous transport effects in Eqs. (1) and (2) couple together the vector and axial densities/currents. A similar situation also happens in the external magnetic field, where the interplay between CME and CSE leads to a gapless collective mode called a *"chiral magnetic wave"* (CMW) [30–33]. In this paper, we show that the vortical effects also induce a new wave mode for vector and axial density fluctuations, which we call a *"chiral vortical wave"* (CVW). We will derive this new wave equation and determine the CVW propagation speed in both the hydrodynamic and kinetic theory frameworks. We then show that the CVW can induce a fermion charge quadrupole distribution from initial vector density. Finally, we will make predictions for possible implications of CVW in heavy ion collisions.

While we will use rotating QGP as a concrete example, the theoretical contents of CVW are in a general fashion and the proposed phenomenon is universal, being able to be realized in various systems with effective chiral fermions. We note that various other chiral effects have been explored in systems from compact stars [33–36] to Weyl and Dirac semimetals [37–41] and spin-orbit coupled cold Fermi gases [42–44] (as reviewed in Refs. [1,2]). It would be feasible and of great interest to realize CVW as a new and

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independent way of manifesting chiral effects in those systems.

II. THE CHIRAL VORTICAL WAVE

Let us start by rewriting the vortical effects (1) and (2) in terms of currents $J_{L/R} = \frac{1}{2}(J_V \mp J_A)$:

$$J_{L/R} = \mp \left(\frac{1}{12} T^2 + \frac{1}{4\pi^2} \mu_{L/R}^2 \right) \boldsymbol{\omega}, \qquad (3)$$

where $\mu_{L/R} = \mu \mp \mu_5$. Intuitively the above vortical effects can be understood as follows: In the presence of global rotation, the underlying fermions experience an effective interaction of the form $\sim -\boldsymbol{\omega} \cdot S$ in their local rest frame, with S the spin of fermions. This causes a *charge-blind* spin polarization effect (as is indeed found in other context [45,46]); namely, the fermions will have their spins preferably aligned with $\boldsymbol{\omega}$. As a result, the right- or left-handed fermions will preferably have their momenta parallel or antiparallel to $\boldsymbol{\omega}$, giving rise to the currents in (3).

Let us then consider the small fluctuations of left- and right-handed densities on top of a uniform equilibrium background. For simplicity, we consider ω to be constant and neglect fluctuations of temperature (which is controlled by linearized hydroequations for the energy-momentum tensor). By combining the continuity equations $\partial_t n_{L/R} + \nabla \cdot J_{L/R} = 0$ [here $n_{L/R} = \frac{1}{2}(J_V^0 \mp J_A^0)$] with Eq. (3), one obtains

$$\partial_t n_{L/R} = \pm \frac{1}{4\pi^2} \omega \partial_x (\mu_{L/R}^2) = \pm \frac{\omega \mu_{L/R}}{2\pi^2} \partial_x \mu_{L/R}, \quad (4)$$

where we have set vorticity along the x-direction $\boldsymbol{\omega} = \omega \hat{x}$ with $\hat{x} \equiv x/|x|$. Clearly there are two modes, one for right-handed density and the other for left-handed density, that propagate in opposite directions. For later convenience, we introduce the susceptibilities for the corresponding densities: $\chi_\mu^{L/R} \equiv \partial n_{L/R}/\partial \mu_{L/R}$.

At this point, two possibilities may occur. For simplicity, we focus on the right-handed mode below. (1) The background fluid is charge neutral, i.e. $\mu_0 = 0$. In this case the density fluctuation $\delta n = \chi_0 \delta \mu$ and is governed by the nonlinear wave equation:

$$\partial_t (\delta n) + \frac{\omega}{4\pi^2 \chi_0^2} \partial_x (\delta n^2) = 0. \qquad (5)$$

This takes the form of the inviscid Burgers' equation [40], whose (implicit) solution can be formally written as

$$\delta n(x, t) = F_i \left(x - \frac{\omega t \delta n}{2\pi^2 \chi_0^2} \right), \qquad (6)$$

with $F_i(x) = \delta n(x, t = 0)$ given by the initial density fluctuation. (2) The background has nonzero charge density $\mu_0 \neq 0$. In this case, one can linearize the equation for the evolution of small density fluctuations on top of the background density and obtain a linear wave equation:

$$\partial_t (\delta n) + \frac{\mu_0 \omega}{2\pi^2 \chi_{\mu_0}} \partial_x (\delta n) = 0. \qquad (7)$$

This is just a usual wave equation describing a propagating mode with a gapless dispersion relation:

$$\nu = V_\Omega |\boldsymbol{k}|, \qquad V_\Omega = \frac{\mu_0 \omega}{2\pi^2 \chi_{\mu_0}}, \qquad (8)$$

where ν is the wave frequency and $\boldsymbol{k} = k\hat{x}$ is the wave vector. This is the *chiral vortical wave (CVW)* with the wave speed V_Ω defined above. More precisely, this is the right-handed wave mode that propagates along the $\boldsymbol{\omega}$ direction. The left-handed wave mode propagates in the opposite direction to $\boldsymbol{\omega}$, with a speed given by a similar formula, albeit replacing μ_0 and χ_{μ_0} with the left-handed quantities.

In short, the CVW found above is essentially a hydro-dynamic density wave arising from slowly varying vector and axial density fluctuations that are coupled together through vortical effects. Possible diffusion effects can also be included by adding to the lhs of (7) terms like $-(D_L \partial_x^2 + D_T \partial_T^2)(\delta n)$, where D_L and D_T are the longitudinal and transverse diffusion constants. If the initial condition of density fluctuations is very "lumpy," then such diffusion effects must be taken into account.

III. CVW FROM CHIRAL KINETIC THEORY

Recently, the physics of chiral anomaly has been incorporated into kinetic theory framework, and the anomalous transport effects such as the CME and CVE [47–52] as well as the CMW [51] were understood in a transparent way in such chiral kinetic theory. It is therefore desirable to understand how the newly found CVW may arise in the chiral kinetic theory framework.

Let us consider a rotating system of noninteracting right-handed (denoted by "+") Weyl fermions as well as their left-handed (denoted by "−") antiparticles. (The discussion for a system of left-handed fermions with their right-handed antifermions will be similar.) Taking a similar approach as in Refs. [49,51], we start from the equations of motion for these fermions in their local rest frame:

$$\sqrt{G_\pm} \dot{x} = \frac{p}{p} \pm \frac{\omega}{p}, \qquad \sqrt{G_\pm} \dot{p} = 2p \times \omega, \qquad (9)$$

where $\sqrt{G_\pm} = 1 \pm p \cdot \omega/p^2$ and $\omega = |\omega|$ is the global rotational angular speed of the system. The corresponding kinetic equations can be written as

$$\partial_t f_\pm + \dot{x} \cdot \partial_x f_\pm + \dot{p} \cdot \partial_p f_\pm = C_\pm [f_+, f_-]. \quad (10)$$

Integrating these equations and using Eq. (9), we obtain

$$\partial_t \int_p \sqrt{G_\pm} f_\pm + \int_p \left(\frac{p}{p} \pm \frac{\omega}{p} \right) \cdot \partial_x f_\pm$$
$$+ \int_p 2(p \times \omega) \cdot \partial_p f_\pm = \int_p \sqrt{G_\pm} C_\pm. \qquad (11)$$

The last term on the left-hand side is zero after integration by parts. Also, $\int_p \equiv \int \frac{d^3 p}{(2\pi)^3}$.

We now examine small fluctuations in the net (vector) density on top of a certain background equilibrium distribution $f_{0\pm}(p; T, \mu_0)$. Similarly to the analysis in Ref. [51], we parameterize the density fluctuations as

$$f_{\pm}(t, \mathbf{x}, \mathbf{p}) = f_{0\pm}(p) + \delta f_{\pm}(t, \mathbf{x}, \mathbf{p}), \qquad (12)$$

$$\delta f_{\pm} = \pm[\partial_p f_{0\pm}(p)] \int d\nu d^3 k e^{i(\nu t - \mathbf{k}\cdot\mathbf{x})} h(\nu, \mathbf{k}, \mathbf{p}), \quad (13)$$

where the $\delta f_{\pm}$'s have been expanded in Fourier modes. Subjecting the above to Eq. (11), and taking a difference to yield the time evolution of net density, we obtain the following relation in linear order of fluctuations:

$$\nu \int_{\mathbf{p}} [\partial_p f_{0+}(p) + \partial_p f_{0-}(p)] h(\nu, \mathbf{k}, \mathbf{p})$$
$$= \mathbf{k} \cdot \int_{\mathbf{p}} \frac{\boldsymbol{\omega}}{p} [\partial_p f_{0+}(p) - \partial_p f_{0-}(p)] h(\nu, \mathbf{k}, \mathbf{p}), \quad (14)$$

where we have used the facts that (a) the equilibrium distribution $f_{0\pm}$ is a space-time-independent fixed point of the collision kernel, (b) $\int_{\mathbf{p}} \mathbf{p} z(p) = 0$ for any $z(p = |\mathbf{p}|)$, and (c) the collision terms from fluctuations vanish because of the charge conservation constraint.

Let us then examine the low-frequency, long-wavelength limit, $\nu \to 0$ and $k \to 0$. The hydrodynamic zero mode in this limit arises from $\delta f_{\pm} \to \pm[\partial_p f_{0\pm}(p)] H(t, \mathbf{x})$, which implies h becoming independent of $\mathbf{p}$. This allows one to perform integrations over $\mathbf{p}$ in (14), and obtain

$$[\chi_{\mu_0} \nu - C\mathbf{k} \cdot \boldsymbol{\omega}] h(\nu, \mathbf{k}) = 0, \qquad (15)$$

where χ_{μ_0} is the thermodynamic susceptibility defined in equilibrium, and $\chi_{\mu_0}(T, \mu_0) = \partial n / \partial \mu|_{T,\mu_0}$ with net charge density $n = \frac{1}{(2\pi)^3} \int_{\mathbf{p}} \sqrt{G}(f_{0+} - f_{0-})$. The constant above is defined by $C = -\int_{\mathbf{p}}(1/p)[\partial_p f_{0+}(p) - \partial_p f_{0-}(p)] = \frac{1}{2\pi^2} \int dp[f_{0+}(p) - f_{0-}(p)]$, and for the Fermi-Dirac distribution $C = \mu_0/(2\pi^2)$. This allows one to immediately identify a hydrodynamic collective excitation that propagates along the vorticity direction, $\mathbf{k} \| \boldsymbol{\omega}$, with the following dispersion relation:

$$\nu = V_\Omega |\mathbf{k}|, \qquad V_\Omega = \frac{\mu_0 \omega}{2\pi^2 \chi_{\mu_0}}. \qquad (16)$$

Notably, the so-obtained CVW speed V_Ω agrees exactly with that in Eq. (8).

IV. CVW-INDUCED CHARGE TRANSPORT

We now discuss interesting charge transport phenomena induced by the CVW. To be concrete, we consider different initial density fluctuations on top of a background medium with vorticity $\boldsymbol{\omega} = \omega\hat{x}$ and CVW speed V_Ω given by (8).

Let us first consider purely axial charge density fluctuation in the initial condition, $F_i^A(x)$ at $t = 0$ (we have

suppressed "trivial" coordinates y, z). This can be cast into initial conditions for right-handed and left-handed density fluctuations, $F_i^{R/L} = \pm F_i^A/2$. The subsequent evolution via the wave equations simply yields $(\delta n)_t^{R/L} = \pm F_i^A(x \mp V_\Omega t)/2$. We are interested in the transport of vector charge density, which is an observable quantity. This can be obtained as follows:

$$(\delta n)_t^V = [F_i^A(x - V_\Omega t) - F_i^A(x + V_\Omega t)]/2$$
$$\approx [-\partial_x F_i^A(x)] V_\Omega t, \qquad (17)$$

where the second line is true for small $V_\Omega t$. This implies a separation effect of vector charge along the vorticity direction: when initial axial charge fluctuation is positive and concentrated around $x = 0$ (e.g. a Gaussian form), then CVW will transport positive/negative vector charges toward the $\pm\hat{x}$ directions, respectively, leading to a *charge dipole moment* in parallel to $\boldsymbol{\omega}$ with strength $|d_\Omega|$ proportional to CVW speed V_Ω and propagation time; when the initial axial fluctuation is negative, the dipole moment flips.

Let us then consider purely vector charge density fluctuation in the initial condition, $F_i^V(x)$ at $t = 0$. Following similar procedure, the vector charge density from CVW evolution is given by

$$(\delta n)_t^V = [F_i^V(x - V_\Omega t) + F_i^V(x + V_\Omega t)]/2$$
$$\approx F_i^V(x) + [\partial_x^2 F_i^V(x)/2](V_\Omega t)^2, \qquad (18)$$

where again the second line is true for small $V_\Omega t$. This transport process leads to a quadrupole moment of vector charge density along $\boldsymbol{\omega}$. Consider the initial fluctuation to be positive and concentrated around $x = 0$ (e.g. a Guassian form); then $[\partial_x^2 F_i^V(x)/2]$ is positive (negative) at large (small) $|x|$, implying the concentration of positive charges away from $x = 0$ toward both directions along $\boldsymbol{\omega}$. The resulting quadrupole $|q_\Omega|$ is proportional to $(V_\Omega t)^2$.

V. EXPERIMENTAL OBSERVABLE IN HEAVY ION COLLISIONS

Our discussions on the CVW and its induced charge transport effects so far are rather general. We now consider its possible experimental manifestation in a concrete system—namely, rotating quark-gluon plasma created in off-central heavy ion collisions. The global rotation points in the out-of-plane direction. In such a QGP, CVW occurs for each light flavor—e.g. u, d quarks and possibly s quarks as well—and transports flavor charges toward the two "tips" of the QGP fireball, leading to a quadrupole charge distribution on the transverse plane. Here we make a first estimate of such effects.

Let us first quantify the quadrupole moment q_Ω^f resulting from CVW for a single quark flavor. We use the participant density from the Glauber model as an initial condition for the flavor charge fluctuation and study the dependence of q_Ω^f on the key parameter V_Ω by solving the CVW
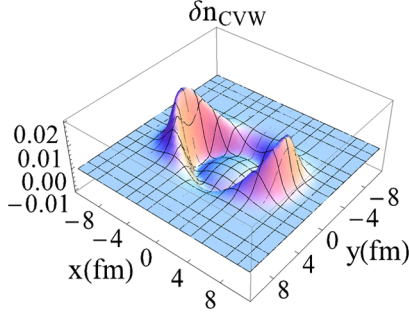
FIG. 1 (color online). CVW-induced flavor charge density profile.

FIG. 2 (color online). Normalized $\bar{\Lambda}$ and Λ elliptic flow splitting, $[v_2^{\bar{\Lambda}} - v_2^{\Lambda}]/[|q_\Omega^f|A_\pm^\Lambda]$, for symmetric two-flavor (2-F) and three-flavor (3-F) cases. The midcentral and the peripheral correspond to for 15%–30% and 60%–92% centrality classes (see Ref. [60]).

equation. An illustration of the CVW-transported flavor charge density distribution at $\tau = 8$ fm (with beam energy $\sqrt{s} = 200$ GeV, impact parameter $b = 7$ fm, initial time $\tau_0 = 0.6$ fm) is shown in Fig. 1: a quadrupole pattern is evident. The quadrupole moment can be obtained by integrating the density distribution $q_\Omega^f = [\int dx dy (\delta n_f) \cos(2\phi_s)]/[\int dx dy (\delta n_f)]$. We have computed this quantity, with the results $q_\Omega^f \simeq -0.03(V_\Omega \Delta \tau)^2$ with $\Delta \tau$ (in fm/c) the propagation time in QGP. The numerical coefficient is for minimum bias events, and it varies at $\sim 15\%$ level across centrality. The minus sign is merely due to the convention of defining azimuthal angle ϕ_s with respect to the in-plane direction.

Clearly we need a plausible estimate of V_Ω. Using the "A Multi-Phase Transport"(AMPT) model [53], our simulations suggest an initial value of ω at about 0.5 fm^{-1} [54–57], and lattice susceptibility at initial temperature $T_0 \sim 350$ MeV is about $\chi_f \sim 3$ fm^{-2} [58,59], with both decreasing as QGP expands. Using a background density μ_0 of 0.1–1 fm^{-1}, one gets an estimate $\langle V_\Omega \rangle \simeq 10^{-2}$–$10^{-3}$, leading to an induced quadrupole $|q_\Omega^f|$ at $\sim 10^{-4}$ level. Such an estimate is very sensitive to μ_0. By going to lower beam energy or by selecting events with large baryon asymmetry, the background μ_0 could be considerably increased, thus magnifying $|q_\Omega^f|$.

A flavor quadrupole implies that more baryons will be formed on the tips than on the equator of the fireball. The stronger in-plane radial flow will thus translate the quadrupole into baryon/antibaryon v_2 splitting. This mechanism is in analogy to the electric charge quadrupole induced by CMW [31]. Suppose at the freeze-out, the flavor-wise chemical potential for quarks contains the CVW-induced quadrupole contribution $\delta \mu_f \propto 2 q_\Omega^f \cos(2\phi_s)$ (with $f = u$, d, s). The corresponding chemical potential for a given type of hadron can be determined from its constituent quark content, e.g. for the Λ baryon, $\delta \mu_\Lambda \propto 2(q_\Omega^u + q_\Omega^d + q_\Omega^s) \cos(2\phi_s)$. We particularly propose to use Λ baryon which is electric-charge neutral, and thus unaffected by possible CMW effects. We then use the STAR blast-wave model [60] to compute the resulting differential flow
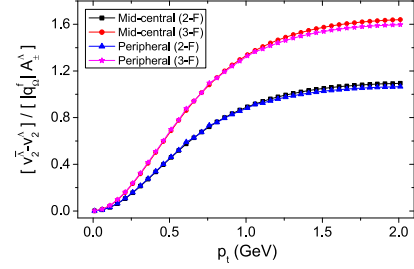
splitting. As it is unclear how much the s-quark mass may reduce their chiral effects, we consider two extreme cases: a symmetric two-flavor (2-F) case $q_\Omega^u = q_\Omega^d = q_\Omega^f$ with $q_\Omega^s = 0$, or a symmetric three-flavor (3-F) case $q_\Omega^u = q_\Omega^d = q_\Omega^s = q_\Omega^f$. From the Cooper-Frye scheme it is easy to see $\Delta v_2 = v_2^{\bar{\Lambda}} - v_2^{\Lambda} \propto |q_\Omega^f|A_\pm^\Lambda$ with $A_\pm^\Lambda = (N^\Lambda - N^{\bar{\Lambda}})/(N^\Lambda + N^{\bar{\Lambda}})$ the Λ-asymmetry that is directly related to background density μ_0 (in analogy to a similar relation in the CMW case [31]). The results for normalized flow splitting $\Delta v_2/[|q_\Omega^f|A_\pm^\Lambda]$ are shown in Fig. 2. Note that while the curves for the two centralities appear close, they have rather different normalization, as the $|q_\Omega^f| \sim V_\Omega^2$ strongly depends on centrality. Note also that the CVW predicts a particular slope for $\Delta v_2 \propto A_\pm^\Lambda$ but may not exclude a finite intercept at $A_\pm^\Lambda = 0$ with either sign. Needless to say, these are crude estimates, and a realistic hydrodynamic modeling of CVW (that accounts for factors like time-dependent vorticity, susceptibility and diffusion) will be done in a future work. Given that, our results suggest that a CVW-induced signal could be detected and may give indications on chiral effects of strange flavor. Experimental measurements of Λ and $\bar{\Lambda}$ v_2 are feasible (see e.g. Ref. [61]), and the predicted flow splitting may be measured for events binned according to their baryonic number asymmetry [62].

VI. SUMMARY

In summary, we have found a new gapless collective excitation in a rotating fluid system with chiral fermions, named the chiral vortical wave. We derive the wave equation for the CVW and determine its speed from both hydrodynamic and chiral kinetic theory. We demonstrate that the CVW can induce a flavor quadrupole in rotating quark-gluon plasma in heavy ion collisions, which in turn split the elliptic flow for Λ baryons. Such a proposal could be tested with future experimental data.

As a final remark, while the proposed CVW bears certain similarity to the CMW, it is a completely new phenomenon that provides an independent way of manifesting chiral

anomaly. In the context of heavy ion collisions, the vorticity lasts significantly longer than the lifetime of the strong magnetic field and may induce a more robust signal. In certain spin-orbit coupled cold Fermi gases that could simulate chiral anomaly effects [44], only the vorticity-driven effects can be easily and directly induced with those charge-neutral atoms. With such unique merit, the CVW has its own significance and interest for the study of anomalous effects in a wide range of physical systems.

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