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Nonlinear evolution of the baryon acoustic oscillation scale in alternative theories of gravity

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The scale of baryon acoustic oscillations (BAO) imprinted in the matter power spectrum provides an almost-perfect standard ruler: it only suffers subpercent deviations from fixed comoving length due to nonlinear effects. We study the BAO shift in the large Horndeski class of gravitational theories and compute its magnitude in momentum space using second-order perturbation theory and a peak-background split. The standard prediction is affected by the modified linear growth, as well as by nonlinear gravitational effects that alter the mode-coupling kernel. For covariant Galileon models, we find a 14%–45% enhancement of the BAO shift with respect to standard gravity and a distinct time evolution depending on the parameters. Despite the larger values, the shift remains well below the forecasted precision of next-generation galaxy surveys. Models that produce significant BAO shift would cause large redshift-space distortions or affect the bispectrum considerably. Our computation therefore validates the use of the BAO scale as a comoving standard ruler for tests of general dark energy models.

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I. MOTIVATION

One of the most exciting promises of modern cosmology is the possibility of testing fundamental physics using the largest scales available to observation [1]. Among other developments, the signatures of baryon acoustic oscillations (BAO) have provided an invaluable test of models for cosmic acceleration through their imprint in the cosmic microwave background [2] and the distribution of large scale structure (LSS) [3] using either galaxies [4–8], the Lyman-α forest or quasars [9,10] (see Refs. [11–13] for reviews). To an excellent approximation, the BAO signal in the LSS provides a comoving standard ruler that traces the expansion of the universe and probes the onset of cosmic acceleration.

Nonlinear corrections are known to introduce a small departure from the perfect standard ruler behavior, systematically shifting the BAO scale toward smaller values at low redshift. This effect has been well studied for cold dark matter cosmologies with a cosmological constant using perturbation theory [14–21] and simulations [22–24] (for earlier works see [25,26]). The result is that the BAO scale imprinted in the matter distribution shrinks by approximately 0.3% at redshift zero [22,24]. However, this value relies on the assumption that gravity is Newtonian in the scales of interest.

Little attention has been devoted to the nonlinear BAO evolution in more general theories of gravity. Since the shift in the BAO scale is comparable to the subpercent level of precision expected by forthcoming galaxy surveys [27] that aim to test such theories, it will be necessary to understand the effects of nonstandard gravity on the BAO scale to correctly interpret the data in the next generation of dark energy experiments.

II. PEAK-BACKGROUND SPLIT COMPUTATION OF THE BAO SHIFT

Sherwin and Zaldarriaga have explained the BAO shift in terms of the effect of long modes on the short scale power spectrum [21]. In their picture, large overdense regions undergo less overall expansion, reducing the size of the physical BAO scale with respect to the average (see also [28]). This effect is not compensated by underdense regions, because cosmic structures in overdense regions undergo more growth and give a larger contribution to the power spectrum. Therefore, local differences in expansion and growth lead to a net shortening of the comoving BAO scale, causing a small departure from the standard ruler behavior. Alternatively, the shift of the BAO scale can also be understood as arising from contributions to the power spectrum which are off-phase with respect to the linear prediction [16,18].

The BAO shift can be studied by comparing the nonlinear power spectrum to a rescaled version of the linear one [18]:

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$$P(k) \approx P_{11}(k/\alpha) = P_{11}(k) - (\alpha - 1)kP'_{11}(k) + \cdots, \quad (1)$$

where the shift can be read from the coefficient of the second term (more sophisticated templates are actually used to obtain the BAO scale from data, but we will stick to this description for simplicity). Here and below the power spectrum is defined as $\langle \delta(\vec{k})\delta(\vec{k}')\rangle \equiv (2\pi)^3\delta_D(\vec{k}+\vec{k}')P(k)$ and $P_{nm} \propto \langle \delta_n \delta_m \rangle$, where we expand the density contrast as $\delta = \delta_1 + \delta_2 + \cdots$.

One can compare Eq. (1) with the prediction from standard perturbation theory

$$P(k) = (P_{11} \cdots + P_{1n}) + (P_{22} \cdots + P_{mn}). \quad (2)$$

All P_{1n} contributions are proportional to $P_{11}(k)$ and thus do not contribute to the second term in Eq. (1) [18]. Only the mode-coupling terms (second parenthesis) do contribute to the shift, with the first of such contributions given by

$$P_{22}(k) = \int \frac{d^3q}{(2\pi)^3} 4[F_2(\vec{k}-\vec{q},\vec{q})]^2 P_{11}(\vec{k}-\vec{q})P_{11}(\vec{q}). \quad (3)$$

Here F_2 is the second-order symmetrized mode-coupling kernel [29]. The rest of the computation in the peak-background split approximation proceeds by expanding in k/q, integrating with a cutoff at $k_{\rm BAO}$ and extracting the coefficient of $kP'_{11}(k)$ from the result (see [21] for further details). The long modes with $q \ll k \sim k_{\rm BAO}$ describe the effect of the large fluctuations on the smaller scales.

The computation can be generalized to alternative theories of gravity by noting that the structure of the kernel F_2 is preserved on subhorizon scales, but each term acquires a time-dependent coefficient $C_i(t)$

$$F_2(\vec{p},\vec{q}) = C_0 + C_1\mu\left(\frac{p}{q}+\frac{q}{p}\right) + C_2\left(\mu^2 - \frac{1}{3}\right), \quad (4)$$

which reduces to the standard constant values, $C_0 = 17/21$, $C_1 = 1/2$ and $C_2 = 2/7$, under matter domination in the case of standard gravity (we drop the time-dependence for notation convenience). Explicit computations in the subhorizon, quasistatic limit of Horndeski theories with nonrelativistic matter determine that the modifications to the kernel coefficients are not independent [30,31] and satisfy

$$C_1 = \frac{1}{2}, \qquad C_0 + \frac{2}{3}C_2 = 1. \quad (5)$$

The BAO shift can be read by plugging the generalized kernel (4), into the mode-coupling power spectrum (3), expanding to leading order in q/k, performing the integration and comparing with Eq. (1). This generalizes the result of Ref. [21] to

$$\alpha - 1 = \frac{2}{5}\left(2C_0 - \frac{1}{2}\right)\langle \delta_L^2 \rangle, \quad (6)$$

where Eqs. (5) have been used to write the result in terms of the monopole C_0. In the above expression the integration over the momentum leads to the long mode variance

$$\langle \delta_L^2 \rangle \equiv \int_0^{k_{\rm BAO}} \frac{dq}{(2\pi)^3} 4\pi q^2 P_{11}(q,t) \approx \sigma_{r_s}^2(t), \quad (7)$$

where we use a cutoff at BAO scale, well estimated by the sound horizon at the drag epoch $k_{\rm BAO} \sim 1/r_s(z_d)$ [32]. Following Ref. [21], we use the square of the variance of the density field on a sphere of radius $r_s(z_d)$ for the computation of the BAO shift: $\langle \delta_\Lambda^2 \rangle \approx \sigma_{r_s}^2(t)$. This gives a slight underestimate with respect to the shift measured in simulations of standard cosmology, but we expect comparison among models to be accurate.

LSS surveys observe galaxies, which are known to be biased with respect to the underlying matter distribution. The effects of nonlinear density-halo bias can be parametrized as

$$\delta_h(x) = b_1\delta(x) + \frac{1}{2}b_2\delta^2(x) + \cdots, \quad (8)$$

where the bias parameters b_1, b_2 relate the matter and the halo overdensity (the above expansion should hold on large scales). The halo-halo power spectrum generalizing Eq. (2) reads

$$P_h(k) = b_1^2(P_{11} + P_{22}) + b_1 b_2 P_{\delta_1^2\delta_2} + \cdots, \quad (9)$$

where second-order terms that do not involve mode coupling have been omitted as they do not contribute to the BAO shift. The first parenthesis contains the linear and mode-coupling matter power spectrum renormalized by the linear bias. The second term mixes second order corrections with nonlinear bias, and affects the BAO shift. This term is given by

$$P_{\delta_1^2\delta_2}(k) = \int \frac{d^3q}{(2\pi)^3} 2F_2(\vec{k}-\vec{q},\vec{q})P_{11}(\vec{k}-\vec{q})P_{11}(q), \quad (10)$$

where the only difference with Eq. (3) is that the kernel appears linearly. As before, one can compute the leading order expansion in q/k and compare with a rescaled version of the linear halo power spectrum [cf. Eq. (1) multipled by b_1^2]. Identifying the $kP'_{11}(k)$ term yields the contribution to the shift, which now reads

$$(\alpha - 1)|_h = \left(\frac{4}{5}C_0 - \frac{1}{5} + \frac{2}{3}\frac{b_2}{b_1}\right)\langle \delta_L^2 \rangle. \quad (11)$$

This result generalizes Eq. (6). Note that the relations (5) for the perturbation theory kernels make the bias contribution to the shift independent of the theory of gravity, i.e. of any departure in the value of C_0. For this reason we will not consider nonlinear bias in the next section.

III. BAO SHIFT IN ALTERNATIVE THEORIES OF GRAVITY

We focus our analysis on theories within the Horndeski Lagrangian [33], which contains many examples of interest for cosmology including Brans-Dicke, $f(R)$, chameleons, kinetic gravity braiding and covariant Galileons. Horndeski's theory also contains the characteristic interactions that appear in consistent theories of massive gravity and higher dimensional theories, and it is thus expected to effectively describe some of their distinctive features [34].[1]

Although our analysis is general, for the sake of simplicity we will present results for a covariant Galileon model [44] (see also [45,46]). We fix the Galileon Lagrangian parameters and the cosmological parameters to the best-fit models obtained by Barreira *et al.* [47] (without massive neutrinos), which have zero cosmological constant. We noticed that the quintic Galileon model we present has a gradient instability in the tensor sector. However we decided to include it in our analysis since it is a good fit for the data and has interesting properties at second-order in perturbation theory. Indeed, as we shall see it produces large modifications of the dark matter kernel, which can be detected by studying the bispectrum with current surveys [48,49].

Simpler scalar-tensor theories [such as Brans-Dicke, chameleons or $f(R)$] lead to very constrained modifications and cannot produce large contributions to the kernel [31]. For such models the only sizeable contributions to the BAO shift stem from the enhancement of linear growth, and would thus be in conflict with measurements of LSS clustering. If one further demands that these theories are screened in the Galaxy or the Solar system, the range of the scalar force is too short to even affect cosmological scales in the linear regime [50]. An exception is given by nonuniversal couplings to matter: Coupled dark matter models can significantly increase the shift of the BAO scale [51].

The generalized Sherwin-Zaldarriaga formula (6) depends on the theory of gravity in two ways: a correction from linear physics, given by $\sigma_{r_s}^2$, and a modification of the mode coupling kernel (4), given by C_0. We compute the evolution of the background, the linear power spectrum and the density contrast $\sigma_{r_s}^2$ using a modified version of the CLASS code [52,53] based on the general description of Horndeski perturbations presented in [54] (see also [55]). The computation of the nonlinear corrections to the kernel follows the approach of Ref. [31] (see also [56]) by taking the subhorizon approximation and the quasistatic approximation (valid for covariant Galileons on the scales of interest [57]).

[1] We will not consider viable extensions of Horndeski's theory [35,36], nor full theories containing interacting gravitons [37–39]. See Refs. [40–43] for reviews on the cosmology of alternative theories of gravity.

TABLE I. Density contrast, mode coupling kernel monopole and BAO shift in the matter power spectrum for reference model and selected Galileon models at redshift zero (cf. Fig. 1). Values in parenthesis indicate the relative deviation with respect to a cosmological constant model. Tracer bias can be added using Eq. (11).

Model	σ_{r_s}		$2C_0$		$\alpha_k - 1$ [%]	
Λ	0.067		1.62		0.20	
Cubic	0.071	(7%)	1.61	(−0.4%)	0.23	(14%)
Quartic	0.073	(9%)	1.58	(−2%)	0.23	(15%)
Quintic	0.071	(7%)	1.92	(19%)	0.29	(45%)

We will also assume that the scale at which the model becomes strongly coupled is smaller than the BAO scale. This is indeed the case for cubic and quartic Galileon models, as suggested by a comparison between fully nonlinear and linearized N-body simulations for Galileons [58,59].

The quantities that determine the BAO shift (6,11) are presented in Table I for the selected models at redshift zero, together with their deviation with respect to the prediction of standard gravity with a cosmological constant. All the models considered tend to increase the density contrast σ_{r_s} due to an enhanced effective force of gravity and the different background expansion. The nonlinear corrections to C_0 are highly dependent on the model parameters, acquiring positive and negative sign and ranging from subpercent in the cubic, percent in the quartic, and becoming fairly large in the quintic example.

The time evolution of the BAO shift and the mode-coupling corrections are displayed in Fig. 1. Departures with respect to standard gravity occur only at low redshift and become largest in the accelerated era when the scalar field energy density drives the cosmic expansion. Besides this general trend, each model is characterized by a specific time dependence. Our results allow us to distinguish between a very soft nonlinear regime in which the mode coupling is mostly determined by interactions of the matter fluid (standard gravity, cubic model) and large nonlinear effects, as in the case of the quintic model, with the quartic case being an intermediate example. This is a consequence of the nonlinear gravitational interactions introduced in Horndeski's theory.

IV. DISCUSSION

Our results show an enhancement of the BAO shift with respect to the standard prediction and provide the first estimate of this effect for modified gravity. It is possible to compare the predicted shift to the forecasted sensitivity of next-generation galaxy surveys. Let us focus on measurements on the BAO scale transverse to the line of sight $\theta_{\rm BAO} = r_s/D_A$, where D_A is the comoving angular diameter distance (comparison with line-of-sight BAO yields similar results). One can compare the two sources of uncertainty

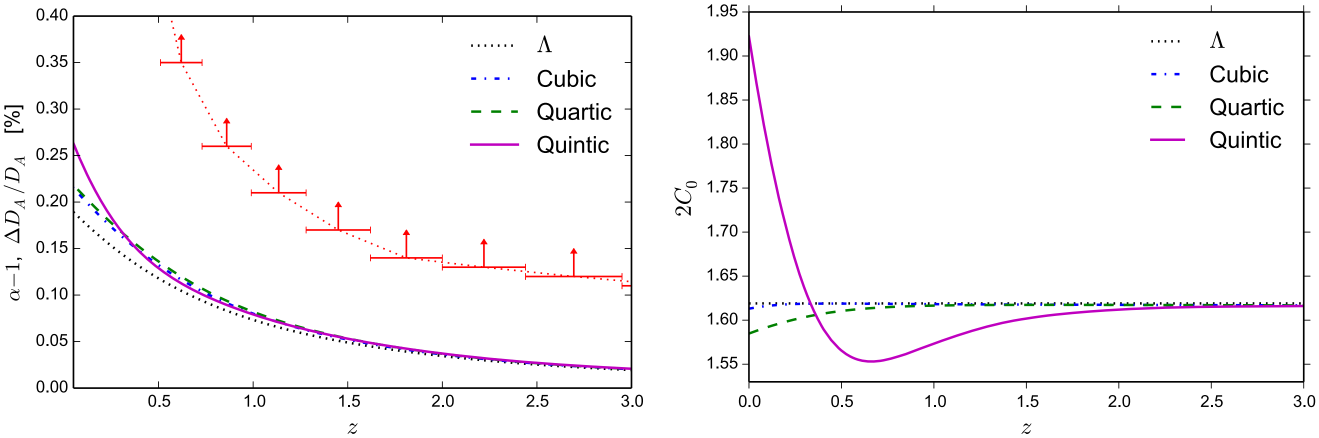

FIG. 1 (color online). Time evolution of the BAO shift in the matter power spectrum (left panel) and integrated kernel (right panel) for standard and Galileon gravity models (cf. Table I). Red lines indicate expected sensitivity of D_A across redshift bins from an optimistic BAO survey [12] [see discussion around Eq. (12)].

$$\frac{\Delta D_A}{D_A} = \frac{D_A}{r_s}\Delta\theta + \frac{\Delta r_s}{r_s}, \tag{12}$$

where the first term is the observational error (assuming known r_s) and the second term is the systematic error induced by the shift, $\Delta r_s/r_s = \alpha - 1$. Weinberg *et al.* have provided an example forecast for an all-sky BAO survey in which the expected error ranges from 2.8% at $z = 0.15$ to 0.1% $z \gtrsim 3.5$ (these data can be found in Table 2 of Ref. [12]).[2] Figure 1 compares both terms in Eq. (12) and shows that the BAO shift is well below the precision for all the examples considered at any fiducial redshift. Note that the forecasted precision is mainly limited by survey volume, implying that more sophisticated observational setups will not be able to reduce the errors considerably. More realistic forecasts based on specific surveys lead to lower precision (see Ref. [27]).

It is very unlikely that models more general than the ones considered here can lead to sufficiently large shifts to bias the BAO scale measurements while remaining compatible with other observations. The theoretical prediction, Eq. (6), allows one to identify two contributions to the shift: the modified linear growth and the nonlinear gravitational effects that modify the mode-coupling kernel. Any theory of gravity with a very large shift requires a considerable enhancement of at least one of these contributions, which can be probed by observables other than BAO.[3]

A large departure of σ_{r_s} would be ruled out by redshift space distortions or other clustering measurements. Similarly, large corrections to the mode coupling kernel would induce large distortions in the bispectrum [note that Eq. (5) implies that a $\gtrsim$23.5% increase in C_0 would change of sign in the quadrupole term in F_2]. We emphasize that these nonlinear gravitational effects are exclusive of fully fledged Horndeski theories (cf. quartic and quintic example Galileons considered here) and very suppressed in simpler scalar-tensor theories [e.g. Brans-Dicke, $f(R)$] or cubic theories (e.g. our cubic example, kinetic gravity braiding [60] and limits of extra-dimensional theories [61]). Most works on higher order perturbation theory for modified gravity have focused on the latter type of models [30,56,62–64].

Our findings validate the use of BAO measurements as a comoving standard ruler for current and next-generation LSS surveys, at all redshifts of interest and even for the most extreme theories of gravity. There are several refinements that can improve our calculation, such as including higher order perturbation theory corrections. Other developments would be necessary in order to better connect these results with observations, such as the inclusion of more sophisticated bias models (which has been shown to affect the magnitude and time evolution [24]) and redshift-space distortions (which typically increase the magnitude of the shift relative to real space). Finally, extending our result would allow us to confirm the validity of density field reconstruction [65,66] of BAO for general theories of gravity.[4] Despite possible refinements, the smallness of the effects ensures the validity of our conclusions regarding the shift.

[2] Their forecast also assumes density field reconstruction improvements in the nonlinear damping by a factor of 2. Since this procedure has not been validated for general theories of gravity, we take the forecasted precision as an optimistic bound.

[3] Another possibility is that a modification of gravity enhances the BAO shift by producing a large value of the nonlinear halo bias b_2/b_1. Such a possibility would however rely on the details of halo formation and its study would require methods other than cosmological perturbation theory.

[4] The validity of BAO reconstruction schemes has been argued to rely exclusively in the equivalence principle [67]. This assumption depends on the theory of gravity: it is valid for the Galileon models considered here but would be violated by chameleon gravity [68].

These are some initial steps in understanding the interplay between extended theories of gravity and the BAO scale imprints on the distribution of LSS. Further work should address other aspects of LSS and BAO in general theories of gravity in order to optimize the performance and model independence of the next-generation of dark energy experiments. This will ultimately shed light on the optimal strategy to test gravitational physics using future LSS surveys and learn more on the connections between fundamental physics and cosmology.

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[1] L. Amendola *et al.* (Euclid Theory Working Group), Cosmology and fundamental physics with the Euclid satellite, Living Rev. Relativity **16**, 6 (2013).
[2] P. Ade *et al.* (Planck), Planck 2015 results. XIV. Dark energy and modified gravity, arXiv:1502.01590.
[3] D. J. Eisenstein and W. Hu, Baryonic features in the matter transfer function, Astrophys. J. **496**, 605 (1998).
[4] S. Cole *et al.* (2dFGRS), The 2dF Galaxy Redshift Survey: Power-spectrum analysis of the final data set and cosmological implications, Mon. Not. R. Astron. Soc. **362**, 505 (2005).
[5] D. J. Eisenstein *et al.* (SDSS), Detection of the baryon acoustic peak in the large-scale correlation function of SDSS luminous red galaxies, Astrophys. J. **633**, 560 (2005).
[6] C. Blake *et al.*, The WiggleZ Dark Energy Survey: Mapping the distance–redshift relation with baryon acoustic oscillations, Mon. Not. R. Astron. Soc. **418**, 1707 (2011).
[7] F. Beutler, C. Blake, M. Colless, D. Heath Jones, L. Staveley-Smith, L. Campbell, Q. Parker, W. Saunders, and F. Watson, The 6dF Galaxy Survey: Baryon acoustic oscillations and the local Hubble constant, Mon. Not. R. Astron. Soc. **416**, 3017 (2011).
[8] L. Anderson *et al.* (BOSS), The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: Baryon acoustic oscillations in the Data Releases 10 and 11 Galaxy samples, Mon. Not. R. Astron. Soc. **441**, 24 (2014).
[9] A. Font-Ribera *et al.* (BOSS), Quasar-Lyman α forest cross-correlation from BOSS DR11: Baryon Acoustic Oscillations, J. Cosmol. Astropart. Phys. 05 (2014) 027.
[10] T. Delubac *et al.* (BOSS), Baryon acoustic oscillations in the Lyα forest of BOSS DR11 quasars, Astron. Astrophys. **574**, A59 (2015).
[11] B. A. Bassett and R. Hlozek, Baryon Acoustic Oscillations, arXiv:0910.5224.
[12] D. H. Weinberg, M. J. Mortonson, D. J. Eisenstein, C. Hirata, A. G. Riess, and E. Rozo, Observational probes of cosmic acceleration, Phys. Rep. **530**, 87 (2013).
[13] E. Aubourg *et al.*, Cosmological implications of baryon acoustic oscillation (BAO) measurements, arXiv:1411.1074.
[14] D. J. Eisenstein, H.-j. Seo, and Martin J. White, On the robustness of the acoustic scale in the low-redshift clustering of matter, Astrophys. J. **664**, 660 (2007).
[15] R. E. Smith, R. Scoccimarro, and R. K. Sheth, Motion of the acoustic peak in the correlation function, Phys. Rev. D **77**, 043525 (2008).
[16] M. Crocce and R. Scoccimarro, Nonlinear evolution of baryon acoustic oscillations, Phys. Rev. D **77**, 023533 (2008).
[17] T. Matsubara, Resumming cosmological perturbations via the Lagrangian picture: One-loop results in real space and in redshift space, Phys. Rev. D **77**, 063530 (2008).
[18] N. Padmanabhan and M. White, Calibrating the baryon oscillation ruler for matter and halos, Phys. Rev. D **80**, 063508 (2009).
[19] Y. Noh, M. White, and N. Padmanabhan, Reconstructing baryon oscillations, Phys. Rev. D **80**, 123501 (2009).
[20] N. McCullagh and A. S. Szalay, Nonlinear behavior of baryon acoustic oscillations from the Zel'dovich approximation using a non-fourier perturbation approach, Astrophys. J. **752**, 21 (2012).
[21] B. D. Sherwin and M. Zaldarriaga, Shift of the baryon acoustic oscillation scale: A simple physical picture, Phys. Rev. D **85**, 103523 (2012).
[22] H.-J. Seo, J. Eckel, D. J. Eisenstein, K. Mehta, M. Metchnik, N. Padmanabhan, P. Pinto, R. Takahashi, M. White, and X. Xu, High-precision predictions for the acoustic scale in the nonlinear regime, Astrophys. J. **720**, 1650 (2010).
[23] Y. Rasera, P.-S. Corasaniti, J.-M. Alimi, V. Bouillot, V. Reverdy, and I. Balmes, Cosmic-variance limited Baryon Acoustic Oscillations from the DEUS-FUR ΛCDM simulation, Mon. Not. R. Astron. Soc. **440**, 1420 (2014).

[24] F. Prada, C. G. Scoccola, C.-H. Chuang, G. Yepes, A. A. Klypin, F.-S. Kitaura, S. Gottlober, and C. Zhao, Hunting down systematics in baryon acoustic oscillations after cosmic high noon, arXiv:1410.4684.
[25] S. Bharadwaj, The evolution of correlation functions in the Zeldovich approximation and its implications for the validity of perturbation theory, Astrophys. J. **472**, 1 (1996).
[26] A. Meiksin, M. J. White, and J. Peacock, Baryonic signatures in large-scale structure, Mon. Not. R. Astron. Soc. **304**, 851 (1999).
[27] A. Font-Ribera, P. McDonald, N. Mostek, B. A. Reid, H.-J. Seo, and A. Slosar, DESI and other Dark Energy experiments in the era of neutrino mass measurements, J. Cosmol. Astropart. Phys. 05 (2014) 023.
[28] M. Zumalacarregui, J. Garcia-Bellido, and P. Ruiz-Lapuente, Tension in the void: Cosmic rulers strain inhomogeneous cosmologies, J. Cosmol. Astropart. Phys. 10 (2012) 009.
[29] F. Bernardeau, S. Colombi, E. Gaztanaga, and R. Scoccimarro, Large-scale structure of the Universe and cosmological perturbation theory, Phys. Rep. **367**, 1 (2002).
[30] Y. Takushima, A. Terukina, and K. Yamamoto, Bispectrum of cosmological density perturbations in the most general second-order scalar-tensor theory, Phys. Rev. D **89**, 104007 (2014).
[31] E. Bellini, R. Jimenez, and L. Verde, Signatures of Horndeski gravity on the dark matter bispectrum, J. Cosmol. Astropart. Phys. 05 (2015) 057.
[32] K. Thepsuriya and A. Lewis, Accuracy of cosmological parameters using the baryon acoustic scale, J. Cosmol. Astropart. Phys. 01 (2015) 034.
[33] G. Horndeski, Second-order scalar-tensor field equations in a four-dimensional space, Int. J. Theor. Phys. **10**, 363 (1974).
[34] L. Heisenberg, R. Kimura, and K. Yamamoto, Cosmology of the proxy theory to massive gravity, Phys. Rev. D **89**, 103008 (2014).
[35] M. Zumalacarregui and J. Garcia-Bellido, Transforming gravity: From derivative couplings to matter to second-order scalar-tensor theories beyond the Horndeski Lagrangian, Phys. Rev. D **89**, 064046 (2014).
[36] J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, New Class of Consistent Scalar-Tensor Theories, Phys. Rev. Lett. **114**, 211101 (2015).
[37] C. de Rham, G. Gabadadze, and A. J. Tolley, Resummation of Massive Gravity, Phys. Rev. Lett. **106**, 231101 (2011).
[38] S. Hassan and R. A. Rosen, Bimetric gravity from ghost-free massive gravity, J. High Energy Phys. 02 (2012) 126.
[39] K. Hinterbichler and R. A. Rosen, Interacting spin-2 fields, J. High Energy Phys. 07 (2012) 047.
[40] L. Amendola and S. Tsujikawa, *Dark Energy: Theory and Observations* (Cambridge University Press, Cambridge, England, 2010).
[41] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Modified gravity and cosmology, Phys. Rep. **513**, 1 (2012).
[42] A. Joyce, B. Jain, J. Khoury, and M. Trodden, Beyond the cosmological standard model, Phys. Rep. **568**, 1 (2015).
[43] K. Koyama, Cosmological Tests of Gravity, arXiv:1504.04623.
[44] C. Deffayet, G. Esposito-Farese, and A. Vikman, Covariant Galileon, Phys. Rev. D **79**, 084003 (2009).
[45] A. Nicolis, R. Rattazzi, and E. Trincherini, Galileon as a local modification of gravity, Phys. Rev. D **79**, 064036 (2009).
[46] C. Deffayet, S. Deser, and G. Esposito-Farese, Generalized Galileons: All scalar models whose curved background extensions maintain second-order field equations and stress tensors, Phys. Rev. D **80**, 064015 (2009).
[47] A. Barreira, B. Li, C. Baugh, and S. Pascoli, The observational status of Galileon gravity after Planck, J. Cosmol. Astropart. Phys. 08 (2014) 059.
[48] H. Gil-Marin *et al.*, The power spectrum and bispectrum of SDSS DR11 BOSS galaxies II: Cosmological interpretation, arXiv:1408.0027.
[49] H. Gil-Marin, J. Norena, L. Verde, W. J. Percival, C. Wagner, M. Manera, and D. P. Schneider, The power spectrum and bispectrum of SDSS DR11 BOSS galaxies–I. Bias and gravity, Mon. Not. R. Astron. Soc. **451**, 539 (2015).
[50] J. Wang, L. Hui, and J. Khoury, No-Go Theorems for Generalized Chameleon Field Theories, Phys. Rev. Lett. **109**, 241301 (2012).
[51] V. D. V. Cervantes, F. Marulli, L. Moscardini, M. Baldi, and A. Cimatti, Exploiting the shift of baryonic acoustic oscillations as a dynamical probe for dark interactions, arXiv:1212.0853.
[52] D. Blas, J. Lesgourgues, and T. Tram, The Cosmic Linear Anisotropy Solving System (CLASS). Part II: Approximation schemes, J. Cosmol. Astropart. Phys. 07 (2011) 034.
[53] E. Bellini, J. Lesgourgues, I. Sawicki, and M. Zumalacarregui, Hi-CLASS: Horndeski in the Cosmic Linear Anisotropy Solving System (in preparation).
[54] E. Bellini and I. Sawicki, Maximal freedom at minimum cost: Linear large-scale structure in general modifications of gravity, J. Cosmol. Astropart. Phys. 07 (2014) 050.
[55] J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, Essential building blocks of dark energy, J. Cosmol. Astropart. Phys. 08 (2013) 025.
[56] N. Bartolo, E. Bellini, D. Bertacca, and S. Matarrese, Matter bispectrum in cubic Galileon cosmologies, J. Cosmol. Astropart. Phys. 03 (2013) 034.
[57] I. Sawicki and E. Bellini, Limits of Quasi-Static Approximation in Modified-Gravity Cosmologies, arXiv:1503.06831.
[58] A. Barreira, B. Li, W. A. Hellwing, C. M. Baugh, and S. Pascoli, Nonlinear structure formation in the cubic Galileon gravity model, J. Cosmol. Astropart. Phys. 10 (2013) 027.
[59] B. Li, A. Barreira, C M Baugh, W. A Hellwing, K. Koyama, S. Pascoli, and G.-B. Zhao, Simulating the quartic Galileon gravity model on adaptively refined meshes, J. Cosmol. Astropart. Phys. 11 (2013) 012.
[60] C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, Imperfect dark energy from kinetic gravity braiding, J. Cosmol. Astropart. Phys. 10 (2010) 026.
[61] G. Dvali, G. Gabadadze, and M. Porrati, 4D gravity on a brane in 5D Minkowski space, Phys. Lett. B **485**, 208 (2000).
[62] R. Scoccimarro, Large-scale structure in brane-induced gravity. I. Perturbation theory, Phys. Rev. D **80**, 104006 (2009).

[63] P. Brax and P. Valageas, Structure formation in modified gravity scenarios, Phys. Rev. D **86**, 063512 (2012).
[64] A. Taruya, T. Nishimichi, F. Bernardeau, T. Hiramatsu, and K. Koyama, Regularized cosmological power spectrum and correlation function in modified gravity models, Phys. Rev. D **90**, 123515 (2014).
[65] N. Padmanabhan, M. White, and J. Cohn, Reconstructing baryon oscillations: A Lagrangian theory perspective, Phys. Rev. D **79**, 063523 (2009).
[66] M. White, Reconstruction within the Zeldovich approximation, Mon. Not. R. Astron. Soc. **450**, 3822 (2015).
[67] T. Baldauf, M. Mirbabayi, M. Simonovic, and M. Zaldarriaga, Equivalence principle and the baryon acoustic peak, Phys. Rev. D **92**, 043514 (2015).
[68] L. Hui, A. Nicolis, and C. Stubbs, Equivalence principle implications of modified gravity models, Phys. Rev. D **80**, 104002 (2009).