

# Origin of the structures observed in $e^+e^-$ annihilation into multipion states around the $\bar{p}p$ threshold

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We analyze the origin of the structures observed in the reactions  $e^+e^- \rightarrow 3(\pi^+\pi^-)$ ,  $2(\pi^+\pi^-\pi^0)$ ,  $\omega\pi^+\pi^-\pi^0$ , and  $e^+e^- \rightarrow 2(\pi^+\pi^-\pi^0)$  around the antiproton-proton ( $\bar{p}p$ ) threshold. We estimate the contribution of the two-step process  $e^+e^- \rightarrow \bar{N}N \rightarrow$  multipions to the total reaction amplitude. The amplitude for  $e^+e^- \rightarrow \bar{N}N$  is constrained from near-threshold data on the  $e^+e^- \rightarrow \bar{p}p$  cross section and the one for  $\bar{N}N \rightarrow$  multipions can be likewise fixed from available experimental information, for all studied  $5\pi$  and  $6\pi$  states. The resulting amplitude for  $e^+e^- \rightarrow$  multipions turns out to be large enough to play a role for the considered  $e^+e^-$  annihilation channels and, in three of the four reactions, even allows us to reproduce the data quantitatively near the  $\bar{N}N$  threshold. The structures seen in the experiments emerge then as a threshold effect due to the opening of the  $\bar{N}N$  channel.

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## I. INTRODUCTION

Recently, results from several high-statistics measurements of  $e^+e^-$  annihilation into multipion states have been published [1–5]. One of the striking features in the data is a pronounced structure in the vicinity of the antinucleon-nucleon ( $\bar{N}N$ ) threshold that appears either as a sharp drop for  $e^+e^- \rightarrow 3(\pi^+\pi^-)$  [2,4] or as a dip for  $e^+e^- \rightarrow 2(\pi^+\pi^-\pi^0)$  [2,5],  $e^+e^- \rightarrow \omega\pi^+\pi^-\pi^0$  [2], and for  $e^+e^- \rightarrow 2(\pi^+\pi^-\pi^0)$  [3] in the reaction cross section. Earlier measurements with lower statistics had already suggested the presence of such a dip, cf. the review [6].

Phenomenological fits to the  $e^+e^-$  data locate the structure at 1.91 GeV [7] or at 1.86–1.88 GeV [2] while the  $\bar{p}p$  threshold is at 1.8765 GeV. Naturally, this very proximity of the  $\bar{N}N$  threshold suggests that the  $\bar{N}N$  channel should have something to do with the appearance of that structure in the multipion production cross sections. A common speculation is that it could be a signal for a  $\bar{p}p$  bound state [4,7] or a subthreshold  $\bar{p}p$  resonance [5]. Such a conjecture seems to be also in line with experimental findings in a related reaction, namely  $e^+e^- \rightarrow \bar{p}p$ , where a near-threshold enhancement seen in the cross section [8,9] is likewise associated with a possible  $\bar{p}p$  bound state, cf. also Ref. [10]. For a discussion of other structures seen in  $e^+e^- \rightarrow \bar{p}p$ , see, e.g., Ref. [11].

Given the complexity of the reaction mechanism one cannot expect a microscopic calculation of those multipion production cross sections soon. Indeed, so far there is not even a calculation that quantifies the impact of the opening of the  $\bar{N}N$  channel on those reactions. The idea that  $e^+e^- \rightarrow 6\pi$  and  $e^+e^- \rightarrow \bar{p}p$  could be closely related is

picked up in Ref. [12] for interpreting the drop/dip. Aspects related to the question of a  $\bar{N}N$  bound state are discussed in Ref. [13] where it is emphasized that ordinary threshold effects like cusps could also explain the structures seen in the multipion channels.

In the present paper we investigate the significance of the  $\bar{N}N$  channel for the  $e^+e^- \rightarrow 5\pi$  and  $e^+e^- \rightarrow 6\pi$  reactions. Specifically, we aim at a reliable estimation of the influence of the  $\bar{N}N$  channel on those multipion production cross sections around the  $\bar{N}N$  threshold. The calculation builds on earlier works published in Refs. [14,15]. This concerns (i) a  $\bar{N}N$  potential constructed in the framework of chiral effective field theory (EFT), that reproduces the amplitudes determined in a partial-wave analysis of  $\bar{p}p$  scattering data [16] from the  $\bar{N}N$  threshold up to laboratory energies of  $T_{\text{lab}} \approx 200 - 250$  MeV [14] and (ii) an analysis of the reaction  $e^+e^- \rightarrow \bar{p}p$  (and  $\bar{p}p \rightarrow e^+e^-$ , respectively) where the effect of the interaction in the  $\bar{N}N$  system was taken into account rigorously [15] and where the experimentally observed near-threshold enhancement in the cross section and the associated steep rise of the electromagnetic form factors in the timelike region is explained solely in terms of the  $\bar{p}p$  interaction. Note that the employed  $\bar{N}N$  interaction is also able to describe the large near-threshold enhancement observed in the reaction  $J/\psi \rightarrow \gamma\bar{p}p$  [17].

## II. FORMALISM

The estimation of the influence of the  $\bar{N}N$  channel is done in the same framework as the studies mentioned above [14,15] and consistently with them. It amounts to solving the following formal set of coupled equations:

$$\begin{aligned}
T_{\bar{N}N \rightarrow \bar{N}N} &= V_{\bar{N}N \rightarrow \bar{N}N} + V_{\bar{N}N \rightarrow \bar{N}N} G_0 T_{\bar{N}N \rightarrow \bar{N}N}, \\
T_{e^+e^- \rightarrow \bar{N}N} &= V_{e^+e^- \rightarrow \bar{N}N} + V_{e^+e^- \rightarrow \bar{N}N} G_0 T_{\bar{N}N \rightarrow \bar{N}N}, \\
T_{\bar{N}N \rightarrow \nu} &= V_{\bar{N}N \rightarrow \nu} + T_{\bar{N}N \rightarrow \bar{N}N} G_0 V_{\bar{N}N \rightarrow \nu}, \tag{1}
\end{aligned}$$

$$\begin{aligned}
T_{e^+e^- \rightarrow \nu} &= A_{e^+e^- \rightarrow \nu} + V_{e^+e^- \rightarrow \bar{N}N} G_0 T_{\bar{N}N \rightarrow \nu} \\
&= A_{e^+e^- \rightarrow \nu} + T_{e^+e^- \rightarrow \bar{N}N} G_0 V_{\bar{N}N \rightarrow \nu}, \tag{2}
\end{aligned}$$

with  $G_0$  the free  $\bar{N}N$  propagator and  $\nu = 3(\pi^+\pi^-)$ , etc. Here the first one is the Lippmann-Schwinger equation from which the  $\bar{N}N$  scattering amplitude is obtained, see Ref. [14] for details. The second equation provides the  $e^+e^- \rightarrow \bar{N}N$  transition amplitude, which was calculated in distorted-wave Born approximation in Ref. [15]. The third equation defines the amplitude for  $\bar{N}N$  annihilation into the (various)  $5\pi$  and  $6\pi$  channels. These amplitudes will be established in the present work. Luckily there is experimental information for all considered multipion channels, i.e., for  $\bar{p}p \rightarrow 3(\pi^+\pi^-)$ ,  $\bar{p}p \rightarrow 2(\pi^+\pi^-\pi^0)$ ,  $\bar{p}p \rightarrow 2(\pi^+\pi^-\pi^0)$ , and  $\bar{p}p \rightarrow \omega\pi^+\pi^-\pi^0$  [18–20], so that the corresponding transition potentials  $V_{\bar{N}N \rightarrow \nu}$  can be constrained by a fit to data. Once  $T_{e^+e^- \rightarrow \bar{N}N}$  ( $V_{e^+e^- \rightarrow \bar{N}N}$ ) and  $T_{e^+e^- \rightarrow \nu}$  ( $V_{e^+e^- \rightarrow \nu}$ ) are fixed the contributions to the  $e^+e^- \rightarrow \nu$  reactions that proceed via an intermediate  $\bar{N}N$  state, cf. the second terms on the right-hand side of Eq. (2), are likewise fixed. Note that the two lines in Eq. (2) are equivalent. The only unknown quantity in the equations above is  $A_{e^+e^- \rightarrow \nu}$ . It stands for all other contributions to  $e^+e^- \rightarrow \nu$ , i.e., practically speaking it represents the background to the loop contribution due to two-step  $e^+e^- \rightarrow \bar{N}N \rightarrow \nu$  transition.

Of course, it is impossible to take into account the full complexity of the  $e^+e^- \rightarrow 5\pi$ ,  $e^+e^- \rightarrow 6\pi$  and  $\bar{N}N \rightarrow 5\pi$ ,  $\bar{N}N \rightarrow 6\pi$  reactions. Thus, in the following we want to describe the simplifications and approximations made in our study. First of all this concerns the reaction dynamics. Following the strategy in Ref. [14], the elementary transition (annihilation) potential for  $\bar{N}N \rightarrow \nu$  is parametrized by

$$V_{\bar{N}N \rightarrow \nu}(q) = \tilde{C}_\nu + C_\nu q^2, \tag{3}$$

i.e., by two contact terms analogous to those that arise up to next-to-next-to-leading order (NNLO) in the treatment of the  $\bar{N}N$  or  $NN$  interaction within chiral EFT. The quantity  $q$  in Eq. (3) is the center-of mass (c.m.) momentum in the  $\bar{N}N$  system. Since the threshold for the production of 5 or 6 pions lies significantly below the one for  $\bar{N}N$  the pions carry—on average—already fairly high momenta. Thus, the dependence of the annihilation potential on those momenta should be small for energies around the  $\bar{N}N$  threshold and it is, therefore, neglected. The constants  $\tilde{C}_\nu$  and  $C_\nu$  are determined by a fit to the  $\bar{N}N \rightarrow \nu$  cross section (and/or branching ratio) for each annihilation channel  $\nu$ .

The term  $A_{e^+e^- \rightarrow \nu}$  is likewise parametrized in the form (3), but as a function of the  $e^+e^-$  c.m. momentum. The arguments for this simplification are the same as above and they are valid again, of course, only for energies around the  $\bar{N}N$  threshold. However, since in the  $e^+e^-$  case this term does represent actually a background amplitude and not a transition potential we allow the corresponding constants to be complex numbers which are fixed by a fit to the  $e^+e^- \rightarrow \nu$  cross sections.

In our study of the electromagnetic form factors in the timelike region [15] we adopted the standard one-photon approximation. In this case there are only two partial waves that can contribute to the  $e^+e^- \rightarrow \bar{N}N$  transition, namely the (tensor) coupled  $^3S_1 - ^3D_1$  partial waves. We make the same assumption in the present work. For  $\bar{N}N \rightarrow 5\pi, 6\pi$  there are no general limitations on the partial waves. However, since we restrict ourselves to energies close to the  $\bar{N}N$  threshold and we expect the annihilation operator to be of rather short range any  $\bar{N}N$  partial waves besides the  $^3S_1$  and the  $^1S_0$  should play a minor role. In the actual calculation we use only the  $^3S_1$  partial wave. Thus, the corresponding transition potential  $V_{\bar{N}N \rightarrow \nu}$  might actually overestimate the true contribution of this partial wave and, therefore, the resulting amplitude for the two step process  $e^+e^- \rightarrow \bar{N}N \rightarrow \nu$  has to be considered as an upper limit. Judging from available branching ratios for  $\bar{p}p \rightarrow \omega\omega$  [18], where near threshold only the  $^1S_0$  can contribute, its contribution [to the  $2(\pi^+\pi^-\pi^0)$  channel] could be in the order of 20% of the one from the  $^3S_1$  partial wave.

Note that there are selection rules for the  $\bar{N}N \rightarrow 5\pi, 6\pi$  transitions, because  $G$ -parity is preserved. For  $n$  pions the  $G$ -parity is defined by  $G = (-1)^n$  while for  $\bar{N}N$  it is given by  $G = (-1)^{L+S+I}$ , where  $L, S, I$  denote the orbital angular momentum, and the total spin and isospin, respectively. Thus, the  $G$ -parity for the six pion final states (i.e., also for  $\omega\pi^+\pi^-\pi^0$ ) is positive which confines the isospin for the  $\bar{N}N$  pair to  $I = 1$  in the  $^3S_1 - ^3D_1$  partial wave. Conversely, the five-pion decay mode can occur only from the  $I = 0$   $^3S_1 - ^3D_1$   $\bar{N}N$  partial wave.

The explicit form of Eq. (2) reads

$$\begin{aligned}
T_{\nu, e^+e^-}(Q, q_e; E) &= A_{\nu, e^+e^-}(Q, q_e) + \sum_{\bar{N}N} \int_0^\infty \frac{dq q^2}{(2\pi)^3} \\
&\times V_{\nu, \bar{N}N}(Q, q) \frac{1}{E - 2E_q + i0^+} T_{\bar{N}N, e^+e^-}(q, q_e; E), \tag{4}
\end{aligned}$$

written here in matrix notation. The sum refers to  $\bar{p}p$  and  $\bar{n}n$  intermediate states. The corresponding expression for  $\bar{p}p \rightarrow \nu$  can be obtained by substituting  $e^+e^-$  by  $\bar{p}p$  in Eq. (4), those for the other amplitudes in Eq. (1) are given

in Refs. [14,15]. The quantity  $Q$  stands here symbolically for the momenta in the  $5\pi$  and  $6\pi$  channels. But since we assumed that the transition potentials do not depend on the pion momenta, cf. Eq. (3),  $Q$  does not enter anywhere into the actual calculation and we do not need to specify this quantity. All amplitudes (and the potentials) can be written and evaluated as functions of the c.m. momenta in the  $\bar{N}N$  ( $q_p$ ) and  $e^+e^-$  ( $q_e$ ) systems and of the total energy  $E = 2\sqrt{m_p^2 + q_p^2} = 2\sqrt{m_e^2 + q_e^2}$ . The quantity  $E_q$  in Eq. (4) is given by  $E_q = \sqrt{m_p^2 + q^2}$ .

Since the amplitudes do not depend on  $Q$  the integration over the multipion phase space can be done separately when the cross sections are calculated. In practice, it amounts only to a multiplicative factor and, moreover, to factors that are the same for  $e^+e^- \rightarrow \nu$  and  $\bar{N}N \rightarrow \nu$  at the same total energy  $E$ . We performed this phase space integration numerically at the initial stage of the present work but it became clear that we can get more or less equivalent results if we simulate that multipion phase space by effective two-body channels with a threshold that coincides with the ones of the multipion systems. In effect the differences in the phase space can be simply absorbed into the constants in the transition potentials, see Eq. (3)—which anyway have to be fitted to the data. All results presented in this manuscript are based on an effective two-body phase space.

Of course, this simulation via effective two-body channels works only for energies around the  $\bar{N}N$  threshold. We cannot extend our calculation down to the threshold of the multipion channels. However, one has to keep in mind that also the validity of our  $\bar{N}N$  interaction is limited to energies not too far away from the  $\bar{N}N$  threshold. Thus, we have to restrict our study to that small region around the threshold anyway.

With the definitions of the  $T$ -matrices above, the cross section is obtained via

$$\sigma_{e^+e^- \rightarrow \nu}(E) = \frac{3E^2\beta}{2^{10}\pi^3} |T_{\nu, e^+e^-}(E)|^2, \quad (5)$$

and similarly for  $\bar{p}p \rightarrow \nu$ . The quantity  $\beta$  denotes the phase space factor for an effective two-body system with equal masses  $M$ ,  $\beta = \sqrt{(E^2 - 4M^2)}/\sqrt{(E^2 - 4m_e^2)}$ , with  $2M = 6m_\pi$ ,  $5m_\pi$ , or  $m_\omega 3m_\pi$ . For  $\bar{p}p \rightarrow \nu$  the electron mass ( $m_e$ ) has to be replaced by the one of the proton.

### III. THE $\bar{N}N \rightarrow 5\pi, 6\pi$ REACTIONS

First we need to fix the constants  $\tilde{C}_\nu$  and  $C_\nu$  in the  $\bar{N}N \rightarrow \nu$  transition potentials. We do this by considering available branching ratios of  $\bar{p}p$  annihilation at rest for the  $3(\pi^+\pi^-)$ ,  $2(\pi^+\pi^-\pi^0)$ ,  $\omega\pi^+\pi^-\pi^0$ , and  $2(\pi^+\pi^-\pi^0)$  channels [18,19]. For the annihilation into  $3(\pi^+\pi^-)$  and  $2(\pi^+\pi^-\pi^0)$  there are, in addition, in flight measurements for energies not too far

from the  $\bar{N}N$  threshold [20]. Following Ref. [21] we evaluate the relative cross sections at low energy and compare those with the measured branching ratios. Specifically, we calculate the cross sections at  $p_{\text{lab}} = 106 \text{ MeV}/c$  ( $T_{\text{lab}} \approx 5 \text{ MeV}$ ) because at this energy the total annihilation cross section is known from experiment [22] and it can be used to calibrate the cross sections for the individual  $5\pi$  and  $6\pi$  channels based on the branching ratios.

Our results for  $\bar{p}p \rightarrow 3(\pi^+\pi^-)$  and  $\bar{p}p \rightarrow 2(\pi^+\pi^-\pi^0)$  are shown in Fig. 1. Note that the lowest “data” point is not from a measurement but deduced from the branching ratios [18] and the total annihilation cross section [22] as discussed in the preceding paragraph.

There are no in flight data for  $\bar{p}p \rightarrow 2(\pi^+\pi^-\pi^0)$  and  $\bar{p}p \rightarrow \omega\pi^+\pi^-\pi^0$ . Here we fit to the central value of the branching ratios, 17.7% [18] and 16.1% [19], respectively, and assume that the energy dependence is the same as for the  $3(\pi^+\pi^-)$  channel. The resulting cross sections at  $p_{\text{lab}} = 106 \text{ MeV}/c$  are 63.2 mb for the  $2(\pi^+\pi^-\pi^0)$  case and

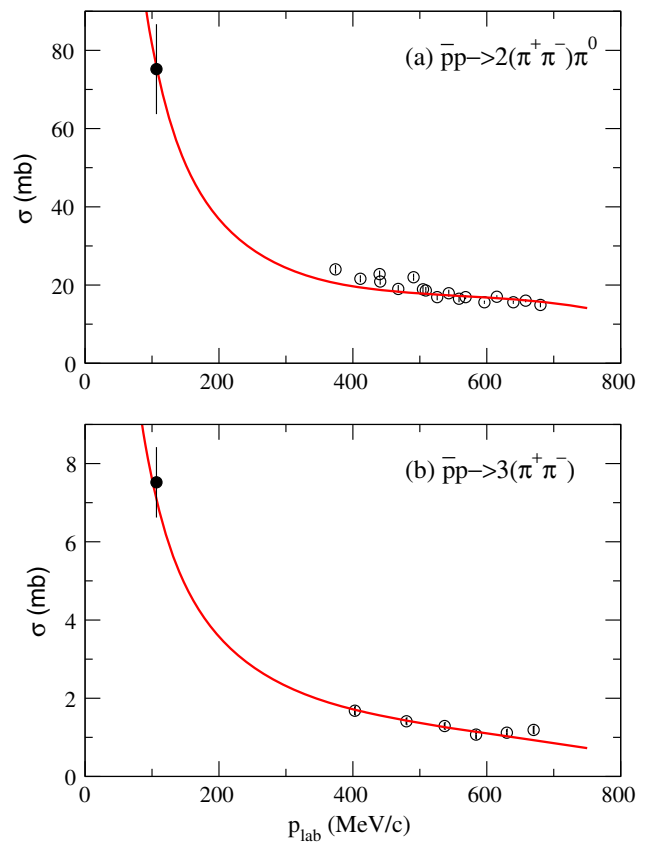


FIG. 1 (color online). Cross section for (a)  $\bar{p}p \rightarrow 2(\pi^+\pi^-\pi^0)$  and (b)  $\bar{p}p \rightarrow 3(\pi^+\pi^-)$ . The solid curves represent our result. The “data” points at 106 MeV/c (filled circles) are deduced from information on the branching ratios of  $\bar{p}p$  annihilation at rest, see text. The in flight data (open circles) are taken from Ref. [20]. Note that the error bars of those data are smaller than the symbol size.

57.5 mb for  $\omega 3\pi$ . Note that the uncertainty in the energy dependence is not too critical. Important is first and foremost the absolute value of those cross sections close to the  $\bar{N}N$  threshold because that value is decisive for the magnitude of the  $e^+e^- \rightarrow \bar{N}N \rightarrow \nu$  two-step contribution and, in turn, for the relevance of the  $\bar{N}N$  intermediate state for the  $e^+e^- \rightarrow \nu$  reaction. Because of this we are also not too concerned about the visible discrepancy between data and our results in case of the  $2(\pi^+\pi^-)\pi^0$  channel at higher momenta, cf. Fig. 1.

The results above are based on the NNLO EFT  $\bar{N}N$  interaction with the cutoff combination  $\{\Lambda, \tilde{\Lambda}\} = \{450, 500\}$  MeV, cf. Ref. [14] for details. Exploratory calculations for the other cutoff combinations considered in Ref. [14] turned out to be very similar. Like for  $\bar{N}N$  scattering itself, much of the cutoff dependence is absorbed by the contact terms [ $\tilde{C}_\nu$  and  $C_\nu$  in Eq. (3)] that are fitted to the data so that the variation of the results for energies of, say,  $\pm 50$  MeV around the  $\bar{N}N$  threshold is rather small. For consistency the momentum dependent regulator function as given in Eq. (2.21) in Ref. [14] is also attached to all momentum dependent quantities here, for example to the transition potential in Eq. (3).

Because of the coupled nature of the  $^3S_1$ - $^3D_1$   $\bar{N}N$  partial wave, in principle, the  $D$  wave should be also included in Eq. (4) and, consequently, also in Eq. (3). Then there would be an additional contact term of the form  $D_\nu q^2$  [14], representing the  $\bar{N}N \rightarrow \nu$  transition potential from the  $\bar{N}N$   $^3D_1$  state, and a summation over the intermediate  $\bar{N}N$   $^3D_1$  state arises, in addition to the integration over the intermediate momentum in Eq. (4). We ignore these complications here because transitions starting from the  $\bar{N}N$   $D$  wave are strongly suppressed for energies around the  $\bar{N}N$  threshold and the contribution from the loop can be anyway effectively included in the contact terms of the transition from the  $\bar{N}N$   $S$ -wave state.

#### IV. RESULTS FOR $e^+e^- \rightarrow 5\pi, 6\pi$

Once the contact terms in  $V_{\bar{N}N \rightarrow \nu}$  are fixed from a fit to the pertinent data the corresponding part of the  $e^+e^- \rightarrow \nu$  amplitude that comes from the transition via an intermediate  $\bar{N}N$  state is also completely fixed, cf. Eq. (2). We then add the background amplitude  $A_{e^+e^- \rightarrow \nu}$ . It is assumed to be of the same functional form as Eq. (3), based on the expectation that this amplitude simulates a possibly very large set of transition processes and, therefore, should have a weak dependence on the total energy in the region of the  $\bar{N}N$  threshold. This feature is implemented by the *ansatz* (3) with  $q$  being interpreted as the c.m. momentum in the  $e^+e^-$  system. We want to emphasize, however, that this is a working hypothesis. In principle, it is possible that a few ‘‘doorway’’ states in the form of  $\rho$ -like (or  $\omega/\phi$ -like) resonances [23] alone provide most of the background. In this case, the corresponding  $A_{e^+e^- \rightarrow \nu}$  could have a

significant energy dependence, specifically, if those states are narrow and close to the  $\bar{N}N$  threshold. In the present study we do not consider this option because we are primarily interested in exploring to what extent the energy dependence induced by the loop contribution due to two-step  $e^+e^- \rightarrow \bar{N}N \rightarrow \nu$  transition is already sufficient to explain the structures seen in the  $e^+e^- \rightarrow \nu$  data. In any case, it should be said that the experimental situation with regard to such  $\rho$ -like resonances in the relevant mass region is rather unclear as discussed in the mini review on the  $\rho(1450)$  and  $\rho(1700)$  in Ref. [23]; see also Ref. [24].

Note that  $A_{e^+e^- \rightarrow \nu}$  can no longer be identified with a transition potential (like for  $\bar{N}N \rightarrow \nu$ ). It has to account for all other contributions to  $e^+e^- \rightarrow \nu$ , besides the one that includes the intermediate  $\bar{N}N$  state. Specifically, this term can have a relative phase as compared to the contribution from the  $\bar{N}N$  loop. Therefore, in this case the parameters can and should be complex. The two complex constants in the analogous Eq. (3) for  $A_{e^+e^- \rightarrow \nu}$  are adjusted in a fit to the cross sections of each of the four  $e^+e^- \rightarrow \nu$  reactions studied in the present investigation. For the fit we considered data in the range  $1750 \text{ MeV} \leq E \leq 1950 \text{ MeV}$ , i.e., in a region that spans more or less symmetrically the  $\bar{N}N$  threshold.

In principle, the  $e^+e^- \rightarrow \bar{p}p$  amplitude that enters the loop contribution in Eq. (4) can be taken straight from Ref. [15] where it was fixed in a fit to the  $e^+e^- \rightarrow \bar{p}p$  cross section. The results in that work were obtained by using a  $\bar{p}p$  amplitude which is the sum of the isospin  $I = 0$  and  $I = 1$  amplitudes, i.e.,  $T_{\bar{p}p} = (T^{I=1} + T^{I=0})/2$ . However, it was found that employing other combinations of  $T^1$  and  $T^0$  lead to very similar results and in all cases an excellent agreement with the energy dependence exhibited by the data could be achieved. Thus, since isospin is not conserved in the reaction  $e^+e^- \rightarrow \bar{p}p$  the actual isospin content of the produced  $\bar{p}p$  could not be fixed. The mentioned selection rules for  $\bar{N}N \rightarrow n\pi$  imply that the  $6\pi$  final state can only be reached from an  $I = 1$   $^3S_1$   $\bar{N}N$  state while 5 pions have to come from the corresponding  $I = 0$  state. Thus, the magnitude of the  $\bar{N}N$  loop contribution to  $e^+e^- \rightarrow \nu$  depends decisively on the isospin content of the intermediate  $\bar{N}N$  state. We did calculations for  $e^+e^- \rightarrow \nu$  with the combination as used in Ref. [15] but it turned out that a slightly larger  $I = 1$  admixture, namely  $T_{\bar{p}p} \approx 0.7T^1 + 0.3T^0$ , is preferable and leads to a somewhat better overall agreement with the experiments and, therefore, we adopt this combination here. The cross section for  $e^+e^- \rightarrow \bar{n}n$  is also known experimentally [25,26], though with somewhat less accuracy. It agrees with the one for  $e^+e^- \rightarrow \bar{p}p$  within the error bars [26]. Therefore, we simply put  $T_{e^+e^- \rightarrow \bar{n}n} = T_{e^+e^- \rightarrow \bar{p}p}$  in the sum in Eq. (4), which is certainly justified as can be seen from the actual  $e^+e^- \rightarrow \bar{n}n$  result presented in Fig. 2.

As discussed above,  $D$ -wave contributions were ignored in case of  $\bar{p}p \rightarrow \nu$ . However, for  $e^+e^- \rightarrow \nu$  around the  $\bar{N}N$

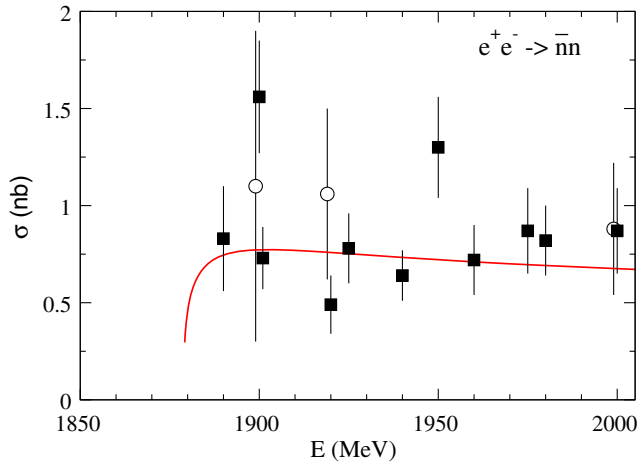


FIG. 2 (color online). Cross section for  $e^+e^- \rightarrow \bar{n}n$ . The solid line represents our result. Data are taken from Refs. [25] (circles) and [26] (squares).

threshold the momentum in the incoming system is no longer small and the  $e^+e^- \ ^3D_1$  component cannot be neglected. However, it can be easily included because the  $e^+e^- \rightarrow \bar{p}p$  transition amplitudes from the  $^3S_1$  and  $^3D_1$   $e^+e^-$  states are proportional to each other, see Eq. (6) of Ref. [15]. Thus, for including the  $D$ -wave contribution we simply have to multiply the  $S$ -wave cross section by a factor 1.5.

Our results are shown in Fig. 3. Obviously, in three of the four considered reactions the contribution from the two-step process  $e^+e^- \rightarrow \bar{N}N \rightarrow \nu$  is large enough to be of relevance and, moreover, together with a suitably adjusted background a rather good description of the cross sections around the  $\bar{N}N$  threshold can be achieved (solid curves). The cross section due to the background alone is indicated by the dash-dotted curves. It is practically constant and does not exhibit any structure. The contribution involving the intermediate  $\bar{N}N$  state generates a distinct structure at

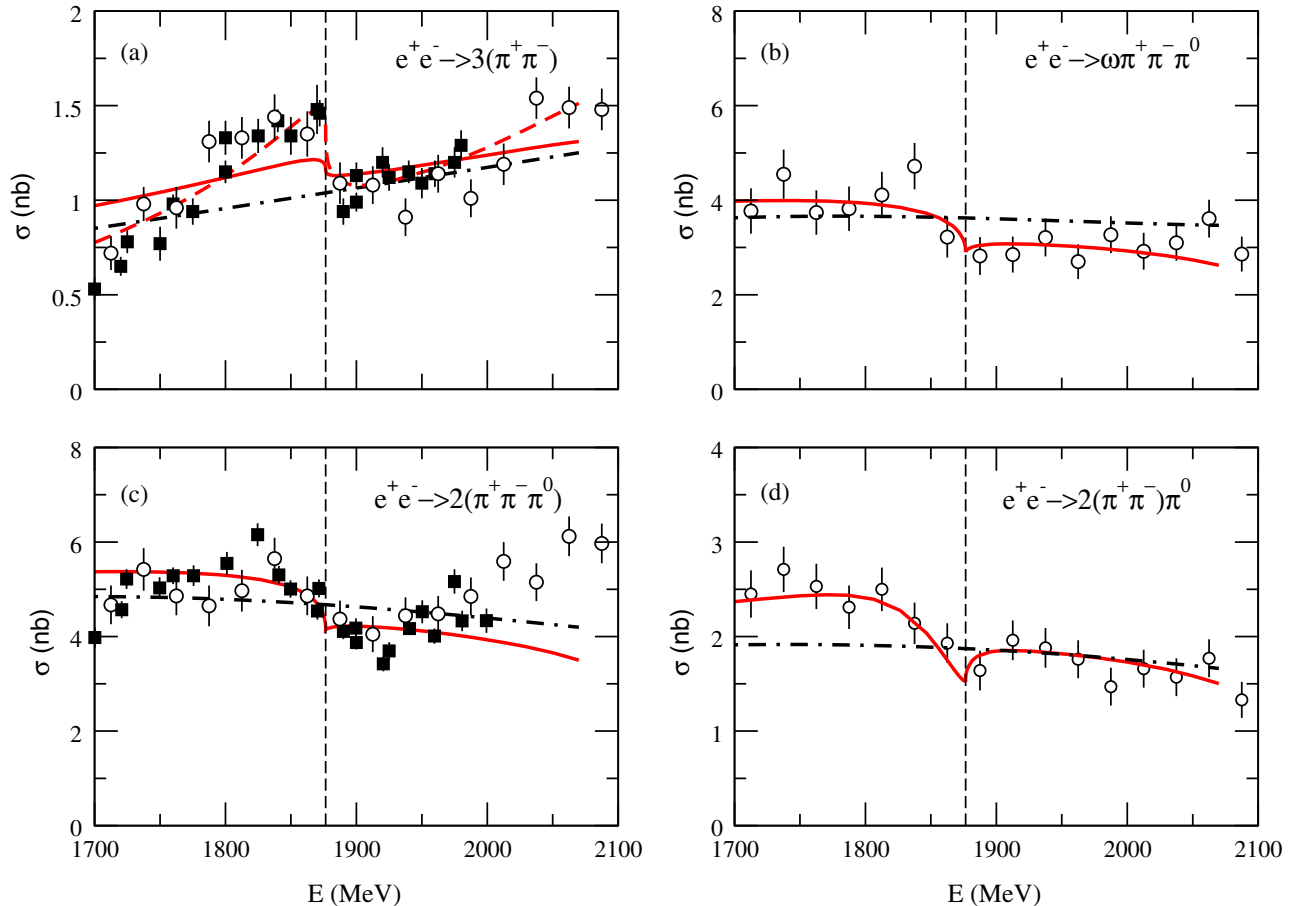


FIG. 3 (color online). Cross section for (a)  $e^+e^- \rightarrow 3(\pi^+\pi^-)$ , (b)  $\omega\pi^+\pi^-\pi^0$ , (c)  $2(\pi^+\pi^-\pi^0)$ , and (d)  $2(\pi^+\pi^-\pi^0)$ . The solid (red) curves represent our full result, including the  $\bar{N}N$  intermediate state, while the dash-dotted (black) curves are based on the background term alone. The vertical lines indicate the  $\bar{p}p$  threshold. The dashed (red) curve in (a) corresponds to amplifying deliberately the  $\bar{N}N$  loop contribution by a factor of four. Data are taken from Refs. [2,3] (circles) and [4,5] (squares).

the  $\bar{N}N$  threshold and is responsible for the fact that the full result is indeed in line with the behavior suggested by the measurements. Of course, in case of the  $2(\pi^+\pi^-\pi^0)$  channel the data [2,5] could hint at a minimum at an energy slightly above the threshold. If so, then a genuine resonance could be predominantly responsible for the dip structure, as suggested for example by the fit presented in Ref. [2]. However, there is also some variation between the two experiments and, specifically, those of Ref. [5] are still preliminary [27]. Obviously, further measurements with improved statistics and also with a better momentum resolution would be quite helpful. This applies certainly also to the other channels.

No satisfactory result could be achieved for the reaction  $e^+e^- \rightarrow 3(\pi^+\pi^-)$ . Here the amplitude due to the intermediate  $\bar{N}N$  state would have to be roughly a factor four larger in order to explain the data, see the dashed line. We emphasize that this curve is shown only for illustrative purposes. At the moment we do not have any physical arguments why that particular amplitude should be increased by a factor four. Indeed, we have examined and explored various uncertainties that could be used to motivate an amplification of the amplitude but without success. For example, assuming that the  $e^+e^- \rightarrow \bar{p}p$  reaction is given by the isospin 1 alone changes the result only marginally and the same is the case if we take into account that the  $e^+e^- \rightarrow \bar{n}n$  cross section could be slightly larger than the one for  $e^+e^- \rightarrow \bar{p}p$  as indicated by the data in Ref. [25].

Finally, let us come to the key question, namely are those structures seen in the experiment a signal for a  $\bar{N}N$  bound state? As discussed in Ref. [14], we did not find any near-threshold poles for our EFT  $\bar{N}N$  interaction in the  $^3S_1 - ^3D_1$  partial wave with  $I = 1$ . However, there is a pole in the  $I = 0$  case and this pole corresponds to a “binding” energy of  $E_B = (+4.8 - i68.2.9)$  MeV for the NNLO interaction employed in the present study [14]. The positive sign of the real part of  $E_B$  indicates that the pole we found is actually located above the  $\bar{N}N$  threshold (in the energy plane). As discussed in Ref. [14], the pole moves below the threshold when we switch off the imaginary part of the potential and that is the reason why we refer to it as bound state.

There is a distinct difference in the  $e^+e^- \rightarrow \nu$  amplitudes due to the  $\bar{N}N$  loop contribution for the two isospin channels, see Fig. 4, and the modulus exhibits indeed the features one expects in case of the absence/presence of a bound state, namely a genuine cusp or a rounded step and a maximum below the threshold. However, the structure in the cross section is strongly influenced and modified by the interference with the (complex) background amplitude as testified by the results in Fig. 3. Thus, at this stage we do not see a convincing evidence for the presence of an  $\bar{N}N$  bound state in the data for  $2(\pi^+\pi^-\pi^0)$  and for the opposite in case of  $\omega\pi^+\pi^-\pi^0$ , say. However, high accuracy data around the  $\bar{N}N$  threshold together with a better theoretical

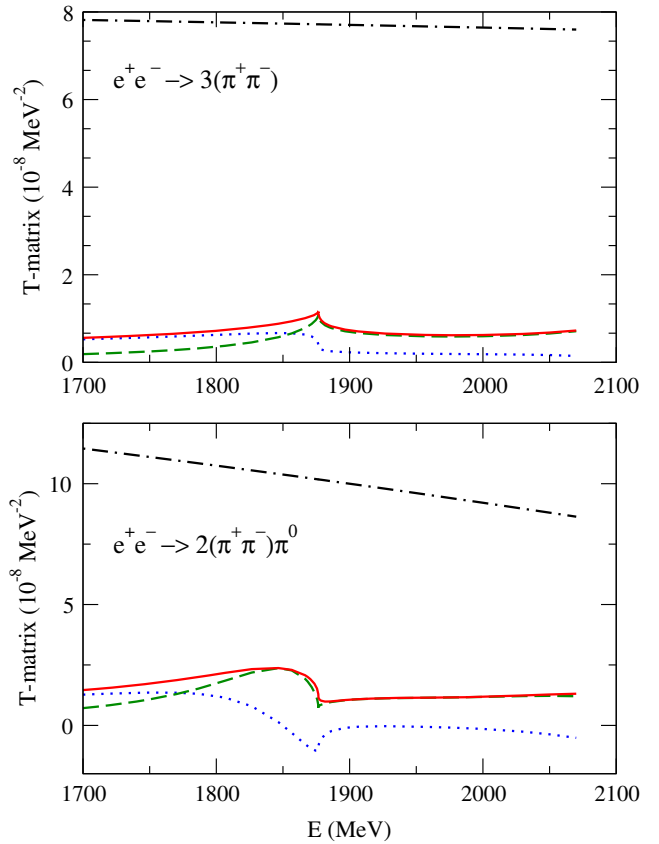


FIG. 4 (color online). Real (dotted/blue curve) and imaginary part (dashed/green), and modulus (solid/red line) of the  $\bar{N}N$  loop contribution, see Eq. (4). The modulus of the background term is shown by the dash-dotted (black) curves.

understanding of the background could certainly change the perspective for more reliable conclusions in the future.

In any case, our results corroborate that one should see an effect of the opening of the  $\bar{N}N$  channel in the cross sections of the considered  $e^+e^- \rightarrow \nu$  reactions. Thus, the observation of a dip or a cusplike structure in that energy region is not really something unusual or exotic. As argued above, our calculation provides a fairly reliable estimate for the amplitude that results from two-step processes with an intermediate  $\bar{N}N$  state. Though all the reactions considered in the present study are obviously dominated by processes that are not related to the  $\bar{N}N$  interaction, cf. Fig. 4, the amplitude due to the coupling to the  $\bar{N}N$  system is large enough so that it can produce sizeable interference effects. In three of the four reactions investigated those interference effects are indeed sufficient to explain the behavior of the measured cross sections in the region around the  $\bar{N}N$  threshold.

Should one expect similar structures also in other annihilation channels such as  $e^+e^- \rightarrow \pi^+\pi^-$ ,  $e^+e^- \rightarrow 2(\pi^+\pi^-)$ , etc.? An educated guess can be made based on the relative magnitude of the branching ratios for the pertinent  $\bar{N}N$  annihilation channels compared to

TABLE I. Branching ratios for  $\bar{p}p$  annihilation at rest [18] and  $e^+e^-$  annihilation cross sections around the  $\bar{N}N$  threshold [28,29].

$\nu$	BR for $\bar{p}p \rightarrow \nu$ [%]	$\sigma_{e^+e^- \rightarrow \nu}$ [nb]
$\pi^+\pi^-$	$0.314 \pm 0.012$	$\approx 1$
$\pi^+\pi^-\pi^0$	$6.7 \pm 1.0$	$\approx 1$
$2(\pi^+\pi^-)$	$5.6 \pm 0.9$	$\approx 6$
$\pi^+\pi^-2\pi^0$	$12.2 \pm 1.8$	$\approx 9$
$2(\pi^+\pi^-)\pi^0$	$21.0 \pm 3.2$	$\approx 2$
$2(\pi^+\pi^-\pi^0)$	$17.7 \pm 2.7$	$\approx 4$
$3(\pi^+\pi^-)$	$2.1 \pm 0.25$	$\approx 1$

annihilation cross sections from the  $e^+e^-$  state. Judging from the branching ratios summarized in Ref. [18] and the compilation of  $e^+e^-$  induced cross sections in Refs. [28,29] the most promising case is certainly the  $\pi^+\pi^-\pi^0$  channel, see Table I. In fact, the available data [30,31] could hint at an anomaly around the  $\bar{N}N$  threshold. In case of the  $4\pi$  channels there is a kink at the  $\bar{N}N$  threshold, see, e.g., Ref. [31]. On the other hand, for the  $\pi^+\pi^-$  case the branching ratio is very small, see Table I, so that we do not expect any noticeable effects there. Indeed, the data for  $e^+e^- \rightarrow \pi^+\pi^-$  [32] support this conjecture.

## V. SUMMARY

We analyzed the origin of the structure observed in the reactions  $e^+e^- \rightarrow 3(\pi^+\pi^-)$ ,  $2(\pi^+\pi^-\pi^0)$ ,  $\omega\pi^+\pi^-\pi^0$ , and  $e^+e^- \rightarrow 2(\pi^+\pi^-)\pi^0$  around the  $\bar{p}p$  threshold in recent *BABAR* and *CMD* measurements. Specifically, we evaluated the contribution of the two-step process  $e^+e^- \rightarrow \bar{N}N \rightarrow$  multipions to the total reaction amplitude. The amplitude for  $e^+e^- \rightarrow \bar{N}N$  was constrained from near-threshold data on the  $e^+e^- \rightarrow \bar{p}p$  cross section and the one for  $\bar{N}N \rightarrow$  multipions was fixed from available experimental information, for all studied  $5\pi$  and  $6\pi$  states.

The resulting amplitude turned out to be large enough to play a role for the considered  $e^+e^-$  annihilation channels and, in three of the four reactions, even allowed us to reproduce the data quantitatively near the  $\bar{N}N$  threshold once the interference with a background amplitude was taken into account. The latter simulates other transition processes that do not involve an  $\bar{N}N$  intermediate state. In case of the reaction  $e^+e^- \rightarrow 3(\pi^+\pi^-)$  there is also a visible effect from the  $\bar{N}N$  channel, however, overall the magnitude of the pertinent amplitude is too small.

In our study the structures seen in the experiments emerge as a threshold effect due to the opening of the  $\bar{N}N$  channel. The question whether that is an evidence for a  $\bar{N}N$  bound state is discussed, but here no firm conclusion could be made. It should be also said that the possibility that genuine resonances could be responsible for the dip observed in the experiments, at least in some of the considered reactions, cannot be excluded based on an analysis like ours. Anyway, our results provide a clear indication for the important role played by the  $\bar{N}N$  channel in those multipion production reactions for energies around its threshold. Consequently, it is certainly mandatory to take into account this effect in any quantitative study of those reactions that attempts to establish the existence of such genuine resonances.

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