Reconciling Planck results with low redshift astronomical measurements

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We show that emerging tension between the direct astronomical measurements at low redshifts and cosmological parameters deduced from the Planck measurements of the cosmic microwave background anisotropies can be alleviated if the dark matter consists of two fractions, the stable part being dominant with a smaller unstable fraction. The latter constitutes ∼10 per cent at the recombination epoch if it has decayed by now.

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I. INTRODUCTION

Measurements of the cosmic microwave background (CMB) fluctuations by WMAP [\[1\]](#page-3-0) and Planck [\[2\]](#page-3-1) collaborations opened a new era in high precision cosmology. These data are well described by the standard spatially flat ΛCDM cosmology with a power law spectrum of adiabatic scalar perturbations, and make a great step towards the precise determination of cosmological parameters, in particular of the Hubble constant $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$, the density parameters Ω_b and Ω_{dm} for the baryon and dark matter (DM) fractions, and therefore of the whole matter density $\Omega_m = \Omega_b + \Omega_{dm} = 1 - \Omega_{\Lambda}$. Recently, the latest results of the Planck Collaboration have been published [\[3\]](#page-3-2) based on the full mission Planck data. They are in excellent agreement with the 2013 data [\[2\]](#page-3-1) but with improved precision.

The Plank data imply a rather low value for the Hubble constant, which is in tension with the direct astronomical measurements of h . The Planck 2015 TT, TE, $EE + lowP$ data, which we take in this article as a benchmark, determine the Hubble constant with 1 percent precision, $h = 0.6727 \pm 0.0066$ [\[3\].](#page-3-2)

Direct astronomical measurements of the Hubble constant indicate larger values. The analysis [\[4\]](#page-3-3) of the Hubble Space Telescope (HST) data based on over 600 cepheids in host galaxies and eight samples of Supernovae Ia (SNe Ia) yields $h = 0.738 \pm 0.024$ including both statistical and systematic errors. Independent analysis of the Carnegie Hubble program [\[5\]](#page-4-0) using the Spitzer Space Telescope data for calibration purposes lead to $h = 0.743 \pm 0.021$. Both of these results are discordant with the Planck result at about the 2.5σ level. Other astronomical estimates also typically imply high values of the Hubble constant. For example, the analysis of the gravitational lensing time delay measurements of the system RXJ1131-1231 implies $h = 0.787 \pm 0.045$ [\[6\].](#page-4-1)

In addition to h , there is tension between other CMB derived observables and their direct low redshift measurements. The Planck results show a tension between the cosmological constraints on σ_8 and Ω_m from the CMB [\[7,8\]](#page-4-2) and from clusters as cosmological probes. Cluster data prefer lower values of these observables deviated at more than the 2σ level; see, e.g., Refs. [\[9,10\].](#page-4-3)

Recently baryon acoustic oscillations (BAO) in the $Ly\alpha$ forest of BOSS DR11 quasars have been studied at redshift $z = 2.34$ [\[11,12\]](#page-4-4). The measured position of the BAO peak determines the angular distance, $D_A(z)$ and expansion rate, $H(z)$. Obtained constraints imply values of D_A and H that are, respectively, 7% low and 7% high compared to the predictions of a flat ΛCDM cosmological model with the best-fit Planck parameters. The significance of this discrepancy is approximately 2.5σ [\[12\].](#page-4-5)

The tension between CMB based determination of several observables by the Planck Collaboration and direct low z measurements is intriguing and deserves attention. The cause of the discrepancy may lie in some calibration errors. On the other hand, it may hint at a deficiency of the standard ΛCDM paradigm. In this paper we show that this discrepancy may be resolved if a certain fraction of dark matter is unstable. Decaying dark matter (DDM) models have been considered previously, see, e.g., the most recent Refs. [\[13,14\],](#page-4-6) with the stringent constraint on DDM decay width Γ. However, in these papers it was assumed that the whole of DM is susceptible to the decay, and it was concluded that the decay time must be larger than 100 Gyr or so. We instead assume that dark matter consists of two fractions, the stable dark matter being dominant while a subdominant unstable part decays between recombination and the present epoch.

II. DECAYING DARK MATTER

A. Planck constraints

To ensure that our model fits the Planck data we accept Planck derived values for all cosmological parameters relevant at recombination. In particular, this means that

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the sum of initial densities of stable and decaying components of DM is fixed so that after formal redshift to the present epoch it would correspond to the value determined by Planck, $\omega_{sdm} + \omega_{ddm} = 0.1198$ [\[3\].](#page-3-2) We assume that unstable component decays into invisible massless particles, without producing too many photons. The initial fraction of the decaying DM in the cosmological mass density is a free parameter in our model:

$$
F \equiv \frac{\omega_{ddm}}{\omega_{sdm} + \omega_{ddm}}.\tag{1}
$$

Alternatively, one can consider a scenario when dark matter consists of two particle species with masses $M + \mu$ and $M - \mu$, and the heavier component decays into the lighter one with emission of invisible massless particles. In this case the dark mass fraction disappearing due to decay is equivalent to $F = \mu/M$. Throughout the paper we normalize the width of the decaying component Γ to km/s/Mpc, i.e., in the same units as H_0 . Γ is another independent cosmological parameter in our model that we also vary for fitting the data. It is bounded from above by the requirement that the unstable fraction does not decay substantially before the last scattering to measurably affect the CMB. Hence, we take $\Gamma < 5000$ in which case this requirement is fulfilled.

Furthermore, we require that the angular diameter distance to the last scattering should be the same for all values of parameters, namely, we fix the sound horizon angle $100 * \theta_s$ to the Planck value 1.04077. This determines Hubble parameter h as a function of F and Γ and guarantees that derived CMB spectra in our model are identical (at high l) to the best-fit Plank spectrum for all values of parameters. Resulting h as a function of Γ is shown in Fig. [1](#page-1-0) for different values of F . Let us remark also that for the choice of parameters as in Fig. [1,](#page-1-0) the age of the

FIG. 1. Hubble parameter h as a function of DM decay width Γ for different values of the DDM fraction F.

Universe $t_0 = \frac{1}{3} H_0^{-1} \Omega_{\Lambda}^{-1/2} \ln[(1 + \Omega_{\Lambda}^{1/2})/(1 - \Omega_{\Lambda}^{1/2})]$ remains nearly the same as predicted by Planck, $t_0 \approx 13.8 \text{ Gyr}$, since increasing H_0 is compensated by increasing dark energy fraction Ω_{Λ} .

Relevant cosmological calculations have been carried out using the CLASS Boltzmann code [\[15,16\]](#page-4-7). The parameter space is explored using the Markov chain Monte Carlo technique with the Monte Python package [\[17\]](#page-4-8). We verified that all CMB spectra are identical at $l \gtrsim 40$. At smaller l the spectra somewhat deviate because the cosmological constant in our model is typically larger as compared to the standard ΛCDM (we consider the spatially flat Universe only). However, corresponding changes are smaller than the cosmic variance. Therefore, we do not constrain model parameters using low *l* Planck data and we use supernova data instead.

B. Adding supernova and HST constraints

For fitting to supernovae observations we use the JLA [\[18\]](#page-4-9) compilation composed of 740 SN Ia. This is the largest data set to date containing samples from low redshift $z \approx$ 0.02 to a large one, $z \approx 1.3$. The data were obtained from the joint analysis of SDSS II and SNLS, improving the analysis by means of a recalibration of light curve fitter SALT2 and in turn reducing possible systematic errors. For "standardization"; of SNe Ia data the linear model for the distance modulus μ is employed with four nuisance parameters in the distance estimates. All necessary data for the analysis were retrieved from [\[19\]](#page-4-10). Resulting best-fit values for all nuisance parameters in our cosmology do not differ notably from the values quoted in Ref. [\[18\]](#page-4-9), derived for ΛCDM.

We further constrain our model using determination of the Hubble parameter with the HST [\[4\]](#page-3-3). Resulting one and two sigma likelihood contours in the plane of Γ and F are shown in Fig. [2](#page-2-0) by solid and dashed lines. We see that the base Λ CDM with $\Gamma = F = 0$ is outside of 2σ contours in our model. The derived likelihood for the Hubble parameter corresponds to $h = 0.716 \pm 0.02$ at one σ . Therefore, with a fraction of decaying dark matter the data of Planck on CMB anisotropies, data on supernova, and HST data can all be reconciled.

C. DDM and BAO

We now turn to the data on baryon acoustic oscillations. The measurement of the characteristic scale of BAO in the correlation function of different matter distribution tracers provides a powerful tool to probe the cosmic expansion and a convincing method for setting cosmological constraints. The BAO peak in the correlation function at a redshift z appears at the angular separation $\Delta \theta = r_d/(1+z)D_A(z)$, where D_A is the angular diameter distance and $r_d = r_s(z_d)$ is the sound horizon at the drag redshift, i.e., at the epoch when baryons decoupled from photons. The BAO feature also appears at the redshift separation $\Delta z = r_d/D_H$, where

FIG. 2 (color online). One and two sigma likelihood contours for our model parameters. Solid and dashed lines correspond to a data set consisting of a JLA sample of SN Ia and HST measurements of h, on top of the best-fit Planck model parameters. The addition of Planck cluster data results in a much narrower shaded area.

 $D_H \equiv c/H(z)$. Therefore, measurement of the BAO peak position at some z constrains the combinations of cosmological parameters that determine D_H/r_d and D_A/r_d at that redshift.

Recently independent constraints on Hr_d and D_A/r_d were obtained using SDSS/BOSS data at $z = 0.35$ [\[20,21\]](#page-4-11), $z = 0.57$ [\[22,23\],](#page-4-12) and $z = 2.34$ [\[12\].](#page-4-5) These data are plotted in Fig. [3.](#page-2-1) Note that the derived constraints for $H(z)$ and D_A are not independent; the correlation coefficient is 0.5. To avoid cluttering in displaying results obtained by different authors at the same redshift, in the right panel we plot the results of Refs. [\[12,21](#page-4-5)–23] for D_A/r_d , while in the left panel the results of Refs. [\[12,20,23\]](#page-4-5) for h are presented.

Again, the solid line corresponds to the ΛCDM model with the best-fit Planck measurements. Two other models, $F = 0.1$ and $F = 0.2$ both with $\Gamma = 2000$, are also shown. We see that the data systematically deviate from the base Λ CDM. Though at $z < 1$ each deviation is about 1σ (and therefore is not considered a problem) they all are in the direction of the DDM models, except for the result of Ref. [\[23\]](#page-4-13) at $z = 0.57$.

We repeated the likelihood analysis procedure of the previous subsection with BAO data added. We used BOSS BAO likelihoods included in the Monte Python package [\[17\]](#page-4-8), latest release 2.1. The result is similar to the one presented in Fig. [2](#page-2-0) but with one and two sigma contours shifted down by about a factor of two. However, the result would clearly depend upon the data set chosen. In Ref. [\[24\]](#page-4-14) the DDM model was analyzed with the inclusion of the latest BAO results only, at $z = 0.57$ [\[23\]](#page-4-13) and $z = 2.34$ [\[12\]](#page-4-5), with pessimistic conclusions. In Fig. [3](#page-2-1) we can see the origin for this conclusion as well. DDM helps to ease tension at $z = 2.34$ both for $H(z)$ and D_A , which are at the 2.5σ level compared to the predictions of the base ΛCDM. However, results of [\[23\]](#page-4-13) at $z = 0.57$, which are also discrepant at the 1σ level, behave differently. While DDM is better fit for $H(z)$, it is not so for D_A ; the latter is represented by an upper (red) data point at $z = 0.57$ in the left panel of Fig. [3](#page-2-1). Overall, DDM does not help much here. As Ref. [\[3\]](#page-3-2) describes, at present it is not clear whether the discrepancy at $z = 2.34$ is caused by systematics in the Ly α BAO measurements (which are more complex and less mature than galaxy BAO measurements) or is an indicator of a new physics.

D. DDM and cluster counts

The decaying dark matter model is capable of resolving tension between the base ΛCDM model and the cluster data

FIG. 3 (color online). Hubble parameter $h(z)$ (left panel) and angular diameter distance D_A (right panel). Model curves are presented for fixed $\Gamma = 2000$ and several values of F. Points at nonzero redshift z are the SDSS BAO data. The HST measurement at $z = 0$ is also shown with the symbol size comparable to the error bars.

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FIG. 4 (color online). Ω_m and σ_8 derived from cluster counts and from the CMB. The line marked DDM shows the trend of these parameters when F and Γ are varied in our model. The white circle represents a model with $F = 0.1$ and $\Gamma = 2000$ as an example.

as well [\[25\].](#page-4-15) This is displayed in Fig. [4](#page-3-4) in the σ_8 and Ω_m parameter plane. The base ΛCDM corresponds to the error cross marked PLANCK CMB. Shaded areas correspond to the parameter regions allowed (at 2σ) by Planck cluster data [\[7,8\]](#page-4-2) and by the extended ROSAT-ESO Flux Limited X-ray Galaxy Cluster Survey [\[10\]](#page-4-16). We should also note that earlier results obtained in [\[9\]](#page-4-3), while in agreement with [\[10\]](#page-4-16), are even farther away from the Planck base ΛCDM model. In the DDM model, when F and Γ are varied, σ_8 and Ω_m closely follow the line marked DDM in Fig. [4](#page-3-4) and cross the region allowed by the cluster data. The white circle on this line represents a model with $F = 0.1$ and $\Gamma = 2000$. With smaller values of F and/or Γ the dot representing a model moves to the right, closer to the base ΛCDM model.

We have added Planck constraints [\[7\]](#page-4-2) from Sunyaev-Zeldovich cluster counts, $\sigma_8(\Omega_m/0.27)^{0.3} = 0.78 \pm 0.01$, to our likelihood analysis (without BAO). The result is shown in Fig. [2](#page-2-0) as the shaded area. (The 1σ area continues actually up to $F \approx 0.25$ and $\Gamma \approx 100$ but this is unresolved at the scale of this figure.) Now the likelihood of base

ΛCDM is vanishingly small compared to a best-fit DDM models. However, as the Planck Collaboration concluded on the cluster counts issue [\[8\],](#page-4-17) it is unclear if this tension arises from low-level systematics in the astrophysical studies, or represents the first glimpse of something more important. We reiterate that the hypothesis of decaying dark matter may help to resolve this tension as well. In fact, from the joint fit shown in Fig. [2](#page-2-0) one can see that the issues of H_0 and σ_8 can be resolved with the same parameter values of the DDM model.

III. CONCLUSIONS

Cosmological parameters deduced from the Planck measurements of the CMB anisotropies with unprecedented accuracy are at some tension with direct astronomical measurements of various parameters at low redshifts. We have shown that Planck-inspired ΛCDM cosmology can be reconciled both with HST measurements and with cluster data within the hypotheses of decaying dark matter. The joint fit to Planck, supernova, HST, and Planck cluster data tells us that if the dark matter decayed between recombination and the present time, then the unstable fraction should be about 10 percent at the recombination epoch. The situation with the BAO discrepancies is less clear at present and we should wait to see in which direction the intrigue will develop.

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Note added.—Recently, Ref. [\[26\]](#page-4-18) appeared in which DDM was also suggested as a resolution to the possible tension between the CMB and weak lensing determinations of σ_8 .

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