Early universe cosmology, effective supergravity, and invariants of algebraic forms

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The presence of light scalars can have profound effects on early universe cosmology, influencing its thermal history as well as paradigms like inflation and baryogenesis. Effective supergravity provides a framework to make quantifiable, model-independent studies of these effects. The Riemannian curvature of the Kähler manifold spanned by scalars belonging to chiral superfields, evaluated along supersymmetry breaking directions, provides an order parameter (in the sense that it must necessarily take certain values) for phenomena as diverse as slow roll modular inflation, nonthermal cosmological histories, and the viability of Affleck-Dine baryogenesis. Within certain classes of UV completions, the order parameter for theories with n scalar moduli is conjectured to be related to invariants of n-ary cubic forms (for example, for models with three moduli, the order parameter is given by a function on the ring of invariants spanned by the Aronhold invariants). Within these completions, and under the caveats spelled out, this may provide an avenue to obtain necessary conditions for the above phenomena that are in principle calculable given nothing but the intersection numbers of a Calabi-Yau compactification geometry. As an additional result, abstract relations between holomorphic sectional and bisectional curvatures are utilized to constrain Affleck-Dine baryogenesis on a wide class of Kähler geometries.

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I. INTRODUCTION

$D = 4$, $\mathcal{N} = 1$ effective supergravity is the setting for much of phenomenology and cosmology, especially work that is "string inspired," with varying degrees of UV completions [1]. In this paper, we will be interested in three cosmological applications: slow roll moduli inflation [2–5], the thermal history of the Universe prior to big bang nucleosynthesis [6], and Affleck-Dine baryogenesis [7–9].

It is remarkable that such a wide range of cosmological phenomena can be described within the same setting. This is because of several key factors that can play an important role in early universe cosmology and are naturally captured within a supergravity framework:

(i) The importance of moduli: Due to their gravitational coupling to other fields, moduli can play an important role in the thermal history of the Universe if they come to dominate the energy density [10–12]. Moreover, moduli potentials are particularly suited for small-field slow roll inflationary models [13–16], due to their flatness in perturbation theory.

(ii) Supersymmetric flat directions: Supersymmetric flat directions play an important role in early universe cosmology. By stabilizing scalar potentials against quantum corrections, supersymmetry helps to satisfy flatness conditions required for slow roll inflation.[1] On the other hand, supersymmetric flat directions in the visible sector participate in robust frameworks like the Affleck-Dine mechanism that can explain the matter- antimatter asymmetry of the Universe.

(iii) Inflation breaks supersymmetry: The vacuum energy during inflation breaks supersymmetry, leading to soft terms ("Hubble-induced terms") in the visible sector Lagrangian.

Within supergravity, the scalar potential V should allow for spontaneous supersymmetry breaking with the following features:

(i) Phenomenology: Acceptable phenomenology requires a point in moduli space where $V \sim 0$, $V' = 0$, and $V'' > 0$ are necessarily true. The first is the requirement of vanishing cosmological constant in the present Universe, the second is the requirement that the vacuum energy is extremized, and the third is the requirement that we live in a (meta)stable Universe.

(ii) Cosmology (inflation): To obtain a viable period of slow roll modular inflation, the scalar potential necessarily needs to satisfy $V \sim H^2$, $V' \sim 0$, and $V'' \lesssim 0$ at some point in field space. The first condition is the requirement of inflationary vacuum energy, while the second and third are required to ensure the smallness of the slow-roll parameters ϵ and η. We reiterate that our interest in this paper will be entirely slow roll modular inflation; in particular, our treatment does not apply to axionic inflation models, brane-antibrane models, visible sector models, etc.

(iii) Cosmology (thermal history): Moduli can dominate the energy density of the Universe and, being gravitationally coupled, can decay late. When the

[1] However, to solve the η problem in supergravity requires extra fine-tuning.

lifetime exceeds the onset of big bang nucleosynthesis, entropy dilution from the decay ruins the successful prediction of the abundances of the light elements; this is the cosmological moduli problem.[2] The lifetime depends on the modulus mass; hence thermal history is determined by points in moduli space that satisfy $V \sim 0$, $V' \sim 0$, and $V'' \sim m_{3/2}^2$. The first two conditions are the cosmological constant and stability criteria, while the last is a statement that the modulus decays around the time of BBN, assuming that supersymmetry is broken at low energies with $m_{3/2}^2 \sim \mathcal{O}(100)$ TeV.

(iv) Cosmology (Affleck-Dine baryogenesis): Unlike other baryogenesis mechanisms, effective supergravity is the natural setting for Affleck-Dine baryogenesis [20–23]. A visible sector baryon number carrying flat direction acquires a tachyonic mass during inflation, rolls to nonzero vacuum expectation value, and acquires a CP-violating decay from the competing effects of Hubble-induced and soft A-terms. In terms of supergravity data, the necessary requirement is that at some point in field space the conditions $V \sim H^2$ and $V''_{vis} < 0$ hold. The first condition corresponds to the inflationary vacuum energy, while the second condition is the requirement that the Hubble-induced mass on some visible sector flat direction is tachyonic.

The full data of a string compactification, including the locations of D-branes, orientifolds, flux quanta, anomaly cancellation, nonlocal effects on the potential, and local visible sector model building would of course settle the question of whether these conditions can be satisfied in a UV complete setting. That is a challenging task (for a survey of these topics, we refer to the excellent reviews listed above).

Since the scenarios listed above depend crucially on certain local analytic properties of the potential V of the scalar components of chiral superfields, it is possible to ask questions purely at the level of local geometry, without referring to a particular UV completion, or even the full global details of the potential. For generic directions in chiral multiplet space, the values of V, V', and V'' depend both on the Kähler potential K and superpotential W, and saying anything predictive amounts to knowing both quantities, i.e., the full data that (along with the gauge kinetic function) determines the supergravity Lagrangian. However, as long as one is satisfied in making *necessary* but not *sufficient* statements, it is indeed possible to obtain a simple, local, geometric order parameter (we will use the term "order parameter" to denote some quantity that must *necessarily* take certain values in order for a phenomenon to take place). A study of the scalar potential shows that this

[2]Originally the Polonyi problem, dating from the earliest theories of supergravity [17–19].

TABLE I. Conditions on the quantities $R[f]$ and $R[f, Q]$ for the various cosmological phenomena described in the text.

Phenomenon	$R[f]$	$R[f, Q]$
Modular Inflation	$< \frac{2}{3}$	$\cdots$
Nonthermal History	$\mathcal{O}(1)$	$\cdots$
A-D Baryogenesis	$\cdots$	> 1

parameter is the local curvature [24–26]. Specifically, it is the component of the Riemann curvature tensor along the Goldstino directions defined by the auxiliary fields that encode the supersymmetry breaking data

$$\text{order parameter}: \; R[f] \equiv R_{i\bar{j}m\bar{n}} f^i f^{\bar{j}} f^m f^{\bar{n}} \equiv \sum_{\substack{i,j,k \equiv \\ \text{all } \cancel{SUSY} \\ \text{directions}}} \frac{R_{i\bar{j}k\bar{l}}}{g_{i\bar{j}} g_{k\bar{l}}}. \tag{1}$$

The necessary conditions on this quantity are listed in Table I. Here, $f^i \equiv F^i/|F|$ is a chiral field with nonvanishing F-term. The important point is that these bounds on the Riemannian curvature have to be satisfied regardless of the details of the UV completion. Moreover, since $R[f]$ is a function of the scalar fields, the value of the quantity in Eq. (1) is implicitly dependent on the superpotential W, which (along with K) determines the allowed field values in the effective theory.

One may ask if it is possible to recast the conditions on the parameter $R[f]$ in terms of equivalent conditions on other parameters (which we call Δ for now) that are entirely field independent, constructed from for example the input parameters that specify the Kähler geometry. The reason this is useful is the following. If a quantity Δ serves as an order parameter, then in principle it is possible to rule out certain cosmological phenomena on classes of manifolds $\mathcal{M}$ based solely on the value of $\Delta(\mathcal{M})$, which is given by the geometric input that specifies the Kähler potential of $\mathcal{M}$, *without* any other physical ingredient at all.

As an example, one can consider the case of Calabi-Yau compactifications, where these input parameters are the intersection numbers. The Kähler potential of Kähler moduli in type IIB Calabi-Yau compactifications is given by

$$K = -2 \ln \mathcal{V}, \tag{2}$$

where $\mathcal{V}$ is the volume form, given in terms of intersection numbers in a basis of two cycles. As a first guess to a plausible Δ, we can require that the conditions on Δ should not change under a change of the basis of two cycles in terms of which the geometric data is presented. This means that Δ should be a function of the ring of covariants of the volume form $\mathcal{V}$. If further one requires the conditions on Δ to be field independent, then Δ should be a function of the ring of *invariants* of $\mathcal{V}$. We note that invariants of a polynomial $\mathcal{V}$ are polynomials in the coefficients (and *only* the coefficients) of the polynomial $\mathcal{V}$ which remain invariant under linear transformation of the variables. For example, the invariants of the Calabi-Yau volume form are polynomials in its intersection numbers whose forms do not change under the rotation of basis divisors. Given a database of Calabi-Yau geometries specified by intersection numbers, one requires no other data in the effective supergravity theory (the matter Kähler metric or the superpotential) to check whether necessary conditions for the cosmological pheomena are satisfied on a given manifold. This makes such quantities suitable for computational studies along the lines of [27–30].

In the specific supergravity theories taken in Sec. IV, we show that this is indeed possible, under several restrictive conditions. If the UV completion is type IIB string theory compactified on a Calabi-Yau, it can be shown explicitly in the case of compactifications with two moduli (neglecting α' and g_s corrections) that the order parameter is an invariant (the discriminant) of the Calabi-Yau volume form. Generalizing to the case of n moduli, we conjecture that the relevant order parameter (again, neglecting α' and g_s corrections) is the discriminant of an n-ary cubic, generated by its ring of invariants. The discriminant for an arbitrary n moduli compactification is a polynomial of degree $n.(2)^{n-1}$. For example, for the case of three moduli, the ring of invariants is generated by the two Aronhold invariants S and T, of degree 4 and 6 respectively, and the discriminant is a homogeneous polynomial of degree 12 in the intersection numbers, given by $\Delta_3 = S^3 - T^2$:

$$\text{order parameter: discriminant}\,\Delta_n. \tag{3}$$

We use these results to study inflation and moduli dynamics. In the simplest case of two moduli, we are able to make definitive statements regarding the possibility of slow roll modular inflation depending on the sign of Δ_2. These results are given in Eq. (46). The case for n moduli is substantially more involved; nevertheless, we conjecture that it is the sign of Δ_n that is important, and give partial results towards this conjecture in Eq. (54). Similarly, for the two moduli case, we prove that moduli masses are bounded by the gravitino mass in Eq. (47), while giving indications that a similar result should hold in the general n moduli case as well.

Apart from the above considerations, we also present a different and complementary method of extracting information at the level of the geometry. At a given point in moduli space, the quantity in Eq. (1) is the sum over holomorphic sectional curvatures of planes in tangent space that are spanned by supersymmetry breaking directions. On the other hand, the induced soft mass (V''_{vis}) along a visible sector field Q^α depends on the quantity

$$R[f,Q] \equiv R_{i\bar{j}Q_\alpha\bar{Q}_{\bar{\beta}}} f^i f^{\bar{j}} Q^\alpha \bar{Q}^{\bar{\beta}} \equiv \sum_{\substack{i,j\equiv \cancel{SUSY} \\ \alpha,\beta\equiv \text{vis}}} \frac{R_{i\bar{j}\alpha\bar{\beta}}}{g_{i\bar{j}}g_{\alpha\bar{\beta}}}. \tag{4}$$

At a given point in the space of moduli and visible sector fields, this is the holomorphic *bisectional* curvature of the planes in tangent space spanned by supersymmetry breaking moduli and visible sector fields Q [31].

In Sec. V, we take the point of view that by exploring abstract relations between holomorphic sectional [Eq. (1)] and bisectional [Eq. (4)] curvatures, one can constrain Hubble-induced soft masses in a model-independent way on classes of manifolds. As an application, we consider the case of Affleck-Dine baryogenesis, which requires Hubble-induced soft masses to be tachyonic ($V''_{vis} < 0$). We find a no-go result: Affleck-Dine baryogenesis and modular inflation are incompatible on complex space forms, which are Kähler manifolds with isotropic holomorphic sectional curvature at every point. This generalizes an easily verifiable result for the case of symmetric coset manifolds.

The rest of the paper is structured as follows. In Sec. II, we review $D = 4, \mathcal{N} = 1$ supergravity and set our notation. In Sec. III, we describe the various cosmological phenomena we are interested in and the relevant bounds on the sectional curvature $R[f]$. In Sec. IV, we describe the connection to algebraic invariants. In Sec. V, we give the relations between holomorphic sectional and bisectional curvatures. We end with our conclusions. Most of the calculations are relegated to several appendixes.

II. MASS RELATIONS IN EFFECTIVE SUPERGRAVITY

The setting for much of our work will be $\mathcal{N} = 1$, $D = 4$ effective supergravity whose main features we briefly review in this section, following [32].

We will in general be interested in supergravities that have an observable or visible sector (which will be a supersymmetric extension of the Standard Model) and a modulus sector. We will generally leave the visible sector unspecified; depending on the UV completion, different extensions of the minimal supersymmetric Standard Model (MSSM) can be constructed. The chiral superfields in the visible sector will be labeled by Q^I, and will include all the quark, lepton, and Higgs superfields of the MSSM, and possibly additional particles [generally with $\mathcal{O}(1)$ TeV masses].

As for the moduli or "hidden" sector, we will denote the chiral superfields by Φ^i. Their vacuum expectation values (vevs) $\langle\Phi^i\rangle$ parametrize continuous families of the string vacua. While we will consider a UV completion later, right now it is enough to keep in mind that the effective potential of the moduli can receive both perturbative and nonperturbative contributions, a combination of which will stabilize them to finite vevs. These contributions can also lead to spontaneous supersymmetry breaking in the modulus sector, signaled by a nonzero vev of an F-term or a D-term. These vevs are the auxiliary components of chiral and vector superfields.

We thus have the following assumptions about the moduli sector:

(i) $V_{\rm eff}(\Phi)$ has a stable minimum, without flat directions.

(ii) At that minimum, $V_{\rm eff}(\langle\Phi\rangle)=0$, that is, the effective cosmological constant vanishes. Of course, this does not "solve" the cosmological constant problem, but rather reflects a fine-tuning of $V_{\rm eff}(\langle\Phi\rangle)$.

(iii) Some of the $\langle F^i\rangle$ in the moduli direction are nonzero.

The Lagrangian of the effective supergravity theory is given in terms of gauge couplings (that are moduli dependent in ways that depend on the specific UV completion), the Kähler function K (gauge-invariant real analytic function of the chiral superfields), and the superpotential W (holomorphic function of the chiral superfields).

We begin with the superpotential for the effective theory of the moduli Φ^i and the observable chiral superfields Q^α, which generally looks like $W_{\rm full}=\hat{W}(\Phi)+W_{\rm matter}$, where

$$W_{\rm matter}(Q^\alpha)=\frac{1}{2}\mu_{\alpha\beta}Q^\alpha Q^\beta+\frac{1}{3}Y_{\alpha\beta\gamma}Q^\alpha Q^\alpha Q^\alpha, \qquad (5)$$

is the classical superpotential. The modulus superpotential can be written schematically as

$$\hat{W}(\Phi)=W_{\rm tree}+W_{n-p}, \qquad (6)$$

where W_{n-p} stands for possible nonperturbative corrections to the superpotential that are crucial to obtaining stabilized moduli. The superpotential does not suffer from renormalization in any order of perturbation theory.

The Kähler function K is responsible for the kinetic terms in the Lagrangian. Expanding in powers of Q^α and $Q^{\bar\alpha}$, we have

$$K_{\rm full}=\hat{K}(\Phi,\bar\Phi)+Z_{\bar\alpha\beta}(\Phi,\bar\Phi)\bar{Q}^{\bar\alpha}Q^\beta + \left(\frac{1}{2}H_{\alpha\beta}(\Phi,\bar\Phi)Q^\alpha Q^\beta+{\rm H.c.}\right)+\cdots, \qquad (7)$$

where the $\cdots$ stand for the higher-order terms; $Z_{\bar\alpha\beta}$ is the Kähler metric for the observable superfields; the Kähler metric for the moduli is given by $K_{\bar{i}j}\equiv\bar\partial_{\bar i}\partial_j K$. In general, neither the higher-order terms nor the $Z_{\bar\alpha\beta}$ are calculable in a model-independent manner.

The effective potential for the moduli, which will be very important for us, is given by

$$V_{\rm eff}(\Phi,\bar\Phi)=\hat{K}_{i\bar j}F^i\bar{F}^{\bar j}-3e^{\hat K}|\hat{W}(\Phi)|^2, \qquad (8)$$

where

$$\bar{F}^{\bar j}=e^{\hat{K}/2}\hat{K}^{\bar j i}(\partial_i\hat{W}+\hat{W}\partial_i\hat{K}), \qquad \hat{K}^{\bar j i}=(\hat{K}_{i\bar j})^{-1}. \qquad (9)$$

At the minimum of Eq. (8), $V_{\rm eff}(\Phi,\bar\Phi)=0$, but (some) $\langle F^i\rangle\neq 0$ and thus supersymmetry is spontaneously broken. The measure of this breakdown is the gravitino mass

$$m_{3/2}=e^{\langle\hat K\rangle/2}|\hat{W}(\langle\Phi\rangle)|=\left\langle\frac{1}{3}\hat{K}_{i\bar j}F^i\bar{F}^{\bar j}\right\rangle^{1/2}. \qquad (10)$$

We note that supersymmetry is also generally broken at any other period of cosmic history when the vacuum energy is nonzero, such as during inflation. We will assume that such spontaneous breaking is also performed by moduli F-terms, keeping the details for the next section.

At this stage, the effective Lagrangian of the observable sector can be written down in a straightforward manner. The Lagrangian of the effective theory for Q^α and Φ^i is first written down, and the dynamical moduli fields, including the auxiliary F-terms, are replaced by their vevs. The flat limit $M_{pl}\to\infty$, while keeping $m_{3/2}$ fixed, is taken.

We will be especially interested in the potential for the observable scalars (which, by abuse of notation, we call Q^α). This is given by

$$\begin{aligned} &V_{\rm eff}(Q,\bar Q)\\ &=\sum_a\frac{g_a^2}{4}(\bar{Q}^{\bar\alpha}Z_{\bar\alpha\beta}T_aQ^\beta)^2+\partial_\alpha W_{\rm eff}Z^{\alpha\bar\beta}\bar\partial_{\bar\beta}\bar{W}_{\rm eff}\\ &\quad+m^2_{\alpha\bar\beta}Q^\alpha\bar{Q}^{\bar\beta}+\left(\frac{1}{3}A_{\alpha\beta\gamma}Q^\alpha Q^\beta Q^\gamma+\frac{1}{2}B_{\alpha\beta}Q^\alpha Q^\beta+{\rm H.c.}\right) \end{aligned} \qquad (11)$$

The first line gives the scalar potential of an effective theory with unbroken rigid supersymmetry. The second line encodes the soft terms. The soft terms are given in terms of moduli vevs and F-terms as follows:

$$\begin{aligned} m^2_{\alpha\bar\beta}&=m^2_{3/2}Z_{\alpha\bar\beta}-F^i\bar{F}^{\bar j}R_{i\bar j\alpha\bar\beta}+V_0,\\ A_{\alpha\beta\gamma}&=F^iD_iY_{\alpha\beta\gamma},\\ B_{\alpha\beta}&=F^iD_i\mu_{\alpha\beta}-m_{3/2}\mu_{\alpha\beta}, \end{aligned} \qquad (12)$$

where

$$R_{i\bar j\alpha\bar\beta}=\partial_i\bar\partial_{\bar j}Z_{\alpha\bar\beta}-\Gamma^\gamma_{i\alpha}Z_{\gamma\bar\delta}\bar\Gamma^{\bar\delta}_{\bar j\bar\beta}, \qquad \Gamma^\gamma_{i\alpha}=Z^{\gamma\bar\beta}\partial_iZ_{\bar\beta\alpha}, \qquad (13)$$

$$D_iY_{\alpha\beta\gamma}=\partial_i\hat{Y}_{\alpha\beta\gamma}+\frac{1}{2}\hat{K}_iY_{\alpha\beta\gamma}-\Gamma^\delta_{i(\alpha}Y_{\beta\gamma)\delta}, \qquad (14)$$

$$D_i\mu_{\alpha\beta}=\partial_i\mu_{\alpha\beta}+\frac{1}{2}\hat{K}_i\mu_{\alpha\beta}-\Gamma^\gamma_{i(\alpha}\mu_{\beta)\gamma}. \qquad (15)$$

All quantities appearing in Eq. (12) are covariant with respect to the supersymmetric reparametrization of matter and moduli fields as well as covariant under Kähler transformations.

III. COSMOLOGICAL PHENOMENA AND GEOMETRIC CONSTRAINTS

Having described the setting of effective supergravity, we now turn to a discussion of the three cosmological applications of Eq. (8) and Eq. (11) mentioned in the Introduction. As a unifying theme, we first show how Eq. (1) emerges as a crucial quantity.

A. A bound on $V''(\Phi)$

The starting point is to consider the mass matrix

$$N = \begin{pmatrix} \nabla^i\nabla_j V & \nabla^i\nabla_{\bar{j}} V \\ \nabla^{\bar{i}}\nabla_j V & \nabla^{\bar{i}}\nabla_{\bar{j}} V \end{pmatrix}, \qquad V \equiv V_{\mathrm{eff}}(\Phi, \bar{\Phi}), \quad (16)$$

where as usual

$$\nabla_i f^k \equiv \partial_i f^k + \Gamma^k_{ij} f^j \quad (17)$$

for any vector f^k.

The lightest modulus mass will be denoted by $m^2_{\Phi,\mathrm{lightest}}$ and is given by

$$m^2_{\Phi,\mathrm{lightest}} = \text{min eigenvalue}\{N\}. \quad (18)$$

Since $m^2_{\Phi,\mathrm{lightest}}$ is defined as the minimum eigenvalue of the matrix N, it always satisfies a bound. For any given unit vector u^I one has

$$m^2_{\Phi,\mathrm{lightest}} \le u_I N^I_J u^J. \quad (19)$$

Choosing u^I cleverly, it is possible to obtain simple expressions. The obvious choice is the "preferred" direction in moduli space, along the SUSY breaking direction

$$u_I = (e^{-\imath\phi} f_i, e^{\imath\phi} f_{\bar{i}})/(\sqrt{2}), \quad (20)$$

where $i = 1\ldots p$ denote all the SUSY breaking moduli, ϕ is an arbitrary phase, and the f_i are aligned along the SUSY breaking directions

$$f_i = F_i/|F|. \quad (21)$$

Taking $u^J = (e^{\imath\phi} f^i, e^{-\imath\phi} f^{\bar{i}})/(\sqrt{2})$, one obtains

$$m^2_{\Phi,\mathrm{lightest}} \le \nabla_i\nabla_{\bar{j}} V f^i f^{\bar{j}} + \mathrm{Re}(e^{2\imath\phi}\nabla_i\nabla_j V f^i f^j). \quad (22)$$

The second piece in Eq. (22) is superpotential dependent and needs to be eliminated. Choosing $\phi = 0$ and $\phi = \pi/2$ and adding the two resulting versions of Eq. (22) achieves this and one obtains

$$m^2_{\Phi,\mathrm{lightest}} \le \nabla_i\nabla_{\bar{j}} V f^i f^{\bar{j}}. \quad (23)$$

It now remains to evaluate the right-hand side of Eq. (23). One obtains

$$m^2_{\Phi,\mathrm{lightest}} \le 2m^2_{3/2} - (V + 3m^2_{3/2}) R_{i\bar{j}k\bar{l}} f^i f^{\bar{j}} f^k f^{\bar{l}} + \frac{1}{V + 3m^2_{3/2}} \nabla_i V \nabla^i V + 4\left(\frac{m^2_{3/2}}{V + 3m^2_{3/2}}\right)^{\frac{1}{2}} \mathrm{Re}(\nabla_i V f^i). \quad (24)$$

We now apply Eq. (24) to different physical contexts.

B. Slow roll modular inflation

For slow roll modular inflation, we note that the slow-roll parameter η is given by

$$\eta = \frac{1}{3H^2 M^2_{pl}} m^2_{\Phi,\mathrm{lightest}}. \quad (25)$$

In Eq. (24), we drop all terms involving $\nabla_i V \sim \sqrt{\epsilon}$, since $\sqrt{\epsilon} < \mathcal{O}(10^{-3})$. We also set $V = 3H^2 M^2_{pl}$. The spectral index is given by

$$n_s = 1 + 2\eta \Rightarrow \eta_{\mathrm{observed}} \sim -0.01. \quad (26)$$

Putting this value on the right-hand side of Eq. (24), we obtain

$$R[f] \le \frac{2}{3} \frac{m^2_{3/2}}{m^2_{3/2} + H^2}. \quad (27)$$

The quantity Eq. (1) is thus bounded from the requirement of the flatness of the inflaton potential required to generate a sufficient number of e-foldings. While the exact value of the bound depends on the relative values of the gravitino mass and H, there is a hard bound:

$$R[f] < \frac{2}{3}. \quad (28)$$

Several comments are in order. Clearly, the condition is not sufficient—obtaining slow-roll parameters along some modular direction that can lead to acceptable inflationary observables requires full knowledge of the potential and higher-order corrections. This will entail knowing the superpotential, for example. However, given a set of Kähler geometries, and asked which ones can, *in principle*, admit modular inflation, the necessary condition Eq. (28) is most useful as a first check.

C. Thermal history

There is another set of applications that one can get from Eq. (24), which also involves the quantity Eq. (1). Since Eq. (24) is a bound on the lightest modulus, this has implications for the thermal history of the Universe.

The coherent oscillations of a modulus Φ about its low-energy minimum lead to the formation of a scalar condensate, which scales like matter and dilutes more slowly that the primordial radiation produced during reheating. Depending on the initial displacement of the modulus, its energy can come to dominate the energy density of the Universe. Moreover, because it is only gravitationally coupled to other fields, its decay rate is

$$\Gamma_\Phi = \frac{c}{2\pi}\frac{m_\Phi^3}{\Lambda^2}, \tag{29}$$

where we expect $\Lambda \sim M_{pl}$ and c depends on the precise coupling in the fundamental Lagrangian, but typically takes values of at most $\mathcal{O}(100)$. Light Standard Model particles that are produced during this decay will "reheat" the universe for a second time. The corresponding reheat temperature is given by $T_r \sim g_*^{-1/4}\sqrt{\Gamma_\Phi M_{pl}}$ or

$$T_{\rm r} = c^{1/2}\left(\frac{10.75}{g_*}\right)^{1/4}\left(\frac{m_\Phi}{50\ {\rm TeV}}\right)^{3/2} T_{\rm BBN}, \tag{30}$$

where $T_{\rm BBN} \simeq 3$ MeV and g_* is the number of relativistic degrees of freedom at $T_{\rm r}$. The reheat temperature must be larger than around 3 MeV to be in agreement with light element abundances as predicted successfully by big bang nucleosynthesis [33].

An interesting departure from a thermal post-inflationary universe occurs if the mass of the lightest modulus is in a window between $\mathcal{O}(10)$ and $\mathcal{O}(1000)$ TeV. For masses that are much higher, little departure from a thermal universe is expected, whereas the lower bound comes from consistency with BBN as discussed before.

The key question is, should one generally expect a modulus in this mass range? To answer this question, we start from Eq. (24) and set

$$\nabla_i V = V = 0. \tag{31}$$

This yields the following bound:

$$m^2_{\Phi,\rm lightest} \le 3m^2_{3/2}\left(\frac{2}{3} - R[f]\right). \tag{32}$$

Low-energy supersymmetry with gravity mediation typically has the gravitino mass $\mathcal{O}(10)$–$\mathcal{O}(1000)$ TeV. Given that, nonthermal histories are obtained when [34]

$$R[f] \sim \mathcal{O}(1). \tag{33}$$

D. Baryogenesis

In this subsection, we discuss the connection between Hubble-induced masses and the quantity in Eq. (4), applying it to the case of Affleck-Dine baryogenesis.

The vacuum energy V_0 during inflation breaks supersymmetry. The Affleck-Dine baryogenesis mechanism relies on this supersymmetry breaking to induce tachyonic soft masses along a supersymmetric flat direction. If the flat direction, which we denote by Q, is initially displaced from its true minimum, it subsequently oscillates when V_0 becomes smaller than the effective mass which is $\sim m_{3/2}$. Depending on the magnitude of the baryon number violating terms in $V(Q)$, a net baryon asymmetry may be produced from the resulting condensate.

The potential for the flat direction Q may be written as

$$V(Q) = (m^2_{\rm soft,inf} + m^2_{\rm soft,final})|Q|^2 + \left(\frac{(A + a_{\rm inf})\lambda Q^n}{nM_P^{n-3}} + {\rm H.c.}\right) + |\lambda|^2\frac{|Q|^{2n-2}}{M_P^{2n-6}}. \tag{34}$$

Here, $m^2_{\rm soft,inf}$ and $a_{\rm inf}$ denote soft parameters induced by supersymmetry breaking during inflation, while $m_{\rm soft,final}$ and A arise from supersymmetry breaking in the final vacuum of the theory. The last term comes from nonrenormalizable superpotential contributions.

If $m_{\rm soft,inf}$ is tachyonic, the field Q acquires a nonzero vacuum expectation value during inflation and tracks an instantaneous minimum thereafter, until $H \sim m_{3/2}$. At this point, the field begins to oscillate around the new minimum $Q = 0$ and the soft A-term becomes comparable to the Hubble-induced $a_{\rm inf}$. The field acquires an angular motion to settle into a new phase and the baryon number violation becomes maximal at this time.

From Eq. (4) and Eq. (12), and using $F^2 = V_0 + 3m^2_{3/2}$, the soft masses can be written as

$$m^2_{\rm soft,inf} = V_0(1 - R[f,Q]) + 3m^2_{3/2}\left(\frac{1}{3} - R[f,Q]\right). \tag{35}$$

Requiring this to be tachyonic, and making the assumption that during inflation $V_0 \gg m^2_{3/2}$, we thus obtain the result that Affleck-Dine baryogenesis is only possible if

$$R[f,Q] > 1. \tag{36}$$

E. Summary

Summarizing the results of the three cases above in terms of $R[f]$ and $R[f,Q]$, we get Table I.

IV. COSMOLOGY AND INVARIANTS OF ALGEBRAIC FORMS

In the previous sections, we have described a particularly simple order parameter for a host of cosmological phenomena:

$$R_{i\bar{j}k\bar{l}} f^i f^{\bar{j}} f^k f^{\bar{l}}, \tag{37}$$

where the indices are summed over the directions in field space along which supersymmetry is broken. We note several features of this order parameter:

(i) It depends on the Kähler potential of the theory, since the curvature tensor is derived from the Kähler potential.
(ii) The expression is field dependent, so it depends on the allowed values of the moduli and hence implicitly on the superpotential data also.
(iii) The expression requires knowledge about the SUSY breaking mechanism, to identify the vectors f^i in moduli space along which SUSY is dominantly broken.
(iv) The expression only makes sense after the correct Kähler coordinates [in which the effective potential takes the form Eq. (8)] are identified. This is a nontrivial task.

The goal is to obtain equivalent conditions on a parameter Δ that is
(i) field independent,
(ii) independent of knowledge of the SUSY breaking mechanism (hence independent of the orientation of the f^i).

We will see that this leads naturally into classical algebraic invariant theory.

To fix the class of Kähler potentials for our effective supergravity, we take the setting of type IIB string theory. We give a full description of the Kähler potential and the identification of Kähler coordinates in Appendix A. The final result is that the Kähler potential is given by the logarithm of the volume form (which is a cubic in the Kähler coordinates τ that correspond to volumes of four cycles in the compactified Calabi-Yau):

$$K = -2\ln\mathcal{V}(\tau). \tag{38}$$

The volume $\mathcal{V}$ is given by Eq. (A4) in terms of the intersection numbers d^{abc}, which are given in Eq. (A4) as well. For completeness, we collect the expressions here:

$$\tau^a = \frac{1}{16}d^{abc}v_b v_c,$$
$$\mathcal{V} = \frac{1}{48}d^{abc}v_a v_b v_c. \tag{39}$$

We note that the v_a denote volumes of two cycles, a basis in which the intersection numbers are naturally expressed. However, they do not constitute the correct Kähler coordinates for the low-energy action, which are provided by the τ^a that are defined through the Legendre transform in Eq. (39). While obtaining the τ^a coordinates explicitly starting out from the v^a is difficult, in practice one can avoid the problem by working implicitly with Eq. (39). This is shown in Appendix B, where finally the Riemannian curvature has been computed. For easy reference, we display the expression below

$$\begin{aligned} R_{ijmn} = &-g_{im}g_{jn} + e^{-2K}(\tilde{d}_{ijk}g^{kl}\tilde{d}_{lmn} + \tilde{d}_{ink}g^{kl}\tilde{d}_{ljm}) \\ &+ g_{in}K_jK_m + g_{jm}K_iK_n + g_{im}K_jK_n + g_{jn}K_iK_m \\ &+ g_{ij}K_mK_n + g_{mn}K_iK_j - 3K_iK_jK_mK_n \\ &- e^{-K}(\tilde{d}_{imj}K_n + \tilde{d}_{imn}K_j + \tilde{d}_{inj}K_m + \tilde{d}_{nmj}K_i). \end{aligned} \tag{40}$$

We now proceed to compute the quantity $R_{i\bar{j}m\bar{n}}f^if^{\bar{j}}f^mf^{\bar{n}}$. To this end, we will find it particularly useful to decompose the vectors f^i into directions along K^i and directions $K^i_\perp$ that are orthogonal to it. Denoting the unit vectors along those two directions by k^i and $k^i_\perp$ respectively, we can write

$$f^i = \sin\theta k^i + \cos\theta k^i_\perp. \tag{41}$$

We note that $k^i_\perp$ is itself a vector in a $h^{(1,1)} - 1$ dimensional space, parametrized by $h^{(1,1)} - 2$ angles which we can denote by $\theta_{\perp,p}$ with $p = 1\ldots h^{(1,1)} - 2$.

This is clearly a good strategy, given the structure of Eq. (40). Using moreover the no-scale property $K_iK^i = 3$, the expression reduces to [24]

$$\frac{2}{3} - R_{i\bar{j}m\bar{n}}f^if^{\bar{j}}f^mf^{\bar{n}} = (-A^iA_i + B), \tag{42}$$

where A^i and B are functions of the angle $(\theta, \theta_{\perp,p})$, the intersection numbers, and the metric. The full forms of these functions are displayed in Appendix C.

Clearly, it is essential to compute the quantity $A^iA_i + B$, which serves as an order parameter for inflation. In fact, we have

$$\begin{aligned} &(-A^iA_i + B)_{\max} > 0 \Rightarrow \text{inflation allowed}, \\ &(-A^iA_i + B)_{\max} < 0 \Rightarrow \text{inflation not allowed}. \end{aligned} \tag{43}$$

For a given geometry specified by the intersection numbers, the maximization has to be carried out with respect to the angles $(\theta, \theta_{\perp,p})$ that have been defined earlier.

This is a nontrivial computation, involving a set of coupled cubic equations in $\tan(\theta, \theta_{\perp,p})$. In the simplest case of two moduli $h^{(1,1)} = 2$, it can be carried out explicitly, with the result that

$$\left(\frac{2}{3} - R_{i\bar{j}m\bar{n}}f^if^{\bar{j}}f^mf^{\bar{n}}\right)_{\max} = k \times \frac{\Delta_2}{24}\frac{(\det g)^3}{e^{4K}} \le 1, \tag{44}$$

where the prefactor k is positive, and the entire right-hand side of the above equation can be shown to be less than one. For a proof of Eq. (44), we refer to Appendix C.

The expression Δ_2 is the discriminant of the volume form (which, for two moduli, is a binary cubic $d^{ijk}v_iv_jv_k$), and is given by

$$\Delta_2 = -27[(d^{000})^2(d^{111})^2 - 3(d^{001})^2(d^{011})^2 + 4(d^{000})(d^{011})^3 + 4(d^{001})^3(d^{111}) - 6(d^{000})(d^{001})(d^{011})(d^{111})]. \quad (45)$$

We note that the subscript in Δ_2 signifies that it is the discriminant of a binary cubic; for a general model with n moduli, $h^{(1,1)} = n$, we will be concerned with the discriminant of an n-ary cubic, which we will denote by Δ_n.

This has two immediate consequences:

(i) for models with $h^{(1,1)} = 2$, the necessary condition for slow roll modular inflation can be stated as a condition on the discriminant of the volume form:

$$\Delta_2 > 0 \Rightarrow \text{inflationallowed}, \quad \Delta_2 < 0 \Rightarrow \text{inflationnot allowed}, \quad (46)$$

and

(ii) for models with $h^{(1,1)} = 2$, the canonically normalized moduli masses are bounded by the gravitino mass, from Eq. (32):

$$\text{moduli mass bound: } m^2_{\Phi,\text{lightest}} \leq 3m^2_{3/2}. \quad (47)$$

A. Algebraic invariants and a generalization to $h^{(1,1)} = n$

The emergence of the discriminant for the case of two moduli is striking. At this point, it is useful to recall some basic facts about classical algebraic invariant theory [35]. The invariant of a binary form of degree d (by definition in two variables, which, for us, are the moduli) is a polynomial in the coefficients (which, for us, are the intersection numbers) that remains invariant under the action of the special linear group acting on the variables.

Specifically, we can consider the following binary form of degree d:

$$f(x,y) = \sum_{k=0}^{d} \binom{d}{k} a_k x^k y^{d-k}. \quad (48)$$

A linear change of variables [under the group $GL_2(C)$], which we label (c_{ij}) is a transformation of the variables x and y, given by

$$x = c_{11}\bar{x} + c_{12}\bar{y}, \qquad y = c_{21}\bar{x} + c_{22}\bar{y} \quad (49)$$

such that the determinant $c_{11}c_{22} - c_{12}c_{21}$ is nonzero. Under the $SL_2(C)$ action, the binary form is transformed into a new form $\bar{f}(\bar{x},\bar{y})$ in the transformed variables $\bar{x}$ and $\bar{y}$, with coefficients $\bar{a}_k$

$$\bar{f}(\bar{x},\bar{y}) = \sum_{k=0}^{d} \binom{d}{k} \bar{a}_k \bar{x}^k \bar{y}^{d-k}. \quad (50)$$

Clearly, the new coefficients $\bar{a}_k$ are polynomials in the original coefficients a_i and the parameters c_{ij}.

A *covariant* of the binary form is defined as a nonconstant polynomial $I(a_0, a_1, \ldots, a_d, x, y)$ such that the following identity holds:

$$I(\bar{a}_0, \bar{a}_1, \ldots, \bar{a}_d, \bar{x}, \bar{y}) = (c_{11}c_{22} - c_{21}c_{12})^g I(a_0, a_1, \ldots, a_d, x, y), \quad (51)$$

where g is a non-negative integer. The prefactor on the right-hand side is one for a special transformation.

A covariant in which the variables x and y do not occur is called an *invariant*. Every invariant of a binary cubic can be written in terms of the discriminant Δ_2, of degree 4, defined previously. Moreover, the algebra of covariants for a binary cubic is generated by the discriminant Δ_2, the form itself, its Hessian, and a covariant of degree 3.

A $GL(2,C)$ transformation in our case corresponds to a transformation on the basis of divisors $Div(CY)$ of the Calabi-Yau. It is expected that the necessary criterion for inflation should not change under a basis transformation. However, the important point is that the order parameter is not a *covariant*, but even more strongly, an *invariant* that is completely independent of the stabilized values of the moduli, and specified only by the intersection numbers.

Given the above, we can probe a possible generalization to the case of $h^{(1,1)} = n$, i.e., n moduli. There is an immediate problem with this. For a form of degree d in n variables, the discriminant is a homogeneous polynomial of degree $n.(d-1)^{n-1}$. For us, $d = 3$ giving

$$\Delta_n \to n.(2)^{n-1} \text{ degree polynomial in } d^{ijk}. \quad (52)$$

On the other hand, from the definition of $R_{i\bar{j}k\bar{l}} f^i f^{\bar{j}} f^k f^{\bar{l}}$ and the general form of A_i and B given in Appendix C, it is clear that at any given local maximum [with respect to the angular dependence $(\theta, \theta_{\perp,p})$] of $(-A^i A_i + B)$ is a degree 4 polynomial in the intersection numbers:

$$\left(\frac{2}{3} - R_{i\bar{j}m\bar{n}} f^i f^{\bar{j}} f^m f^{\bar{n}}\right)_{\text{local max}} \to \text{fourth degree polynomial in } d^{ijk}. \quad (53)$$

In the limit that all the divisors are identified and only two independent cycles remain, the result must reduce to the invariant Δ_2. Starting with a higher-order covariant, it is difficult to see how the moduli dependence will drop out. Thus, the natural solution is to start with a higher-order *invariant*, like the discriminant, which naturally reduces to its lower dimensional value upon identification of intersection numbers. To match the polynomial degree, we conjecture that a *product* of local maxima (which is a subset of the total number of extrema, but contains the global maximum) in the n moduli case should give the higher-order discriminant:

$$\prod_{a=1}^{n.2^{n-3}} \left(\frac{2}{3} - R_{i\bar{j}m\bar{n}} f^i f^{\bar{j}} f^m f^{\bar{n}}\right)_{\text{crit},a^{th}} \propto \Delta_n \frac{(\det g)^{3n.2^{n-3}}}{e^{4n.2^{n-3}K}}, \tag{54}$$

This result is weaker than the $n = 2$ case, where Δ_2 was directly serving as an order parameter in Eq. (46). Moreover, we are unable to calculate the sign of the coefficient on the right-hand side. We note that a similar conjecture was arrived at in the case of metastable vacua in heterotic string compactifications [36].

However, we can still see that

$$\left(\frac{2}{3} - R_{i\bar{j}m\bar{n}} f^i f^{\bar{j}} f^m f^{\bar{n}}\right)_{a^{th}} \sim (\Delta_n)^{\frac{1}{n.2^{n-3}}} \frac{(\det g)^3}{e^{4K}} \sim \mathcal{O}(1)$$
$$\Rightarrow R[f] \sim \mathcal{O}(1). \tag{55}$$

Thus, similar to the case of two moduli, one obtains

$$m^2_{\Phi,\text{lightest}} \lesssim 3m^2_{3/2}. \tag{56}$$

It would be very interesting to work out the exact sign in Eq. (54), as well as advance a rigourous proof. We leave that for the future [37].

V. SECTIONAL AND BISECTIONAL CURVATURES

We have seen in the previous sections that the quantities $R[f]$ and $R[f,Q]$ serve as important order parameters for cosmology within the setting of effective supergravity. In particular, the aim was to express these quantities in as general a form as possible.

In this section, we define these quantities carefully and look for abstract relations between them. $R[f]$ serves as an order parameter for slow roll modular inflation, while $R[f,Q]$ controls soft masses induced by supersymmetry breaking (during inflation, for example) along visible sector fields. Thus, relations between them will constrain Affleck-Dine baryogenesis, which relies critically on Hubble-induced soft terms.

We begin with a careful definition of the quantities $R[f]$ and $R[f,Q]$. We consider a Kähler manifold of complex dimension n, with R denoting its Riemannian curvature tensor. At each point x of M, R is a quadrilinear map

$$T_x(M) \times T_x(M) \times T_x(M) \times T_x(M) \to \mathbb{R}. \tag{57}$$

Here, $T_x(M)$ denotes the tangent space at the point x on the manifold M, while $\mathbb{R}$ denotes the real numbers.

We can now consider a plane f in the tangent space $T_x(M)$, with an orthonormal basis (X,Y). The sectional curvature is given by a function on the Grassmann bundle of two planes in the tangent space at x. Specifically, for the plane f at the point x on M, the function is given by

$$K[f] = R(X,Y,X,Y). \tag{58}$$

The sectional curvature depends on the point x where it is evaluated, and the plane f it is defined for, but not on the choice of basis vectors (X,Y).

We denote the (almost) complex structure of M by J. The set of J-invariant planes constitutes a holomorphic bundle over M with fiber $P_{n-1}(C)$. The restriction of the sectional curvature to this complex projective bundle is called the holomorphic sectional curvature:

$$\mathbb{H}[f] = R(X,JX,X,JX) = -\frac{R_{X\bar{X}X\bar{X}}}{g_{X\bar{X}}g_{X\bar{X}}}. \tag{59}$$

The bisectional curvature is defined similarly. For two J-invariant planes f (with unit vector X) and $\mathcal{Q}$ (with unit vector Q) in $T_x(M)$, the holomorphic bisectional curvature is given by

$$\mathbb{H}[f,Q] = R(X,JX,Q,JQ) = -\frac{R_{X\bar{X}Q\bar{Q}}}{g_{X\bar{X}}g_{Q\bar{Q}}}. \tag{60}$$

Like the sectional curvature, this quantity too is independent of the particular choice of basis vectors. Moreover, one trivially has

$$\mathbb{H}[f,f] = \mathbb{H}[f]. \tag{61}$$

Moreover, in terms of the quantities $R[f]$ and $R[f,Q]$ that were previously defined, one has

$$\mathbb{H}[f] = -R[f],$$
$$\mathbb{H}[f,Q] = -R[f,Q]. \tag{62}$$

We will be particularly interested in relations between the holomorphic sectional and bisectional curvature.

At a given point x in the manifold, for orthonormal directions X and Q, the holomorphic bisectional curvature $\mathbb{H}[f,Q]$ between the planes $(Q,\bar{Q})$ and $(X,\bar{X})$ is a linear combination of holomorphic sectional curvatures of certain planes:

$$\mathbb{H}[f,Q] = \frac{1}{4}\left\{\sum_{a=1}^{4}\mathbb{H}[\lambda_a] - \mathbb{H}[f] - \mathbb{H}[Q]\right\}, \tag{63}$$

where the λ_a denote certain holomorphic and anti-holomorphic sections associated with the section spanned by the pair (Q,X). For the special case where the holomorphic sectional curvatures are simply constant for all choices of planes in tangent space at x

$$R_{j\bar{j}j\bar{j}} = \text{constant} \quad \forall\ [\text{span}(\partial_j, \partial_{\bar{j}}) \in T_x(\mathcal{M})], \tag{64}$$

one obtains

$$\mathbb{H}[f] = \text{const}\,(c) \Rightarrow \frac{|c|}{2} \le |\mathbb{H}[f,Q]| \le |c|. \tag{65}$$

For orthonormal planes, the lower bound is exactly satisfied.

We note that a manifold of this type is called a complex space form. If, in addition, the isotropy of the holomorphic sectional curvature in tangent space holds for all x belonging to the Kähler manifold, we say that the manifold has constant holomorphic sectional curvature, of which a maximally symmetric coset space is an example. This is a statement about special components of the curvature tensor; namely, a manifold has constant holomorphic sectional curvature when

$$R_{j\bar{j}j\bar{j}} = \text{constant} \quad \forall\ [x \in \mathcal{M}, \text{span}(j\bar{j}) \in T_x(\mathcal{M})]. \tag{66}$$

Taking the scale of inflation to be high, the conditions for inflation and baryogenesis are

$$\mathbb{H}[f] > 0,$$
$$\mathbb{H}[f,Q] \lesssim -1. \tag{67}$$

There is a clear contradiction and we thus have the following no-go result: accommodating *both* slow roll modular inflation and Affleck-Dine baryogenesis is impossible on complex space forms. This is trivial to check explicitly in the special example of maximally symmetric coset spaces.

VI. CONCLUSIONS

In this paper, we have attempted to construct a universal order parameter within effective supergravity for slow roll modular inflation, nonthermal cosmological histories, and Affleck-Dine baryogenesis. Our starting point was the fact that the local curvature properties of the Kähler manifold spanned by scalars belonging to chiral superfields play a vital role in determining the viability of these diverse phenomena. The Riemannian curvature tensor, evaluated along supersymmetry breaking directions, must necessarily take certain values, summarized in Table I.

Next, we have attempted to recast the conditions on the Riemannian curvature in terms of equivalent conditions on other parameters that are entirely field independent, constructed from the input parameters that specify the Kähler geometry. For type IIB Calabi-Yau compactifications, in the case of two moduli, we have proven that the order parameter is an invariant (the discriminant) of the Calabi-Yau volume form, neglecting α' and g_s corrections. Generalizing to the case of n moduli, we have conjectured that the relevant order parameter is the discriminant of an n-ary cubic, generated by its ring of invariants.

We have utilized these results in the case of two moduli to make definitive statements regarding the possibility of slow roll modular inflation depending on the sign of Δ_2. These results are given in Eq. (46). In the case of n moduli we conjecture that it is the sign of Δ_n that is important, and give partial results towards this conjecture in Eq. (54). In the case of two moduli, we are also able to directly prove that the lightest modulus mass is bounded by three times the gravitino mass.

The results in this paper may be useful to rule out certain cosmological phenomena on classes of manifolds based solely on the geometric input that specifies the Kähler potential. As such, they may be useful for computational studies along the lines of [27–30].

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APPENDIX A: MODULI SPACE AND KAHLER COORDINATES IN TYPE IIB STRING THEORY

Our goal in this appendix is to provide details for the emergence of the coordinates τ^i, which parametrize the volumes of four cycles in the internal Calabi-Yau, as the correct Kähler coordinates in type IIB string theory. This provides the background for the assumptions in Sec. IV. Along the way, we describe the moduli space of the theory, preparing the ground for the discussion on nonthermal cosmologies in Sec. III C.

We closely follow the reviews of [38,39], extracting the main results.

1. Calabi-Yau moduli space

The forms of a Calabi-Yau moduli space are the following:

(i) one (constant) harmonic 0-form;
(ii) one (3,0) form and one (0,3) form, labeled Ω_{CY} and $\bar{\Omega}_{CY}$, respectively;
(iii) a set of $h^{(1,1)}$ harmonic (1,1) and (2,2) forms. The cohomology basis of the (1,1) forms [respectively, the (2,2) forms] is denoted by $w_a(\tilde{w_a})$, with $a = 1, .., h^{(1,1)}$;
(iv) a set of $h^{(2,1)}$ harmonic (2,1) and (1,2)-forms. The cohomology basis of the (2,1) forms [respectively, the (1,2) forms] is denoted by $\chi_k(\tilde{\chi_k})$, with $k = 1, .., h^{(2,1)}$;
(v) one (3,3) form, the volume $\mathcal{V}$.

For simplicity, we will also sometimes group the basis cycles as follows: $H^{(0)} \oplus H^{(1,1)}$ with basis $w_A = (1, w_a)$, $A = 0, .., h^{(1,1)}$; $H^{(3)}$ with basis (α_K, β^K), $K = 0, .., h^{(2,1)}$.

The four-dimensional effective action before fluxes or orientifolding corresponds to an $\mathcal{N} = 2$ ungauged supergravity theory. The strategy is to expand (six-dimensional) internal space deformations of the various fields in the above basis cycles. The (four-dimensional) coefficients for each term of such an expansion will then correspond to visible spacetime fields. These fields are the moduli.

Denoting the internal space coordinates collectively as y, and four dimensional spacetime as x, we then have

$$\begin{aligned} \phi(x,y) &= \phi(x), \\ g_{i\bar{j}}(x,y) &= i v^a(x)(\omega_a)_{i\bar{j}}(y), \\ g_{ij}(x,y) &= i\bar{z}^k(x)\left(\frac{(\bar{\chi}_k)_{i\bar{k}\bar{l}}\Omega^{\bar{k}\bar{l}}{}_j}{|\Omega|^2}\right)(y), \\ B_2(x,y) &= B_2(x) + b^a(x)\omega_a(y). \end{aligned} \tag{A1}$$

In the above, $g_{i\bar{j}}$ denotes the metric. From the Neveu-Schwartz sector, we thus obtain a total of $2(h^{(1,1)}+1)+h^{(2,1)}$ x-dependent fields or moduli.

A similar expansion can be carried out for the fields belonging to the Ramond-Ramond (RR) sector, which we display below only for type IIB:

$$\begin{aligned} C_0(x,y) &= C_0(x), \\ C_2(x,y) &= C_2(x) + c^a(x)\omega_a(y), \\ C_4(x,y) &= V_1^K(x)\alpha_K(y) + \rho_a(x)\tilde{\omega}^a(y). \end{aligned} \tag{A2}$$

Moreover, the Kähler form J is parametrized as

$$J = \sum_{a=1}^{h^{(1,1)}} v^a \omega_a, \tag{A3}$$

which endows the v^a with a natural interpretation as volumes of two cycles.

The type IIB moduli are arranged into $\mathcal{N}=2$ multiplets. Of the fields shown above, the metric deformations v^a and the deformations b^a get arranged into a hypermultiplet of dimension $h^{(1,1)}$. Similarly, the moduli z^k go to a $h^{(2,1)}$ dimensional vector multiplet, while the fields B_2 and ϕ go to a one dimensional tensor multiplet. The four-dimensional metric $g_{\mu\nu}$ (which we have not shown) belongs to a gravity multiplet. All these multiplets are completed by fields coming from the RR sector.

We also display some important quantities that can be obtained from the basis two-cycles w^a, the Kähler form J, and the moduli v^a. These quantities include the intersection numbers of the geometry d^{abc}, the volume form $\mathcal{V}$, and volumes of four-cycles τ^a:

$$\begin{aligned} d^{abc} &= \int \omega^a \wedge \omega^b \wedge \omega^c, \\ d^{ab} &= \frac{1}{8}\int \omega^a \wedge \omega^b \wedge J = \frac{1}{8} d^{abc} v_c \\ \tau^a &= \frac{1}{16}\int \omega^a \wedge J \wedge J = \frac{1}{16} d^{abc} v_b v_c, \\ \mathcal{V} &= \frac{1}{48}\int J \wedge J \wedge J = \frac{1}{48} d^{abc} v_a v_b v_c. \end{aligned} \tag{A4}$$

The $\mathcal{N}=2$ compactification moduli space is thus given by $\mathcal{M}_h \times \mathcal{M}_v$, where $\mathcal{M}_h$ denotes the hypermultiplet moduli space while $\mathcal{M}_v$ is the vector multiplet moduli space. $\mathcal{M}_h$ is a quaternionic manifold while $\mathcal{M}_v$ is a special Kähler manifold. The dilaton field is a hypermultiplet component. Thus, the geometry of $\mathcal{M}_h$ receives both α' and g_s corrections. $\mathcal{M}_v$, on the other hand, is exact at tree level in both α' and g_s. The hypermultiplet moduli space $\mathcal{M}_h$ contains a subspace $\mathcal{M}_h^0$ parametrized by vacuum expectation values of Neveu-Schwartz fields, with the RR moduli being set to zero. We have displayed this parametrization above. At string tree level the subspace $\mathcal{M}_h^0$ has a special Kähler structure.

We next turn to the reduction of this theory to a $\mathcal{N}=1$ effective supergravity theory, obtained by orientifolding.

2. Moduli space for $\mathcal{N}=1$ supergravity

The $\mathcal{N}=1$ theory is obtained by gauging a discrete symmetry of the form $(-1)^{\epsilon F_L}\Omega\sigma$ where Ω denotes world-sheet parity, F_L is the left-moving fermion number, and ϵ takes values 0,1 depending on the model. Moreover, $\sigma\colon CY \to CY$ is a holomorphic involution of the Calabi-Yau manifold CY which preserves the holomorphic three-form Ω_{CY} up to sign $\sigma^*\Omega_{CY} = (-1)^\epsilon \Omega_{CY}$. The value $\epsilon = 1$ corresponds to theories with O3/O7 planes.

The massless spectrum of $\mathcal{N}=1$ orientifold compactifications is naturally organized in vector and chiral multiplets. For orientifolds with O3/O7 planes, there are $h_-^{2,1}$ chiral multiplets which correspond to the invariant complex structure deformations (denoted above by z^k), $h_+^{1,1}$ chiral multiplets that correspond to invariant complexified Kähler deformations (formed of v^a and ρ_a), and $h_-^{1,1}$ chiral multiplets that parametrize the expectation values of the two-form fields B_2 (denoted above by b^a) and a similar form C_2 coming from the RR sector (denoted above by c^a). This field content is displayed in Table II.

Very importantly for all calculations that follow, the moduli space of the $N=1$ theory is a Kähler manifold. For small string coupling and large compactification radius the moduli space is a direct product of the complex structure moduli, complexified Kähler moduli and a dilaton-axion factor.

By definition, correct Kähler coordinates are those in which the effective action takes the standard $\mathcal{N}=1$ form:

$$\begin{aligned} S^{(4)}_{\mathcal{N}=1} = -\int_{M_4} \frac{1}{2}R * \mathbf{1} + K_{I\bar{J}} DM^I \wedge * D\bar{M}^{\bar{J}} \\ + \frac{1}{2}\mathrm{Re} f_{\alpha\beta} F^\alpha \wedge * F^\beta + \frac{1}{2}\mathrm{Im} f_{\alpha\beta} F^\alpha \wedge F^\beta + V * \mathbf{1}. \end{aligned} \tag{A5}$$

Here M^I denote the complex scalars in the chiral multiplets. The potential V is given in terms of the superpotential W and the D-terms D_α by

TABLE II. Type IIB moduli arranged in $\mathcal{N}=1$ multiplets for O3/O7 and O5/O9 orientifolds.

	O3/O7		O5/O9	
Gravity multiplet	1	$g_{\mu\nu}$	1	$g_{\mu\nu}$
Vector multiplets	$h_+^{(2,1)}$	V_1^α	$h_-^{(2,1)}$	V_1^k
Chiral multiplets	$h_-^{(2,1)}$	z^k	$h_+^{(2,1)}$	z^α
	$h_+^{(1,1)}$	(v^α,ρ_α)	$h_+^{(1,1)}$	(v^α,c^α)
	$h_-^{(1,1)}$	(b^a,c^a)	$h_-^{(1,1)}$	(b^a,ρ_a)
	1	(ϕ,C_0)	1	(ϕ,C_2)

$$V=e^K(K^{I\bar{J}}D_IWD_{\bar{J}}\bar{W}-3|W|^2)+\frac{1}{2}(\mathrm{Re}\,f)^{-1\alpha\beta}D_\alpha D_\beta, \tag{A6}$$

with the Kähler covariant derivatives, defined as

$$D_IW=\partial_IW+W\partial_IK. \tag{A7}$$

In type IIB, the Kähler coordinates depend on what kind of orientifold projection is performed. For O3/O7 projections, these are the complex structure moduli z^k and

$$\xi=C_0+ie^{-\phi},\qquad G^a=c^a-\xi b^a,$$
$$T^\alpha=\tau^\alpha+i\rho^\alpha-\frac{i}{2(\xi-\bar{\xi})}d^{\alpha bc}G_b(G-\bar{G})_c, \tag{A8}$$

where the intersection numbers $d^{\alpha bc}$ and τ^α have been defined before.

The Kähler potential is

$$K_{\mathrm{O3/O7}}=-2\ln\mathcal{V}(T,G,\xi)-\ln\left[-i\int\Omega(z)\wedge\bar{\Omega}(\bar{z})\right]$$
$$-\ln[-i(\xi-\bar{\xi})]. \tag{A9}$$

APPENDIX B: COMPUTING THE RIEMANNIAN CURVATURE

In this section, we present details of the computation of the Riemannian curvature tensor in type IIB Kähler coordinates.

We will denote all derivatives with respect to the Kähler coordinates T^i by a lower index. Thus, for example,

$$\frac{\partial}{\partial T^i}=\frac{\partial}{\partial\bar{T}^i}=\frac{1}{2}\frac{\partial}{\partial\tau^i}. \tag{B1}$$

For a Kähler manifold, all geometric data such as the metric, connection, and curvature can be obtained by taking repeated derivatives of the Kähler potential. For the Kähler potential relevant for us, this amounts to derivatives of the volume form. One immediately obtains

$$\partial_{\tau^i}\mathcal{V}\equiv\mathcal{V}_i=\frac{1}{32}d^{jkl}v_jv_k\frac{\partial v_l}{\partial\tau^i}=\frac{1}{4}v_j\frac{\partial\tau^j}{\partial v_l}\frac{\partial v_l}{\partial\tau^i}=\frac{1}{4}v_i, \tag{B2}$$

and from there, the first derivative of the Kähler potential:

$$\partial_{\tau^i}K\equiv K_i=-2\frac{\mathcal{V}_i}{\mathcal{V}}=-\frac{v_i}{2\mathcal{V}}. \tag{B3}$$

Interestingly, we are able to obtain the information without explicitly solving for the τ^i in terms of the v^i. The price we have to pay, however, is that the answer contains both sets of coordinates.

Several useful relations that can be expressed purely in the τ^i coordinates are

$$\tau^iK_i=-3/2,$$
$$K^i=-2\tau^i,$$
$$K^iK_i=3, \tag{B4}$$

The third relation is especially important, since it clarifies the no-scale structure of the geometry.

We now press forward to a computation of the metric. To this end, we define the following two parameters:

$$d^{ij}\equiv\frac{\partial\tau_i}{\partial v^j}=\frac{1}{8}d^{ijk}v_k,\qquad d_{ij}\equiv\frac{\partial v_i}{\partial\tau^j}. \tag{B5}$$

We note that the first is just the definition from before, while the second relation with the lowered indices d_{ij} is only formally defined. Obtaining it explicitly involves inverting the Legendre transformation between τ^i and v^i in Eq. (A4), which is in general difficult.

The Kähler metric and its inverse can now be formally defined given all the above relations:

$$g_{ij}=\frac{1}{2}K_iK_j-\frac{1}{4}e^{K/2}d_{ij},\qquad g^{ij}=4\tau^i\tau^j-4e^{-K/2}d^{ij}. \tag{B6}$$

We note that while the metric g_{ij} is only formally defined in terms of the quantity d_{ij}, the inverse metric g^{ij} is written explicitly and involves a combination of τ^i and v^i dependence (the latter coming from d^{ij}). The inverse metric can also be recast into an equivalent form using the no-scale property:

$$g^{ij}=e^{-K}d^{ijk}K_k+K^iK^j. \tag{B7}$$

For later reference, we also define the following quantity:

$$\tilde{d}_{ijk}\equiv g_{ip}g_{jq}g_{kl}d^{pql}. \tag{B8}$$

We note that our intersection numbers d^{pql} are defined with raised indices; the quantity $\tilde{d}_{ijk}$ above is purely formal.

We now go on to a computation of the curvature tensor, for which we need the third and fourth derivatives of the metric. However, as we have seen above, it is the inverse metric g^{ij} that is more amenable to a direct computation. We thus find

$$\partial_k g^{ij} = e^{-K} d^{ijm} g_{mk} - (g^{ij} - K^i K^j) K_k - \delta^i_k K^j - \delta^j_k K^i,$$
$$\partial_{mn} g^{ij} = -e^{-2K} d^{ijp} g_{pq} d^{qrs} g_{rm} g_{sn} + \delta^i_m \delta^j_n + \delta^i_n \delta^j_m. \qquad \text{(B9)}$$

From here, it is possible to express K_{ijm} and the Riemann tensor as

$$K_{ijm} = -g_{ip}(\partial_j g^{pq}) g_{qm},$$
$$R_{ijmn} = -g_{ip} g_{qj} (\partial_{mn} g^{pq}) + g_{ir} (\partial_m g^{rp}) g_{pq} (\partial_n g^{qs}) g_{sj}. \qquad \text{(B10)}$$

Inserting the relevant values of the third and fourth derivatives yields [24,40,41]

$$K_{ijm} = e^{-K} \tilde{d}_{ijm} - g_{ij} K_m - g_{im} K_j - g_{jm} K_i + K_i K_j K_k,$$
$$R_{ijmn} = -g_{im} g_{jn} + e^{-2K} (\tilde{d}_{ijk} g^{kl} \tilde{d}_{lmn} + \tilde{d}_{ink} g^{kl} \tilde{d}_{ljm}) + g_{in} K_j K_m + g_{jm} K_i K_n + g_{im} K_j K_n + g_{jn} K_i K_m + g_{ij} K_m K_n + g_{mn} K_i K_j - 3 K_i K_j K_m K_n - e^{-K} (\tilde{d}_{imj} K_n + \tilde{d}_{imn} K_j + \tilde{d}_{inj} K_m + \tilde{d}_{nmj} K_i). \qquad \text{(B11)}$$

APPENDIX C: COMPUTING A_i AND B

In this appendix, we will give full expressions for the quantities A_i and B in Eq. (42). Going on, we will compute them explicitly in the case of two moduli, and understand how the discriminant Δ_2 appears in this case, proving Eq. (44). We will then give an outline of the full computation in the n moduli case, motivating Eq. (54).

We first decompose the Goldstino unit vectors in the following way,[3] following the notation of the main text:

$$f_i = \sin\theta k_i + \cos\theta k^\perp_i. \qquad \text{(C1)}$$

Using this decomposition directly in the expression for the curvature tensor Eq. (B11), we obtain

$$\frac{2}{3} - R_{i\bar{j}m\bar{n}} f^i f^{\bar{j}} f^m f^{\bar{n}} = (-A^i A_i + B), \qquad \text{(C2)}$$

[3]We note that one can also define a relative phase $e^{i\delta}$ between the basis vectors k_i and $k^\perp_i$. In the case of two moduli, it can be shown that this phase reduces to unity when the function is maximized with respect to δ. While a similar computation is lacking for a larger number of moduli, we will assume $e^{i\delta} = 1$ is satisfied at all critical points, for simplicity.

where

$$A_i = 2\sqrt{2} \sin\theta k^\perp_i - \frac{1}{\sqrt{2}} e^{-K} P_{ij} d^{jmn} k^\perp_m k^\perp_n,$$
$$B = \left(g^{im} g^{jn} - \frac{3}{2} e^{-2K} d^{ijp} P_{pq} d^{qmn} \right) k^\perp_i k^\perp_j k^\perp_m k^\perp_n. \qquad \text{(C3)}$$

1. Two moduli: $h^{(1,1)} = 2$

The simplest nontrivial case is that of two moduli, $h^{(1,1)} = 2$. We will see that the main features of the computation are explicit here.

The space perpendicular to k^i is one dimensional and is spanned by $k^{\perp i}$. The projection operator appearing in Eq. (C3) is simply given by

$$P^{ij} = g^{ij} - k^i k^j = k^{\perp i} k^{\perp j}. \qquad \text{(C4)}$$

The expressions for A_i and B become

$$\frac{1}{\cos\theta^2} A_i = k^\perp_i \left[\frac{2\sqrt{2}}{\sqrt{3}} \tan\theta - \frac{1}{\sqrt{2}} e^{-K} d^{pqr} k^\perp_p k^\perp_q k^\perp_r \right], \qquad \text{(C5)}$$

$$\frac{1}{\cos\theta^4} B = \left[1 - \frac{3}{2} (e^{-K} d^{pqr} k^\perp_p k^\perp_q k^\perp_r)^2 \right]. \qquad \text{(C6)}$$

We will show, in the next subsection, that the quantity appearing in Eq. (C5) simplifies to

$$1 - \frac{3}{2} (e^{-K} d^{pqr} k^\perp_p k^\perp_q k^\perp_r)^2 = \frac{\Delta_2}{24} \frac{(\det g)^3}{e^{4K}} (\le 1). \qquad \text{(C7)}$$

The inequality ≤ 1 comes from inspection of the left side of the equation.

Putting all of this together, we get

$$\frac{1}{\cos\theta^4} (-A^i A_i + B) = \left(\frac{\Delta_2}{24} \frac{(\det g)^3}{e^{4K}} \right) - \frac{8}{3} \left(\tan\theta - \sqrt{\frac{1 - (\frac{\Delta_2}{24} \frac{(\det g)^3}{e^{4K}})}{8}} \right)^2. \qquad \text{(C8)}$$

We now need to extremize with respect to θ:

$$\partial_\theta (-A^i A_i + B) = 0. \qquad \text{(C9)}$$

Solving Eq. (C9) leads to a cubic equation in $\tan\theta$, whose approximate solution leads to the vanishing of the square term in the above equation. One finally obtains

$$(-A^iA_i+B)_{\max}\sim\frac{64(\frac{\Delta_2}{24}\frac{(\det g)^3}{e^{4K}})}{(9-(\frac{\Delta_2}{24}\frac{(\det g)^3}{e^{4K}}))^2}. \tag{C10}$$

We thus conclude that

$$\left(\frac{2}{3}-R_{i\bar{j}m\bar{n}}f^if^{\bar{j}}f^mf^{\bar{n}}\right)_{\max}=k\times\frac{\Delta_2}{24}\frac{(\det g)^3}{e^{4K}}\leq 1, \tag{C11}$$

which is Eq. (44).

2. Two moduli: How Δ_2 emerges

The purpose of this subsection is to prove Eq. (C7). The simplest way to accomplish this is to perform computations in the so-called canonical frame of divisors, and then transform back to the general frame [42–45].

The canonical frame is defined as follows. A real invertible matrix U is introduced such that

$$\mathfrak{v}_i=U_i^jv_j. \tag{C12}$$

Simultaneously, the intersection numbers are transformed as

$$\mathfrak{d}^{ijk}=\alpha(U^{-1})^i_l(U^{-1})^j_m(U^{-1})^k_n d^{lmn}. \tag{C13}$$

This leaves the Kähler potential unchanged up to an irrelevant shift

$$\mathfrak{K}=K-\ln\alpha^2. \tag{C14}$$

The transformation U is chosen such that in the canonical frame, the intersection numbers, metric, and two-cycle volumes take the following form:

$$\mathfrak{v}_i=2\sqrt{3}\delta_i^0,\qquad \mathfrak{g}^{ij}=\delta^{ij},\qquad \mathfrak{K}=0. \tag{C15}$$

This is always possible by counting parameters. We note that the transformation U has simply been introduced as a calculational device, taking advantage of the fact that it is a Kähler manifold.

From the above constraints, one gets $K^i=-\sqrt{3}\delta_0^i$, and the intersection numbers in the canonical basis are given by

$$\mathfrak{d}^{000}=\frac{2}{\sqrt{3}},\qquad \mathfrak{d}^{00a}=0,\qquad \mathfrak{d}^{0ab}=\frac{1}{\sqrt{3}}\delta^{ab},$$
$$\mathfrak{d}^{abc}=\text{free}, \tag{C16}$$

with $a,b,c=1,\ldots,h^{(1,1)}-1$.

The Riemann tensor can be worked out in the canonical frame to be

$$R_{0000}=\frac{2}{3},\qquad R_{000a}=0,\qquad R_{00ab}=\frac{2}{3}\delta_{ab},$$
$$R_{0abc}=\frac{1}{\sqrt{3}}\mathfrak{d}^{abc},$$
$$R_{abcd}=-\delta_{ac}\delta_{bd}+\frac{1}{3}\delta_{ab}\delta_{cd}+\frac{1}{3}\delta_{ad}\delta_{bc}+\mathfrak{d}^{abe}\mathfrak{d}^{ecd}+\mathfrak{d}^{ade}\mathfrak{d}^{ebc}. \tag{C17}$$

We now specialize the above equations to the simplest case of two moduli $h^{(1,1)}=n=2$. Moreover, we have

$$\mathfrak{k}_i=\frac{\mathfrak{K}_i}{\sqrt{\mathfrak{K}^i\mathfrak{K}_i}}=(-1,0),$$
$$\mathfrak{k}_i^{\perp}=(0,1). \tag{C18}$$

Thus,

$$\mathfrak{f}^i=(\sin\theta,\cos\theta). \tag{C19}$$

With the simplified expressions, we can easily derive A_i and B in the canonical frame. We obtain

$$\frac{1}{\cos\theta^4}(-A^iA_i+B)=\left(1-\frac{3}{2}\mathfrak{d}_{111}^2\right)-\frac{8}{3}\left(\tan\theta-\sqrt{\frac{1-(1-\frac{3}{2}\mathfrak{d}_{111}^2)}{8}}\right)^2. \tag{C20}$$

At this point, we note that the discriminant in the canonical frame is easily computed to be

$$\Delta_{2,\mathrm{can}}=24-36\mathfrak{d}_{111}^2. \tag{C21}$$

We can thus recast Eq. (C20) as

$$\frac{1}{\cos\theta^4}(-A^iA_i+B)=\frac{\Delta_{2,\mathrm{can}}}{24}-\frac{8}{3}\left(\tan\theta-\sqrt{\frac{1-\frac{\Delta_{2,\mathrm{can}}}{24}}{8}}\right)^2. \tag{C22}$$

It now remains to transform back to the general frame. From the definition of the canonical frame Eq. (C13), and using the expression for the discriminant of a general cubic in Eq. (45), one obtains the following relation between the canonical frame discriminant and the general one [44]

$$\Delta_{2,\mathrm{can}}=\alpha^4(\det U)^{-6}\Delta_2. \tag{C23}$$

One also has, from the definition of the canonical frame,

$$\mathfrak{v}^i = \alpha (U^{-1})^i_j \tau^j,$$
$$\mathfrak{g}^{ij} = \alpha^2 (U^{-1})^i_p (U^{-1})^j_q g^{pq},$$
$$\Rightarrow (\det U)^{-6} = \alpha^{-12} (\det g)^3. \quad \text{(C24)}$$

Since $e^{\mathfrak{K}} = e^K \alpha^{-2}$, we can combine all of the above to finally get

$$\Delta_{2,\mathrm{can}} = \Delta_2 \times \frac{(\det g)^3}{e^{4K}}. \quad \text{(C25)}$$

Plugging this back to Eq. (C22), we get Eq. (44).

3. $h^{(1,1)} = n$ moduli case

In this subsection, we make some preliminary attempts at solving the case of $h^{(1,1)} = n$ moduli, towards a derivation of Eq. (54). The strategy is to explore the structure of A_i and B, and obtain the generalizations of Eqs. (92), (94), and (95).

As before, we first decompose the Goldstino direction in components along orthonormal directions spanning the space orthogonal to K_i

$$K_i k^{\perp i}_\alpha = 0, \qquad k^{\perp i}_\alpha k^{\perp i}_\beta = \delta_{\alpha\beta},$$
$$\overline{k^{\perp i}}_\alpha = k^{\perp i}_\alpha \quad \text{for } \alpha, \beta = 1, \ldots, p-1. \quad \text{(C26)}$$

The projector P^{ij} onto the orthogonal complement of K^i can be written as

$$P^{ij} = \sum_{\alpha=1}^{p-1} k^{\perp i}_\alpha k^{\perp j}_\alpha. \quad \text{(C27)}$$

A general unit vector $K^{\perp i}$ orthogonal to K^i can be parametrized as

$$K^{\perp i} = \sum_{\alpha=1}^{p-1} e^{\mathrm{Im}\varphi_\alpha} c_\alpha k^{\perp i}_\alpha \quad \text{(C28)}$$

with real phases φ_α and real c_α satisfying

$$\sum_{\alpha=1}^{p-1} c_\alpha^2 = 1. \quad \text{(C29)}$$

B can now be written as

$$B = 1 - \frac{3}{2} e^{-2K} \sum_{\alpha\beta\gamma\delta\eta} c_\beta c_\gamma c_\delta c_\eta D_{\alpha\beta\gamma} D_{\alpha\delta\eta}, \quad \text{(C30)}$$

where the symmetric rank 3 tensor reads

$$D_{\alpha\beta\gamma} := d_{ijk} k^{\perp i}_\alpha k^{\perp j}_\beta k^{\perp k}_\gamma. \quad \text{(C31)}$$

At this point, it is simplest to go to the canonical frame, where something like (C8) can be derived. Specifically, we define

$$\mathfrak{b}^{\mathcal{O}}_{abcd} \equiv \left[\frac{1}{3}\delta^{ab}\delta^{cd} - \frac{1}{2}\mathfrak{d}^{abe}\mathfrak{d}^{ecd}\right] + \left[\frac{1}{3}\delta^{ac}\delta^{bd} - \frac{1}{2}\mathfrak{d}^{ace}\mathfrak{d}^{ebd}\right] + \left[\frac{1}{3}\delta^{ad}\delta^{bc} - \frac{1}{2}\mathfrak{d}^{ade}\mathfrak{d}^{ebc}\right] \quad \text{(C32)}$$

and

$$\mathfrak{B} \equiv \mathfrak{b}^{\mathcal{O}}_{abcd} f^a f^b f^c f^d. \quad \text{(C33)}$$

Then, in the canonical frame, we can write

$$\left(\frac{2}{3} - R_{i\bar{j}m\bar{n}} f^i f^{\bar{j}} f^m f^{\bar{n}}\right) = \mathfrak{B} - \frac{8}{3}\sum_e \left[f^0 f^e + \frac{\sqrt{3}}{4} f^a f^b \mathfrak{d}^{abe}\right]^2, \quad \text{(C34)}$$

where we have

$$\mathfrak{B} \equiv \mathfrak{B}(\mathfrak{c}_\alpha) \equiv \mathfrak{B}(\theta, \theta_p). \quad \text{(C35)}$$

We have displayed the fact that $\mathfrak{B}$ s a function of the expansion coefficients c_α of the Goldstino directions f^i, or equivalently of the angles θ_p that fix the f^i.

In the case of two moduli, the angular dependence was simple and was displayed in Eq. (C8). The maximization procedure led to the approximate vanishing of the square term. In this case, the angular dependence is more complicated, and a set of coupled cubic equations in $\tan\theta_p$ must be solved. Nevertheless, we can conjecture that at least a subset of maxima, including the global one, corresponds to the case when the square term in Eq. (C34) vanishes. Thus, we have

$$\left(\frac{2}{3} - R_{i\bar{j}m\bar{n}} f^i f^{\bar{j}} f^m f^{\bar{n}}\right)_{\text{local max}} = \mathfrak{B}_{\text{local max}}. \quad \text{(C36)}$$

It now remains to determine the structure of $\mathfrak{B}_{\text{local max}}$. It should be clear from Eq. (C32) and Eq. (C33), as well as the case of two moduli, that any given local maximum $\mathfrak{B}_{\text{local max}}$ is a fourth degree polynomial in intersection numbers. Moreover, the factors of $\det g$ and e^K work out the same way as the two moduli case [Eqs. (110)–(112)], when going from the canonical to the general frame. Thus, we arrive at conjecture Eq. (54).

APPENDIX D: INVARIANTS AOF THREE AND FOUR MODULI

In the case of three moduli, we have

$$\prod_{a=1}^{3} \left(\frac{2}{3} - R_{i\bar{j}m\bar{n}} f^i f^{\bar{j}} f^m f^{\bar{n}}\right)_{\text{crit},a^{\text{th}}} \propto \Delta_3 \frac{(\det g)^9}{e^{12K}}. \quad \text{(D1)}$$

The ring of invariants of ternary cubics is generated by the Aronhold invariants S and T, which are homogeneous polynomials of degree 4 and 6, respectively, in the coefficients of the cubic [46]. The discriminant is a homogeneous polynomial of degree 12, given by

$$\Delta_3 = S^3 - T^2. \tag{D2}$$

For convenience, we give the polynomials S and T below. For the cubic $f(x,y,z)$

$$f(x,y,z) = ax^3 + by^3 + cz^3 + 3dx^2y + 3ey^2z + 3fz^2x + 3gxy^2 + 3hyz^2 + 3izx^2 + 6jxyz \tag{D3}$$

we have

$$\begin{aligned} S = {} & agec - agh^2 - ajbc + ajeh + afbh \\ & - afe^2 - d^2ec + d^2h^2 + dibc - dieh \\ & + dgjc - dgfh - 2dj^2h + 3djfe - df^2b \\ & - i^2bh + i^2e^2 - ig^2c + 3igjh - igfe - 2ij^2e \\ & + ijfb + g^2f^2 - 2gj^2f + j^4 \end{aligned} \tag{D4}$$

and

$$\begin{aligned} T = {} & a^2b^2c^2 - 3a^2e^2h^2 - 6a^2behc + 4a^2bh^3 + 4a^2e^3c - 6adgbc^2 + 18adgehc - 12adgh^3 + 12adjbhc \\ & - 24adje^2c + 12adjeh^2 - 12adfbh^2 + 6adfbec + 6adfe^2h + 6aigbhc - 12aige^2c + 6aigeh^2 + 12aijbec \\ & + 12aije^2h - 6aifb^2c + 18aifbeh - 24ag^2jhc - 24aijbh^2 - 12aife^3 + 4ag^3c^2 - 12ag^2fec + 24ag^2fh^2 \\ & + 36agj^2ec + 12agj^2h^2 + 12agjfbc - 60agjfeh - 12agf^2bh + 24agf^2e^2 - 20aj^3bc - 12aj^3eh \\ & + 36aj^2fbh + 12aj^2fe^2 - 24ajf^2be + 4af^3b^2 + 4d^3bc^2 - 12d^3ehc + 8d^3h^3 + 24d^2ie^2c - 12d^2ieh^2 \\ & + 12d^2gjhc + 6d^2gfec - 24d^2j^2h^2 - 12d^2ibhc - 3d^2g^2c^2 - 24g^2j^2f^2 + 24gj^4f - 12d^2gfh^2 + 12d^2j^2ec \\ & - 24d^2jfbc - 27d^2f^2e^2 + 36d^2jfeh + 24d^2f^2bh + 24di^2bh^2 - 12di^2bec - 12di^2e^2h + 6dig^2hc - 60digjec \\ & + 36digjh^2 + 18digfbc - 6digfeh + 36dij^2bc - 12dij^2eh - 60dijfbh + 36dijfe^2 + 6dif^2be + 12dg^2jfc \\ & - 12dgj^3c - 12dgj^2fh + 36dgjf^2e12dgf^3b + 24dj^4h + 12dj^2f^2b + 4i^3b^2c + 24i^2g^2ec - 27i^2g^2h^2 \\ & - 36dj^3fe - 12i^3beh + 8i^3e^3 - 24i^2gjbc + 36i^2gjeh + 6i^2gfbh + 12i^2j^2bh - 3i^2f^2b^2 - 12dg^2f^2h \\ & - 12i^2gfe^2 - 24i^2j^2e^2 + 12i^2jfbe - 12ig^3fc + 12ig^2j^2c + 36ig^2jfh - 12ig^2f^2e - 36igj^3h \\ & - 12igj^2fe + 12igjf^2b + 24ij^4e - 12ij^3fb + 8g^3f^3 - 8j^6. \end{aligned} \tag{D5}$$

Similar to the case of three moduli, the ring of invariants for the case of a cubic in four variables is generated by homogeneous polynomials $I_8, I_{16}, I_{24}, I_{32}, I_{40}, I_{100}$, where the subscript denotes the degree of the polynomial in the coefficients of the cubic. The discriminant of a quaternary cubic is given by

$$\Delta_4 = (I_8^2 - 64I_{16})^2 - 2^{11}(I_8I_{24} + 8I_{32}). \tag{D6}$$

For a form of degree d in b variables, the discriminant $\Delta_{d,b}$ is a homogeneous polynomial of degree $b \cdot (d-1)^{b-1}$.

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